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A Functional Approach to (Parallelised) Monte Carlo Simulation

Advanced R for Econometricians

Final Project

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1 Introduction

Monte Carlo simulates complex probabilistic events using simple random events, such as the tossing of a pair of dice to simulate the casino's overall business model. In Monte Carlo computing, a pseudo-random number generator is repeatedly called which returns a real number in [0, 1], and the results are used to generate a distribution of samples that is a fair representation of the target probability distribution under study (**Barbu_2022**). Monte Carlo Method is combined with programming in modern research and contributes to various studies in statistics, economics, and many other science fields. The paper make a progress on developing a collection of different wrapper functions. The main function provides a convenient interface for Monte Carlo simulations and allows users to create a parameter grid and to iterate homogenous function calls over the parameter grid. It also offers an informative summary statistics including visualization with ggplot-methods and an option to use a parallelization process by using `furrr` package. The paper proceeds as follows. Chapter 2 describes preprocesses to establish the Monte Carlo simulation function. The preprocesses includes functions to create grid and dataset with random variables in user-defined distributions, along with functions that provide summary statistics. Chanter 3 details the main Monte Carlo simulation function which consists of functions in the chapter 2. Chapter 4 presents examples with the main function. Finally, chapter 5 concludes. Each chapter contains a simple example to show that a function is applicable to as many cases as possible. If there is a restriction, It will be covered and discussed as well.

2 Preprocess: Creating Helper functions

2.1 Function for creating grid

`create_grid` is one of the functions to improve performance of the main Monte Carlo simulation function. That creates a hyper-parameter grid with all permutations of the given parameters. Hyper-parameters are the variables that are required to be introduced before implementing a learning algorithm. It is typically unknown in advance about the hyper-parameters that should be harmonized, their valid ranges and which values in these ranges are most likely to yield a high performance(**Rana_2022**). Users can make their combination of hyper-parameters, then apply it into MC simulation in the main function.

```
create_grid <- function(parameters, nrep){
  input <- parameters
  storage <- list()
  name_vec <- c()

  for(i in 1:length(input)){ #1:3
    a <- as.numeric(input[[i]][[2]])
    b <- as.numeric(input[[i]][[3]])
    c <- as.numeric(input[[i]][[4]])
```

```
    output <- seq(from=a, to=b, by=c)
    storage[[i]] <-  output
    name_vec[i] <- input[[i]][[1]]
  }

  grid <- expand_grid(unlist(storage[1])
                      , unlist(storage[2])
                      , unlist(storage[3])
                      , unlist(storage[4])
                      , unlist(storage[5])
                      , c(1:nrep))

  names(grid) <- c(name_vec, "rep")

  return(grid)
}
```

Users have to input parameters as a list as following:

```
parameter_list <- list(c("variable name 1", from, to, by)
                      ,c("variable name 2", from, to, by)
                      ,c("variable name 3", from, to, by)
                      ,c("variable name 4", from, to, by))
```

`parameter_list` works with a minimum of 1 and a maximum of 4 variables. The structure of arguments is similar to `seq()` in R: A line of arguments is composed with variable name, the start and the end of sequence, the steps. It would be fairly easy to adapt this helper function for more parameters, but it is assumed that a grid with up to 4 parameters offers enough complexity for the simulation. The function basically takes the information of the input parameter list and creates a grid with `tidyr::expand_grid()`. The structure of `create_grid` makes sure that the columns are located after the corresponding variable and creates a different row for each number of repetition(`nrep`).

`create_grid()` Example:

```
#four parameters
param_list0 <- list(c("n", 10, 20, 10)
                    ,c("mu", 0, 0.5, 0.25)
                    ,c("sd", 0, 0.3, 0.1)
                    ,c("gender", 0, 1, 1))

head(create_grid(param_list0, nrep=3), n=2)
```

```
## # A tibble: 2 x 5
##       n    mu    sd gender   rep
##   <dbl> <dbl> <dbl>  <dbl> <int>
## 1    10     0     0      0     1
## 2    10     0     0      0     2
```

2.2 Data generation function

`data_generation()` takes `grid` and `simulation` as inputs. Users can define a probability distribution of data by entering the name of R packages into the function, such as normal distribution(`rnorm`) and uniform distribution(`runif`).

##################################?draw of data points? The draw of data points for each row of the parameter grid gets stored as a seperate element in a list.

`data_generation()` chooses a mapping function itself based on the number of parameters. The table below shows Mapping function, Mapping function for parallelization and the number of parameters that is used in `data_generation()`:

Map	Map for Parallelization	Number of parameters
map()	future_map	$n = 1$
map2()	future_map2	$n = 2$
pmap	future_pmap	$n \geq 3$

`options = furrr_options(seed = TRUE)` is for reproducible random number generation(RNG) process. This argument takes control of the RNG process for parallelization and generates the same numbers according to the given seed. More details can be found by running the command `?furrr_options` in RStudio.

```
data_generation <- function(simulation, grid){ #this is for use inside the function

  if(ncol(grid)==2){
    var1 <- c(unlist(grid[,1]))
    if(cores>1){
      data <- future_map(var1, simulation,.options = furrr_options(seed = TRUE))
    }else{
      data <- map(var1, simulation)
    }
  }

  if(ncol(grid)==3){
    var1 <- c(unlist(grid[,1]))
    var2 <- c(unlist(grid[,2]))
```

```
    if(cores>1){
      data <- future_map2(var1, var2, simulation,.options = furrr_options(seed = TRUE))
    } else{
      data <- map2(var1, var2, simulation)
    }
  }

  if(ncol(grid)==4){ #need to implement more than 3?!
    var1 <- c(unlist(grid[,1]))
    var2 <- c(unlist(grid[,2]))
    var3 <- c(unlist(grid[,3]))
    list1 <- list(var1,var2,var3)
    if(cores>1){
      data <- future_pmap(list1, .f=simulation,.options = furrr_options(seed = TRUE))
    }else{
      data <- pmap(list1, .f=simulation)
    }
  }

  return(data)
}
```

Monte Carlo simulations can become quickly very demanding in terms of computing time. In that case, Parallel processing is useful. Processes are parallel if at any time both of them are simultaneously executed, for instance, processes are executed by separate, distributed processors interconnected by communication channel(**Czech_2017**). `data_generation_parallelised` uses `furrr_options()` that is a parallel processing function in `furrr` package.

`data_generation()` Example: The example below demonstrates a non-parallel processing function with a Poisson distribution. The time difference between parallel and non-parallel functions will be dealt with in the Chapter 3.

```
cores <- 1
#param_list1 <- list(c("n", 10, 20, 10))

param_list2 <- list(c("n", 10, 20, 10)
                    ,c("lambda", 0.5, 1, 0.5))
#create_grid(param_list1, nrep=10)
#create_grid(param_list1, nrep=1)

'grid1 <- create_grid(param_list1, nrep=3)
tail(data_generation(simulation=rnorm, grid=grid1),1)'
```

```
## [1] "grid1 <- create_grid(param_list1, nrep=3)\ntail(data_generation(simulation=rnorm, gr
```

```r
grid2 <- create_grid(param_list2, nrep=1)
sim1 <- data_generation(simulation=rpois, grid=grid2)


names(sim1) <- c("n=10, lamda = 0.5","n=10, lamda = 1",
                 "n=20, lamda = 0.5","n=20, lamda = 1")
grid2
```

```
## # A tibble: 4 x 3
##        n lambda   rep
##    <dbl>  <dbl> <int>
## 1     10    0.5     1
## 2     10    1       1
## 3     20    0.5     1
## 4     20    1       1
```

```r
sim1
```

```
## $`n=10, lamda = 0.5`
##  [1] 0 1 0 1 2 0 0 1 0 0
##
## $`n=10, lamda = 1`
##  [1] 3 1 1 1 0 2 0 0 0 3
##
## $`n=20, lamda = 0.5`
##  [1] 1 1 1 3 1 1 0 0 0 0 2 1 1 1 0 0 1 0 0 0
##
## $`n=20, lamda = 1`
##  [1] 0 1 1 1 0 0 0 1 0 2 0 1 2 0 1 0 0 2 2 1
```

`sim1` contains the generated data by `data_generation` with grid2 and poisson distribution as inputs. The below function shows the application of `data_generation` to uniform and poisson distribution:

```r
# Application to Uniform distribution
param_list_runif <- list(c("n", 10, 30, 10)
                         ,c("min", 0, 0, 0)
                         ,c("max", 1, 1, 0))


grid_unif <- create_grid(param_list_runif, nrep=3)
tail(data_generation(simulation=runif, grid=grid_unif),1)
```

```
## $n9
##  [1] 0.48204261 0.25296493 0.21625479 0.67437639 0.04766363 0.70085309
##  [7] 0.35188864 0.40894400 0.82095132 0.91885735 0.28252833 0.96110479
## [13] 0.72839443 0.68637508 0.05284394 0.39522013 0.47784538 0.56025326
## [19] 0.69826159 0.91568354 0.61835123 0.42842151 0.54208037 0.05847849
## [25] 0.26085686 0.39715195 0.19774474 0.83192756 0.15288722 0.80341854
```

```
# Application to Poisson distribution

param_list_rpois <- list(c("n", 10, 30, 10)
                         , c("lambda", 0, 10, 1))

grid_pois <- create_grid(param_list_rpois, nrep=3)
head(grid_pois,2) # nrow(grid_pois) = 99
```

```
## # A tibble: 2 x 3
##       n lambda   rep
##   <dbl>  <dbl> <int>
## 1    10      0     1
## 2    10      0     2
```

```
head(data_generation(simulation=rpois, grid=grid_pois),1)
```

```
## $n1
##  [1] 0 0 0 0 0 0 0 0 0 0
```

2.3 Summary function

So far, the paper explain the way to create a *raw* data that is distributed as users choose by using `create_grid` and `data_generation`. Based on the raw data, this chapter introduces `summary_function()` that produces summary statistics that users require. The function basically extracts the user-defined summary function (`sum_fun`) from the raw data using a `sapply()`-loop. Results are stored in a $(nrow(grid) * 1)$-dimensional matrix, which is combined with a parameter grid in the next step.

```
#summary function for one input
summary_function <- function(sum_fun, data_input){

  count <- length(data_input)
  summary_matrix <- matrix(nrow=count, ncol=1)

  for(i in 1:count){
```

```
    input <- list(data_input[[i]])
    output <- sapply(sum_fun, do.call, input)
    summary_matrix[i] <- output
  }
  #output <- as.data.frame(summary_matrix)
  #names(output) <- sum_fun
  colnames(summary_matrix) <- sum_fun
  return(summary_matrix)
}
```

`summary_function` Example: `summary_data` shows `mean` of the generated data which is normally distributed and corresponds given with the n, μ and the standard deviation

```
param_list3 <- list(c("n", 10, 20, 10)
                    ,c("mu", 1, 2, 0.25)
                    ,c("sd", 0.5, 1, 0.1))
grid_test <- create_grid(param_list3, nrep=3)
test_data <- data_generation(simulation=rnorm, grid=grid_test)
summary_data <- summary_function(sum_fun=list("mean"), data_input=test_data)
head(summary_data,2)
```

```
##           mean
## [1,] 1.0165396
## [2,] 0.8156316
```

2.4 Summary array funcation

Even though the example above includes a fairly small range of parameter grid, the simulation returns 180 summaried data points. In `main_function`, the results from the previous step get merged with the parameter grid into one data frame. ######################## data wrangling processes?? This way to store the data allows users to apply further data wrangling processes. A multi-dimensional array is more suitable for printing the output in a tidy and clear way. `create_array_function()` takes all relevant data from the previous steps (parameter grid and the results of the Monte Carlo simulation) and transforms it into an array with the correct dimensions. #################### correct dimension?

```
create_array_function <- function(comb, parameters, nrep){
  storage <- list()
  name_vec <- c()

  for(i in 1:length(parameters)){
    #this creates the sequences of parameters
```

```
  a <- as.numeric(parameters[[i]][[2]])
  b <- as.numeric(parameters[[i]][[3]])
  c <- as.numeric(parameters[[i]][[4]])
  output <- seq(from=a, to=b, by=c)
  storage[[i]] <-  output
  name_vec[i] <- parameters[[i]][[1]]
  #this just stores the names of the variables
}


matrix.numeration <-  paste("rep","=", 1:nrep, sep = "")

if(length(parameters)==1){
  comb_ordered <-  comb %>% arrange(comb[,2])
  seq1 <- c(unlist(storage[1]))

  row.names <- paste(name_vec[1],"=",seq1, sep = "")

  dimension_array <- c(length(seq1), nrep)
  dim_names_list <- list(row.names, matrix.numeration)
}

if(length(parameters)==2){
  comb_ordered <-  comb %>% arrange(comb[,2])  %>% arrange(comb[,3])
  seq1 <- c(unlist(storage[1]))
  seq2 <- c(unlist(storage[2]))

  row.names <- paste(name_vec[1],"=",seq1, sep = "")
  column.names <-  paste(name_vec[2],"=",seq2, sep = "")

  dimension_array <- c(length(seq1), length(seq2), nrep)
  dim_names_list <- list(row.names, column.names, matrix.numeration)
}

if(length(parameters)==3){
  comb_ordered <-  comb %>% arrange(comb[,2])  %>%
    arrange(comb[,3]) %>% arrange(comb[,4])
  seq1 <- c(unlist(storage[1]))
  seq2 <- c(unlist(storage[2]))
  seq3 <- c(unlist(storage[3]))

  row.names <- paste(name_vec[1],"=",seq1, sep = "")
```

```r
    column.names <-  paste(name_vec[2],"=",seq2, sep = "")
    matrix.names1 <-  paste(name_vec[3],"=",seq3, sep = "")

    dimension_array <- c(length(seq1), length(seq2), length(seq3), nrep)
    dim_names_list <- list(row.names, column.names,
                           matrix.names1, matrix.numeration)

  }



  array1 <- array(comb_ordered[,ncol(comb)]
                  #change to automatically adjust dim
                  , dim = dimension_array
                  , dim_names_list)
  return(array1)
}
```

In order to test `create_array_function`, we need to set up an altered version of `main_function`. The `main_function` is discussed in the next chapter.

