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A Functional Approach to (Parallelised) Monte Carlo Simulation

Advanced R for Econometricians

Final Project

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1 Introduction

The Monte Carlo method is a simulation method for calculating the probabilistic value of the desired function using random numbers. A repeated pseudo-random number generator estimates sample statistics and returns a probability distribution of the sample, representing the parameter(**Barbu_2020**). Monte Carlo methods are combined with programming in modern research and contribute to various studies in statistics, economics, and many other scientific fields. This paper progresses on developing a collection of different wrapper functions to partially automatize the process of running a Monte Carlo simulation. The main function provides a convenient interface for Monte Carlo simulations and allows users to create a parameter grid and iterate homogenous function calls over the parameter grid. It also offers informative summary statistics, including visualization with ggplot2-methods and an option to use a parallelization process using the `furrr` package. The paper proceeds as follows: Chapter 2 describes pre-processes to establish the Monte Carlo simulation function. The pre-processes include functions to create a parameter grid and the respective data points drawn from random variables from a user-defined distribution, along with functions that provide summary statistics. Chapter 3 details the main Monte Carlo simulation function, which consists of the helper functions introduced in chapter 2. Chapter 4 presents specific examples for the simulation process. Finally, chapter 5 summarizes the work presented in this paper.

2 Preprocess: Creating Helper functions

The preprocess comprises 5 helper functions: `create_grid()`, `data_generation()`, `summary_function()`, `create_array_function()`, and `output_function()`. The functions are named in a way that the underlying purpose is directly clear. These functions are the building blocks of the `main_function()`, which takes the user input and runs the Monte Carlo simulation by itself.

2.1 create_grid()

The first helper function introduced is the `create_grid()`-function, which automatically creates a hyper-parameter grid over all permutations specified by the user. It is one of the functions to improve the performance of the main Monte Carlo simulation function. Although a is mainly not known in advance, it is important to find an appropriate range of a parameter to improve performance of the functions(**Rana_2022**).

```
create_grid <- function(parameters, nrep){
  input <- parameters
  storage <- list()
  name_vec <- c()

  for(i in 1:length(input)){ #1:3
```

```r
    a <- as.numeric(input[[i]][[2]])
    b <- as.numeric(input[[i]][[3]])
    c <- as.numeric(input[[i]][[4]])
    output <- seq(from=a, to=b, by=c)
    storage[[i]] <-  output
    name_vec[i] <- input[[i]][[1]]
  }

  grid <- expand_grid(unlist(storage[1])
                      , unlist(storage[2])
                      , unlist(storage[3])
                      , unlist(storage[4])
                      , unlist(storage[5])
                      , c(1:nrep))

  names(grid) <- c(name_vec, "rep")

  return(grid)
}
```

Users have to input the parameters in a specific format:

```
parameter_list <- list(c("variable name 1", from, to, by)
                      ,c("variable name 2", from, to, by)
                      ,c("variable name 3", from, to, by)
                      ,c("variable name 4", from, to, by))
```

`parameter_list` works with a minimum of 1 and a maximum of 4 variables. The structure of arguments is similar to `seq()` in R: Each vector contained in the list needs four arguments specified, that is, the function name, the start of the sequence, the end of the sequence, and the steps, by which the interval gets divided. It would be easy to adapt this helper function for more parameters, but it is assumed that a grid with up to 4 parameters offers enough complexity for the simulation. The function basically takes the information of the input parameter list and creates a grid with `tidyr::expand_grid()`. The argument `nrep` specifies how many repetitions per parameter constellation are created, where a separate row in the parameter grid gets created for each repetition.

`create_grid()` Example:

```r
#four parameters
param_list0 <- list(c("n", 10, 20, 10)
                    ,c("mu", 0, 0.5, 0.25)
                    ,c("sd", 0, 0.3, 0.1)
```

```
                    ,c("gender", 0, 1, 1))

head(create_grid(param_list0, nrep=3), n=10)
```

```
## # A tibble: 10 x 5
##         n    mu    sd gender   rep
##     <dbl> <dbl> <dbl>  <dbl> <int>
##  1     10     0   0        0     1
##  2     10     0   0        0     2
##  3     10     0   0        0     3
##  4     10     0   0        1     1
##  5     10     0   0        1     2
##  6     10     0   0        1     3
##  7     10     0   0.1      0     1
##  8     10     0   0.1      0     2
##  9     10     0   0.1      0     3
## 10     10     0   0.1      1     1
```

2.2 data_generation()

The second helper function called `data_generation()` takes the arguments `grid` and `simulation` as inputs. `simulation` is the argument for the user-defined function for the data generation process, while `grid` is the parameter grid previously created. The user can choose between a variety of probability distributions by entering the name of the function, for example, `rnorm` for the normal distribution or `runif` for the uniform distribution.

The n data points created for each set of parameters are stored as separate elements in a list since this format is a very flexible way of storing data.

`data_generation()` chooses a mapping function based on the number of parameters depending on if parallelization should be used.The table below shows the ‘:

Map	Map for Parallelization	Number of parameters
`purrr:map()`	`furrrr:future_map()`	$n = 1$
`purrr:map2()`	`furrrr:future_map2()`	$n = 2$
`purrr:pmap()`	`furrrr:future_pmap()`	$n \geq 3$

`options = furrr_options(seed = TRUE)` is for reproducible random number generation(RNG) processes. This argument takes control of the RNG process for parallelization and always generates the same numbers according to the given seed. More details can be found by running the command `?furrr_options` in RStudio.

```r
data_generation <- function(simulation, grid){

  if(ncol(grid)==2){
    var1 <- c(unlist(grid[,1]))
    if(cores>1){
      data <- future_map(var1, simulation,.options = furrr_options(seed = TRUE))
    }else{
      data <- map(var1, simulation)
    }
  }

  if(ncol(grid)==3){
    var1 <- c(unlist(grid[,1]))
    var2 <- c(unlist(grid[,2]))
    if(cores>1){
      data <- future_map2(var1, var2, simulation,.options = furrr_options(seed = TRUE))
    } else{
      data <- map2(var1, var2, simulation)
    }
  }

  if(ncol(grid)==4){
    var1 <- c(unlist(grid[,1]))
    var2 <- c(unlist(grid[,2]))
    var3 <- c(unlist(grid[,3]))
    list1 <- list(var1,var2,var3)
    if(cores>1){
      data <- future_pmap(list1, .f=simulation,.options = furrr_options(seed = TRUE))
    }else{
      data <- pmap(list1, .f=simulation)
    }
  }

  return(data)
}
```

Monte Carlo simulations can become very demanding in terms of computing time. In that case, parallel processing may be used to reduce the time of the simulation. In parallel processes, each process is executed simultaneously but separately. Interconnections are proceeded through communication channel(**Czech_2017**).

`data_generation()` automatically used the proper function from the `furrrr`-package if the user specified more than one core. Otherwise, the function will stick to the respective mapping function from the **purrr**-package.

data_generation() Example: The example below demonstrates a non-parallel processing function with a Poisson distribution. The time difference between parallel and non-parallel functions will be dealt with in Chapter 3.

```
cores <- 1


param_list2 <- list(c("n", 10, 20, 10)
                  ,c("lambda", 0.5, 1, 0.5))



grid2 <- create_grid(param_list2, nrep=1)
sim1 <- data_generation(simulation=rpois, grid=grid2)


names(sim1) <- c("n=10, lamda = 0.5","n=10, lamda = 1",
                 "n=20, lamda = 0.5","n=20, lamda = 1")
grid2
```

```
## # A tibble: 4 x 3
##       n lambda   rep
##   <dbl>  <dbl> <int>
## 1    10    0.5     1
## 2    10    1       1
## 3    20    0.5     1
## 4    20    1       1
```

```
sim1
```

```
## $`n=10, lamda = 0.5`
##  [1] 0 1 0 1 2 0 0 1 0 0
##
## $`n=10, lamda = 1`
##  [1] 3 1 1 1 0 2 0 0 0 3
##
## $`n=20, lamda = 0.5`
##  [1] 1 1 1 3 1 1 0 0 0 0 2 1 1 1 0 0 1 0 0 0
##
## $`n=20, lamda = 1`
##  [1] 0 1 1 1 0 0 0 1 0 2 0 1 2 0 1 0 0 2 2 1
```

`sim1` contains the generated data by `data_generation()` with `grid2` and poisson distribution as input. The below function shows the application of `data_generation()` to Uniform and Poisson distribution:

```
# Application to Uniform distribution
param_list_runif <- list(c("n", 10, 30, 10)
                         ,c("min", 0, 0, 0)
                         ,c("max", 1, 1, 0))



grid_unif <- create_grid(param_list_runif, nrep=3)
tail(data_generation(simulation=runif, grid=grid_unif),1)
```

```
## $n9
##  [1] 0.48204261 0.25296493 0.21625479 0.67437639 0.04766363 0.70085309
##  [7] 0.35188864 0.40894400 0.82095132 0.91885735 0.28252833 0.96110479
## [13] 0.72839443 0.68637508 0.05284394 0.39522013 0.47784538 0.56025326
## [19] 0.69826159 0.91568354 0.61835123 0.42842151 0.54208037 0.05847849
## [25] 0.26085686 0.39715195 0.19774474 0.83192756 0.15288722 0.80341854
```

```
# Application to Poisson distribution

param_list_rpois <- list(c("n", 10, 30, 10)
                         , c("lambda", 0, 10, 1))

grid_pois <- create_grid(param_list_rpois, nrep=3)
head(grid_pois,2) # nrow(grid_pois) = 99
```

```
## # A tibble: 2 x 3
##       n lambda   rep
##   <dbl>  <dbl> <int>
## 1    10      0     1
## 2    10      0     2
```

```
head(data_generation(simulation=rpois, grid=grid_pois),1)
```

```
## $n1
##  [1] 0 0 0 0 0 0 0 0 0 0
```

As explained before, data is stored in a list, where data points for each set of parameters are stored under a different variable. The first variable (saved under `$n1`) are based on 10 draws from a Uniform distribution with min = 0 and max = 2. Each variable in the list relates to a row in the parameter grid.

2.3 summary_function()

So far, the paper explains how to create *raw* data that is distributed as users choose by using `create_grid` and `data_generation`. Based on the raw data, this chapter introduces `summary_function()`, which produces summary statistics that users require. The function extracts the user-defined summary function (`sum_fun`) from the raw data using a `sapply()`-loop. Results are stored in a $(nrow(grid) * 1)$-dimensional matrix, which is combined with a parameter grid in the next step.

```
summary_function <- function(sum_fun, data_input){

  count <- length(data_input)
  summary_matrix <- matrix(nrow=count, ncol=1)

  for(i in 1:count){
    input <- list(data_input[[i]])
    output <- sapply(sum_fun, do.call, input)
    summary_matrix[i] <- output
  }

  colnames(summary_matrix) <- sum_fun
  return(summary_matrix)
}
```

`summary_function` Example: For this example, data is generated using the normal distribution, with the respective parameters n, μ, and the standard deviation specified in the parameter list. The list containing the raw data using `data_generation()` is stored in the object `test_data`, which is the data input for `summary_function()`. The arithmetic mean, using R´s built-in `mean()` - function is used as the second input to calculate the mean of all the data points created under a unique set of parameters. This output relates exactly to each row of the used parameter grid. Later, these summary statistics merged with the parameter grid in a data frame.

```
param_list3 <- list(c("n", 10, 20, 10)
                   ,c("mu", 1, 2, 0.25)
                   ,c("sd", 0.5, 1, 0.1))

grid_test <- create_grid(param_list3, nrep=3)
test_data <- data_generation(simulation=rnorm, grid=grid_test)
summary_data <- summary_function(sum_fun=list("mean"), data_input=test_data)
head(summary_data)
```

```
##           mean
## [1,] 1.0165396
```

```
## [2,] 0.8156316
## [3,] 1.0361343
## [4,] 1.0129142
## [5,] 1.0714922
## [6,] 1.1017460
```

```
nrow(summary_data)
```

```
## [1] 180
```

2.4 create_array_function()

Even though the example above includes a relatively small range of parameter grids, the simulation returns 180 summarize data points. In `main_function`, the results from the previous step get merged with the parameter grid into one data frame. This way of storing the data allows users to apply further data wrangling processes. A multidimensional array is more suitable for printing the output in a tidy and clear way. `create_array_function()` takes all relevant data from the previous steps (parameter grid and the results of the Monte Carlo simulation) and transforms it into an array with the correct dimensions.

```
create_array_function <- function(comb, parameters, nrep){
  storage <- list()
  name_vec <- c()

  for(i in 1:length(parameters)){
    a <- as.numeric(parameters[[i]][[2]])
    b <- as.numeric(parameters[[i]][[3]])
    c <- as.numeric(parameters[[i]][[4]])
    output <- seq(from=a, to=b, by=c)
    storage[[i]] <-  output
    name_vec[i] <- parameters[[i]][[1]]
  }


  matrix.numeration <-  paste("rep","=", 1:nrep, sep = "")

  if(length(parameters)==1){
    comb_ordered <-  comb %>% arrange(comb[,2])
    seq1 <- c(unlist(storage[1]))

    row.names <- paste(name_vec[1],"=",seq1, sep = "")
```

```
    dimension_array <- c(length(seq1), nrep)
    dim_names_list <- list(row.names, matrix.numeration)
  }

  if(length(parameters)==2){
    comb_ordered <-  comb %>% arrange(comb[,2])  %>% arrange(comb[,3])
    seq1 <- c(unlist(storage[1]))
    seq2 <- c(unlist(storage[2]))

    row.names <- paste(name_vec[1],"=",seq1, sep = "")
    column.names <-  paste(name_vec[2],"=",seq2, sep = "")

    dimension_array <- c(length(seq1), length(seq2), nrep)
    dim_names_list <- list(row.names, column.names, matrix.numeration)
  }

  if(length(parameters)==3){
    comb_ordered <-  comb %>% arrange(comb[,2])  %>%
      arrange(comb[,3]) %>% arrange(comb[,4])
    seq1 <- c(unlist(storage[1]))
    seq2 <- c(unlist(storage[2]))
    seq3 <- c(unlist(storage[3]))

    row.names <- paste(name_vec[1],"=",seq1, sep = "")
    column.names <-  paste(name_vec[2],"=",seq2, sep = "")
    matrix.names1 <-  paste(name_vec[3],"=",seq3, sep = "")

    dimension_array <- c(length(seq1), length(seq2), length(seq3), nrep)
    dim_names_list <- list(row.names, column.names,
                           matrix.names1, matrix.numeration)

  }



  array1 <- array(comb_ordered[,ncol(comb)]
                  , dim = dimension_array
                  , dim_names_list)
  return(array1)
}
```

In order to test `create_array_function`, we need to set up an altered version of `main_function`. The `main_function` is discussed in the next chapter. Also, a slightly modified version of the example using the `rnorm()` is used, where the parameter grid spans over a larger sequence