`create_array_function` Example:

```r
# PREP TEST `create_array_function`
main_function_array_test <-  function(parameters #list of parameters
                                      , nrep #number of repetitions
                                      , simulation #data generation
                                      , sum_fun){ #summary statistics

  grid <- create_grid(parameters, nrep) #Step 1: create grid
  raw_data <- data_generation(simulation, grid) #Step 2: simulate data
  summary <- summary_function(sum_fun, data_input=raw_data) #Step 3: Summary statistics
  comb <- cbind(grid, summary) #Step 4: Combine results with parameters
  array_1 <- create_array_function(comb, parameters, nrep) #Step 5: Create array

  return(comb)
}

param_list3x <- list(c("n", 10, 20, 10)
                     ,c("mu", 0, 5, 1)
                     ,c("sd", 0, 3, 1))

comb1 <- main_function_array_test(parameters=param_list3x
                                  , nrep = 3
```

```r
                                        , simulation = rnorm
                                        , sum_fun="mean")

head(comb1, 2)
```

```
##    n mu sd rep mean
## 1 10  0  0   1    0
## 2 10  0  0   2    0
```

```r
################################ need to reduce a range of grid and nrep?
create_array_function(comb=comb1, parameters=param_list3x, nrep=3)
```

```
## , , sd=0, rep=1
##
##      mu=0 mu=1 mu=2 mu=3 mu=4 mu=5
## n=10    0    1    2    3    4    5
## n=20    0    1    2    3    4    5
##
## , , sd=1, rep=1
##
##            mu=0     mu=1     mu=2     mu=3     mu=4     mu=5
## n=10  0.3592213 1.856067 1.597645 2.631208 3.822198 5.589992
## n=20 -0.2024437 1.095379 2.207984 2.853540 3.701126 5.105401
##
## , , sd=2, rep=1
##
##            mu=0     mu=1     mu=2     mu=3     mu=4     mu=5
## n=10 -0.8768202 1.201310 1.904537 2.897435 3.509705 4.542338
## n=20 -0.6888252 1.395944 2.148185 2.966625 4.063234 5.059694
##
## , , sd=3, rep=1
##
##            mu=0     mu=1     mu=2     mu=3     mu=4     mu=5
## n=10 -0.3415539 1.284343 2.100330 2.699081 3.838544 3.959394
## n=20  0.7880765 1.004376 2.048212 2.978085 3.923330 4.814248
##
## , , sd=0, rep=2
##
##      mu=0 mu=1 mu=2 mu=3 mu=4 mu=5
## n=10    0    1    2    3    4    5
## n=20    0    1    2    3    4    5
##
```

```
## , , sd=1, rep=2
##
##             mu=0      mu=1     mu=2     mu=3     mu=4     mu=5
## n=10 -0.2452999 1.0835364 2.283412 3.593855 4.122586 4.401167
## n=20  0.4255240 0.8355293 1.804812 3.007484 3.752047 5.015939
##
## , , sd=2, rep=2
##
##              mu=0      mu=1     mu=2     mu=3     mu=4     mu=5
## n=10 -0.05139362 0.2426633 1.729941 1.968998 4.163818 4.394397
## n=20 -0.47500354 0.9687178 1.951855 2.666098 4.016383 4.273766
##
## , , sd=3, rep=2
##
##           mu=0      mu=1     mu=2     mu=3     mu=4     mu=5
## n=10 1.3917130 1.3173227 1.810426 3.500988 4.481365 7.218648
## n=20 0.5305719 0.5736219 1.831536 3.345939 4.261094 6.304353
##
## , , sd=0, rep=3
##
##      mu=0 mu=1 mu=2 mu=3 mu=4 mu=5
## n=10    0    1    2    3    4    5
## n=20    0    1    2    3    4    5
##
## , , sd=1, rep=3
##
##             mu=0     mu=1     mu=2     mu=3     mu=4     mu=5
## n=10  0.3012704 1.752093 1.451813 2.815746 3.860342 4.933741
## n=20 -0.1036849 1.066900 1.727783 2.844167 4.284351 4.876313
##
## , , sd=2, rep=3
##
##           mu=0      mu=1     mu=2     mu=3     mu=4     mu=5
## n=10 0.2150016 0.3657670 2.492447 3.937548 4.332272 5.816431
## n=20 0.4667103 0.8742835 1.616299 3.028465 3.845771 5.456615
##
## , , sd=3, rep=3
##
##             mu=0      mu=1     mu=2     mu=3     mu=4     mu=5
## n=10 -0.4774057 0.1070965 1.041310 2.135599 4.200848 4.426807
## n=20 -0.6590698 0.7566315 2.341365 4.161599 3.833062 4.154558
```

Through `create_array_function`, users can obtain all combinations corresponding each grid, the number of repetition, and the summary function that users require, such as `mean` in this case.

2.5 Average function

```
average_function <- function(grid_for_avg, summary, nrep){
  grid_for_avg <- grid_for_avg[-ncol(grid_for_avg)] #remove column for reps
  n_rows <- nrow(grid_for_avg)
  n_col <- ncol(grid_for_avg)

  for(i in 1:n_rows){
    start <- 1 + (i-1)*nrep
    end <- i*nrep
    grid_for_avg[i, n_col+1] <- mean(summary[start:end, ])
  }

  grid_plus_mc <- data.frame(grid_for_avg)

  colnames(grid_plus_mc)[n_col+1] <- "avg"

  return(grid_plus_mc)

}
```

2.6 Output Function

Goal is to create a function, that takes the Monte Carlo simulation results and all parameter input and converts it onto output format that prints nicely into the console. `Output_function` created for this purpose to store simulation results. `array_1`,`average_over_reps`,`parameters`,`cores`,`simulation`, `nrep`,`cpt` are used as input parameters in the `output_function`. Except the parameter `cpt`, other parameters are defined and explanied before this section. `cpt` parameters will be explanied in the section `Main Function`. It is basically saving the execution time of the simulation. Lets go to the each code line and explain them briefly.

Firstly, `out` object is created to store simulation results, averaged results and summary of the result in list `list()` format. The name `Eco` implemented as a class of the list `out`, since the spesific class had to implemented for the simulation results as a part of the task in visualisation. The results from `array_1` saved here as `out$results` also result for `average_over_reps` saved as `out$average`. Next, same class name is assigned to to `out$results` and `out$average` also default classes of the both list objects kept as a class. The reason is to prevent the future error while using the ggplot2 methods for simulation results. Because ggplot2 methos works only some spesific classses (data.frame ,etc.). After that the reporting part is created as a report of simulation result by using `cat()` function. At the end function returned to the object `out`.

```
output_function <- function(array_1,average_over_reps,parameters,cores,simulation,
                            nrep,cpt){

  out <- list()#Create a emptly list to store simulation result and average result.
  class(out) <- "Eco"  #We have to implement a class. I just gave a random name. "Eco"
  out$results <- array_1 # Saved the simulation result
  out$average <- average_over_reps # And this is the result from average function. All the r
  #Because output_function will return the list "out".

  #To us ggplot function Alex has created a average function that takes average of the simul
  class(out$average) <- c("Eco",class(out$average))#Again, name the class of the average res
  class(out$results) <- c("Eco",class(out$results))#Also same for the simulation result.

  if(cores>1){
    parallel = "Multisession"
  } else {
    parallel = "Sequential"
  }
  #This part is just a report. It will be shown at the end of the simulation result.
  text <-  cat("\n",
           "Repetition(nrep)      : ",nrep,"\n\n",
           "Parallelization Type  : ",parallel,"\n\n",
           "Number of Cores Used in  Parallelization : ",cores," out of",detectCores(),"\n\r
           "Input Parameters : ",paste(parameters),"\n\n",
           "Simulation Length :",length(array_1),"\n",
           "Minumum :",min(array_1),"\n",
           "Maximum :",max(array_1),"\n",
           "Mean    :", mean(array_1),"\n",
           "Median  :",median(array_1),"\n\n",
           "Execution Time of Monte Carlo Simulation",as.numeric(cpt),"secs \n\n",
           "Name of The Class :",class(out))

  return(out)
}
```

Output of `output_function` will be as same as the `main_function`.That's no further example is needed here. It will be covered in the next topic called "Main function".

2.7 Main Function

The "main_function" is a function that consist of the helper functions that created above. Here, all the helper functions are included and additionaly some commands and functions added to improve the simulation results. Here only additional arguments will be explained, since the helper functions are explained before.

First, `if()&else()` commands are added to check the number of cores are used in the main function is bigger than maximum number of the cores or not. Logically the computer cannot use the cores that doesn't exist. `max.cores` is a numeric object that stores the maximum number of the cores in the CPU. By using the function `detectCores()` from "parallel" packeage ,maximum number of the cores are stored in `max.cores`. The next is to check if the seed is provided by user or not. If the seed is not provided by the user , the function `sample.int()`generates a random number and uses it as a seed for reproducibility of the simulation. After the function `set.seed()` , `Sys.time()`function is implemented to check execution time of the simulation. `startTime` saves the startind time of the simulation ans endTime saves the ending time of the simulation. At the end , `startTime`is subtracted from `endtTime` and `cpt`is created to store execution time. As explained before ,cpt is used in `output_function` as a part of summary. Lastly, `plan()` function is used for the parallelisation to run the methods "sequential" or "multisession". "Sequential" runs the simulation with 1 core which means no parallelisation is used and "Multisession" runs the simulation in parallel by using the number of cores that provided by user. For more details please run the command `?future::plan` in RStudio.

```
main_function <-  function(parameters #list of parameters
                          , nrep #number of repetitions
                          , simulation #data genereation
                          , sum_fun #summary statistics
                          ,seed = NULL#Reproducibility
                          ,cores=NULL){



  #Number of cores
  max.cores <- detectCores()
  if(cores>max.cores){
    stop("Number of Cores cannot be bigger than total number of cores")
  }
  if(!is.null(seed)) {#Reproducibility
    set.seed(seed)}#If seed provided then set.seed takes the number
  else {
    warning("No seed provided!", call. = FALSE)
    seed <- sample.int(10000, 1)#if its not provided then we generate random seed
    set.seed(seed)
    message("Random seed = ", seed, "\n")}


  startTime <- Sys.time()#Starting time
```

```r
  grid <- create_grid(parameters, nrep) #Step 1: create grid

  if(cores > 1){
    plan(multisession,workers = cores)
  } else{
    plan(sequential)
  }
  suppressMessages(raw_data <- data_generation(simulation, grid))

  summary <- summary_function(sum_fun, data_input=raw_data) #Step 3: Summary statistics

  average_over_reps <- average_function(grid_for_avg=create_grid(parameters, 1), summary, nr

  comb <- cbind(grid, summary) #Step 4: Combine resuluts with parameters

  array_1 <- create_array_function(comb, parameters, nrep) #Step 5: Create array

  endTime <- Sys.time()#Endtime

  cpt <- endTime - startTime#Execution time

  summary_1 <- output_function(array_1,average_over_reps,parameters,cores,simulation,
                               nrep,cpt)

return(summary_1)
}
```