`create_array_function` Example:

```r
# PREP TEST `create_array_function`
main_function_array_test <-  function(parameters #list of parameters
                                     , nrep #number of repetitions
                                     , simulation #data genereation
                                     , sum_fun){ #summary statistics

  grid <- create_grid(parameters, nrep) #Step 1: create grid
  raw_data <- data_generation(simulation, grid) #Step 2: simlate data
  summary <- summary_function(sum_fun, data_input=raw_data) #Step 3: Summary statistics
  comb <- cbind(grid, summary) #Step 4: Combine resuluts with parameters
  array_1 <- create_array_function(comb, parameters, nrep) #Step 5: Create array

  return(comb)
}





param_list3x <- list(c("n", 10, 60, 10)
                     ,c("mu", 0, 6, 1)
                     ,c("sd", 1, 2, 1))

comb1 <- main_function_array_test(parameters=param_list3x
                                  , nrep = 2
                                  , simulation = rnorm
                                  , sum_fun="mean")

array_test <- create_array_function(comb=comb1, parameters=param_list3x, nrep=2)

array_test
```

```
## , , sd=1, rep=1
##
##             mu=0      mu=1     mu=2     mu=3     mu=4     mu=5     mu=6
## n=10   0.35922134 0.9743032 1.840865 3.100655 4.105774 4.451813 6.033443
## n=20   0.20480046 0.9723919 2.328471 3.274243 3.655587 5.176857 6.066900
## n=30  -0.06908261 1.0305645 1.808347 3.067782 4.208634 5.106752 6.004960
## n=40   0.19792602 0.8905866 2.056054 3.070807 3.920055 4.953902 6.352097
```

```
## n=50  0.04315490 1.2481016 1.873798 3.060249 4.082757 5.154856 6.049549
## n=60 -0.16232605 1.1374502 2.063770 3.111954 4.011165 4.960842 6.121272
##
## , , sd=2, rep=1
##
##             mu=0      mu=1     mu=2     mu=3     mu=4     mu=5     mu=6
## n=10  0.60254072 0.7722974 2.167073 2.365767 3.195291 4.729941 5.360873
## n=20  0.36846743 1.2480449 1.468367 3.851048 4.466710 5.190758 5.968718
## n=30  0.20187932 1.1789963 1.987329 2.342386 3.856807 4.881965 6.448827
## n=40  0.01754790 1.2318360 2.335579 2.154241 3.974455 5.191231 5.995017
## n=50 -0.47032636 1.0030714 2.005420 2.809839 3.990456 5.256817 5.909269
## n=60 -0.05194949 1.0987541 1.713785 2.670093 3.713659 5.066787 6.306739
##
## , , sd=1, rep=2
##
##             mu=0      mu=1     mu=2     mu=3     mu=4     mu=5     mu=6
## n=10 -0.24529993 1.1075008 2.856067 2.621332 3.702366 4.952268 5.936809
## n=20 -0.28339174 0.8075972 1.667454 2.797556 3.762498 4.780310 6.197972
## n=30 -0.07877761 0.8748818 1.995114 3.262357 3.766495 4.891683 6.055742
## n=40 -0.13103886 1.1819030 1.964973 3.028790 4.215424 4.729398 5.984479
## n=50 -0.10724463 0.7486847 1.884428 3.037365 3.724010 4.797619 5.653058
## n=60  0.04126511 1.0762207 2.011426 3.077304 4.025145 4.827894 5.702232
##
## , , sd=2, rep=2
##
##            mu=0      mu=1     mu=2     mu=3     mu=4     mu=5     mu=6
## n=10 -0.8768202 1.9278087 3.504186 3.189562 4.566824 5.492447 5.262416
## n=20 -0.1211375 1.1066363 2.061346 2.792630 4.525384 4.671059 5.874283
## n=30 -0.5477753 0.9192767 1.786796 3.307646 4.150965 4.978085 5.715300
## n=40  0.3841224 0.8326288 2.353355 2.693947 3.662984 5.679677 5.544140
## n=50 -0.2656664 1.4745724 2.541997 3.405539 3.997244 5.182033 6.003846
## n=60  0.5448446 1.2357074 2.123715 2.729288 3.809622 5.218432 5.743393
```

Under these specifications, the data was transformed into a 4-dimensional array. Sample size n and the sample mean μ are the variables at the side of each row and column. Also, it was specified that two repetitions are done for each set of parameters, while two values ($\sigma = \{1, 2\}$) were given for the standard deviation. The output contains the correct amount of tables (4). This output style allows the user to oversee a wide variety of different parameter constellations easily. Through `create_array_function`, users can obtain all combinations corresponding to each grid, the number of repetitions, and the summary function that users require, such as `mean` in this case.

2.5 average_function()

The example above used two repetitions for each parameter constellation to keep the output simple. However, in practice, the user would probably repeat the simulations over a higher number of repetitions, which would also drastically increase the size of the array. `average_function()` calculates the average over all repetitions and stores in an array with a dimension, that is reduced by one since the dimension for repetitions is not relevant anymore.

```
average_function <- function(grid_for_avg, summary, nrep){
  grid_for_avg <- grid_for_avg[-ncol(grid_for_avg)] #remove column for reps
  n_rows <- nrow(grid_for_avg)
  n_col <- ncol(grid_for_avg)

  for(i in 1:n_rows){
    start <- 1 + (i-1)*nrep
    end <- i*nrep
    grid_for_avg[i, n_col+1] <- mean(summary[start:end, ])
  }

  grid_plus_mc <- data.frame(grid_for_avg)

  colnames(grid_plus_mc)[n_col+1] <- "avg"

  return(grid_plus_mc)

}
```

2.6 output_function()

`Output_function` is the last part of pre-process. This function takes results and parameters of Monte Carlo simulation as inputs and converts them into output format on the console. Thus, users can win the tidy form of a simulation summary.

`array_1`,`average_over_reps`,`parameters`,`cores`,`simulation`, `nrep`,`cpt` are used as input parameters in the `output_function`. `cpt` means execution time for simulation. More detail will be addressed in the next chapter.

Regarding the structure `output_function`, (1) `out` is an empty list for storing simulation results, averaged results, and summary. (2) `Eco` is the class name of `out`. The class should be assigned to visualize simulation results with `ggplot2` methods. (3) Results from `array_1` and `average_over_reps` are assigned as `out$results` and `out$average` respectively. (4) These results have the classes with their own names: `out$results` and `out$average`, thereby preventing future error when visualizing simulation results with `ggplot2` methods. `ggplot2` methods require specific classes, such as `data.frame`. (5) `cat` is useful for producing output in user-defined functions. It converts its arguments to character vectors, concatenates them to a single character vector, appends the given '*sep*' = *string*(*s*) to each element, and then outputs them(**rdocumentation**). (6) The neat output is returned at the end.

```
output_function <- function(array_1,average_over_reps,parameters,cores,simulation,
                            nrep,cpt){
  # (1)
  out <- list()
  # (2)
  class(out) <- "Eco"
  # (3)
  out$results <- array_1
  out$average <- average_over_reps
  # (4)
  class(out$average) <- c("Eco",class(out$average))
  class(out$results) <- c("Eco",class(out$results))
  # (5)
  if(cores>1){
    parallel = "Multisession"
  } else {
    parallel = "Sequential"
  }
  text <-  cat("\n",
           "Repetition(nrep)      : ",nrep,"\n\n",
           "Parallelization Type  : ",parallel,"\n\n",
           "Number of Cores Used in  Parallelization : ",cores,
           " out of",detectCores(),"\n\n",
           "Input Parameters : ",paste(parameters),"\n\n",
           "Simulation Length :",length(array_1),"\n",
           "Minumum :",min(array_1),"\n",
           "Maximum :",max(array_1),"\n",
           "Mean    :", mean(array_1),"\n",
           "Median  :",median(array_1),"\n\n",
           "Execution Time of Monte Carlo Simulation",
           as.numeric(cpt),"secs \n\n",
           "Name of The Class :",class(out))
  # (6)
  return(out)
}
```

`output_function` represents the output of the main Monte Carlo simulation function in the next chapter.

3 Main Function: Monte Carlo simulation function

The `main_function` is built up by the helper functions in chapter 2 and includes additional arguments to improve the performance of the function. (1) `parameters` is a parameter list, `nrep` is the number of repetitions, simulation is a `data_generation`, `sum_fun` is a summary function that users require, `seed set` is for reproducibility of simulation, and `cores` is the number of cores that exist in the CPU. (2) `if and else` commands check how many cores are used in the main function and whether cores are more than the maximum number of cores. By using `detectCores()` in the parallel package, the maximum number of the cores in the CPU is saved in the object `max.cores`. (3) Setting the seed is an important step to get the same result for randomization. `main_function` either takes the seed that users decide or set the random seed when there is no seed provided. `sample.int()` generates a random number and uses it as a seed for reproducibility of the simulation. (4) `Sys.time()` function confirms the execution time of the simulation. `startTime` and `endTime` save the simulation's starting time and ending time, respectively. The difference between them is the execution time `cpt`, which is also included in summary statistics. (5) `plan()` is used to parallelize simulation. `plan()` has two arguments, "sequential" or "multisession". "Sequential" runs non-parallel processing with one core. On the other hand, users can define the number of cores with the "Multisession" argument. Lastly, (6) main_function produces tidy summary statistics with output_function in chapter 2.

```
#(1)
main_function <-  function(parameters
                           , nrep
                           , simulation
                           , sum_fun
                           ,seed = NULL
                           ,cores=NULL){



  #(2)
  max.cores <- detectCores()
  if(cores>max.cores){
    stop("Number of Cores cannot be bigger than total number of cores")
  }
  #(3)
  if(!is.null(seed)) {
    set.seed(seed)}
  else {
    warning("No seed provided!", call. = FALSE)
    seed <- sample.int(10000, 1)
    set.seed(seed)
    message("Random seed = ", seed, "\n")}
```

```
  #(4) and (5)
  startTime <- Sys.time()#Starting time




  grid <- create_grid(parameters, nrep)

  if(cores > 1){
    plan(multisession,workers = cores)
  } else{
    plan(sequential)
  }
  suppressMessages(raw_data <- data_generation(simulation, grid))

  summary <- summary_function(sum_fun, data_input=raw_data)

  average_over_reps <- average_function(grid_for_avg=create_grid(parameters, 1), summary, nr

  comb <- cbind(grid, summary)

  array_1 <- create_array_function(comb, parameters, nrep)

  endTime <- Sys.time()

  cpt <- endTime - startTime

  #(6)
  summary_1 <- output_function(array_1,average_over_reps,parameters,cores,simulation,
                               nrep,cpt)

return(summary_1)
}
```

The tests below present the performance of main_function with normal distribution.

3.1 Test1:Summary Performance

```
param_list3x <- list(c("n", 10, 100, 10)
                     ,c("mu", 0, 10, 1)
                     ,c("sd", 0, 5, 1))
```

```r
test_me <- main_function(parameters=param_list3x
              , nrep = 5
              , simulation = rnorm
              , sum_fun="mean"
              ,seed=123
              ,cores=1)
```

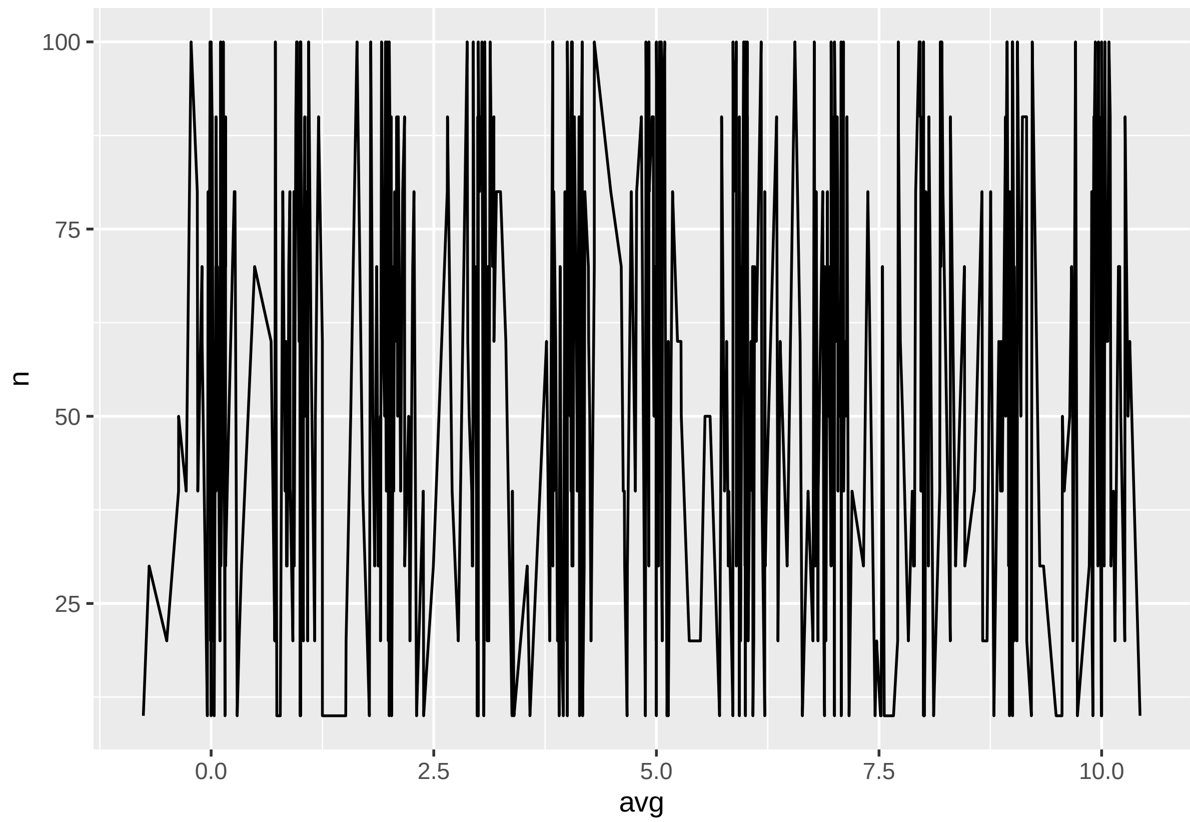
```
##
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Sequential
##
##  Number of Cores Used in  Parallelization :  1  out of 8
##
##  Input Parameters :  c("n", "10", "100", "10") c("mu", "0", "10", "1") c("sd", "0", "5",
##
##  Simulation Length : 3300
##  Minumum : -1.844634
##  Maximum : 13.43665
##  Mean    : 5.001471
##  Median  : 5
##
##  Execution Time of Monte Carlo Simulation 0.354063 secs
##
##  Name of The Class : Eco
```

All summary components, results, average result, and simulation summary are clear.

3.2 Test2: Visualization performance

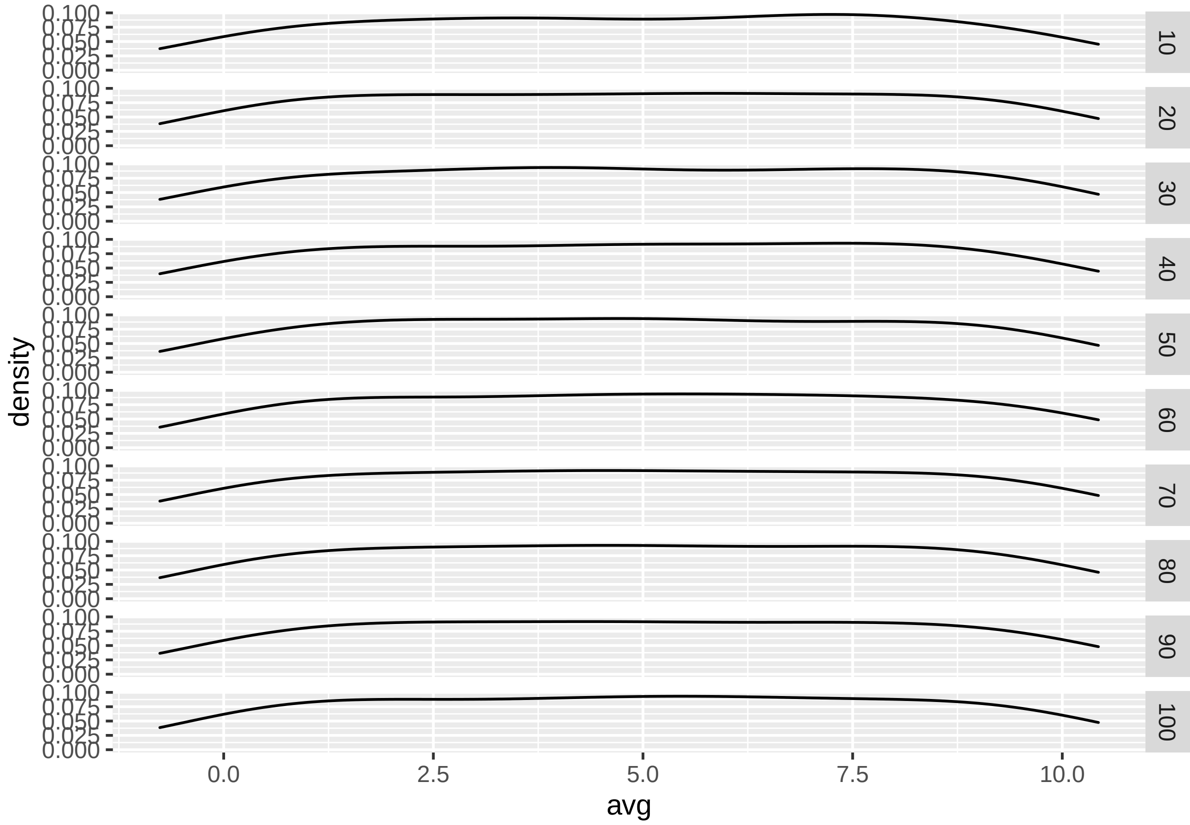
- Visualizing average

```r
ggplot(test_me$average,aes(x=avg,y=n))+geom_line()
```



- Visualizing average by using `facet_grid()`

```
ggplot(test_me$average,aes(x=avg))+facet_grid(n~.)+geom_density()
```



As given graphs above, the simulation result works well with `ggplot2` methods.

4 Examples

As an example of Monte Carlo Simulation, OLS and GLS coefficients(/beta2) are simulated with/without parallelization to show the execution time of the parallel process.

```
ols_f <- function(n,mu,sd){
  e <- rnorm(n,mu,sd)
  x <- runif(n)
  y <- 0.5*x + e
  ols.hat <- t(x) %*% y / t(x)%*%x
  return("ols"=ols.hat)}

gls_f <- function(n,mu,sd){
  e <- rnorm(n,mu,sd)
  x <- runif(n)
  y <- 0.5*x + e
  v.inv <- diag(1/(1:n))
  c <- chol(v.inv)
  cy <- c %*% y
  cx <- c %*% x
  gls_hat <- t(cx) %*% cy / t(cx)%*%cx
  return("gls"=gls_hat)

  param_list <- list(c("n",100,1000,100),c("mu",0,1,0.25),c("sd",1,2,.5))
}
```

As shown above, simple OLS and GLS functions are defined to find /beta2 coefficients. However, the execution time of the GLS function would be much longer than OLS function since "The Cholesky Decomposition" is applied to GLS function.

- OLS simulation without parallel processing:

```
 param_list <- list(c("n",100,1000,100),c("mu",0,1,0.25),c("sd",1,2,.5))
ols <- main_function(parameters = param_list,nrep=5,simulation = ols_f,sum_fun="mean",seed=1
```

```
##
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Sequential
##
```

```
##  Number of Cores Used in  Parallelization :  1  out of 8
##
##  Input Parameters :  c("n", "100", "1000", "100") c("mu", "0", "1", "0.25") c("sd", "1",
##
##  Simulation Length : 750
##  Minumum : 0.08611764
##  Maximum : 2.598461
##  Mean    : 1.251827
##  Median  : 1.246948
##
##  Execution Time of Monte Carlo Simulation 0.2100129 secs
##
##  Name of The Class : Eco
```

```
ols
```

```
## $results
## , , sd=1, rep=1
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.6824267 0.7726517 1.390428 1.352546 2.033425
## n=200  0.4786798 0.7268500 1.551036 1.469126 2.016927
## n=300  0.5430408 0.6925565 1.412873 1.516396 2.020209
## n=400  0.4916201 0.9335676 1.240309 1.588818 1.939011
## n=500  0.4665197 0.8658038 1.302298 1.621465 2.071480
## n=600  0.4135505 1.0087766 1.263624 1.564343 1.873183
## n=700  0.6062303 0.8060657 1.423163 1.677571 1.915217
## n=800  0.5215150 0.8373901 1.384251 1.536015 1.845518
## n=900  0.4377088 0.8522425 1.323030 1.625229 2.113246
## n=1000 0.4272311 0.9598533 1.171133 1.607383 2.007037
##
## , , sd=1.5, rep=1
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.2325414 0.8709612 1.007916 1.187153 1.646406
## n=200  0.2684302 0.5956376 1.066756 1.572480 1.780621
## n=300  0.3463280 0.5074475 1.249930 1.526170 2.281022
## n=400  0.5532419 0.6727324 1.069168 1.587438 2.040513
## n=500  0.5088940 0.9917867 1.254739 1.687302 2.191762
## n=600  0.5817868 1.0516531 1.388582 1.787617 2.039833
## n=700  0.4085830 0.9266206 1.177307 1.618533 1.795094
## n=800  0.4788177 0.7297284 1.261184 1.635941 1.990439
## n=900  0.4293894 0.7521720 1.281461 1.550926 1.969204
```

```
## n=1000 0.4379182 0.7061176 1.281116 1.604609 1.875328
##
## , , sd=2, rep=1
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.6237415 0.8008305 1.639701 1.897970 2.513486
## n=200  0.6110261 1.2155626 1.815507 1.725145 2.033192
## n=300  0.4066040 0.8631502 1.239882 1.684178 2.106188
## n=400  0.4235146 0.6718906 1.072217 1.373997 2.000179
## n=500  0.4979046 1.0646659 1.248795 1.624880 2.036934
## n=600  0.4167380 0.9594940 1.222973 1.538267 1.972503
## n=700  0.4474413 0.7043636 1.318657 1.298269 2.086695
## n=800  0.3558261 0.9032952 1.340988 1.480559 1.852471
## n=900  0.4184591 0.8440885 1.312351 1.481778 1.959888
## n=1000 0.5482327 0.8486597 1.325121 1.557654 2.174067
##
## , , sd=1, rep=2
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5699594 0.6347600 1.240574 1.409578 2.102247
## n=200  0.3411974 0.9859774 1.127496 1.654986 2.005527
## n=300  0.5568138 0.7710960 1.309760 1.640136 1.868297
## n=400  0.5892899 0.8630584 1.420656 1.488524 2.106760
## n=500  0.4565660 0.9875469 1.169863 1.642037 2.125870
## n=600  0.6074920 0.8311391 1.144387 1.561809 2.127146
## n=700  0.5311162 0.8793250 1.213227 1.758464 2.019087
## n=800  0.4907978 0.8137087 1.243065 1.544169 1.890569
## n=900  0.5006696 0.9126527 1.185916 1.551420 2.011440
## n=1000 0.5132049 0.8425093 1.323331 1.665086 2.067430
##
## , , sd=1.5, rep=2
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4100172 1.0050342 1.3586338 1.594119 2.030257
## n=200  0.7621120 1.1129076 1.2050693 1.766159 2.052134
## n=300  0.4495498 0.9131922 1.3998834 1.844595 2.048539
## n=400  0.5189146 0.9392205 0.9085613 1.502305 2.159537
## n=500  0.7089984 0.6966299 1.1168839 1.472302 1.942223
## n=600  0.3991638 0.7265354 1.1572410 1.748560 1.735354
## n=700  0.4141086 0.7800295 1.3853238 1.765437 1.860513
## n=800  0.3797112 0.8637205 1.1967238 1.669677 1.904555
## n=900  0.4946693 0.9968749 1.3024325 1.747231 2.093768
```

```
## n=1000 0.6315810 0.8282354 1.1035843 1.541674 1.946093
##
## , , sd=2, rep=2
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.2670268 1.0417428 0.9765324 1.779177 2.139938
## n=200  0.2064741 0.8966665 1.2011619 1.562213 1.929578
## n=300  0.2953656 0.8280923 1.4380720 1.301385 2.089569
## n=400  0.7044943 0.8050854 1.4651276 1.320472 2.051137
## n=500  0.4125427 0.8432624 0.9844522 1.838044 2.144216
## n=600  0.7200719 0.9085732 1.0035755 1.809966 2.205790
## n=700  0.5352158 0.8738860 1.2227186 1.564884 2.114450
## n=800  0.3849922 0.7982686 1.2963208 1.625716 1.980291
## n=900  0.3505717 0.7606954 1.0653161 1.494651 1.986403
## n=1000 0.5160520 0.8734081 1.3842792 1.766722 2.085573
##
## , , sd=1, rep=3
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5968253 1.0425781 1.207359 1.494982 1.755818
## n=200  0.6013728 0.8169956 1.089477 1.759701 2.050444
## n=300  0.4185383 0.8722548 1.202220 1.632546 2.054984
## n=400  0.5761413 0.9439121 1.215469 1.611440 2.019500
## n=500  0.6625234 0.8265606 1.301594 1.551010 1.973081
## n=600  0.6338517 0.9970815 1.257844 1.716586 2.047527
## n=700  0.4954136 0.8538731 1.202218 1.656197 1.921142
## n=800  0.4847987 0.8606272 1.280399 1.681161 1.939804
## n=900  0.4581839 0.8963640 1.363804 1.640401 2.005699
## n=1000 0.4964781 0.9341992 1.243164 1.696117 1.985360
##
## , , sd=1.5, rep=3
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5659700 0.7903221 1.254210 1.886166 1.790293
## n=200  0.5821258 0.6511703 1.343681 1.782181 1.894019
## n=300  0.6550444 0.9367739 1.380756 1.705998 2.028275
## n=400  0.7048194 0.9633662 1.120887 1.688024 1.811344
## n=500  0.3923555 0.7626086 1.104116 1.537444 2.047353
## n=600  0.4717144 0.8688271 1.175261 1.826726 1.983990
## n=700  0.3871651 0.6201482 1.305263 1.669981 2.042779
## n=800  0.4834246 0.9357010 1.305622 1.523736 2.081715
## n=900  0.4433156 0.7916531 1.359010 1.580634 1.940405
```

```
## n=1000 0.4946876 0.8067392 1.480151 1.486763 1.900808
##
## , , sd=2, rep=3
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4237920 0.5144245 0.4718755 2.004104 2.186295
## n=200  0.5777614 0.7299343 1.6298909 1.499530 1.929751
## n=300  0.6276662 0.8600749 0.9626478 2.011661 2.175176
## n=400  0.5785860 0.9158230 1.1544399 1.389809 2.169336
## n=500  0.2516702 0.9907678 1.0602298 1.532465 1.682624
## n=600  0.5165529 0.8616768 1.5189127 1.652623 1.941119
## n=700  0.4674128 0.7479103 1.2133761 1.738886 1.996975
## n=800  0.5090721 0.8062749 1.3959268 1.542798 1.994769
## n=900  0.4197560 0.8280591 1.2129929 1.504396 1.947569
## n=1000 0.6527630 0.8873246 1.2662086 1.656169 2.115128
##
## , , sd=1, rep=4
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7359835 1.0087000 1.1157127 1.851686 2.286974
## n=200  0.5466285 0.8691083 0.9809655 1.677466 1.914859
## n=300  0.4092951 0.8421824 1.1834134 1.463671 2.105769
## n=400  0.4870693 0.8787365 1.3442915 1.641625 2.038680
## n=500  0.6247941 0.9737552 1.1814126 1.583607 1.981457
## n=600  0.5543342 0.9385104 1.2433161 1.676785 1.916412
## n=700  0.4178732 0.8808745 1.1952781 1.530920 1.960542
## n=800  0.5618498 0.8943310 1.2475019 1.686068 1.946561
## n=900  0.5052058 0.8785736 1.2679218 1.571624 2.009535
## n=1000 0.4550461 0.8622264 1.2189931 1.640547 2.027112
##
## , , sd=1.5, rep=4
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4449720 0.8435500 1.5231357 1.528876 2.461235