Lets test the main function by using `rnorm` Monter Carlo simulation.

```r
param_list3x <- list(c("n", 10, 100, 10)
                     ,c("mu", 0, 10, 1)
                     ,c("sd", 0, 5, 1))

test_me <- main_function(parameters=param_list3x
              , nrep = 5
              , simulation = rnorm
              , sum_fun="mean"
              ,seed=123
              ,cores=1)
```

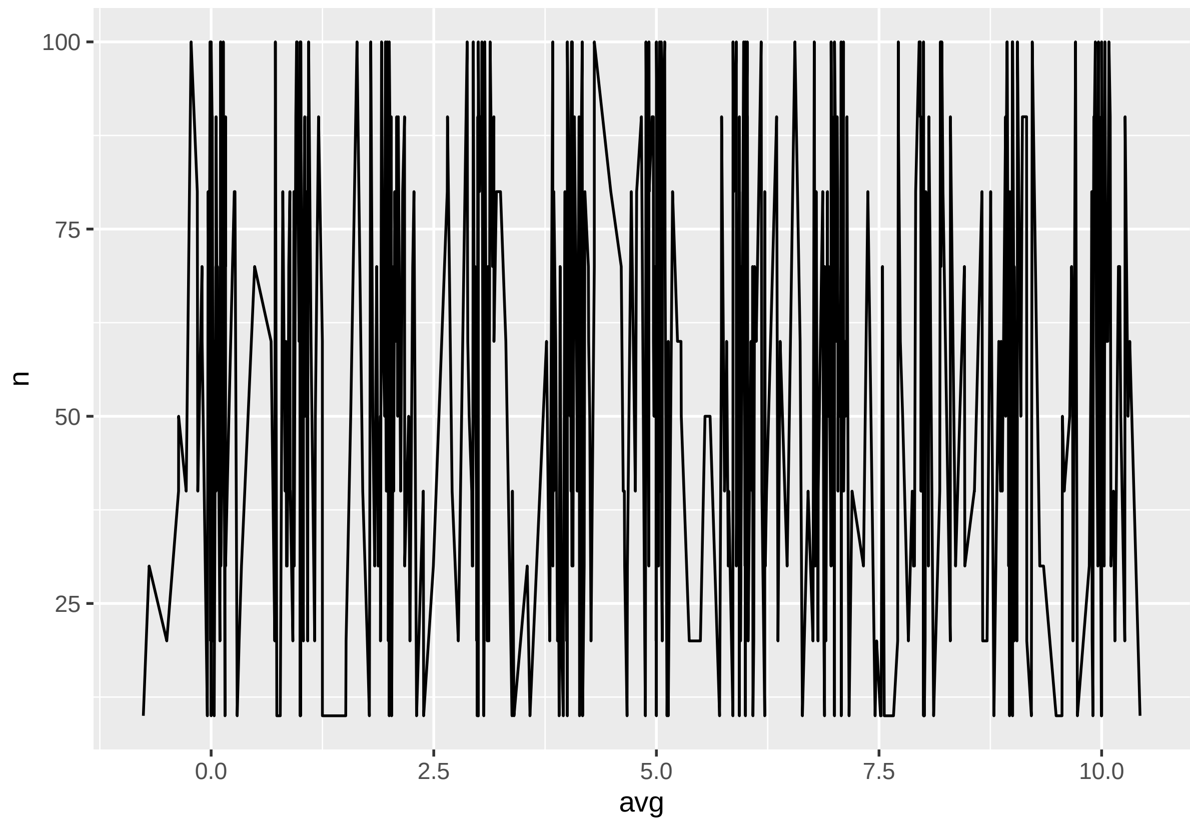
```
##
```

```
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Sequential
##
##  Number of Cores Used in  Parallelization :  1  out of 8
##
##  Input Parameters :  c("n", "10", "100", "10") c("mu", "0", "10", "1") c("sd", "0", "5",
##
##  Simulation Length : 3300
##  Minumum : -1.844634
##  Maximum : 13.43665
##  Mean    : 5.001471
##  Median  : 5
##
##  Execution Time of Monte Carlo Simulation 0.3788722 secs
##
##  Name of The Class : Eco
```

Here there are simulation results, average of simulation result and a summary about simulation. Now, lets check the `ggplot2` methods for this simulation results. `out$average` is created for visualisation purpose since working with arrays sometimes are trouble.`Ggplot2` methods can be used without any problem by taking average of the simulation.

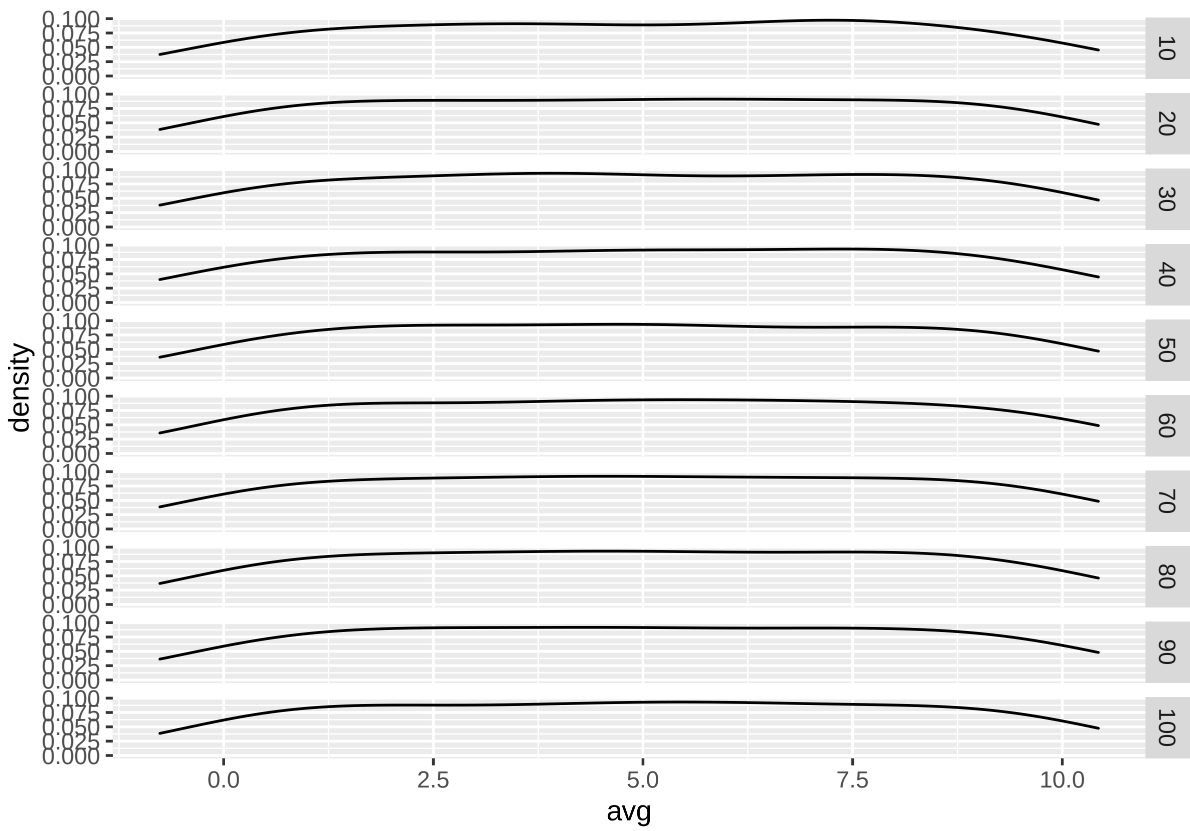
Lets try it.

```
ggplot(test_me$average,aes(x=avg,y=n))+geom_line()
```



Also with `facet_grid()` function.

```
ggplot(test_me$average,aes(x=avg))+facet_grid(n~.)+geom_density()
```



As proven above ,the simulation result works well with ggplot2 methods.

3 Examples

As an example to Monte Carlo Simulation , `OLS` and `GLS` *beta*2 coeffients are simulated by using parallelisation and also without parallelisation to show execution time of the parallel process.

```
ols_f <- function(n,mu,sd){
  e <- rnorm(n,mu,sd)
  x <- runif(n)
  y <- 0.5*x + e
  ols.hat <- t(x) %*% y / t(x)%*%x
  return("ols"=ols.hat)}

gls_f <- function(n,mu,sd){
  e <- rnorm(n,mu,sd)
  x <- runif(n)
  y <- 0.5*x + e
  v.inv <- diag(1/(1:n))
  c <- chol(v.inv)
  cy <- c %*% y
  cx <- c %*% x
  gls_hat <- t(cx) %*% cy / t(cx)%*%cx
  return("gls"=gls_hat)

  param_list <- list(c("n",100,1000,100),c("mu",0,1,0.25),c("sd",1,2,.5))
}
```

As seen as above, simple `OLS` and `GLS` functions are defined to find *beta*2 coefficients. Here, the execution time of the `GLS` function would be much more longer than `OLS` function.The reason for that in `GLS` function `The Cholesky Decomposition` is used.Lets run the simulation for `OLS` function.

Total execution time of `OLS` simulation is 8.46 seconds. Only one core is used.

```
 param_list <- list(c("n",100,1000,100),c("mu",0,1,0.25),c("sd",1,2,.5))
ols <- main_function(parameters = param_list,nrep=5,simulation = ols_f,sum_fun="mean",seed=1
```

```
##
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Sequential
##
```

```
##  Number of Cores Used in  Parallelization :  1  out of 8
##
##  Input Parameters :  c("n", "100", "1000", "100") c("mu", "0", "1", "0.25") c("sd", "1",
##
##  Simulation Length : 750
##  Minumum : 0.08611764
##  Maximum : 2.598461
##  Mean    : 1.251827
##  Median  : 1.246948
##
##  Execution Time of Monte Carlo Simulation 0.213414 secs
##
##  Name of The Class : Eco
```

```
ols
```

```
## $results
## , , sd=1, rep=1
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.6824267 0.7726517 1.390428 1.352546 2.033425
## n=200  0.4786798 0.7268500 1.551036 1.469126 2.016927
## n=300  0.5430408 0.6925565 1.412873 1.516396 2.020209
## n=400  0.4916201 0.9335676 1.240309 1.588818 1.939011
## n=500  0.4665197 0.8658038 1.302298 1.621465 2.071480
## n=600  0.4135505 1.0087766 1.263624 1.564343 1.873183
## n=700  0.6062303 0.8060657 1.423163 1.677571 1.915217
## n=800  0.5215150 0.8373901 1.384251 1.536015 1.845518
## n=900  0.4377088 0.8522425 1.323030 1.625229 2.113246
## n=1000 0.4272311 0.9598533 1.171133 1.607383 2.007037
##
## , , sd=1.5, rep=1
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.2325414 0.8709612 1.007916 1.187153 1.646406
## n=200  0.2684302 0.5956376 1.066756 1.572480 1.780621
## n=300  0.3463280 0.5074475 1.249930 1.526170 2.281022
## n=400  0.5532419 0.6727324 1.069168 1.587438 2.040513
## n=500  0.5088940 0.9917867 1.254739 1.687302 2.191762
## n=600  0.5817868 1.0516531 1.388582 1.787617 2.039833
## n=700  0.4085830 0.9266206 1.177307 1.618533 1.795094
## n=800  0.4788177 0.7297284 1.261184 1.635941 1.990439
## n=900  0.4293894 0.7521720 1.281461 1.550926 1.969204
```

```
## n=1000 0.4379182 0.7061176 1.281116 1.604609 1.875328
##
## , , sd=2, rep=1
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.6237415 0.8008305 1.639701 1.897970 2.513486
## n=200  0.6110261 1.2155626 1.815507 1.725145 2.033192
## n=300  0.4066040 0.8631502 1.239882 1.684178 2.106188
## n=400  0.4235146 0.6718906 1.072217 1.373997 2.000179
## n=500  0.4979046 1.0646659 1.248795 1.624880 2.036934
## n=600  0.4167380 0.9594940 1.222973 1.538267 1.972503
## n=700  0.4474413 0.7043636 1.318657 1.298269 2.086695
## n=800  0.3558261 0.9032952 1.340988 1.480559 1.852471
## n=900  0.4184591 0.8440885 1.312351 1.481778 1.959888
## n=1000 0.5482327 0.8486597 1.325121 1.557654 2.174067
##
## , , sd=1, rep=2
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5699594 0.6347600 1.240574 1.409578 2.102247
## n=200  0.3411974 0.9859774 1.127496 1.654986 2.005527
## n=300  0.5568138 0.7710960 1.309760 1.640136 1.868297
## n=400  0.5892899 0.8630584 1.420656 1.488524 2.106760
## n=500  0.4565660 0.9875469 1.169863 1.642037 2.125870
## n=600  0.6074920 0.8311391 1.144387 1.561809 2.127146
## n=700  0.5311162 0.8793250 1.213227 1.758464 2.019087
## n=800  0.4907978 0.8137087 1.243065 1.544169 1.890569
## n=900  0.5006696 0.9126527 1.185916 1.551420 2.011440
## n=1000 0.5132049 0.8425093 1.323331 1.665086 2.067430
##
## , , sd=1.5, rep=2
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4100172 1.0050342 1.3586338 1.594119 2.030257
## n=200  0.7621120 1.1129076 1.2050693 1.766159 2.052134
## n=300  0.4495498 0.9131922 1.3998834 1.844595 2.048539
## n=400  0.5189146 0.9392205 0.9085613 1.502305 2.159537
## n=500  0.7089984 0.6966299 1.1168839 1.472302 1.942223
## n=600  0.3991638 0.7265354 1.1572410 1.748560 1.735354
## n=700  0.4141086 0.7800295 1.3853238 1.765437 1.860513
## n=800  0.3797112 0.8637205 1.1967238 1.669677 1.904555
## n=900  0.4946693 0.9968749 1.3024325 1.747231 2.093768
```

```
## n=1000 0.6315810 0.8282354 1.1035843 1.541674 1.946093
##
## , , sd=2, rep=2
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.2670268 1.0417428 0.9765324 1.779177 2.139938
## n=200  0.2064741 0.8966665 1.2011619 1.562213 1.929578
## n=300  0.2953656 0.8280923 1.4380720 1.301385 2.089569
## n=400  0.7044943 0.8050854 1.4651276 1.320472 2.051137
## n=500  0.4125427 0.8432624 0.9844522 1.838044 2.144216
## n=600  0.7200719 0.9085732 1.0035755 1.809966 2.205790
## n=700  0.5352158 0.8738860 1.2227186 1.564884 2.114450
## n=800  0.3849922 0.7982686 1.2963208 1.625716 1.980291
## n=900  0.3505717 0.7606954 1.0653161 1.494651 1.986403
## n=1000 0.5160520 0.8734081 1.3842792 1.766722 2.085573
##
## , , sd=1, rep=3
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5968253 1.0425781 1.207359 1.494982 1.755818
## n=200  0.6013728 0.8169956 1.089477 1.759701 2.050444
## n=300  0.4185383 0.8722548 1.202220 1.632546 2.054984
## n=400  0.5761413 0.9439121 1.215469 1.611440 2.019500
## n=500  0.6625234 0.8265606 1.301594 1.551010 1.973081
## n=600  0.6338517 0.9970815 1.257844 1.716586 2.047527
## n=700  0.4954136 0.8538731 1.202218 1.656197 1.921142
## n=800  0.4847987 0.8606272 1.280399 1.681161 1.939804
## n=900  0.4581839 0.8963640 1.363804 1.640401 2.005699
## n=1000 0.4964781 0.9341992 1.243164 1.696117 1.985360
##
## , , sd=1.5, rep=3
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.5659700 0.7903221 1.254210 1.886166 1.790293
## n=200  0.5821258 0.6511703 1.343681 1.782181 1.894019
## n=300  0.6550444 0.9367739 1.380756 1.705998 2.028275
## n=400  0.7048194 0.9633662 1.120887 1.688024 1.811344
## n=500  0.3923555 0.7626086 1.104116 1.537444 2.047353
## n=600  0.4717144 0.8688271 1.175261 1.826726 1.983990
## n=700  0.3871651 0.6201482 1.305263 1.669981 2.042779
## n=800  0.4834246 0.9357010 1.305622 1.523736 2.081715
## n=900  0.4433156 0.7916531 1.359010 1.580634 1.940405
```

```
## n=1000 0.4946876 0.8067392 1.480151 1.486763 1.900808
##
## , , sd=2, rep=3
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4237920 0.5144245 0.4718755 2.004104 2.186295
## n=200  0.5777614 0.7299343 1.6298909 1.499530 1.929751
## n=300  0.6276662 0.8600749 0.9626478 2.011661 2.175176
## n=400  0.5785860 0.9158230 1.1544399 1.389809 2.169336
## n=500  0.2516702 0.9907678 1.0602298 1.532465 1.682624
## n=600  0.5165529 0.8616768 1.5189127 1.652623 1.941119
## n=700  0.4674128 0.7479103 1.2133761 1.738886 1.996975
## n=800  0.5090721 0.8062749 1.3959268 1.542798 1.994769
## n=900  0.4197560 0.8280591 1.2129929 1.504396 1.947569
## n=1000 0.6527630 0.8873246 1.2662086 1.656169 2.115128
##
## , , sd=1, rep=4
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7359835 1.0087000 1.1157127 1.851686 2.286974
## n=200  0.5466285 0.8691083 0.9809655 1.677466 1.914859
## n=300  0.4092951 0.8421824 1.1834134 1.463671 2.105769
## n=400  0.4870693 0.8787365 1.3442915 1.641625 2.038680
## n=500  0.6247941 0.9737552 1.1814126 1.583607 1.981457
## n=600  0.5543342 0.9385104 1.2433161 1.676785 1.916412
## n=700  0.4178732 0.8808745 1.1952781 1.530920 1.960542
## n=800  0.5618498 0.8943310 1.2475019 1.686068 1.946561
## n=900  0.5052058 0.8785736 1.2679218 1.571624 2.009535
## n=1000 0.4550461 0.8622264 1.2189931 1.640547 2.027112
##
## , , sd=1.5, rep=4
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.4449720 0.8435500 1.5231357 1.528876 2.461235
## n=200  0.3352203 0.8038153 0.9593336 1.823112 2.149320
## n=300  0.4930487 0.6791231 1.0272955 1.574880 2.304307
## n=400  0.3806721 1.1714969 1.4056411 1.543264 2.258722
## n=500  0.4589277 0.7928819 1.5044037 1.632672 1.829121
## n=600  0.4884702 0.6958911 1.2883971 1.606977 2.109358
## n=700  0.5038252 0.8396545 1.4939591 1.598702 2.198315
## n=800  0.5515391 0.8256058 1.2838778 1.644795 2.020077
## n=900  0.4102779 0.7769137 1.2511958 1.567744 1.925168
```

```
## n=1000 0.6885496 0.9734467 1.2688095 1.525439 1.946295
##
## , , sd=2, rep=4
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.5717011 0.8796835 2.1004676 1.351107 2.598461
## n=200  1.1810891 0.4725781 0.9070217 1.490537 2.444452
## n=300  0.5900949 1.0569764 1.3134750 1.766744 2.089249
## n=400  0.7380901 0.4503391 1.0841655 1.704982 1.998472
## n=500  0.6158445 1.0384364 1.6255235 1.911106 1.868147
## n=600  0.6300586 1.1140343 1.1895053 1.822727 1.933571
## n=700  0.3625215 0.9113498 1.2522415 1.654996 1.981563
## n=800  0.4427674 0.8570775 1.1367413 1.796147 2.103874
## n=900  0.3816606 0.8423425 1.3168243 1.712630 2.102760
## n=1000 0.4265370 0.9495205 1.3021528 1.603875 1.875588
##
## , , sd=1, rep=5
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.2288956 0.9259850 1.345124 1.767008 1.904891
## n=200  0.4782098 0.9611524 1.326272 1.689215 2.050280
## n=300  0.4520819 0.9920557 1.238380 1.753581 2.087970
## n=400  0.4573970 1.1576259 1.246390 1.497974 2.019858
## n=500  0.5434649 0.9485599 1.191573 1.631702 2.049623
## n=600  0.4632066 0.8962950 1.266618 1.657375 1.924914
## n=700  0.3907963 0.8164093 1.155234 1.615976 1.949318
## n=800  0.4930138 0.9572131 1.295102 1.684709 1.936302
## n=900  0.4979275 0.8736681 1.214887 1.584737 1.985072
## n=1000 0.5301261 0.8977935 1.313722 1.630559 1.935747
##
## , , sd=1.5, rep=5
##
##              mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.4688968 0.9435859 1.380401 1.586464 2.192555
## n=200  0.7831954 0.8507725 1.104056 1.524999 1.987862
## n=300  0.4957244 0.9953593 1.100958 1.596888 2.032147
## n=400  0.6377009 1.1108968 1.039524 1.621890 1.989716
## n=500  0.2757090 0.7425858 1.221020 1.687816 1.782811
## n=600  0.4674703 0.9888671 1.462157 1.805296 2.070940
## n=700  0.3602414 0.8718152 1.369915 1.868538 1.981321
## n=800  0.3886475 1.1178513 1.246394 1.612441 2.065348
## n=900  0.4814983 0.8016443 1.148795 1.443979 2.099358
```

```
## n=1000 0.4683020 0.8041379 1.013566 1.662726 1.873865
##
## , , sd=2, rep=5
##
##              mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.08611764 0.9398614 0.9146867 1.669708 2.319204
## n=200  0.67138321 0.6832920 1.0754579 1.433456 2.068870
## n=300  0.44312742 1.2584762 1.3301563 1.650963 2.100398
## n=400  0.66455181 0.8364694 1.2167647 1.894041 1.989744
## n=500  0.41708387 0.8419375 1.1330271 1.713047 2.102631
## n=600  0.60004510 0.7267214 1.2282300 1.533603 2.044685
## n=700  0.42035536 0.8611750 1.3460957 1.744825 2.228779
## n=800  0.59792089 1.0213057 0.9896550 1.553520 1.600474
## n=900  0.51242473 0.7970967 1.1751429 1.646252 2.039864
## n=1000 0.56705287 0.8053208 1.2585926 1.673913 1.889833
##
## attr(,"class")
## [1] "Eco"   "array"
##
## $average
##        n   mu  sd       avg
## 1    100 0.00 1.0 0.5628181
## 2    100 0.00 1.5 0.4244795
## 3    100 0.00 2.0 0.3944758
## 4    100 0.25 1.0 0.8769350
## 5    100 0.25 1.5 0.8906907
## 6    100 0.25 2.0 0.8353085
## 7    100 0.50 1.0 1.2598397
## 8    100 0.50 1.5 1.3048593
## 9    100 0.50 2.0 1.2206527
## 10   100 0.75 1.0 1.5751601
## 11   100 0.75 1.5 1.5565558
## 12   100 0.75 2.0 1.7404132
## 13   100 1.00 1.0 2.0166709
## 14   100 1.00 1.5 2.0241493
## 15   100 1.00 2.0 2.3514768
## 16   200 0.00 1.0 0.4892177
## 17   200 0.00 1.5 0.5462168
## 18   200 0.00 2.0 0.6495468
## 19   200 0.25 1.0 0.8720167
## 20   200 0.25 1.5 0.8028606
## 21   200 0.25 2.0 0.7996067
```

```
## 22   200 0.50 1.0 1.2150492
## 23   200 0.50 1.5 1.1357791
## 24   200 0.50 2.0 1.3258080
## 25   200 0.75 1.0 1.6500989
## 26   200 0.75 1.5 1.6937863
## 27   200 0.75 2.0 1.5421762
## 28   200 1.00 1.0 2.0076074
## 29   200 1.00 1.5 1.9727911
## 30   200 1.00 2.0 2.0811684
## 31   300 0.00 1.0 0.4759540
## 32   300 0.00 1.5 0.4879391
## 33   300 0.00 2.0 0.4725716
## 34   300 0.25 1.0 0.8340291
## 35   300 0.25 1.5 0.8063792
## 36   300 0.25 2.0 0.9733540
## 37   300 0.50 1.0 1.2693292
## 38   300 0.50 1.5 1.2317647
## 39   300 0.50 2.0 1.2568465
## 40   300 0.75 1.0 1.6012659
## 41   300 0.75 1.5 1.6497061
## 42   300 0.75 2.0 1.6829863
## 43   300 1.00 1.0 2.0274458
## 44   300 1.00 1.5 2.1388580
## 45   300 1.00 2.0 2.1121160
## 46   400 0.00 1.0 0.5203035
## 47   400 0.00 1.5 0.5590698
## 48   400 0.00 2.0 0.6218474
## 49   400 0.25 1.0 0.9553801
## 50   400 0.25 1.5 0.9715426
## 51   400 0.25 2.0 0.7359215
## 52   400 0.50 1.0 1.2934233
## 53   400 0.50 1.5 1.1087563
## 54   400 0.50 2.0 1.1985429
## 55   400 0.75 1.0 1.5656764
## 56   400 0.75 1.5 1.5885842
## 57   400 0.75 2.0 1.5366603
## 58   400 1.00 1.0 2.0247618
## 59   400 1.00 1.5 2.0519664
## 60   400 1.00 2.0 2.0417733
## 61   500 0.00 1.0 0.5507736
## 62   500 0.00 1.5 0.4689769
## 63   500 0.00 2.0 0.4390092
```

```
## 64   500 0.25 1.0 0.9204453
## 65   500 0.25 1.5 0.7972986
## 66   500 0.25 2.0 0.9558140
## 67   500 0.50 1.0 1.2293479
## 68   500 0.50 1.5 1.2402327
## 69   500 0.50 2.0 1.2104054
## 70   500 0.75 1.0 1.6059639
## 71   500 0.75 1.5 1.6035074
## 72   500 0.75 2.0 1.7239084
## 73   500 1.00 1.0 2.0403021
## 74   500 1.00 1.5 1.9586539
## 75   500 1.00 2.0 1.9669104
## 76   600 0.00 1.0 0.5344870
## 77   600 0.00 1.5 0.4817211
## 78   600 0.00 2.0 0.5766933
## 79   600 0.25 1.0 0.9343605
## 80   600 0.25 1.5 0.8663548
## 81   600 0.25 2.0 0.9141000
## 82   600 0.50 1.0 1.2351578
## 83   600 0.50 1.5 1.2943275
## 84   600 0.50 2.0 1.2326393
## 85   600 0.75 1.0 1.6353797
## 86   600 0.75 1.5 1.7550350
## 87   600 0.75 2.0 1.6714372
## 88   600 1.00 1.0 1.9778364
## 89   600 1.00 1.5 1.9878951
## 90   600 1.00 2.0 2.0195340
## 91   700 0.00 1.0 0.4882859
## 92   700 0.00 1.5 0.4147847
## 93   700 0.00 2.0 0.4465893
## 94   700 0.25 1.0 0.8473095
## 95   700 0.25 1.5 0.8076536
## 96   700 0.25 2.0 0.8197369
## 97   700 0.50 1.0 1.2378238
## 98   700 0.50 1.5 1.3463535
## 99   700 0.50 2.0 1.2706179
## 100  700 0.75 1.0 1.6478257
## 101  700 0.75 1.5 1.7042384
## 102  700 0.75 2.0 1.6003720
## 103  700 1.00 1.0 1.9530612
## 104  700 1.00 1.5 1.9756043
## 105  700 1.00 2.0 2.0816926
```

```
## 106  800 0.00 1.0 0.5103950
## 107  800 0.00 1.5 0.4564280
## 108  800 0.00 2.0 0.4581157
## 109  800 0.25 1.0 0.8726540
## 110  800 0.25 1.5 0.8945214
## 111  800 0.25 2.0 0.8772444
## 112  800 0.50 1.0 1.2900639
## 113  800 0.50 1.5 1.2587603
## 114  800 0.50 2.0 1.2319263
## 115  800 0.75 1.0 1.6264243
## 116  800 0.75 1.5 1.6173179
## 117  800 0.75 2.0 1.5997479
## 118  800 1.00 1.0 1.9117509
## 119  800 1.00 1.5 2.0124269
## 120  800 1.00 2.0 1.9063757
## 121  900 0.00 1.0 0.4799391
## 122  900 0.00 1.5 0.4518301
## 123  900 0.00 2.0 0.4165744
## 124  900 0.25 1.0 0.8827002
## 125  900 0.25 1.5 0.8238516
## 126  900 0.25 2.0 0.8144564
## 127  900 0.50 1.0 1.2711118
## 128  900 0.50 1.5 1.2685788
## 129  900 0.50 2.0 1.2165253
## 130  900 0.75 1.0 1.5946823
## 131  900 0.75 1.5 1.5781026
## 132  900 0.75 2.0 1.5679413
## 133  900 1.00 1.0 2.0249983
## 134  900 1.00 1.5 2.0055806
## 135  900 1.00 2.0 2.0072967
## 136 1000 0.00 1.0 0.4844173
## 137 1000 0.00 1.5 0.5442077
## 138 1000 0.00 2.0 0.5421275
## 139 1000 0.25 1.0 0.8993163
## 140 1000 0.25 1.5 0.8237354
## 141 1000 0.25 2.0 0.8728468
## 142 1000 0.50 1.0 1.2540685
## 143 1000 0.50 1.5 1.2294453
## 144 1000 0.50 2.0 1.3072707
## 145 1000 0.75 1.0 1.6479383
## 146 1000 0.75 1.5 1.5642421
## 147 1000 0.75 2.0 1.6516665
```

```
## 148 1000 1.00 1.0 2.0045373
## 149 1000 1.00 1.5 1.9084779
## 150 1000 1.00 2.0 2.0280377
##
## attr(,"class")
## [1] "Eco"
```