## n=200  0.3352203 0.8038153 0.9593336 1.823112 2.149320
## n=300  0.4930487 0.6791231 1.0272955 1.574880 2.304307
## n=400  0.3806721 1.1714969 1.4056411 1.543264 2.258722
## n=500  0.4589277 0.7928819 1.5044037 1.632672 1.829121
## n=600  0.4884702 0.6958911 1.2883971 1.606977 2.109358
## n=700  0.5038252 0.8396545 1.4939591 1.598702 2.198315
## n=800  0.5515391 0.8256058 1.2838778 1.644795 2.020077
## n=900  0.4102779 0.7769137 1.2511958 1.567744 1.925168
```

```
## n=1000 0.6885496 0.9734467 1.2688095 1.525439 1.946295
##
## , , sd=2, rep=4
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.5717011 0.8796835 2.1004676 1.351107 2.598461
## n=200  1.1810891 0.4725781 0.9070217 1.490537 2.444452
## n=300  0.5900949 1.0569764 1.3134750 1.766744 2.089249
## n=400  0.7380901 0.4503391 1.0841655 1.704982 1.998472
## n=500  0.6158445 1.0384364 1.6255235 1.911106 1.868147
## n=600  0.6300586 1.1140343 1.1895053 1.822727 1.933571
## n=700  0.3625215 0.9113498 1.2522415 1.654996 1.981563
## n=800  0.4427674 0.8570775 1.1367413 1.796147 2.103874
## n=900  0.3816606 0.8423425 1.3168243 1.712630 2.102760
## n=1000 0.4265370 0.9495205 1.3021528 1.603875 1.875588
##
## , , sd=1, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.2288956 0.9259850 1.345124 1.767008 1.904891
## n=200  0.4782098 0.9611524 1.326272 1.689215 2.050280
## n=300  0.4520819 0.9920557 1.238380 1.753581 2.087970
## n=400  0.4573970 1.1576259 1.246390 1.497974 2.019858
## n=500  0.5434649 0.9485599 1.191573 1.631702 2.049623
## n=600  0.4632066 0.8962950 1.266618 1.657375 1.924914
## n=700  0.3907963 0.8164093 1.155234 1.615976 1.949318
## n=800  0.4930138 0.9572131 1.295102 1.684709 1.936302
## n=900  0.4979275 0.8736681 1.214887 1.584737 1.985072
## n=1000 0.5301261 0.8977935 1.313722 1.630559 1.935747
##
## , , sd=1.5, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.4688968 0.9435859 1.380401 1.586464 2.192555
## n=200  0.7831954 0.8507725 1.104056 1.524999 1.987862
## n=300  0.4957244 0.9953593 1.100958 1.596888 2.032147
## n=400  0.6377009 1.1108968 1.039524 1.621890 1.989716
## n=500  0.2757090 0.7425858 1.221020 1.687816 1.782811
## n=600  0.4674703 0.9888671 1.462157 1.805296 2.070940
## n=700  0.3602414 0.8718152 1.369915 1.868538 1.981321
## n=800  0.3886475 1.1178513 1.246394 1.612441 2.065348
## n=900  0.4814983 0.8016443 1.148795 1.443979 2.099358
```

```
## n=1000 0.4683020 0.8041379 1.013566 1.662726 1.873865
##
## , , sd=2, rep=5
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.08611764 0.9398614 0.9146867 1.669708 2.319204
## n=200  0.67138321 0.6832920 1.0754579 1.433456 2.068870
## n=300  0.44312742 1.2584762 1.3301563 1.650963 2.100398
## n=400  0.66455181 0.8364694 1.2167647 1.894041 1.989744
## n=500  0.41708387 0.8419375 1.1330271 1.713047 2.102631
## n=600  0.60004510 0.7267214 1.2282300 1.533603 2.044685
## n=700  0.42035536 0.8611750 1.3460957 1.744825 2.228779
## n=800  0.59792089 1.0213057 0.9896550 1.553520 1.600474
## n=900  0.51242473 0.7970967 1.1751429 1.646252 2.039864
## n=1000 0.56705287 0.8053208 1.2585926 1.673913 1.889833
##
## attr(,"class")
## [1] "Eco"   "array"
##
## $average
##         n   mu  sd       avg
## 1     100 0.00 1.0 0.5628181
## 2     100 0.00 1.5 0.4244795
## 3     100 0.00 2.0 0.3944758
## 4     100 0.25 1.0 0.8769350
## 5     100 0.25 1.5 0.8906907
## 6     100 0.25 2.0 0.8353085
## 7     100 0.50 1.0 1.2598397
## 8     100 0.50 1.5 1.3048593
## 9     100 0.50 2.0 1.2206527
## 10    100 0.75 1.0 1.5751601
## 11    100 0.75 1.5 1.5565558
## 12    100 0.75 2.0 1.7404132
## 13    100 1.00 1.0 2.0166709
## 14    100 1.00 1.5 2.0241493
## 15    100 1.00 2.0 2.3514768
## 16    200 0.00 1.0 0.4892177
## 17    200 0.00 1.5 0.5462168
## 18    200 0.00 2.0 0.6495468
## 19    200 0.25 1.0 0.8720167
## 20    200 0.25 1.5 0.8028606
## 21    200 0.25 2.0 0.7996067
```

```
## 22   200 0.50 1.0 1.2150492
## 23   200 0.50 1.5 1.1357791
## 24   200 0.50 2.0 1.3258080
## 25   200 0.75 1.0 1.6500989
## 26   200 0.75 1.5 1.6937863
## 27   200 0.75 2.0 1.5421762
## 28   200 1.00 1.0 2.0076074
## 29   200 1.00 1.5 1.9727911
## 30   200 1.00 2.0 2.0811684
## 31   300 0.00 1.0 0.4759540
## 32   300 0.00 1.5 0.4879391
## 33   300 0.00 2.0 0.4725716
## 34   300 0.25 1.0 0.8340291
## 35   300 0.25 1.5 0.8063792
## 36   300 0.25 2.0 0.9733540
## 37   300 0.50 1.0 1.2693292
## 38   300 0.50 1.5 1.2317647
## 39   300 0.50 2.0 1.2568465
## 40   300 0.75 1.0 1.6012659
## 41   300 0.75 1.5 1.6497061
## 42   300 0.75 2.0 1.6829863
## 43   300 1.00 1.0 2.0274458
## 44   300 1.00 1.5 2.1388580
## 45   300 1.00 2.0 2.1121160
## 46   400 0.00 1.0 0.5203035
## 47   400 0.00 1.5 0.5590698
## 48   400 0.00 2.0 0.6218474
## 49   400 0.25 1.0 0.9553801
## 50   400 0.25 1.5 0.9715426
## 51   400 0.25 2.0 0.7359215
## 52   400 0.50 1.0 1.2934233
## 53   400 0.50 1.5 1.1087563
## 54   400 0.50 2.0 1.1985429
## 55   400 0.75 1.0 1.5656764
## 56   400 0.75 1.5 1.5885842
## 57   400 0.75 2.0 1.5366603
## 58   400 1.00 1.0 2.0247618
## 59   400 1.00 1.5 2.0519664
## 60   400 1.00 2.0 2.0417733
## 61   500 0.00 1.0 0.5507736
## 62   500 0.00 1.5 0.4689769
## 63   500 0.00 2.0 0.4390092
```

```
## 64   500 0.25 1.0 0.9204453
## 65   500 0.25 1.5 0.7972986
## 66   500 0.25 2.0 0.9558140
## 67   500 0.50 1.0 1.2293479
## 68   500 0.50 1.5 1.2402327
## 69   500 0.50 2.0 1.2104054
## 70   500 0.75 1.0 1.6059639
## 71   500 0.75 1.5 1.6035074
## 72   500 0.75 2.0 1.7239084
## 73   500 1.00 1.0 2.0403021
## 74   500 1.00 1.5 1.9586539
## 75   500 1.00 2.0 1.9669104
## 76   600 0.00 1.0 0.5344870
## 77   600 0.00 1.5 0.4817211
## 78   600 0.00 2.0 0.5766933
## 79   600 0.25 1.0 0.9343605
## 80   600 0.25 1.5 0.8663548
## 81   600 0.25 2.0 0.9141000
## 82   600 0.50 1.0 1.2351578
## 83   600 0.50 1.5 1.2943275
## 84   600 0.50 2.0 1.2326393
## 85   600 0.75 1.0 1.6353797
## 86   600 0.75 1.5 1.7550350
## 87   600 0.75 2.0 1.6714372
## 88   600 1.00 1.0 1.9778364
## 89   600 1.00 1.5 1.9878951
## 90   600 1.00 2.0 2.0195340
## 91   700 0.00 1.0 0.4882859
## 92   700 0.00 1.5 0.4147847
## 93   700 0.00 2.0 0.4465893
## 94   700 0.25 1.0 0.8473095
## 95   700 0.25 1.5 0.8076536
## 96   700 0.25 2.0 0.8197369
## 97   700 0.50 1.0 1.2378238
## 98   700 0.50 1.5 1.3463535
## 99   700 0.50 2.0 1.2706179
## 100  700 0.75 1.0 1.6478257
## 101  700 0.75 1.5 1.7042384
## 102  700 0.75 2.0 1.6003720
## 103  700 1.00 1.0 1.9530612
## 104  700 1.00 1.5 1.9756043
## 105  700 1.00 2.0 2.0816926
```

```
## 106  800 0.00 1.0 0.5103950
## 107  800 0.00 1.5 0.4564280
## 108  800 0.00 2.0 0.4581157
## 109  800 0.25 1.0 0.8726540
## 110  800 0.25 1.5 0.8945214
## 111  800 0.25 2.0 0.8772444
## 112  800 0.50 1.0 1.2900639
## 113  800 0.50 1.5 1.2587603
## 114  800 0.50 2.0 1.2319263
## 115  800 0.75 1.0 1.6264243
## 116  800 0.75 1.5 1.6173179
## 117  800 0.75 2.0 1.5997479
## 118  800 1.00 1.0 1.9117509
## 119  800 1.00 1.5 2.0124269
## 120  800 1.00 2.0 1.9063757
## 121  900 0.00 1.0 0.4799391
## 122  900 0.00 1.5 0.4518301
## 123  900 0.00 2.0 0.4165744
## 124  900 0.25 1.0 0.8827002
## 125  900 0.25 1.5 0.8238516
## 126  900 0.25 2.0 0.8144564
## 127  900 0.50 1.0 1.2711118
## 128  900 0.50 1.5 1.2685788
## 129  900 0.50 2.0 1.2165253
## 130  900 0.75 1.0 1.5946823
## 131  900 0.75 1.5 1.5781026
## 132  900 0.75 2.0 1.5679413
## 133  900 1.00 1.0 2.0249983
## 134  900 1.00 1.5 2.0055806
## 135  900 1.00 2.0 2.0072967
## 136 1000 0.00 1.0 0.4844173
## 137 1000 0.00 1.5 0.5442077
## 138 1000 0.00 2.0 0.5421275
## 139 1000 0.25 1.0 0.8993163
## 140 1000 0.25 1.5 0.8237354
## 141 1000 0.25 2.0 0.8728468
## 142 1000 0.50 1.0 1.2540685
## 143 1000 0.50 1.5 1.2294453
## 144 1000 0.50 2.0 1.3072707
## 145 1000 0.75 1.0 1.6479383
## 146 1000 0.75 1.5 1.5642421
## 147 1000 0.75 2.0 1.6516665
```

```
## 148 1000 1.00 1.0 2.0045373
## 149 1000 1.00 1.5 1.9084779
## 150 1000 1.00 2.0 2.0280377
##
## attr(,"class")
## [1] "Eco"
```