Same function ,`OLS` is now used in parallel with 8 cores. Execution time of the simulation is 3.85 seconds.

Now lets run `GLS` function in parallel and check the execution time. Execution time of the simulation is 16.84 seconds (with 8 cores).

```
gls <- main_function(parameters = param_list,nrep=5,simulation = gls_f,sum_fun="mean",seed=1
```

```
##
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Multisession
##
##  Number of Cores Used in  Parallelization :  8  out of 8
##
##  Input Parameters :  c("n", "100", "1000", "100") c("mu", "0", "1", "0.25") c("sd", "1",
##
##  Simulation Length : 750
##  Minumum : -0.9971
##  Maximum : 3.383637
##  Mean     : 1.260196
##  Median   : 1.284057
##
##  Execution Time of Monte Carlo Simulation 57.62402 secs
##
##  Name of The Class : Eco
```

```
gls
```

```
## $results
## , , sd=1, rep=1
##
##              mu=0    mu=0.25     mu=0.5   mu=0.75      mu=1
## n=100   0.7554538 0.5016876 1.7356494 1.517285 2.639110
## n=200   0.3903311 0.6408042 1.5194431 1.417126 3.255660
## n=300   0.8317798 0.8173362 1.2688679 1.366244 2.007371
```

```
## n=400  0.0664591 0.7887018 1.1664611 1.262650 1.601207
## n=500  0.7349604 0.8108023 1.5108027 1.624144 2.316178
## n=600  0.5483240 1.3362425 0.7466396 1.539507 1.775243
## n=700  1.1797722 0.7454456 1.5538655 1.596843 1.851457
## n=800  0.7388026 0.4151828 1.2355987 2.121480 1.617091
## n=900  0.3375558 0.5516673 0.9502070 1.037452 2.600048
## n=1000 0.2812896 0.8840261 1.4690083 1.667241 2.124077
##
## , , sd=1.5, rep=1
##
##               mu=0    mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100   1.07566007 0.97703407 0.3943886 1.0410972 1.357513
## n=200   0.52383817 1.45057710 1.6489542 1.5278402 1.404216
## n=300   0.71906564 0.40330063 2.2602221 0.8222903 2.070699
## n=400   1.04256675 1.22061498 0.7880425 1.1109698 1.966145
## n=500  -0.07188688 1.41714029 1.4742135 1.9709113 3.005006
## n=600   0.87050124 1.16986970 0.8726417 1.1228528 1.821454
## n=700  -0.25848227 0.49203602 1.0950518 1.6813917 1.675552
## n=800   0.39160576 0.03376035 1.3268501 1.6087353 2.523682
## n=900   0.01281269 0.82989922 1.1917932 2.1448010 2.249365
## n=1000  1.00643264 0.23351513 1.7264966 1.7730168 1.716113
##
## , , sd=2, rep=1
##
##              mu=0   mu=0.25   mu=0.5   mu=0.75     mu=1
## n=100   0.1989645 0.9356581 1.650652 1.4279687 2.842366
## n=200   0.6770072 0.8534528 1.886844 1.1371663 2.086750
## n=300   0.9021613 1.3441820 1.592364 1.5281411 1.918323
## n=400   0.3678666 0.6749776 1.494146 1.0124086 2.125654
## n=500  -0.3538140 1.4641727 1.702553 2.0690877 1.771409
## n=600   0.2521821 0.8727736 1.281437 1.6461148 1.787155
## n=700  -0.5283604 1.1573474 1.301522 1.6284337 2.040922
## n=800   0.7162405 1.7569821 1.089098 1.8510202 2.008889
## n=900  -0.1466358 1.2663258 0.669074 0.9226198 1.983874
## n=1000  0.4922426 0.5592436 1.209339 0.5571104 2.177159
##
## , , sd=1, rep=2
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.93794659 0.3042632 0.8558104 1.395916 2.488330
## n=200   0.46865388 0.8026164 0.6673771 1.754976 2.196959
## n=300   0.21497718 0.6857743 1.4560339 1.591513 1.441236
```

```
## n=400   0.09099797 0.1139196 0.9989419 1.530588 1.901867
## n=500   0.26214018 1.0189866 0.9483061 1.929226 2.028602
## n=600   0.36515870 1.1206859 1.3254734 2.154065 2.398600
## n=700   0.84564182 0.6471012 1.4065464 1.768395 2.419217
## n=800   0.85872788 0.7206308 1.3704309 1.246906 2.218637
## n=900  -0.14990817 1.2264243 1.6872835 1.774656 2.095731
## n=1000  0.96179814 0.6660319 1.2241939 1.846178 2.384093
##
## , , sd=1.5, rep=2
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7971403 0.6846694 1.4747453 1.376802 1.289544
## n=200  0.8722172 1.5062628 0.8667520 1.455997 3.187951
## n=300  1.1649019 1.7666435 0.9995124 1.495770 1.687811
## n=400  0.8440923 0.5933976 0.8222845 1.806278 2.256190
## n=500  0.5786570 1.1103728 1.9378712 1.644442 2.421673
## n=600  0.7493392 1.1661280 1.2064266 1.645635 1.537470
## n=700  0.8992186 1.0192023 1.0847103 1.817849 1.520385
## n=800  0.5199020 0.9399592 1.7241364 1.260431 1.886227
## n=900  1.0116356 1.1141843 1.4591379 2.097425 2.420061
## n=1000 1.1299320 0.6024359 0.3630059 2.020767 1.720216
##
## , , sd=2, rep=2
##
##               mu=0     mu=0.25     mu=0.5    mu=0.75     mu=1
## n=100  -0.01919950  0.10389729 -0.7181519  2.1975727 2.688336
## n=200   1.02896044 -0.06349928  1.7339415  1.9835128 1.816331
## n=300  -0.18610242  1.60434976  1.6070085 -0.3271213 1.931503
## n=400   1.41709802  1.20750797  1.5523077  1.6424103 1.171583
## n=500   1.23639159  0.54645719  1.6774379  1.3658300 3.363879
## n=600   0.39037287  1.42422862  0.5759413  2.1932974 1.854756
## n=700  -0.34830352  0.85511492  1.2548947  2.4213875 2.016490
## n=800  -0.08188814  1.27109005  1.7893126  0.9299721 1.717897
## n=900  -0.07990839  0.66567655  1.8002084  1.7681873 1.740396
## n=1000  0.88195270  0.50717811  1.2185675  2.1915303 1.744440
##
## , , sd=1, rep=3
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.2500595 1.3250897 0.9547569 2.293587 1.819439
## n=200  0.5471549 0.9566464 1.3296827 1.775252 1.804901
## n=300  1.0456930 1.4996928 1.4615289 1.399473 2.154331
```

```
## n=400  0.6342614 0.5517242 1.4421203 1.787110 2.553724
## n=500  0.5785579 1.0079606 1.1506459 1.573662 1.915940
## n=600  0.3589415 0.7270598 1.5240060 1.239381 1.971919
## n=700  0.7345540 0.2262226 1.2611827 1.103715 2.039746
## n=800  0.5015531 1.9152089 1.6893004 1.561901 2.198732
## n=900  0.1721376 1.0750490 1.1722128 1.835734 1.946778
## n=1000 0.5581321 0.7179216 1.0656947 1.773012 2.400244
##
## , , sd=1.5, rep=3
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.75515002 1.0532149 1.3675520 2.458531 2.511075
## n=200  -0.04934648 0.5496542 2.0479018 1.675846 1.142718
## n=300   1.00181000 1.6369489 0.7753943 2.638377 1.829983
## n=400   0.38708014 1.2533585 1.4714135 1.435519 2.018202
## n=500   0.37250451 0.7996507 1.2444854 1.892440 1.427025
## n=600   0.26431978 1.3546424 1.2058852 1.796262 2.147378
## n=700   1.12511433 0.6271908 0.8411113 1.638469 1.970271
## n=800   0.52208688 0.6764603 1.6023554 1.526346 1.906584
## n=900   0.22266670 0.5468360 1.5856621 2.134440 1.515266
## n=1000  0.20373892 1.9340035 1.2128314 1.337933 1.380179
##
## , , sd=2, rep=3
##
##              mu=0    mu=0.25      mu=0.5  mu=0.75      mu=1
## n=100  -0.5134825 -0.9971000  0.02609116 2.483622 2.1737654
## n=200   0.8564846  0.5270190  2.19874069 1.115522 2.0483072
## n=300   0.1357223  1.1346463  0.67885853 1.989233 2.3651937
## n=400  -0.1132654  1.5174377 -0.39911038 2.430706 2.7068852
## n=500   0.2692164  0.9373865  0.45341757 1.488270 2.0855372
## n=600   0.2767981  1.9185888  1.10986631 1.779951 2.2288551
## n=700   0.4175472  0.7176046  0.78665976 1.500209 2.3208915
## n=800   0.2236755  0.7400305  1.63497839 2.006660 2.2390314
## n=900   1.3383189  1.7077645  1.90542412 2.962268 1.5069244
## n=1000  0.2337000  0.4510541  0.72055597 2.240282 0.9580619
##
## , , sd=1, rep=4
##
##             mu=0     mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  1.3469860 -0.05560687 1.0932454 1.903516 2.156825
## n=200  0.3208913  0.96409270 1.0863966 1.386434 1.835235
## n=300  0.6078219  0.39871289 1.3616965 2.100878 1.933667
```

```
## n=400  0.6771807  1.05598360 1.5240453 2.261056 2.531636
## n=500  0.4919199  1.10827112 1.5332686 1.517603 1.843585
## n=600  0.5637876  0.85089642 1.8731358 1.617511 2.200501
## n=700  0.3582705  0.99200307 0.8214109 1.560444 2.128348
## n=800  0.5240053  0.94117803 1.2620739 1.546574 1.873964
## n=900  0.6295721  0.83325772 0.9917639 1.333128 1.971597
## n=1000 0.6248638  1.00187859 1.3589893 1.665165 1.630167
##
## , , sd=1.5, rep=4
##
##              mu=0     mu=0.25    mu=0.5   mu=0.75      mu=1
## n=100  0.14190040  2.01821137 0.8517502 2.0724785 2.762258
## n=200  0.82223920  0.95236727 2.3576613 2.7525450 2.656997
## n=300  0.41468489  0.81135204 1.7321312 2.2129173 1.586342
## n=400  0.47456049  1.28481297 1.6045288 1.6357618 2.517795
## n=500  0.19596935 -0.09776629 1.3816823 1.1821549 1.806055
## n=600  0.04919382  1.33474572 1.2062848 0.0429013 2.320981
## n=700  0.34423613  0.48523985 1.9450566 1.5101308 2.451672
## n=800  1.32565822  0.84774132 1.2387826 2.3976442 1.870370
## n=900  0.47854733  0.51916320 1.1726107 1.6816128 1.705770
## n=1000 0.82817973  0.80225623 1.5175430 1.4264566 1.863020
##
## , , sd=2, rep=4
##
##             mu=0    mu=0.25     mu=0.5  mu=0.75      mu=1
## n=100  0.8525966  1.1749948 3.38363655 1.248533 3.057232
## n=200  0.5182802  0.4838462 1.41934260 2.241456 2.358571
## n=300  1.0733668  2.0578513 0.96974173 2.245360 1.290691
## n=400  0.8011078  0.4410973 0.90802926 1.296971 2.153393
## n=500  0.2470769  0.7742284 2.22385975 2.482988 1.599441
## n=600  1.1095269  0.4747139 0.06967184 1.576253 1.038778
## n=700  0.3443959 -0.7980158 0.88632336 1.763624 1.642792
## n=800  0.3212259 -0.1889027 1.12345703 2.337993 1.622583
## n=900  0.8488329  1.3243651 0.42830492 1.324446 2.120547
## n=1000 1.6411901  0.1763252 1.43712633 1.095596 1.415895
##
## , , sd=1, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.1106945 1.3324074 1.283149 1.795427 1.479256
## n=200  0.5031188 1.5027343 1.460474 1.921770 2.617298
## n=300  0.3730896 0.6590489 1.300485 1.664153 2.229097
```

```
## n=400  0.4404047 1.6079649 1.600920 1.597121 1.219695
## n=500  0.9142947 0.8821124 0.993282 2.142944 2.017532
## n=600  0.7455402 1.3534103 1.252659 1.654961 2.019734
## n=700  0.1307611 0.5045976 1.915240 1.719891 1.816085
## n=800  0.0348579 0.4676373 1.687917 1.773396 1.871487
## n=900  0.5683141 1.0347100 1.329680 1.971053 2.031339
## n=1000 0.2944263 0.7265625 1.342830 1.397584 1.678541
##
## , , sd=1.5, rep=5
##
##              mu=0     mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100   0.3492151 0.941764185 1.9927388 1.1271039 2.343358
## n=200   0.9322249 0.009236562 1.6216833 1.5246716 2.126926
## n=300  -0.2061382 0.527637441 1.1785690 1.2855802 1.955373
## n=400  -0.2080310 0.865475626 1.5810396 1.8863548 2.607916
## n=500   1.0591201 1.269133681 1.3106256 1.6893725 2.221825
## n=600   1.0529351 0.935277255 0.8883463 0.4731693 2.883712
## n=700  -0.1566764 1.060909722 1.3480097 1.5789494 2.593332
## n=800   0.3458820 0.824462226 1.0043539 1.3977273 1.529786
## n=900   0.4578455 0.443865375 1.0857179 1.1477187 1.641970
## n=1000  0.7074108 0.556170334 1.5201499 1.3028712 2.580936
##
## , , sd=2, rep=5
##
##               mu=0    mu=0.25    mu=0.5    mu=0.75     mu=1
## n=100   0.35838129  1.9134632 0.6236571  1.8050059 2.775072
## n=200   1.13555545 -0.3462561 0.5517002  1.2833010 2.082454
## n=300  -0.12779402  0.8027196 0.2661467  1.1407255 2.706796
## n=400   0.84449836  1.5107112 1.3472610  2.1182648 2.410341
## n=500   0.58335229  0.3130528 2.2292441  0.5206100 1.407526
## n=600   1.00687281  0.8306865 0.8167926 -0.4327573 1.897837
## n=700   0.90763719  0.7547756 1.2602192  1.9296762 2.212971
## n=800   1.16899109  0.5890770 1.1466515  0.6023830 1.086282
## n=900  -0.53156942  0.2064395 1.3326650  0.1082137 1.878707
## n=1000 -0.06566813  1.5126884 1.9075839  1.1067314 1.610853
##
## attr(,"class")
## [1] "Eco"   "array"
##
## $average
##        n   mu  sd       avg
## 1    100 0.00 1.0 0.6802281
```

```
## 2    100 0.00 1.5 0.6238132
## 3    100 0.00 2.0 0.1754521
## 4    100 0.25 1.0 0.6815682
## 5    100 0.25 1.5 1.1349788
## 6    100 0.25 2.0 0.6261827
## 7    100 0.50 1.0 1.1845221
## 8    100 0.50 1.5 1.2162350
## 9    100 0.50 2.0 0.9931770
## 10   100 0.75 1.0 1.7811463
## 11   100 0.75 1.5 1.6152025
## 12   100 0.75 2.0 1.8325404
## 13   100 1.00 1.0 2.1165920
## 14   100 1.00 1.5 2.0527497
## 15   100 1.00 2.0 2.7073544
## 16   200 0.00 1.0 0.4460300
## 17   200 0.00 1.5 0.6202346
## 18   200 0.00 2.0 0.8432576
## 19   200 0.25 1.0 0.9733788
## 20   200 0.25 1.5 0.8936196
## 21   200 0.25 2.0 0.2909125
## 22   200 0.50 1.0 1.2126747
## 23   200 0.50 1.5 1.7085905
## 24   200 0.50 2.0 1.5581137
## 25   200 0.75 1.0 1.6511116
## 26   200 0.75 1.5 1.7873799
## 27   200 0.75 2.0 1.5521916
## 28   200 1.00 1.0 2.3420107
## 29   200 1.00 1.5 2.1037616
## 30   200 1.00 2.0 2.0784825
## 31   300 0.00 1.0 0.6146723
## 32   300 0.00 1.5 0.6188648
## 33   300 0.00 2.0 0.3594708
## 34   300 0.25 1.0 0.8121130
## 35   300 0.25 1.5 1.0291765
## 36   300 0.25 2.0 1.3887498
## 37   300 0.50 1.0 1.3697225
## 38   300 0.50 1.5 1.3891658
## 39   300 0.50 2.0 1.0228240
## 40   300 0.75 1.0 1.6244521
## 41   300 0.75 1.5 1.6909870
## 42   300 0.75 2.0 1.3152675
## 43   300 1.00 1.0 1.9531402
```

```
## 44   300 1.00 1.5 1.8260415
## 45   300 1.00 2.0 2.0425014
## 46   400 0.00 1.0 0.3818608
## 47   400 0.00 1.5 0.5080537
## 48   400 0.00 2.0 0.6634611
## 49   400 0.25 1.0 0.8236588
## 50   400 0.25 1.5 1.0435319
## 51   400 0.25 2.0 1.0703464
## 52   400 0.50 1.0 1.3464978
## 53   400 0.50 1.5 1.2534618
## 54   400 0.50 2.0 0.9805268
## 55   400 0.75 1.0 1.6877048
## 56   400 0.75 1.5 1.5749766
## 57   400 0.75 2.0 1.7001523
## 58   400 1.00 1.0 1.9616256
## 59   400 1.00 1.5 2.2732495
## 60   400 1.00 2.0 2.1135711
## 61   500 0.00 1.0 0.5963746
## 62   500 0.00 1.5 0.4268728
## 63   500 0.00 2.0 0.3964446
## 64   500 0.25 1.0 0.9656266
## 65   500 0.25 1.5 0.8997062
## 66   500 0.25 2.0 0.8070595
## 67   500 0.50 1.0 1.2272611
## 68   500 0.50 1.5 1.4697756
## 69   500 0.50 2.0 1.6573025
## 70   500 0.75 1.0 1.7575159
## 71   500 0.75 1.5 1.6758640
## 72   500 0.75 2.0 1.5853571
## 73   500 1.00 1.0 2.0243674
## 74   500 1.00 1.5 2.1763168
## 75   500 1.00 2.0 2.0455584
## 76   600 0.00 1.0 0.5163504
## 77   600 0.00 1.5 0.5972578
## 78   600 0.00 2.0 0.6071506
## 79   600 0.25 1.0 1.0776590
## 80   600 0.25 1.5 1.1921326
## 81   600 0.25 2.0 1.1041983
## 82   600 0.50 1.0 1.3443827
## 83   600 0.50 1.5 1.0759169
## 84   600 0.50 2.0 0.7707418
## 85   600 0.75 1.0 1.6410851
```

```
## 86    600 0.75 1.5 1.0161640
## 87    600 0.75 2.0 1.3525717
## 88    600 1.00 1.0 2.0731992
## 89    600 1.00 1.5 2.1421991
## 90    600 1.00 2.0 1.7614761
## 91    700 0.00 1.0 0.6497999
## 92    700 0.00 1.5 0.3906821
## 93    700 0.00 2.0 0.1585833
## 94    700 0.25 1.0 0.6230740
## 95    700 0.25 1.5 0.7369157
## 96    700 0.25 2.0 0.5373653
## 97    700 0.50 1.0 1.3916491
## 98    700 0.50 1.5 1.2627879
## 99    700 0.50 2.0 1.0979239
## 100   700 0.75 1.0 1.5498575
## 101   700 0.75 1.5 1.6453580
## 102   700 0.75 2.0 1.8486661
## 103   700 1.00 1.0 2.0509706
## 104   700 1.00 1.5 2.0422424
## 105   700 1.00 2.0 2.0468134
## 106   800 0.00 1.0 0.5315894
## 107   800 0.00 1.5 0.6210270
## 108   800 0.00 2.0 0.4696490
## 109   800 0.25 1.0 0.8919676
## 110   800 0.25 1.5 0.6644767
## 111   800 0.25 2.0 0.8336554
## 112   800 0.50 1.0 1.4490643
## 113   800 0.50 1.5 1.3792957
## 114   800 0.50 2.0 1.3566995
## 115   800 0.75 1.0 1.6500513
## 116   800 0.75 1.5 1.6381768
## 117   800 0.75 2.0 1.5456057
## 118   800 1.00 1.0 1.9559821
## 119   800 1.00 1.5 1.9433300
## 120   800 1.00 2.0 1.7349365
## 121   900 0.00 1.0 0.3115343
## 122   900 0.00 1.5 0.4367016
## 123   900 0.00 2.0 0.2858076
## 124   900 0.25 1.0 0.9442217
## 125   900 0.25 1.5 0.6907896
## 126   900 0.25 2.0 1.0341143
## 127   900 0.50 1.0 1.2262295
```

```
## 128  900 0.50 1.5 1.2989843
## 129  900 0.50 2.0 1.2271353
## 130  900 0.75 1.0 1.5904047
## 131  900 0.75 1.5 1.8411995
## 132  900 0.75 2.0 1.4171470
## 133  900 1.00 1.0 2.1290987
## 134  900 1.00 1.5 1.9064864
## 135  900 1.00 2.0 1.8460896
## 136 1000 0.00 1.0 0.5441020
## 137 1000 0.00 1.5 0.7751388
## 138 1000 0.00 2.0 0.6366835
## 139 1000 0.25 1.0 0.7992841
## 140 1000 0.25 1.5 0.8256762
## 141 1000 0.25 2.0 0.6412979
## 142 1000 0.50 1.0 1.2921433
## 143 1000 0.50 1.5 1.2680054
## 144 1000 0.50 2.0 1.2986345
## 145 1000 0.75 1.0 1.6698362
## 146 1000 0.75 1.5 1.5722088
## 147 1000 0.75 2.0 1.4382501
## 148 1000 1.00 1.0 2.0434242
## 149 1000 1.00 1.5 1.8520927
## 150 1000 1.00 2.0 1.5812816
##
## attr(,"class")
## [1] "Eco"
```