The total execution time of OLS simulation is 8.46 seconds when only one core is used.

- OLS simulation with parallel processing:

```
ols <- main_function(parameters = param_list,nrep=5,simulation = ols_f,sum_fun="mean",seed=1
```

```
##
##  Repetition(nrep)       :  5
##
##  Parallelization Type   :  Multisession
##
##  Number of Cores Used in  Parallelization :  4  out of 8
##
##  Input Parameters :  c("n", "100", "1000", "100") c("mu", "0", "1", "0.25") c("sd", "1",
##
##  Simulation Length : 750
##  Minumum : 0.08611764
##  Maximum : 2.598461
##  Mean     : 1.251827
##  Median   : 1.246948
##
##  Execution Time of Monte Carlo Simulation 1.780556 secs
##
##  Name of The Class : Eco
```

```
ols
```

```
## $results
## , , sd=1, rep=1
##
##                mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100   0.6824267 0.7726517 1.390428 1.352546 2.033425
## n=200   0.4786798 0.7268500 1.551036 1.469126 2.016927
## n=300   0.5430408 0.6925565 1.412873 1.516396 2.020209
## n=400   0.4916201 0.9335676 1.240309 1.588818 1.939011
```

```
## n=500  0.4665197 0.8658038 1.302298 1.621465 2.071480
## n=600  0.4135505 1.0087766 1.263624 1.564343 1.873183
## n=700  0.6062303 0.8060657 1.423163 1.677571 1.915217
## n=800  0.5215150 0.8373901 1.384251 1.536015 1.845518
## n=900  0.4377088 0.8522425 1.323030 1.625229 2.113246
## n=1000 0.4272311 0.9598533 1.171133 1.607383 2.007037
##
## , , sd=1.5, rep=1
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.2325414 0.8709612 1.007916 1.187153 1.646406
## n=200  0.2684302 0.5956376 1.066756 1.572480 1.780621
## n=300  0.3463280 0.5074475 1.249930 1.526170 2.281022
## n=400  0.5532419 0.6727324 1.069168 1.587438 2.040513
## n=500  0.5088940 0.9917867 1.254739 1.687302 2.191762
## n=600  0.5817868 1.0516531 1.388582 1.787617 2.039833
## n=700  0.4085830 0.9266206 1.177307 1.618533 1.795094
## n=800  0.4788177 0.7297284 1.261184 1.635941 1.990439
## n=900  0.4293894 0.7521720 1.281461 1.550926 1.969204
## n=1000 0.4379182 0.7061176 1.281116 1.604609 1.875328
##
## , , sd=2, rep=1
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.6237415 0.8008305 1.639701 1.897970 2.513486
## n=200  0.6110261 1.2155626 1.815507 1.725145 2.033192
## n=300  0.4066040 0.8631502 1.239882 1.684178 2.106188
## n=400  0.4235146 0.6718906 1.072217 1.373997 2.000179
## n=500  0.4979046 1.0646659 1.248795 1.624880 2.036934
## n=600  0.4167380 0.9594940 1.222973 1.538267 1.972503
## n=700  0.4474413 0.7043636 1.318657 1.298269 2.086695
## n=800  0.3558261 0.9032952 1.340988 1.480559 1.852471
## n=900  0.4184591 0.8440885 1.312351 1.481778 1.959888
## n=1000 0.5482327 0.8486597 1.325121 1.557654 2.174067
##
## , , sd=1, rep=2
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5699594 0.6347600 1.240574 1.409578 2.102247
## n=200  0.3411974 0.9859774 1.127496 1.654986 2.005527
## n=300  0.5568138 0.7710960 1.309760 1.640136 1.868297
## n=400  0.5892899 0.8630584 1.420656 1.488524 2.106760
```

```
## n=500  0.4565660 0.9875469 1.169863 1.642037 2.125870
## n=600  0.6074920 0.8311391 1.144387 1.561809 2.127146
## n=700  0.5311162 0.8793250 1.213227 1.758464 2.019087
## n=800  0.4907978 0.8137087 1.243065 1.544169 1.890569
## n=900  0.5006696 0.9126527 1.185916 1.551420 2.011440
## n=1000 0.5132049 0.8425093 1.323331 1.665086 2.067430
##
## , , sd=1.5, rep=2
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4100172 1.0050342 1.3586338 1.594119 2.030257
## n=200  0.7621120 1.1129076 1.2050693 1.766159 2.052134
## n=300  0.4495498 0.9131922 1.3998834 1.844595 2.048539
## n=400  0.5189146 0.9392205 0.9085613 1.502305 2.159537
## n=500  0.7089984 0.6966299 1.1168839 1.472302 1.942223
## n=600  0.3991638 0.7265354 1.1572410 1.748560 1.735354
## n=700  0.4141086 0.7800295 1.3853238 1.765437 1.860513
## n=800  0.3797112 0.8637205 1.1967238 1.669677 1.904555
## n=900  0.4946693 0.9968749 1.3024325 1.747231 2.093768
## n=1000 0.6315810 0.8282354 1.1035843 1.541674 1.946093
##
## , , sd=2, rep=2
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.2670268 1.0417428 0.9765324 1.779177 2.139938
## n=200  0.2064741 0.8966665 1.2011619 1.562213 1.929578
## n=300  0.2953656 0.8280923 1.4380720 1.301385 2.089569
## n=400  0.7044943 0.8050854 1.4651276 1.320472 2.051137
## n=500  0.4125427 0.8432624 0.9844522 1.838044 2.144216
## n=600  0.7200719 0.9085732 1.0035755 1.809966 2.205790
## n=700  0.5352158 0.8738860 1.2227186 1.564884 2.114450
## n=800  0.3849922 0.7982686 1.2963208 1.625716 1.980291
## n=900  0.3505717 0.7606954 1.0653161 1.494651 1.986403
## n=1000 0.5160520 0.8734081 1.3842792 1.766722 2.085573
##
## , , sd=1, rep=3
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5968253 1.0425781 1.207359 1.494982 1.755818
## n=200  0.6013728 0.8169956 1.089477 1.759701 2.050444
## n=300  0.4185383 0.8722548 1.202220 1.632546 2.054984
## n=400  0.5761413 0.9439121 1.215469 1.611440 2.019500
```

```
## n=500  0.6625234 0.8265606 1.301594 1.551010 1.973081
## n=600  0.6338517 0.9970815 1.257844 1.716586 2.047527
## n=700  0.4954136 0.8538731 1.202218 1.656197 1.921142
## n=800  0.4847987 0.8606272 1.280399 1.681161 1.939804
## n=900  0.4581839 0.8963640 1.363804 1.640401 2.005699
## n=1000 0.4964781 0.9341992 1.243164 1.696117 1.985360
##
## , , sd=1.5, rep=3
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5659700 0.7903221 1.254210 1.886166 1.790293
## n=200  0.5821258 0.6511703 1.343681 1.782181 1.894019
## n=300  0.6550444 0.9367739 1.380756 1.705998 2.028275
## n=400  0.7048194 0.9633662 1.120887 1.688024 1.811344
## n=500  0.3923555 0.7626086 1.104116 1.537444 2.047353
## n=600  0.4717144 0.8688271 1.175261 1.826726 1.983990
## n=700  0.3871651 0.6201482 1.305263 1.669981 2.042779
## n=800  0.4834246 0.9357010 1.305622 1.523736 2.081715
## n=900  0.4433156 0.7916531 1.359010 1.580634 1.940405
## n=1000 0.4946876 0.8067392 1.480151 1.486763 1.900808
##
## , , sd=2, rep=3
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4237920 0.5144245 0.4718755 2.004104 2.186295
## n=200  0.5777614 0.7299343 1.6298909 1.499530 1.929751
## n=300  0.6276662 0.8600749 0.9626478 2.011661 2.175176
## n=400  0.5785860 0.9158230 1.1544399 1.389809 2.169336
## n=500  0.2516702 0.9907678 1.0602298 1.532465 1.682624
## n=600  0.5165529 0.8616768 1.5189127 1.652623 1.941119
## n=700  0.4674128 0.7479103 1.2133761 1.738886 1.996975
## n=800  0.5090721 0.8062749 1.3959268 1.542798 1.994769
## n=900  0.4197560 0.8280591 1.2129929 1.504396 1.947569
## n=1000 0.6527630 0.8873246 1.2662086 1.656169 2.115128
##
## , , sd=1, rep=4
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7359835 1.0087000 1.1157127 1.851686 2.286974
## n=200  0.5466285 0.8691083 0.9809655 1.677466 1.914859
## n=300  0.4092951 0.8421824 1.1834134 1.463671 2.105769
## n=400  0.4870693 0.8787365 1.3442915 1.641625 2.038680
```

```
## n=500  0.6247941 0.9737552 1.1814126 1.583607 1.981457
## n=600  0.5543342 0.9385104 1.2433161 1.676785 1.916412
## n=700  0.4178732 0.8808745 1.1952781 1.530920 1.960542
## n=800  0.5618498 0.8943310 1.2475019 1.686068 1.946561
## n=900  0.5052058 0.8785736 1.2679218 1.571624 2.009535
## n=1000 0.4550461 0.8622264 1.2189931 1.640547 2.027112
##
## , , sd=1.5, rep=4
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4449720 0.8435500 1.5231357 1.528876 2.461235
## n=200  0.3352203 0.8038153 0.9593336 1.823112 2.149320
## n=300  0.4930487 0.6791231 1.0272955 1.574880 2.304307
## n=400  0.3806721 1.1714969 1.4056411 1.543264 2.258722
## n=500  0.4589277 0.7928819 1.5044037 1.632672 1.829121
## n=600  0.4884702 0.6958911 1.2883971 1.606977 2.109358
## n=700  0.5038252 0.8396545 1.4939591 1.598702 2.198315
## n=800  0.5515391 0.8256058 1.2838778 1.644795 2.020077
## n=900  0.4102779 0.7769137 1.2511958 1.567744 1.925168
## n=1000 0.6885496 0.9734467 1.2688095 1.525439 1.946295
##
## , , sd=2, rep=4
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.5717011 0.8796835 2.1004676 1.351107 2.598461
## n=200  1.1810891 0.4725781 0.9070217 1.490537 2.444452
## n=300  0.5900949 1.0569764 1.3134750 1.766744 2.089249
## n=400  0.7380901 0.4503391 1.0841655 1.704982 1.998472
## n=500  0.6158445 1.0384364 1.6255235 1.911106 1.868147
## n=600  0.6300586 1.1140343 1.1895053 1.822727 1.933571
## n=700  0.3625215 0.9113498 1.2522415 1.654996 1.981563
## n=800  0.4427674 0.8570775 1.1367413 1.796147 2.103874
## n=900  0.3816606 0.8423425 1.3168243 1.712630 2.102760
## n=1000 0.4265370 0.9495205 1.3021528 1.603875 1.875588
##
## , , sd=1, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.2288956 0.9259850 1.345124 1.767008 1.904891
## n=200  0.4782098 0.9611524 1.326272 1.689215 2.050280
## n=300  0.4520819 0.9920557 1.238380 1.753581 2.087970
## n=400  0.4573970 1.1576259 1.246390 1.497974 2.019858
```

```
## n=500  0.5434649 0.9485599 1.191573 1.631702 2.049623
## n=600  0.4632066 0.8962950 1.266618 1.657375 1.924914
## n=700  0.3907963 0.8164093 1.155234 1.615976 1.949318
## n=800  0.4930138 0.9572131 1.295102 1.684709 1.936302
## n=900  0.4979275 0.8736681 1.214887 1.584737 1.985072
## n=1000 0.5301261 0.8977935 1.313722 1.630559 1.935747
##
## , , sd=1.5, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.4688968 0.9435859 1.380401 1.586464 2.192555
## n=200  0.7831954 0.8507725 1.104056 1.524999 1.987862
## n=300  0.4957244 0.9953593 1.100958 1.596888 2.032147
## n=400  0.6377009 1.1108968 1.039524 1.621890 1.989716
## n=500  0.2757090 0.7425858 1.221020 1.687816 1.782811
## n=600  0.4674703 0.9888671 1.462157 1.805296 2.070940
## n=700  0.3602414 0.8718152 1.369915 1.868538 1.981321
## n=800  0.3886475 1.1178513 1.246394 1.612441 2.065348
## n=900  0.4814983 0.8016443 1.148795 1.443979 2.099358
## n=1000 0.4683020 0.8041379 1.013566 1.662726 1.873865
##
## , , sd=2, rep=5
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.08611764 0.9398614 0.9146867 1.669708 2.319204
## n=200  0.67138321 0.6832920 1.0754579 1.433456 2.068870
## n=300  0.44312742 1.2584762 1.3301563 1.650963 2.100398
## n=400  0.66455181 0.8364694 1.2167647 1.894041 1.989744
## n=500  0.41708387 0.8419375 1.1330271 1.713047 2.102631
## n=600  0.60004510 0.7267214 1.2282300 1.533603 2.044685
## n=700  0.42035536 0.8611750 1.3460957 1.744825 2.228779
## n=800  0.59792089 1.0213057 0.9896550 1.553520 1.600474
## n=900  0.51242473 0.7970967 1.1751429 1.646252 2.039864
## n=1000 0.56705287 0.8053208 1.2585926 1.673913 1.889833
##
## attr(,"class")
## [1] "Eco"   "array"
##
## $average
##        n   mu  sd       avg
## 1    100 0.00 1.0 0.5628181
## 2    100 0.00 1.5 0.4244795
```

```
## 3     100 0.00 2.0 0.3944758
## 4     100 0.25 1.0 0.8769350
## 5     100 0.25 1.5 0.8906907
## 6     100 0.25 2.0 0.8353085
## 7     100 0.50 1.0 1.2598397
## 8     100 0.50 1.5 1.3048593
## 9     100 0.50 2.0 1.2206527
## 10    100 0.75 1.0 1.5751601
## 11    100 0.75 1.5 1.5565558
## 12    100 0.75 2.0 1.7404132
## 13    100 1.00 1.0 2.0166709
## 14    100 1.00 1.5 2.0241493
## 15    100 1.00 2.0 2.3514768
## 16    200 0.00 1.0 0.4892177
## 17    200 0.00 1.5 0.5462168
## 18    200 0.00 2.0 0.6495468
## 19    200 0.25 1.0 0.8720167
## 20    200 0.25 1.5 0.8028606
## 21    200 0.25 2.0 0.7996067
## 22    200 0.50 1.0 1.2150492
## 23    200 0.50 1.5 1.1357791
## 24    200 0.50 2.0 1.3258080
## 25    200 0.75 1.0 1.6500989
## 26    200 0.75 1.5 1.6937863
## 27    200 0.75 2.0 1.5421762
## 28    200 1.00 1.0 2.0076074
## 29    200 1.00 1.5 1.9727911
## 30    200 1.00 2.0 2.0811684
## 31    300 0.00 1.0 0.4759540
## 32    300 0.00 1.5 0.4879391
## 33    300 0.00 2.0 0.4725716
## 34    300 0.25 1.0 0.8340291
## 35    300 0.25 1.5 0.8063792
## 36    300 0.25 2.0 0.9733540
## 37    300 0.50 1.0 1.2693292
## 38    300 0.50 1.5 1.2317647
## 39    300 0.50 2.0 1.2568465
## 40    300 0.75 1.0 1.6012659
## 41    300 0.75 1.5 1.6497061
## 42    300 0.75 2.0 1.6829863
## 43    300 1.00 1.0 2.0274458
## 44    300 1.00 1.5 2.1388580
```

```
## 45   300 1.00 2.0 2.1121160
## 46   400 0.00 1.0 0.5203035
## 47   400 0.00 1.5 0.5590698
## 48   400 0.00 2.0 0.6218474
## 49   400 0.25 1.0 0.9553801
## 50   400 0.25 1.5 0.9715426
## 51   400 0.25 2.0 0.7359215
## 52   400 0.50 1.0 1.2934233
## 53   400 0.50 1.5 1.1087563
## 54   400 0.50 2.0 1.1985429
## 55   400 0.75 1.0 1.5656764
## 56   400 0.75 1.5 1.5885842
## 57   400 0.75 2.0 1.5366603
## 58   400 1.00 1.0 2.0247618
## 59   400 1.00 1.5 2.0519664
## 60   400 1.00 2.0 2.0417733
## 61   500 0.00 1.0 0.5507736
## 62   500 0.00 1.5 0.4689769
## 63   500 0.00 2.0 0.4390092
## 64   500 0.25 1.0 0.9204453
## 65   500 0.25 1.5 0.7972986
## 66   500 0.25 2.0 0.9558140
## 67   500 0.50 1.0 1.2293479
## 68   500 0.50 1.5 1.2402327
## 69   500 0.50 2.0 1.2104054
## 70   500 0.75 1.0 1.6059639
## 71   500 0.75 1.5 1.6035074
## 72   500 0.75 2.0 1.7239084
## 73   500 1.00 1.0 2.0403021
## 74   500 1.00 1.5 1.9586539
## 75   500 1.00 2.0 1.9669104
## 76   600 0.00 1.0 0.5344870
## 77   600 0.00 1.5 0.4817211
## 78   600 0.00 2.0 0.5766933
## 79   600 0.25 1.0 0.9343605
## 80   600 0.25 1.5 0.8663548
## 81   600 0.25 2.0 0.9141000
## 82   600 0.50 1.0 1.2351578
## 83   600 0.50 1.5 1.2943275
## 84   600 0.50 2.0 1.2326393
## 85   600 0.75 1.0 1.6353797
## 86   600 0.75 1.5 1.7550350
```

```
## 87   600 0.75 2.0 1.6714372
## 88   600 1.00 1.0 1.9778364
## 89   600 1.00 1.5 1.9878951
## 90   600 1.00 2.0 2.0195340
## 91   700 0.00 1.0 0.4882859
## 92   700 0.00 1.5 0.4147847
## 93   700 0.00 2.0 0.4465893
## 94   700 0.25 1.0 0.8473095
## 95   700 0.25 1.5 0.8076536
## 96   700 0.25 2.0 0.8197369
## 97   700 0.50 1.0 1.2378238
## 98   700 0.50 1.5 1.3463535
## 99   700 0.50 2.0 1.2706179
## 100  700 0.75 1.0 1.6478257
## 101  700 0.75 1.5 1.7042384
## 102  700 0.75 2.0 1.6003720
## 103  700 1.00 1.0 1.9530612
## 104  700 1.00 1.5 1.9756043
## 105  700 1.00 2.0 2.0816926
## 106  800 0.00 1.0 0.5103950
## 107  800 0.00 1.5 0.4564280
## 108  800 0.00 2.0 0.4581157
## 109  800 0.25 1.0 0.8726540
## 110  800 0.25 1.5 0.8945214
## 111  800 0.25 2.0 0.8772444
## 112  800 0.50 1.0 1.2900639
## 113  800 0.50 1.5 1.2587603
## 114  800 0.50 2.0 1.2319263
## 115  800 0.75 1.0 1.6264243
## 116  800 0.75 1.5 1.6173179
## 117  800 0.75 2.0 1.5997479
## 118  800 1.00 1.0 1.9117509
## 119  800 1.00 1.5 2.0124269
## 120  800 1.00 2.0 1.9063757
## 121  900 0.00 1.0 0.4799391
## 122  900 0.00 1.5 0.4518301
## 123  900 0.00 2.0 0.4165744
## 124  900 0.25 1.0 0.8827002
## 125  900 0.25 1.5 0.8238516
## 126  900 0.25 2.0 0.8144564
## 127  900 0.50 1.0 1.2711118
## 128  900 0.50 1.5 1.2685788
```

```
## 129   900 0.50 2.0 1.2165253
## 130   900 0.75 1.0 1.5946823
## 131   900 0.75 1.5 1.5781026
## 132   900 0.75 2.0 1.5679413
## 133   900 1.00 1.0 2.0249983
## 134   900 1.00 1.5 2.0055806
## 135   900 1.00 2.0 2.0072967
## 136  1000 0.00 1.0 0.4844173
## 137  1000 0.00 1.5 0.5442077
## 138  1000 0.00 2.0 0.5421275
## 139  1000 0.25 1.0 0.8993163
## 140  1000 0.25 1.5 0.8237354
## 141  1000 0.25 2.0 0.8728468
## 142  1000 0.50 1.0 1.2540685
## 143  1000 0.50 1.5 1.2294453
## 144  1000 0.50 2.0 1.3072707
## 145  1000 0.75 1.0 1.6479383
## 146  1000 0.75 1.5 1.5642421
## 147  1000 0.75 2.0 1.6516665
## 148  1000 1.00 1.0 2.0045373
## 149  1000 1.00 1.5 1.9084779
## 150  1000 1.00 2.0 2.0280377
##
## attr(,"class")
## [1] "Eco"
```

The total execution time of OLS simulation is 12 seconds when only four core is used.