Now without parallelization (with 1 core)

```
gls <- main_function(parameters = param_list,nrep=5,simulation = gls_f,sum_fun="mean",seed=1
```

```
##
##  Repetition(nrep)      :  5
##
##  Parallelization Type  :  Sequential
##
##  Number of Cores Used in  Parallelization :  1  out of 8
##
##  Input Parameters :  c("n", "100", "1000", "100") c("mu", "0", "1", "0.25") c("sd", "1", 
##
##  Simulation Length : 750
##  Minumum : -0.9971
##  Maximum : 3.383637
```

```
##  Mean    : 1.260196
##  Median  : 1.284057
##
##  Execution Time of Monte Carlo Simulation 52.99871 secs
##
##  Name of The Class : Eco
```

```
gls
```

```
## $results
## , , sd=1, rep=1
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7554538 0.5016876 1.7356494 1.517285 2.639110
## n=200  0.3903311 0.6408042 1.5194431 1.417126 3.255660
## n=300  0.8317798 0.8173362 1.2688679 1.366244 2.007371
## n=400  0.0664591 0.7887018 1.1664611 1.262650 1.601207
## n=500  0.7349604 0.8108023 1.5108027 1.624144 2.316178
## n=600  0.5483240 1.3362425 0.7466396 1.539507 1.775243
## n=700  1.1797722 0.7454456 1.5538655 1.596843 1.851457
## n=800  0.7388026 0.4151828 1.2355987 2.121480 1.617091
## n=900  0.3375558 0.5516673 0.9502070 1.037452 2.600048
## n=1000 0.2812896 0.8840261 1.4690083 1.667241 2.124077
##
## , , sd=1.5, rep=1
##
##                mu=0    mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100   1.07566007 0.97703407 0.3943886 1.0410972 1.357513
## n=200   0.52383817 1.45057710 1.6489542 1.5278402 1.404216
## n=300   0.71906564 0.40330063 2.2602221 0.8222903 2.070699
## n=400   1.04256675 1.22061498 0.7880425 1.1109698 1.966145
## n=500  -0.07188688 1.41714029 1.4742135 1.9709113 3.005006
## n=600   0.87050124 1.16986970 0.8726417 1.1228528 1.821454
## n=700  -0.25848227 0.49203602 1.0950518 1.6813917 1.675552
## n=800   0.39160576 0.03376035 1.3268501 1.6087353 2.523682
## n=900   0.01281269 0.82989922 1.1917932 2.1448010 2.249365
## n=1000  1.00643264 0.23351513 1.7264966 1.7730168 1.716113
##
## , , sd=2, rep=1
##
##               mu=0   mu=0.25   mu=0.5   mu=0.75     mu=1
## n=100   0.1989645 0.9356581 1.650652 1.4279687 2.842366
## n=200   0.6770072 0.8534528 1.886844 1.1371663 2.086750
```

```
## n=300   0.9021613 1.3441820 1.592364 1.5281411 1.918323
## n=400   0.3678666 0.6749776 1.494146 1.0124086 2.125654
## n=500  -0.3538140 1.4641727 1.702553 2.0690877 1.771409
## n=600   0.2521821 0.8727736 1.281437 1.6461148 1.787155
## n=700  -0.5283604 1.1573474 1.301522 1.6284337 2.040922
## n=800   0.7162405 1.7569821 1.089098 1.8510202 2.008889
## n=900  -0.1466358 1.2663258 0.669074 0.9226198 1.983874
## n=1000  0.4922426 0.5592436 1.209339 0.5571104 2.177159
##
## , , sd=1, rep=2
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.93794659 0.3042632 0.8558104 1.395916 2.488330
## n=200   0.46865388 0.8026164 0.6673771 1.754976 2.196959
## n=300   0.21497718 0.6857743 1.4560339 1.591513 1.441236
## n=400   0.09099797 0.1139196 0.9989419 1.530588 1.901867
## n=500   0.26214018 1.0189866 0.9483061 1.929226 2.028602
## n=600   0.36515870 1.1206859 1.3254734 2.154065 2.398600
## n=700   0.84564182 0.6471012 1.4065464 1.768395 2.419217
## n=800   0.85872788 0.7206308 1.3704309 1.246906 2.218637
## n=900  -0.14990817 1.2264243 1.6872835 1.774656 2.095731
## n=1000  0.96179814 0.6660319 1.2241939 1.846178 2.384093
##
## , , sd=1.5, rep=2
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.7971403 0.6846694 1.4747453 1.376802 1.289544
## n=200  0.8722172 1.5062628 0.8667520 1.455997 3.187951
## n=300  1.1649019 1.7666435 0.9995124 1.495770 1.687811
## n=400  0.8440923 0.5933976 0.8222845 1.806278 2.256190
## n=500  0.5786570 1.1103728 1.9378712 1.644442 2.421673
## n=600  0.7493392 1.1661280 1.2064266 1.645635 1.537470
## n=700  0.8992186 1.0192023 1.0847103 1.817849 1.520385
## n=800  0.5199020 0.9399592 1.7241364 1.260431 1.886227
## n=900  1.0116356 1.1141843 1.4591379 2.097425 2.420061
## n=1000 1.1299320 0.6024359 0.3630059 2.020767 1.720216
##
## , , sd=2, rep=2
##
##               mu=0     mu=0.25     mu=0.5    mu=0.75     mu=1
## n=100  -0.01919950  0.10389729 -0.7181519  2.1975727 2.688336
## n=200   1.02896044 -0.06349928  1.7339415  1.9835128 1.816331
```

```
## n=300  -0.18610242  1.60434976  1.6070085 -0.3271213 1.931503
## n=400   1.41709802  1.20750797  1.5523077  1.6424103 1.171583
## n=500   1.23639159  0.54645719  1.6774379  1.3658300 3.363879
## n=600   0.39037287  1.42422862  0.5759413  2.1932974 1.854756
## n=700  -0.34830352  0.85511492  1.2548947  2.4213875 2.016490
## n=800  -0.08188814  1.27109005  1.7893126  0.9299721 1.717897
## n=900  -0.07990839  0.66567655  1.8002084  1.7681873 1.740396
## n=1000  0.88195270  0.50717811  1.2185675  2.1915303 1.744440
##
## , , sd=1, rep=3
##
##             mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  0.2500595 1.3250897 0.9547569 2.293587 1.819439
## n=200  0.5471549 0.9566464 1.3296827 1.775252 1.804901
## n=300  1.0456930 1.4996928 1.4615289 1.399473 2.154331
## n=400  0.6342614 0.5517242 1.4421203 1.787110 2.553724
## n=500  0.5785579 1.0079606 1.1506459 1.573662 1.915940
## n=600  0.3589415 0.7270598 1.5240060 1.239381 1.971919
## n=700  0.7345540 0.2262226 1.2611827 1.103715 2.039746
## n=800  0.5015531 1.9152089 1.6893004 1.561901 2.198732
## n=900  0.1721376 1.0750490 1.1722128 1.835734 1.946778
## n=1000 0.5581321 0.7179216 1.0656947 1.773012 2.400244
##
## , , sd=1.5, rep=3
##
##               mu=0   mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100   0.75515002 1.0532149 1.3675520 2.458531 2.511075
## n=200  -0.04934648 0.5496542 2.0479018 1.675846 1.142718
## n=300   1.00181000 1.6369489 0.7753943 2.638377 1.829983
## n=400   0.38708014 1.2533585 1.4714135 1.435519 2.018202
## n=500   0.37250451 0.7996507 1.2444854 1.892440 1.427025
## n=600   0.26431978 1.3546424 1.2058852 1.796262 2.147378
## n=700   1.12511433 0.6271908 0.8411113 1.638469 1.970271
## n=800   0.52208688 0.6764603 1.6023554 1.526346 1.906584
## n=900   0.22266670 0.5468360 1.5856621 2.134440 1.515266
## n=1000  0.20373892 1.9340035 1.2128314 1.337933 1.380179
##
## , , sd=2, rep=3
##
##              mu=0    mu=0.25      mu=0.5  mu=0.75      mu=1
## n=100  -0.5134825 -0.9971000  0.02609116 2.483622 2.1737654
## n=200   0.8564846  0.5270190  2.19874069 1.115522 2.0483072
```

```
## n=300   0.1357223  1.1346463  0.67885853 1.989233 2.3651937
## n=400  -0.1132654  1.5174377 -0.39911038 2.430706 2.7068852
## n=500   0.2692164  0.9373865  0.45341757 1.488270 2.0855372
## n=600   0.2767981  1.9185888  1.10986631 1.779951 2.2288551
## n=700   0.4175472  0.7176046  0.78665976 1.500209 2.3208915
## n=800   0.2236755  0.7400305  1.63497839 2.006660 2.2390314
## n=900   1.3383189  1.7077645  1.90542412 2.962268 1.5069244
## n=1000  0.2337000  0.4510541  0.72055597 2.240282 0.9580619
##
## , , sd=1, rep=4
##
##              mu=0     mu=0.25    mu=0.5  mu=0.75     mu=1
## n=100  1.3469860 -0.05560687 1.0932454 1.903516 2.156825
## n=200  0.3208913  0.96409270 1.0863966 1.386434 1.835235
## n=300  0.6078219  0.39871289 1.3616965 2.100878 1.933667
## n=400  0.6771807  1.05598360 1.5240453 2.261056 2.531636
## n=500  0.4919199  1.10827112 1.5332686 1.517603 1.843585
## n=600  0.5637876  0.85089642 1.8731358 1.617511 2.200501
## n=700  0.3582705  0.99200307 0.8214109 1.560444 2.128348
## n=800  0.5240053  0.94117803 1.2620739 1.546574 1.873964
## n=900  0.6295721  0.83325772 0.9917639 1.333128 1.971597
## n=1000 0.6248638  1.00187859 1.3589893 1.665165 1.630167
##
## , , sd=1.5, rep=4
##
##               mu=0     mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100  0.14190040  2.01821137 0.8517502 2.0724785 2.762258
## n=200  0.82223920  0.95236727 2.3576613 2.7525450 2.656997
## n=300  0.41468489  0.81135204 1.7321312 2.2129173 1.586342
## n=400  0.47456049  1.28481297 1.6045288 1.6357618 2.517795
## n=500  0.19596935 -0.09776629 1.3816823 1.1821549 1.806055
## n=600  0.04919382  1.33474572 1.2062848 0.0429013 2.320981
## n=700  0.34423613  0.48523985 1.9450566 1.5101308 2.451672
## n=800  1.32565822  0.84774132 1.2387826 2.3976442 1.870370
## n=900  0.47854733  0.51916320 1.1726107 1.6816128 1.705770
## n=1000 0.82817973  0.80225623 1.5175430 1.4264566 1.863020
##
## , , sd=2, rep=4
##
##              mu=0    mu=0.25     mu=0.5  mu=0.75     mu=1
## n=100  0.8525966  1.1749948 3.38363655 1.248533 3.057232
## n=200  0.5182802  0.4838462 1.41934260 2.241456 2.358571
```

```
## n=300  1.0733668  2.0578513 0.96974173 2.245360 1.290691
## n=400  0.8011078  0.4410973 0.90802926 1.296971 2.153393
## n=500  0.2470769  0.7742284 2.22385975 2.482988 1.599441
## n=600  1.1095269  0.4747139 0.06967184 1.576253 1.038778
## n=700  0.3443959 -0.7980158 0.88632336 1.763624 1.642792
## n=800  0.3212259 -0.1889027 1.12345703 2.337993 1.622583
## n=900  0.8488329  1.3243651 0.42830492 1.324446 2.120547
## n=1000 1.6411901  0.1763252 1.43712633 1.095596 1.415895
##
## , , sd=1, rep=5
##
##             mu=0   mu=0.25   mu=0.5  mu=0.75     mu=1
## n=100  0.1106945 1.3324074 1.283149 1.795427 1.479256
## n=200  0.5031188 1.5027343 1.460474 1.921770 2.617298
## n=300  0.3730896 0.6590489 1.300485 1.664153 2.229097
## n=400  0.4404047 1.6079649 1.600920 1.597121 1.219695
## n=500  0.9142947 0.8821124 0.993282 2.142944 2.017532
## n=600  0.7455402 1.3534103 1.252659 1.654961 2.019734
## n=700  0.1307611 0.5045976 1.915240 1.719891 1.816085
## n=800  0.0348579 0.4676373 1.687917 1.773396 1.871487
## n=900  0.5683141 1.0347100 1.329680 1.971053 2.031339
## n=1000 0.2944263 0.7265625 1.342830 1.397584 1.678541
##
## , , sd=1.5, rep=5
##
##              mu=0     mu=0.25    mu=0.5   mu=0.75     mu=1
## n=100   0.3492151 0.941764185 1.9927388 1.1271039 2.343358
## n=200   0.9322249 0.009236562 1.6216833 1.5246716 2.126926
## n=300  -0.2061382 0.527637441 1.1785690 1.2855802 1.955373
## n=400  -0.2080310 0.865475626 1.5810396 1.8863548 2.607916
## n=500   1.0591201 1.269133681 1.3106256 1.6893725 2.221825
## n=600   1.0529351 0.935277255 0.8883463 0.4731693 2.883712
## n=700  -0.1566764 1.060909722 1.3480097 1.5789494 2.593332
## n=800   0.3458820 0.824462226 1.0043539 1.3977273 1.529786
## n=900   0.4578455 0.443865375 1.0857179 1.1477187 1.641970
## n=1000  0.7074108 0.556170334 1.5201499 1.3028712 2.580936
##
## , , sd=2, rep=5
##
##               mu=0    mu=0.25    mu=0.5    mu=0.75     mu=1
## n=100   0.35838129  1.9134632 0.6236571  1.8050059 2.775072
## n=200   1.13555545 -0.3462561 0.5517002  1.2833010 2.082454
```

```
## n=300  -0.12779402  0.8027196 0.2661467  1.1407255 2.706796
## n=400   0.84449836  1.5107112 1.3472610  2.1182648 2.410341
## n=500   0.58335229  0.3130528 2.2292441  0.5206100 1.407526
## n=600   1.00687281  0.8306865 0.8167926 -0.4327573 1.897837
## n=700   0.90763719  0.7547756 1.2602192  1.9296762 2.212971
## n=800   1.16899109  0.5890770 1.1466515  0.6023830 1.086282
## n=900  -0.53156942  0.2064395 1.3326650  0.1082137 1.878707
## n=1000 -0.06566813  1.5126884 1.9075839  1.1067314 1.610853
##
## attr(,"class")
## [1] "Eco"   "array"
##
## $average
##        n   mu  sd       avg
## 1    100 0.00 1.0 0.6802281
## 2    100 0.00 1.5 0.6238132
## 3    100 0.00 2.0 0.1754521
## 4    100 0.25 1.0 0.6815682
## 5    100 0.25 1.5 1.1349788
## 6    100 0.25 2.0 0.6261827
## 7    100 0.50 1.0 1.1845221
## 8    100 0.50 1.5 1.2162350
## 9    100 0.50 2.0 0.9931770
## 10   100 0.75 1.0 1.7811463
## 11   100 0.75 1.5 1.6152025
## 12   100 0.75 2.0 1.8325404
## 13   100 1.00 1.0 2.1165920
## 14   100 1.00 1.5 2.0527497
## 15   100 1.00 2.0 2.7073544
## 16   200 0.00 1.0 0.4460300
## 17   200 0.00 1.5 0.6202346
## 18   200 0.00 2.0 0.8432576
## 19   200 0.25 1.0 0.9733788
## 20   200 0.25 1.5 0.8936196
## 21   200 0.25 2.0 0.2909125
## 22   200 0.50 1.0 1.2126747
## 23   200 0.50 1.5 1.7085905
## 24   200 0.50 2.0 1.5581137
## 25   200 0.75 1.0 1.6511116
## 26   200 0.75 1.5 1.7873799
## 27   200 0.75 2.0 1.5521916
## 28   200 1.00 1.0 2.3420107
```

```
## 29   200 1.00 1.5 2.1037616
## 30   200 1.00 2.0 2.0784825
## 31   300 0.00 1.0 0.6146723
## 32   300 0.00 1.5 0.6188648
## 33   300 0.00 2.0 0.3594708
## 34   300 0.25 1.0 0.8121130
## 35   300 0.25 1.5 1.0291765
## 36   300 0.25 2.0 1.3887498
## 37   300 0.50 1.0 1.3697225
## 38   300 0.50 1.5 1.3891658
## 39   300 0.50 2.0 1.0228240
## 40   300 0.75 1.0 1.6244521
## 41   300 0.75 1.5 1.6909870
## 42   300 0.75 2.0 1.3152675
## 43   300 1.00 1.0 1.9531402
## 44   300 1.00 1.5 1.8260415
## 45   300 1.00 2.0 2.0425014
## 46   400 0.00 1.0 0.3818608
## 47   400 0.00 1.5 0.5080537
## 48   400 0.00 2.0 0.6634611
## 49   400 0.25 1.0 0.8236588
## 50   400 0.25 1.5 1.0435319
## 51   400 0.25 2.0 1.0703464
## 52   400 0.50 1.0 1.3464978
## 53   400 0.50 1.5 1.2534618
## 54   400 0.50 2.0 0.9805268
## 55   400 0.75 1.0 1.6877048
## 56   400 0.75 1.5 1.5749766
## 57   400 0.75 2.0 1.7001523
## 58   400 1.00 1.0 1.9616256
## 59   400 1.00 1.5 2.2732495
## 60   400 1.00 2.0 2.1135711
## 61   500 0.00 1.0 0.5963746
## 62   500 0.00 1.5 0.4268728
## 63   500 0.00 2.0 0.3964446
## 64   500 0.25 1.0 0.9656266
## 65   500 0.25 1.5 0.8997062
## 66   500 0.25 2.0 0.8070595
## 67   500 0.50 1.0 1.2272611
## 68   500 0.50 1.5 1.4697756
## 69   500 0.50 2.0 1.6573025
## 70   500 0.75 1.0 1.7575159
```

```
## 71   500 0.75 1.5 1.6758640
## 72   500 0.75 2.0 1.5853571
## 73   500 1.00 1.0 2.0243674
## 74   500 1.00 1.5 2.1763168
## 75   500 1.00 2.0 2.0455584
## 76   600 0.00 1.0 0.5163504
## 77   600 0.00 1.5 0.5972578
## 78   600 0.00 2.0 0.6071506
## 79   600 0.25 1.0 1.0776590
## 80   600 0.25 1.5 1.1921326
## 81   600 0.25 2.0 1.1041983
## 82   600 0.50 1.0 1.3443827
## 83   600 0.50 1.5 1.0759169
## 84   600 0.50 2.0 0.7707418
## 85   600 0.75 1.0 1.6410851
## 86   600 0.75 1.5 1.0161640
## 87   600 0.75 2.0 1.3525717
## 88   600 1.00 1.0 2.0731992
## 89   600 1.00 1.5 2.1421991
## 90   600 1.00 2.0 1.7614761
## 91   700 0.00 1.0 0.6497999
## 92   700 0.00 1.5 0.3906821
## 93   700 0.00 2.0 0.1585833
## 94   700 0.25 1.0 0.6230740
## 95   700 0.25 1.5 0.7369157
## 96   700 0.25 2.0 0.5373653
## 97   700 0.50 1.0 1.3916491
## 98   700 0.50 1.5 1.2627879
## 99   700 0.50 2.0 1.0979239
## 100  700 0.75 1.0 1.5498575
## 101  700 0.75 1.5 1.6453580
## 102  700 0.75 2.0 1.8486661
## 103  700 1.00 1.0 2.0509706
## 104  700 1.00 1.5 2.0422424
## 105  700 1.00 2.0 2.0468134
## 106  800 0.00 1.0 0.5315894
## 107  800 0.00 1.5 0.6210270
## 108  800 0.00 2.0 0.4696490
## 109  800 0.25 1.0 0.8919676
## 110  800 0.25 1.5 0.6644767
## 111  800 0.25 2.0 0.8336554
## 112  800 0.50 1.0 1.4490643
```