- GLS with parallel processing:

```
gls <- main_function(parameters = param_list,nrep=5,simulation = gls_f,sum_fun="mean",seed=1
```

```
##
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Multisession
##
##  Number of Cores Used in  Parallelization :  4  out of 8
##
##  Input Parameters :  c("n", "100", "1000", "100") c("mu", "0", "1", "0.25") c("sd", "1",
##
##  Simulation Length : 750
```

```
##  Minumum : -0.9971
##  Maximum : 3.383637
##  Mean    : 1.260196
##  Median  : 1.284057
##
##  Execution Time of Monte Carlo Simulation 56.02841 secs
##
##  Name of The Class : Eco
```

```
gls
```

```
## $results
## , , sd=1, rep=1
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7554538 0.5016876 1.7356494 1.517285 2.639110
## n=200  0.3903311 0.6408042 1.5194431 1.417126 3.255660
## n=300  0.8317798 0.8173362 1.2688679 1.366244 2.007371
## n=400  0.0664591 0.7887018 1.1664611 1.262650 1.601207
## n=500  0.7349604 0.8108023 1.5108027 1.624144 2.316178
## n=600  0.5483240 1.3362425 0.7466396 1.539507 1.775243
## n=700  1.1797722 0.7454456 1.5538655 1.596843 1.851457
## n=800  0.7388026 0.4151828 1.2355987 2.121480 1.617091
## n=900  0.3375558 0.5516673 0.9502070 1.037452 2.600048
## n=1000 0.2812896 0.8840261 1.4690083 1.667241 2.124077
##
## , , sd=1.5, rep=1
##
##               mu=0    mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100   1.07566007 0.97703407 0.3943886 1.0410972 1.357513
## n=200   0.52383817 1.45057710 1.6489542 1.5278402 1.404216
## n=300   0.71906564 0.40330063 2.2602221 0.8222903 2.070699
## n=400   1.04256675 1.22061498 0.7880425 1.1109698 1.966145
## n=500  -0.07188688 1.41714029 1.4742135 1.9709113 3.005006
## n=600   0.87050124 1.16986970 0.8726417 1.1228528 1.821454
## n=700  -0.25848227 0.49203602 1.0950518 1.6813917 1.675552
## n=800   0.39160576 0.03376035 1.3268501 1.6087353 2.523682
## n=900   0.01281269 0.82989922 1.1917932 2.1448010 2.249365
## n=1000  1.00643264 0.23351513 1.7264966 1.7730168 1.716113
##
## , , sd=2, rep=1
##
##              mu=0   mu=0.25   mu=0.5   mu=0.75     mu=1
```

```
## n=100   0.1989645 0.9356581 1.650652 1.4279687 2.842366
## n=200   0.6770072 0.8534528 1.886844 1.1371663 2.086750
## n=300   0.9021613 1.3441820 1.592364 1.5281411 1.918323
## n=400   0.3678666 0.6749776 1.494146 1.0124086 2.125654
## n=500  -0.3538140 1.4641727 1.702553 2.0690877 1.771409
## n=600   0.2521821 0.8727736 1.281437 1.6461148 1.787155
## n=700  -0.5283604 1.1573474 1.301522 1.6284337 2.040922
## n=800   0.7162405 1.7569821 1.089098 1.8510202 2.008889
## n=900  -0.1466358 1.2663258 0.669074 0.9226198 1.983874
## n=1000  0.4922426 0.5592436 1.209339 0.5571104 2.177159
##
## , , sd=1, rep=2
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.93794659 0.3042632 0.8558104 1.395916 2.488330
## n=200   0.46865388 0.8026164 0.6673771 1.754976 2.196959
## n=300   0.21497718 0.6857743 1.4560339 1.591513 1.441236
## n=400   0.09099797 0.1139196 0.9989419 1.530588 1.901867
## n=500   0.26214018 1.0189866 0.9483061 1.929226 2.028602
## n=600   0.36515870 1.1206859 1.3254734 2.154065 2.398600
## n=700   0.84564182 0.6471012 1.4065464 1.768395 2.419217
## n=800   0.85872788 0.7206308 1.3704309 1.246906 2.218637
## n=900  -0.14990817 1.2264243 1.6872835 1.774656 2.095731
## n=1000  0.96179814 0.6660319 1.2241939 1.846178 2.384093
##
## , , sd=1.5, rep=2
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7971403 0.6846694 1.4747453 1.376802 1.289544
## n=200  0.8722172 1.5062628 0.8667520 1.455997 3.187951
## n=300  1.1649019 1.7666435 0.9995124 1.495770 1.687811
## n=400  0.8440923 0.5933976 0.8222845 1.806278 2.256190
## n=500  0.5786570 1.1103728 1.9378712 1.644442 2.421673
## n=600  0.7493392 1.1661280 1.2064266 1.645635 1.537470
## n=700  0.8992186 1.0192023 1.0847103 1.817849 1.520385
## n=800  0.5199020 0.9399592 1.7241364 1.260431 1.886227
## n=900  1.0116356 1.1141843 1.4591379 2.097425 2.420061
## n=1000 1.1299320 0.6024359 0.3630059 2.020767 1.720216
##
## , , sd=2, rep=2
##
##               mu=0     mu=0.25     mu=0.5    mu=0.75     mu=1
```

```
## n=100  -0.01919950  0.10389729 -0.7181519  2.1975727 2.688336
## n=200   1.02896044 -0.06349928  1.7339415  1.9835128 1.816331
## n=300  -0.18610242  1.60434976  1.6070085 -0.3271213 1.931503
## n=400   1.41709802  1.20750797  1.5523077  1.6424103 1.171583
## n=500   1.23639159  0.54645719  1.6774379  1.3658300 3.363879
## n=600   0.39037287  1.42422862  0.5759413  2.1932974 1.854756
## n=700  -0.34830352  0.85511492  1.2548947  2.4213875 2.016490
## n=800  -0.08188814  1.27109005  1.7893126  0.9299721 1.717897
## n=900  -0.07990839  0.66567655  1.8002084  1.7681873 1.740396
## n=1000  0.88195270  0.50717811  1.2185675  2.1915303 1.744440
##
## , , sd=1, rep=3
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.2500595 1.3250897 0.9547569 2.293587 1.819439
## n=200  0.5471549 0.9566464 1.3296827 1.775252 1.804901
## n=300  1.0456930 1.4996928 1.4615289 1.399473 2.154331
## n=400  0.6342614 0.5517242 1.4421203 1.787110 2.553724
## n=500  0.5785579 1.0079606 1.1506459 1.573662 1.915940
## n=600  0.3589415 0.7270598 1.5240060 1.239381 1.971919
## n=700  0.7345540 0.2262226 1.2611827 1.103715 2.039746
## n=800  0.5015531 1.9152089 1.6893004 1.561901 2.198732
## n=900  0.1721376 1.0750490 1.1722128 1.835734 1.946778
## n=1000 0.5581321 0.7179216 1.0656947 1.773012 2.400244
##
## , , sd=1.5, rep=3
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.75515002 1.0532149 1.3675520 2.458531 2.511075
## n=200  -0.04934648 0.5496542 2.0479018 1.675846 1.142718
## n=300   1.00181000 1.6369489 0.7753943 2.638377 1.829983
## n=400   0.38708014 1.2533585 1.4714135 1.435519 2.018202
## n=500   0.37250451 0.7996507 1.2444854 1.892440 1.427025
## n=600   0.26431978 1.3546424 1.2058852 1.796262 2.147378
## n=700   1.12511433 0.6271908 0.8411113 1.638469 1.970271
## n=800   0.52208688 0.6764603 1.6023554 1.526346 1.906584
## n=900   0.22266670 0.5468360 1.5856621 2.134440 1.515266
## n=1000  0.20373892 1.9340035 1.2128314 1.337933 1.380179
##
## , , sd=2, rep=3
##
##              mu=0    mu=0.25      mu=0.5  mu=0.75      mu=1
```

```
## n=100  -0.5134825 -0.9971000  0.02609116 2.483622 2.1737654
## n=200   0.8564846  0.5270190  2.19874069 1.115522 2.0483072
## n=300   0.1357223  1.1346463  0.67885853 1.989233 2.3651937
## n=400  -0.1132654  1.5174377 -0.39911038 2.430706 2.7068852
## n=500   0.2692164  0.9373865  0.45341757 1.488270 2.0855372
## n=600   0.2767981  1.9185888  1.10986631 1.779951 2.2288551
## n=700   0.4175472  0.7176046  0.78665976 1.500209 2.3208915
## n=800   0.2236755  0.7400305  1.63497839 2.006660 2.2390314
## n=900   1.3383189  1.7077645  1.90542412 2.962268 1.5069244
## n=1000  0.2337000  0.4510541  0.72055597 2.240282 0.9580619
##
## , , sd=1, rep=4
##
##             mu=0     mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  1.3469860 -0.05560687 1.0932454 1.903516 2.156825
## n=200  0.3208913  0.96409270 1.0863966 1.386434 1.835235
## n=300  0.6078219  0.39871289 1.3616965 2.100878 1.933667
## n=400  0.6771807  1.05598360 1.5240453 2.261056 2.531636
## n=500  0.4919199  1.10827112 1.5332686 1.517603 1.843585
## n=600  0.5637876  0.85089642 1.8731358 1.617511 2.200501
## n=700  0.3582705  0.99200307 0.8214109 1.560444 2.128348
## n=800  0.5240053  0.94117803 1.2620739 1.546574 1.873964
## n=900  0.6295721  0.83325772 0.9917639 1.333128 1.971597
## n=1000 0.6248638  1.00187859 1.3589893 1.665165 1.630167
##
## , , sd=1.5, rep=4
##
##              mu=0     mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100  0.14190040  2.01821137 0.8517502 2.0724785 2.762258
## n=200  0.82223920  0.95236727 2.3576613 2.7525450 2.656997
## n=300  0.41468489  0.81135204 1.7321312 2.2129173 1.586342
## n=400  0.47456049  1.28481297 1.6045288 1.6357618 2.517795
## n=500  0.19596935 -0.09776629 1.3816823 1.1821549 1.806055
## n=600  0.04919382  1.33474572 1.2062848 0.0429013 2.320981
## n=700  0.34423613  0.48523985 1.9450566 1.5101308 2.451672
## n=800  1.32565822  0.84774132 1.2387826 2.3976442 1.870370
## n=900  0.47854733  0.51916320 1.1726107 1.6816128 1.705770
## n=1000 0.82817973  0.80225623 1.5175430 1.4264566 1.863020
##
## , , sd=2, rep=4
##
##             mu=0    mu=0.25     mu=0.5  mu=0.75     mu=1
```

```
## n=100  0.8525966  1.1749948 3.38363655 1.248533 3.057232
## n=200  0.5182802  0.4838462 1.41934260 2.241456 2.358571
## n=300  1.0733668  2.0578513 0.96974173 2.245360 1.290691
## n=400  0.8011078  0.4410973 0.90802926 1.296971 2.153393
## n=500  0.2470769  0.7742284 2.22385975 2.482988 1.599441
## n=600  1.1095269  0.4747139 0.06967184 1.576253 1.038778
## n=700  0.3443959 -0.7980158 0.88632336 1.763624 1.642792
## n=800  0.3212259 -0.1889027 1.12345703 2.337993 1.622583
## n=900  0.8488329  1.3243651 0.42830492 1.324446 2.120547
## n=1000 1.6411901  0.1763252 1.43712633 1.095596 1.415895
##
## , , sd=1, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.1106945 1.3324074 1.283149 1.795427 1.479256
## n=200  0.5031188 1.5027343 1.460474 1.921770 2.617298
## n=300  0.3730896 0.6590489 1.300485 1.664153 2.229097
## n=400  0.4404047 1.6079649 1.600920 1.597121 1.219695
## n=500  0.9142947 0.8821124 0.993282 2.142944 2.017532
## n=600  0.7455402 1.3534103 1.252659 1.654961 2.019734
## n=700  0.1307611 0.5045976 1.915240 1.719891 1.816085
## n=800  0.0348579 0.4676373 1.687917 1.773396 1.871487
## n=900  0.5683141 1.0347100 1.329680 1.971053 2.031339
## n=1000 0.2944263 0.7265625 1.342830 1.397584 1.678541
##
## , , sd=1.5, rep=5
##
##              mu=0     mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100   0.3492151 0.941764185 1.9927388 1.1271039 2.343358
## n=200   0.9322249 0.009236562 1.6216833 1.5246716 2.126926
## n=300  -0.2061382 0.527637441 1.1785690 1.2855802 1.955373
## n=400  -0.2080310 0.865475626 1.5810396 1.8863548 2.607916
## n=500   1.0591201 1.269133681 1.3106256 1.6893725 2.221825
## n=600   1.0529351 0.935277255 0.8883463 0.4731693 2.883712
## n=700  -0.1566764 1.060909722 1.3480097 1.5789494 2.593332
## n=800   0.3458820 0.824462226 1.0043539 1.3977273 1.529786
## n=900   0.4578455 0.443865375 1.0857179 1.1477187 1.641970
## n=1000  0.7074108 0.556170334 1.5201499 1.3028712 2.580936
##
## , , sd=2, rep=5
##
##               mu=0    mu=0.25    mu=0.5    mu=0.75     mu=1
```

```
## n=100    0.35838129  1.9134632 0.6236571  1.8050059 2.775072
## n=200    1.13555545 -0.3462561 0.5517002  1.2833010 2.082454
## n=300   -0.12779402  0.8027196 0.2661467  1.1407255 2.706796
## n=400    0.84449836  1.5107112 1.3472610  2.1182648 2.410341
## n=500    0.58335229  0.3130528 2.2292441  0.5206100 1.407526
## n=600    1.00687281  0.8306865 0.8167926 -0.4327573 1.897837
## n=700    0.90763719  0.7547756 1.2602192  1.9296762 2.212971
## n=800    1.16899109  0.5890770 1.1466515  0.6023830 1.086282
## n=900   -0.53156942  0.2064395 1.3326650  0.1082137 1.878707
## n=1000 -0.06566813  1.5126884 1.9075839  1.1067314 1.610853
##
## attr(,"class")
## [1] "Eco"   "array"
##
## $average
##         n   mu  sd       avg
## 1     100 0.00 1.0 0.6802281
## 2     100 0.00 1.5 0.6238132
## 3     100 0.00 2.0 0.1754521
## 4     100 0.25 1.0 0.6815682
## 5     100 0.25 1.5 1.1349788
## 6     100 0.25 2.0 0.6261827
## 7     100 0.50 1.0 1.1845221
## 8     100 0.50 1.5 1.2162350
## 9     100 0.50 2.0 0.9931770
## 10    100 0.75 1.0 1.7811463
## 11    100 0.75 1.5 1.6152025
## 12    100 0.75 2.0 1.8325404
## 13    100 1.00 1.0 2.1165920
## 14    100 1.00 1.5 2.0527497
## 15    100 1.00 2.0 2.7073544
## 16    200 0.00 1.0 0.4460300
## 17    200 0.00 1.5 0.6202346
## 18    200 0.00 2.0 0.8432576
## 19    200 0.25 1.0 0.9733788
## 20    200 0.25 1.5 0.8936196
## 21    200 0.25 2.0 0.2909125
## 22    200 0.50 1.0 1.2126747
## 23    200 0.50 1.5 1.7085905
## 24    200 0.50 2.0 1.5581137
## 25    200 0.75 1.0 1.6511116
## 26    200 0.75 1.5 1.7873799
```

```
## 27   200 0.75 2.0 1.5521916
## 28   200 1.00 1.0 2.3420107
## 29   200 1.00 1.5 2.1037616
## 30   200 1.00 2.0 2.0784825
## 31   300 0.00 1.0 0.6146723
## 32   300 0.00 1.5 0.6188648
## 33   300 0.00 2.0 0.3594708
## 34   300 0.25 1.0 0.8121130
## 35   300 0.25 1.5 1.0291765
## 36   300 0.25 2.0 1.3887498
## 37   300 0.50 1.0 1.3697225
## 38   300 0.50 1.5 1.3891658
## 39   300 0.50 2.0 1.0228240
## 40   300 0.75 1.0 1.6244521
## 41   300 0.75 1.5 1.6909870
## 42   300 0.75 2.0 1.3152675
## 43   300 1.00 1.0 1.9531402
## 44   300 1.00 1.5 1.8260415
## 45   300 1.00 2.0 2.0425014
## 46   400 0.00 1.0 0.3818608
## 47   400 0.00 1.5 0.5080537
## 48   400 0.00 2.0 0.6634611
## 49   400 0.25 1.0 0.8236588
## 50   400 0.25 1.5 1.0435319
## 51   400 0.25 2.0 1.0703464
## 52   400 0.50 1.0 1.3464978
## 53   400 0.50 1.5 1.2534618
## 54   400 0.50 2.0 0.9805268
## 55   400 0.75 1.0 1.6877048
## 56   400 0.75 1.5 1.5749766
## 57   400 0.75 2.0 1.7001523
## 58   400 1.00 1.0 1.9616256
## 59   400 1.00 1.5 2.2732495
## 60   400 1.00 2.0 2.1135711
## 61   500 0.00 1.0 0.5963746
## 62   500 0.00 1.5 0.4268728
## 63   500 0.00 2.0 0.3964446
## 64   500 0.25 1.0 0.9656266
## 65   500 0.25 1.5 0.8997062
## 66   500 0.25 2.0 0.8070595
## 67   500 0.50 1.0 1.2272611
## 68   500 0.50 1.5 1.4697756
```

```
## 69    500 0.50 2.0 1.6573025
## 70    500 0.75 1.0 1.7575159
## 71    500 0.75 1.5 1.6758640
## 72    500 0.75 2.0 1.5853571
## 73    500 1.00 1.0 2.0243674
## 74    500 1.00 1.5 2.1763168
## 75    500 1.00 2.0 2.0455584
## 76    600 0.00 1.0 0.5163504
## 77    600 0.00 1.5 0.5972578
## 78    600 0.00 2.0 0.6071506
## 79    600 0.25 1.0 1.0776590
## 80    600 0.25 1.5 1.1921326
## 81    600 0.25 2.0 1.1041983
## 82    600 0.50 1.0 1.3443827
## 83    600 0.50 1.5 1.0759169
## 84    600 0.50 2.0 0.7707418
## 85    600 0.75 1.0 1.6410851
## 86    600 0.75 1.5 1.0161640
## 87    600 0.75 2.0 1.3525717
## 88    600 1.00 1.0 2.0731992
## 89    600 1.00 1.5 2.1421991
## 90    600 1.00 2.0 1.7614761
## 91    700 0.00 1.0 0.6497999
## 92    700 0.00 1.5 0.3906821
## 93    700 0.00 2.0 0.1585833
## 94    700 0.25 1.0 0.6230740
## 95    700 0.25 1.5 0.7369157
## 96    700 0.25 2.0 0.5373653
## 97    700 0.50 1.0 1.3916491
## 98    700 0.50 1.5 1.2627879
## 99    700 0.50 2.0 1.0979239
## 100   700 0.75 1.0 1.5498575
## 101   700 0.75 1.5 1.6453580
## 102   700 0.75 2.0 1.8486661
## 103   700 1.00 1.0 2.0509706
## 104   700 1.00 1.5 2.0422424
## 105   700 1.00 2.0 2.0468134
## 106   800 0.00 1.0 0.5315894
## 107   800 0.00 1.5 0.6210270
## 108   800 0.00 2.0 0.4696490
## 109   800 0.25 1.0 0.8919676
## 110   800 0.25 1.5 0.6644767
```

```
## 111  800 0.25 2.0 0.8336554
## 112  800 0.50 1.0 1.4490643
## 113  800 0.50 1.5 1.3792957
## 114  800 0.50 2.0 1.3566995
## 115  800 0.75 1.0 1.6500513
## 116  800 0.75 1.5 1.6381768
## 117  800 0.75 2.0 1.5456057
## 118  800 1.00 1.0 1.9559821
## 119  800 1.00 1.5 1.9433300
## 120  800 1.00 2.0 1.7349365
## 121  900 0.00 1.0 0.3115343
## 122  900 0.00 1.5 0.4367016
## 123  900 0.00 2.0 0.2858076
## 124  900 0.25 1.0 0.9442217
## 125  900 0.25 1.5 0.6907896
## 126  900 0.25 2.0 1.0341143
## 127  900 0.50 1.0 1.2262295
## 128  900 0.50 1.5 1.2989843
## 129  900 0.50 2.0 1.2271353
## 130  900 0.75 1.0 1.5904047
## 131  900 0.75 1.5 1.8411995
## 132  900 0.75 2.0 1.4171470
## 133  900 1.00 1.0 2.1290987
## 134  900 1.00 1.5 1.9064864
## 135  900 1.00 2.0 1.8460896
## 136 1000 0.00 1.0 0.5441020
## 137 1000 0.00 1.5 0.7751388
## 138 1000 0.00 2.0 0.6366835
## 139 1000 0.25 1.0 0.7992841
## 140 1000 0.25 1.5 0.8256762
## 141 1000 0.25 2.0 0.6412979
## 142 1000 0.50 1.0 1.2921433
## 143 1000 0.50 1.5 1.2680054
## 144 1000 0.50 2.0 1.2986345
## 145 1000 0.75 1.0 1.6698362
## 146 1000 0.75 1.5 1.5722088
## 147 1000 0.75 2.0 1.4382501
## 148 1000 1.00 1.0 2.0434242
## 149 1000 1.00 1.5 1.8520927
## 150 1000 1.00 2.0 1.5812816
##
## attr(,"class")
```

```
## [1] "Eco"
```

- GLS without parallel processing:

```r
gls <- main_function(parameters = param_list,nrep=5,simulation = gls_f,sum_fun="mean",seed=1
```

```
##
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Sequential
##
##  Number of Cores Used in  Parallelization :  1  out of 8
##
##  Input Parameters :  c("n", "100", "1000", "100") c("mu", "0", "1", "0.25") c("sd", "1",
##
##  Simulation Length : 750
##  Minumum : -0.9971
##  Maximum : 3.383637
##  Mean    : 1.260196
##  Median  : 1.284057
##
##  Execution Time of Monte Carlo Simulation 54.28675 secs
##
##  Name of The Class : Eco
```

```r
gls
```

```
## $results
## , , sd=1, rep=1
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7554538 0.5016876 1.7356494 1.517285 2.639110
## n=200  0.3903311 0.6408042 1.5194431 1.417126 3.255660
## n=300  0.8317798 0.8173362 1.2688679 1.366244 2.007371
## n=400  0.0664591 0.7887018 1.1664611 1.262650 1.601207
## n=500  0.7349604 0.8108023 1.5108027 1.624144 2.316178
## n=600  0.5483240 1.3362425 0.7466396 1.539507 1.775243
## n=700  1.1797722 0.7454456 1.5538655 1.596843 1.851457
## n=800  0.7388026 0.4151828 1.2355987 2.121480 1.617091
## n=900  0.3375558 0.5516673 0.9502070 1.037452 2.600048
## n=1000 0.2812896 0.8840261 1.4690083 1.667241 2.124077
##
```

```
## , , sd=1.5, rep=1
##
##               mu=0    mu=0.25    mu=0.5   mu=0.75      mu=1
## n=100   1.07566007 0.97703407 0.3943886 1.0410972 1.357513
## n=200   0.52383817 1.45057710 1.6489542 1.5278402 1.404216
## n=300   0.71906564 0.40330063 2.2602221 0.8222903 2.070699
## n=400   1.04256675 1.22061498 0.7880425 1.1109698 1.966145
## n=500  -0.07188688 1.41714029 1.4742135 1.9709113 3.005006
## n=600   0.87050124 1.16986970 0.8726417 1.1228528 1.821454
## n=700  -0.25848227 0.49203602 1.0950518 1.6813917 1.675552
## n=800   0.39160576 0.03376035 1.3268501 1.6087353 2.523682
## n=900   0.01281269 0.82989922 1.1917932 2.1448010 2.249365
## n=1000  1.00643264 0.23351513 1.7264966 1.7730168 1.716113
##
## , , sd=2, rep=1
##
##              mu=0   mu=0.25   mu=0.5   mu=0.75     mu=1
## n=100   0.1989645 0.9356581 1.650652 1.4279687 2.842366
## n=200   0.6770072 0.8534528 1.886844 1.1371663 2.086750
## n=300   0.9021613 1.3441820 1.592364 1.5281411 1.918323
## n=400   0.3678666 0.6749776 1.494146 1.0124086 2.125654
## n=500  -0.3538140 1.4641727 1.702553 2.0690877 1.771409
## n=600   0.2521821 0.8727736 1.281437 1.6461148 1.787155
## n=700  -0.5283604 1.1573474 1.301522 1.6284337 2.040922
## n=800   0.7162405 1.7569821 1.089098 1.8510202 2.008889
## n=900  -0.1466358 1.2663258 0.669074 0.9226198 1.983874
## n=1000  0.4922426 0.5592436 1.209339 0.5571104 2.177159
##
## , , sd=1, rep=2
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.93794659 0.3042632 0.8558104 1.395916 2.488330
## n=200   0.46865388 0.8026164 0.6673771 1.754976 2.196959
## n=300   0.21497718 0.6857743 1.4560339 1.591513 1.441236
## n=400   0.09099797 0.1139196 0.9989419 1.530588 1.901867
## n=500   0.26214018 1.0189866 0.9483061 1.929226 2.028602
## n=600   0.36515870 1.1206859 1.3254734 2.154065 2.398600
## n=700   0.84564182 0.6471012 1.4065464 1.768395 2.419217
## n=800   0.85872788 0.7206308 1.3704309 1.246906 2.218637
## n=900  -0.14990817 1.2264243 1.6872835 1.774656 2.095731
## n=1000  0.96179814 0.6660319 1.2241939 1.846178 2.384093
##
```

```
## , , sd=1.5, rep=2
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7971403 0.6846694 1.4747453 1.376802 1.289544
## n=200  0.8722172 1.5062628 0.8667520 1.455997 3.187951
## n=300  1.1649019 1.7666435 0.9995124 1.495770 1.687811
## n=400  0.8440923 0.5933976 0.8222845 1.806278 2.256190
## n=500  0.5786570 1.1103728 1.9378712 1.644442 2.421673
## n=600  0.7493392 1.1661280 1.2064266 1.645635 1.537470
## n=700  0.8992186 1.0192023 1.0847103 1.817849 1.520385
## n=800  0.5199020 0.9399592 1.7241364 1.260431 1.886227
## n=900  1.0116356 1.1141843 1.4591379 2.097425 2.420061
## n=1000 1.1299320 0.6024359 0.3630059 2.020767 1.720216
##
## , , sd=2, rep=2
##
##                mu=0     mu=0.25     mu=0.5    mu=0.75     mu=1
## n=100  -0.01919950  0.10389729 -0.7181519  2.1975727 2.688336
## n=200   1.02896044 -0.06349928  1.7339415  1.9835128 1.816331
## n=300  -0.18610242  1.60434976  1.6070085 -0.3271213 1.931503
## n=400   1.41709802  1.20750797  1.5523077  1.6424103 1.171583
## n=500   1.23639159  0.54645719  1.6774379  1.3658300 3.363879
## n=600   0.39037287  1.42422862  0.5759413  2.1932974 1.854756
## n=700  -0.34830352  0.85511492  1.2548947  2.4213875 2.016490
## n=800  -0.08188814  1.27109005  1.7893126  0.9299721 1.717897
## n=900  -0.07990839  0.66567655  1.8002084  1.7681873 1.740396
## n=1000  0.88195270  0.50717811  1.2185675  2.1915303 1.744440
##
## , , sd=1, rep=3
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.2500595 1.3250897 0.9547569 2.293587 1.819439
## n=200  0.5471549 0.9566464 1.3296827 1.775252 1.804901
## n=300  1.0456930 1.4996928 1.4615289 1.399473 2.154331
## n=400  0.6342614 0.5517242 1.4421203 1.787110 2.553724
## n=500  0.5785579 1.0079606 1.1506459 1.573662 1.915940
## n=600  0.3589415 0.7270598 1.5240060 1.239381 1.971919
## n=700  0.7345540 0.2262226 1.2611827 1.103715 2.039746
## n=800  0.5015531 1.9152089 1.6893004 1.561901 2.198732
## n=900  0.1721376 1.0750490 1.1722128 1.835734 1.946778
## n=1000 0.5581321 0.7179216 1.0656947 1.773012 2.400244
##
```

```
## , , sd=1.5, rep=3
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.75515002 1.0532149 1.3675520 2.458531 2.511075
## n=200  -0.04934648 0.5496542 2.0479018 1.675846 1.142718
## n=300   1.00181000 1.6369489 0.7753943 2.638377 1.829983
## n=400   0.38708014 1.2533585 1.4714135 1.435519 2.018202
## n=500   0.37250451 0.7996507 1.2444854 1.892440 1.427025
## n=600   0.26431978 1.3546424 1.2058852 1.796262 2.147378
## n=700   1.12511433 0.6271908 0.8411113 1.638469 1.970271
## n=800   0.52208688 0.6764603 1.6023554 1.526346 1.906584
## n=900   0.22266670 0.5468360 1.5856621 2.134440 1.515266
## n=1000  0.20373892 1.9340035 1.2128314 1.337933 1.380179
##
## , , sd=2, rep=3
##
##             mu=0    mu=0.25      mu=0.5  mu=0.75      mu=1
## n=100  -0.5134825 -0.9971000  0.02609116 2.483622 2.1737654
## n=200   0.8564846  0.5270190  2.19874069 1.115522 2.0483072
## n=300   0.1357223  1.1346463  0.67885853 1.989233 2.3651937
## n=400  -0.1132654  1.5174377 -0.39911038 2.430706 2.7068852
## n=500   0.2692164  0.9373865  0.45341757 1.488270 2.0855372
## n=600   0.2767981  1.9185888  1.10986631 1.779951 2.2288551
## n=700   0.4175472  0.7176046  0.78665976 1.500209 2.3208915
## n=800   0.2236755  0.7400305  1.63497839 2.006660 2.2390314
## n=900   1.3383189  1.7077645  1.90542412 2.962268 1.5069244
## n=1000  0.2337000  0.4510541  0.72055597 2.240282 0.9580619
##
## , , sd=1, rep=4
##
##            mu=0     mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  1.3469860 -0.05560687 1.0932454 1.903516 2.156825
## n=200  0.3208913  0.96409270 1.0863966 1.386434 1.835235
## n=300  0.6078219  0.39871289 1.3616965 2.100878 1.933667
## n=400  0.6771807  1.05598360 1.5240453 2.261056 2.531636
## n=500  0.4919199  1.10827112 1.5332686 1.517603 1.843585
## n=600  0.5637876  0.85089642 1.8731358 1.617511 2.200501
## n=700  0.3582705  0.99200307 0.8214109 1.560444 2.128348
## n=800  0.5240053  0.94117803 1.2620739 1.546574 1.873964
## n=900  0.6295721  0.83325772 0.9917639 1.333128 1.971597
## n=1000 0.6248638  1.00187859 1.3589893 1.665165 1.630167
##
```

```
## , , sd=1.5, rep=4
##
##              mu=0     mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100  0.14190040  2.01821137 0.8517502 2.0724785 2.762258
## n=200  0.82223920  0.95236727 2.3576613 2.7525450 2.656997
## n=300  0.41468489  0.81135204 1.7321312 2.2129173 1.586342
## n=400  0.47456049  1.28481297 1.6045288 1.6357618 2.517795
## n=500  0.19596935 -0.09776629 1.3816823 1.1821549 1.806055
## n=600  0.04919382  1.33474572 1.2062848 0.0429013 2.320981
## n=700  0.34423613  0.48523985 1.9450566 1.5101308 2.451672
## n=800  1.32565822  0.84774132 1.2387826 2.3976442 1.870370
## n=900  0.47854733  0.51916320 1.1726107 1.6816128 1.705770
## n=1000 0.82817973  0.80225623 1.5175430 1.4264566 1.863020
##
## , , sd=2, rep=4
##
##             mu=0    mu=0.25     mu=0.5  mu=0.75     mu=1
## n=100  0.8525966  1.1749948 3.38363655 1.248533 3.057232
## n=200  0.5182802  0.4838462 1.41934260 2.241456 2.358571
## n=300  1.0733668  2.0578513 0.96974173 2.245360 1.290691
## n=400  0.8011078  0.4410973 0.90802926 1.296971 2.153393
## n=500  0.2470769  0.7742284 2.22385975 2.482988 1.599441
## n=600  1.1095269  0.4747139 0.06967184 1.576253 1.038778
## n=700  0.3443959 -0.7980158 0.88632336 1.763624 1.642792
## n=800  0.3212259 -0.1889027 1.12345703 2.337993 1.622583
## n=900  0.8488329  1.3243651 0.42830492 1.324446 2.120547
## n=1000 1.6411901  0.1763252 1.43712633 1.095596 1.415895
##
## , , sd=1, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.1106945 1.3324074 1.283149 1.795427 1.479256
## n=200  0.5031188 1.5027343 1.460474 1.921770 2.617298
## n=300  0.3730896 0.6590489 1.300485 1.664153 2.229097
## n=400  0.4404047 1.6079649 1.600920 1.597121 1.219695
## n=500  0.9142947 0.8821124 0.993282 2.142944 2.017532
## n=600  0.7455402 1.3534103 1.252659 1.654961 2.019734
## n=700  0.1307611 0.5045976 1.915240 1.719891 1.816085
## n=800  0.0348579 0.4676373 1.687917 1.773396 1.871487
## n=900  0.5683141 1.0347100 1.329680 1.971053 2.031339
## n=1000 0.2944263 0.7265625 1.342830 1.397584 1.678541
##
```

```
## , , sd=1.5, rep=5
##
##               mu=0     mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100   0.3492151 0.941764185 1.9927388 1.1271039 2.343358
## n=200   0.9322249 0.009236562 1.6216833 1.5246716 2.126926
## n=300  -0.2061382 0.527637441 1.1785690 1.2855802 1.955373
## n=400  -0.2080310 0.865475626 1.5810396 1.8863548 2.607916
## n=500   1.0591201 1.269133681 1.3106256 1.6893725 2.221825
## n=600   1.0529351 0.935277255 0.8883463 0.4731693 2.883712
## n=700  -0.1566764 1.060909722 1.3480097 1.5789494 2.593332
## n=800   0.3458820 0.824462226 1.0043539 1.3977273 1.529786
## n=900   0.4578455 0.443865375 1.0857179 1.1477187 1.641970
## n=1000  0.7074108 0.556170334 1.5201499 1.3028712 2.580936
##
## , , sd=2, rep=5
##
##                mu=0    mu=0.25    mu=0.5    mu=0.75     mu=1
## n=100   0.35838129  1.9134632 0.6236571  1.8050059 2.775072
## n=200   1.13555545 -0.3462561 0.5517002  1.2833010 2.082454
## n=300  -0.12779402  0.8027196 0.2661467  1.1407255 2.706796
## n=400   0.84449836  1.5107112 1.3472610  2.1182648 2.410341
## n=500   0.58335229  0.3130528 2.2292441  0.5206100 1.407526
## n=600   1.00687281  0.8306865 0.8167926 -0.4327573 1.897837
## n=700   0.90763719  0.7547756 1.2602192  1.9296762 2.212971
## n=800   1.16899109  0.5890770 1.1466515  0.6023830 1.086282
## n=900  -0.53156942  0.2064395 1.3326650  0.1082137 1.878707
## n=1000 -0.06566813  1.5126884 1.9075839  1.1067314 1.610853
##
## attr(,"class")
## [1] "Eco"   "array"
##
## $average
##         n   mu  sd       avg
## 1     100 0.00 1.0 0.6802281
## 2     100 0.00 1.5 0.6238132
## 3     100 0.00 2.0 0.1754521
## 4     100 0.25 1.0 0.6815682
## 5     100 0.25 1.5 1.1349788
## 6     100 0.25 2.0 0.6261827
## 7     100 0.50 1.0 1.1845221
## 8     100 0.50 1.5 1.2162350
## 9     100 0.50 2.0 0.9931770
```

```
## 10    100 0.75 1.0 1.7811463
## 11    100 0.75 1.5 1.6152025
## 12    100 0.75 2.0 1.8325404
## 13    100 1.00 1.0 2.1165920
## 14    100 1.00 1.5 2.0527497
## 15    100 1.00 2.0 2.7073544
## 16    200 0.00 1.0 0.4460300
## 17    200 0.00 1.5 0.6202346
## 18    200 0.00 2.0 0.8432576
## 19    200 0.25 1.0 0.9733788
## 20    200 0.25 1.5 0.8936196
## 21    200 0.25 2.0 0.2909125
## 22    200 0.50 1.0 1.2126747
## 23    200 0.50 1.5 1.7085905
## 24    200 0.50 2.0 1.5581137
## 25    200 0.75 1.0 1.6511116
## 26    200 0.75 1.5 1.7873799
## 27    200 0.75 2.0 1.5521916
## 28    200 1.00 1.0 2.3420107
## 29    200 1.00 1.5 2.1037616
## 30    200 1.00 2.0 2.0784825
## 31    300 0.00 1.0 0.6146723
## 32    300 0.00 1.5 0.6188648
## 33    300 0.00 2.0 0.3594708
## 34    300 0.25 1.0 0.8121130
## 35    300 0.25 1.5 1.0291765
## 36    300 0.25 2.0 1.3887498
## 37    300 0.50 1.0 1.3697225
## 38    300 0.50 1.5 1.3891658
## 39    300 0.50 2.0 1.0228240
## 40    300 0.75 1.0 1.6244521
## 41    300 0.75 1.5 1.6909870
## 42    300 0.75 2.0 1.3152675
## 43    300 1.00 1.0 1.9531402
## 44    300 1.00 1.5 1.8260415
## 45    300 1.00 2.0 2.0425014
## 46    400 0.00 1.0 0.3818608
## 47    400 0.00 1.5 0.5080537
## 48    400 0.00 2.0 0.6634611
## 49    400 0.25 1.0 0.8236588
## 50    400 0.25 1.5 1.0435319
## 51    400 0.25 2.0 1.0703464
```

```
## 52   400 0.50 1.0 1.3464978
## 53   400 0.50 1.5 1.2534618
## 54   400 0.50 2.0 0.9805268
## 55   400 0.75 1.0 1.6877048
## 56   400 0.75 1.5 1.5749766
## 57   400 0.75 2.0 1.7001523
## 58   400 1.00 1.0 1.9616256
## 59   400 1.00 1.5 2.2732495
## 60   400 1.00 2.0 2.1135711
## 61   500 0.00 1.0 0.5963746
## 62   500 0.00 1.5 0.4268728
## 63   500 0.00 2.0 0.3964446
## 64   500 0.25 1.0 0.9656266
## 65   500 0.25 1.5 0.8997062
## 66   500 0.25 2.0 0.8070595
## 67   500 0.50 1.0 1.2272611
## 68   500 0.50 1.5 1.4697756
## 69   500 0.50 2.0 1.6573025
## 70   500 0.75 1.0 1.7575159
## 71   500 0.75 1.5 1.6758640
## 72   500 0.75 2.0 1.5853571
## 73   500 1.00 1.0 2.0243674
## 74   500 1.00 1.5 2.1763168
## 75   500 1.00 2.0 2.0455584
## 76   600 0.00 1.0 0.5163504
## 77   600 0.00 1.5 0.5972578
## 78   600 0.00 2.0 0.6071506
## 79   600 0.25 1.0 1.0776590
## 80   600 0.25 1.5 1.1921326
## 81   600 0.25 2.0 1.1041983
## 82   600 0.50 1.0 1.3443827
## 83   600 0.50 1.5 1.0759169
## 84   600 0.50 2.0 0.7707418
## 85   600 0.75 1.0 1.6410851
## 86   600 0.75 1.5 1.0161640
## 87   600 0.75 2.0 1.3525717
## 88   600 1.00 1.0 2.0731992
## 89   600 1.00 1.5 2.1421991
## 90   600 1.00 2.0 1.7614761
## 91   700 0.00 1.0 0.6497999
## 92   700 0.00 1.5 0.3906821
## 93   700 0.00 2.0 0.1585833
```

```
## 94   700 0.25 1.0 0.6230740
## 95   700 0.25 1.5 0.7369157
## 96   700 0.25 2.0 0.5373653
## 97   700 0.50 1.0 1.3916491
## 98   700 0.50 1.5 1.2627879
## 99   700 0.50 2.0 1.0979239
## 100  700 0.75 1.0 1.5498575
## 101  700 0.75 1.5 1.6453580
## 102  700 0.75 2.0 1.8486661
## 103  700 1.00 1.0 2.0509706
## 104  700 1.00 1.5 2.0422424
## 105  700 1.00 2.0 2.0468134
## 106  800 0.00 1.0 0.5315894
## 107  800 0.00 1.5 0.6210270
## 108  800 0.00 2.0 0.4696490
## 109  800 0.25 1.0 0.8919676
## 110  800 0.25 1.5 0.6644767
## 111  800 0.25 2.0 0.8336554
## 112  800 0.50 1.0 1.4490643
## 113  800 0.50 1.5 1.3792957
## 114  800 0.50 2.0 1.3566995
## 115  800 0.75 1.0 1.6500513
## 116  800 0.75 1.5 1.6381768
## 117  800 0.75 2.0 1.5456057
## 118  800 1.00 1.0 1.9559821
## 119  800 1.00 1.5 1.9433300
## 120  800 1.00 2.0 1.7349365
## 121  900 0.00 1.0 0.3115343
## 122  900 0.00 1.5 0.4367016
## 123  900 0.00 2.0 0.2858076
## 124  900 0.25 1.0 0.9442217
## 125  900 0.25 1.5 0.6907896
## 126  900 0.25 2.0 1.0341143
## 127  900 0.50 1.0 1.2262295
## 128  900 0.50 1.5 1.2989843
## 129  900 0.50 2.0 1.2271353
## 130  900 0.75 1.0 1.5904047
## 131  900 0.75 1.5 1.8411995
## 132  900 0.75 2.0 1.4171470
## 133  900 1.00 1.0 2.1290987
## 134  900 1.00 1.5 1.9064864
## 135  900 1.00 2.0 1.8460896
```

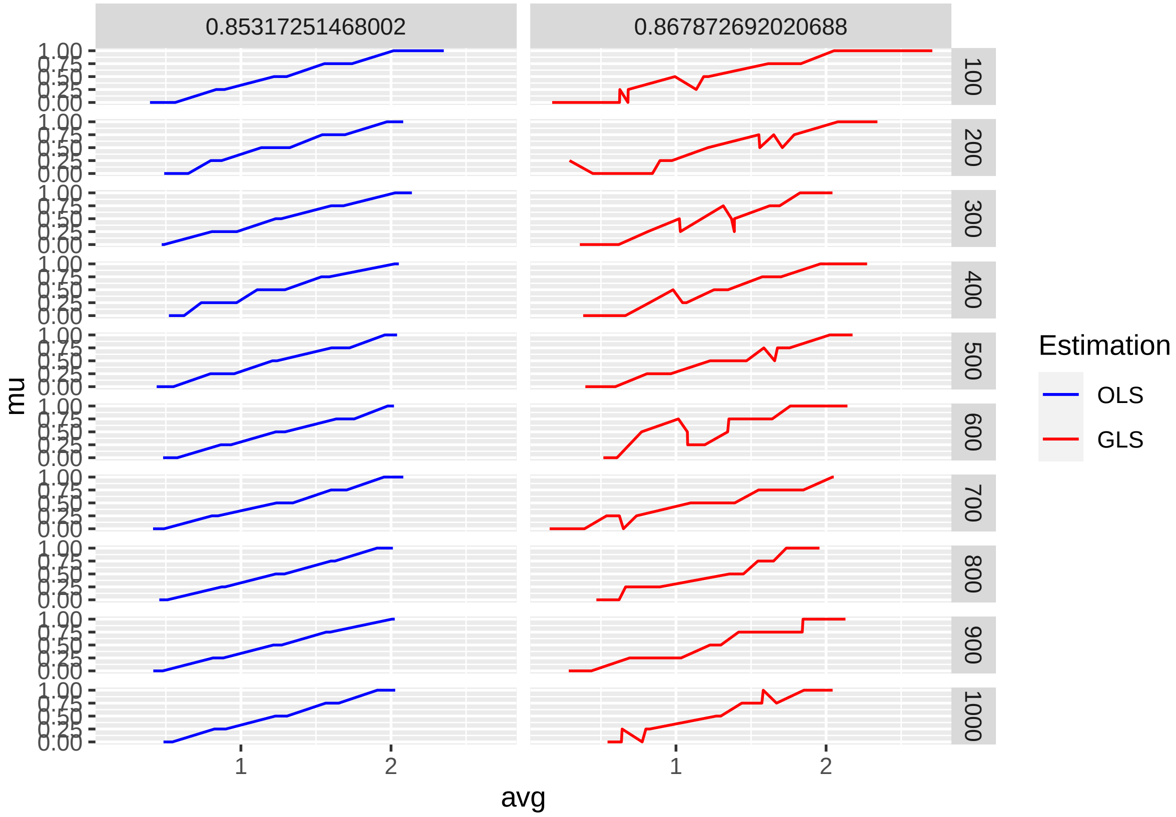
```
## 136 1000 0.00 1.0 0.5441020
## 137 1000 0.00 1.5 0.7751388
## 138 1000 0.00 2.0 0.6366835
## 139 1000 0.25 1.0 0.7992841
## 140 1000 0.25 1.5 0.8256762
## 141 1000 0.25 2.0 0.6412979
## 142 1000 0.50 1.0 1.2921433
## 143 1000 0.50 1.5 1.2680054
## 144 1000 0.50 2.0 1.2986345
## 145 1000 0.75 1.0 1.6698362
## 146 1000 0.75 1.5 1.5722088
## 147 1000 0.75 2.0 1.4382501
## 148 1000 1.00 1.0 2.0434242
## 149 1000 1.00 1.5 1.8520927
## 150 1000 1.00 2.0 1.5812816
##
## attr(,"class")
## [1] "Eco"
```

As seen in the summary part, the total execution time of the simulation took 36.35 seconds which also proves that the parallel process works well. Therefore, execution times of the simulation might differ on other computers.

- Visualizing MSE(Mean Square Error) in OLS and GLS simulations: MSE is calculated by `out$average$mse` for each simulation.

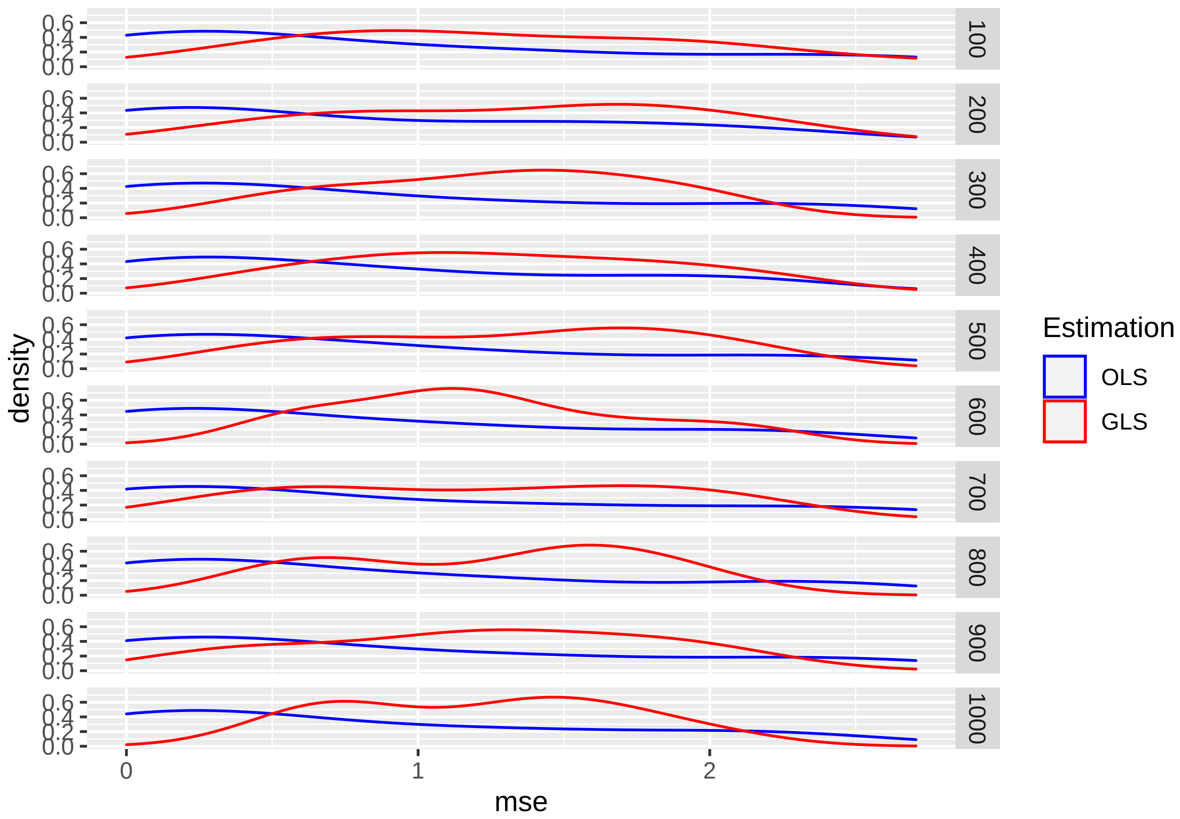
```
gls$average <-  gls$average %>% mutate(mse =(2-avg)^2 )
ols$average <-  ols$average %>% mutate(mse =(2-avg)^2 )

ols$average %>% ggplot(aes(x=avg,y=mu,col="OLS"))+
  facet_grid(n~mean(mse))+geom_line()+
 geom_line(data=gls$average,aes(x=avg,y=mu,col="GLS"))+
  scale_color_manual(name = "Estimation", values = c("OLS" = "blue", "GLS" = "red"))
```



- Density of MSE of „2 in OLS and GLS simulations.

```
ggplot(ols$average,aes(x=mse,col="OLS"))+facet_grid(n~.)+geom_density()+
  geom_density(data=gls$average,aes(x=avg,col="GLS"))+
  scale_color_manual(name = "Estimation", values = c("OLS" = "blue", "GLS" = "red"))
```



5 Conclusion

The above section illustrates the power of our implemented model and gives the fairly easy-to-use tool that still allows for various specifications in terms of used parameters, data generation processes, and summary functions. Researchers who use Monte Carlo studies regularly may save much time using a tool like this in the long run.

By nature, there may be cases where our implementation does not satisfy the user's needs to the fullest, but for a wide variety of examples we showed, it worked well and served the goal we aimed for. Furthermore, our functional programming approach allows for easy and flexible adjustments in case the use of our functions should be expanded, f.e. if a grid of more than 3 (or 4?) parameters is needed.

Theoretically, this work could be implemented as an R package to share with the R community. However, since the `MonteCarlo()` function of the `vignette` package already provides a well-working alternative to our project. Therefore, besides some minor differences, there is no need to do that.

6 Contributions

	Alexander Langnau	Öcal Kaptan	Sunyoung Ji
Planning	0	0	0
Create_grid	0	0	0
Data_generation	0	0	0
Summary_function	0	0	0
Create_array_function	0	0	0
Average_function	0	0	0
Output_function	0	0	0
ggplot part	0	0	0
Data_generation	0	0	0
Formatting	0	0	0
Writing the report	0	0	0
Proof-reading	0	0	0

7 References

Adrian G. Barbu, Song Chun Zhu, "Monte Carlo Methods", Springer Singapore, 2020, pp.1-4, doi:https://doi.org/10.1007/978-981-13-2971-5

M. Rana, N. Kumar, H. Sharma and K. Pandey, "Effect of Hyper-Parameter Tuning on the Performance of Augmented Random Search," 2022 1st International Conference on Informatics (ICI), 2022, pp. 47-52, doi: 10.1109/ICI53355.2022.9786888. Czech, Z. "In Introduction to Parallel Computing", Cambridge University Press, 2017, pp. 1-34, doi:10.1017/9781316795835.002

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Essen, den ____________________ ______________________________

Alexander Langnau, Öcal Kaptan, Sunyoung Ji