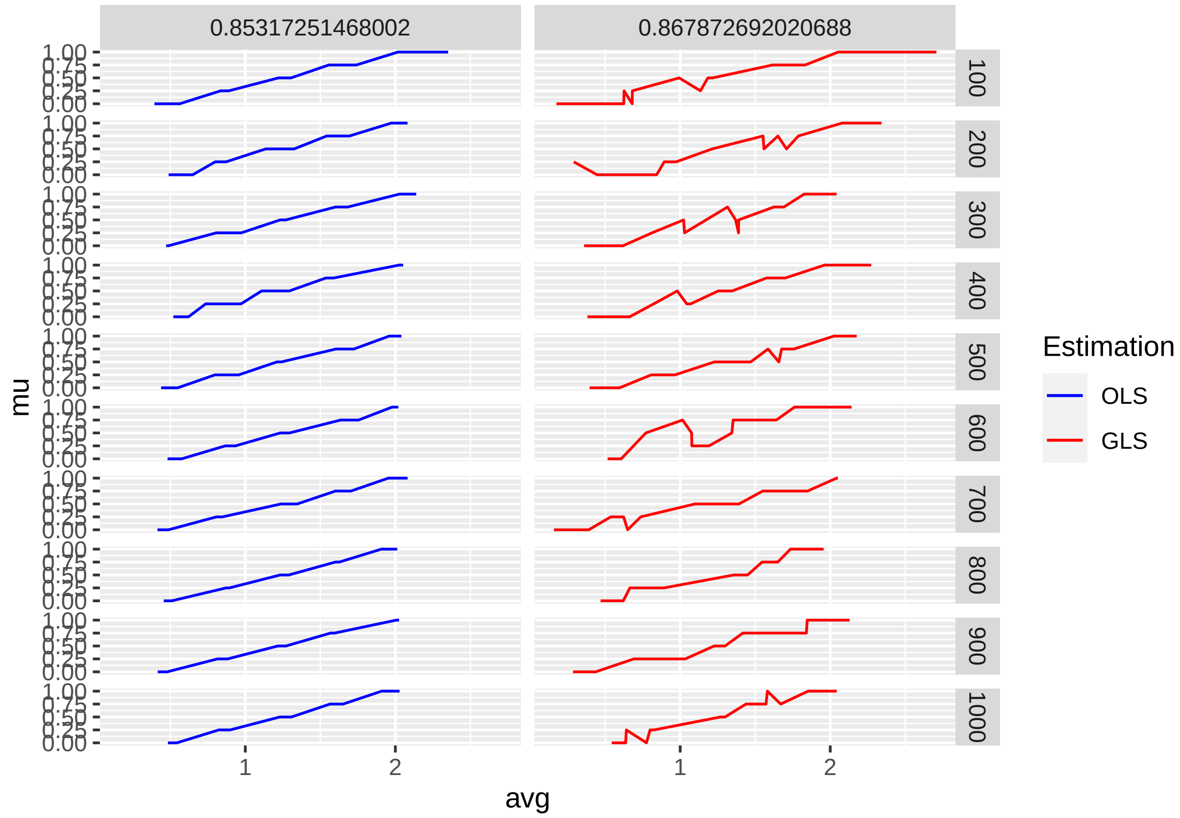
```
## 113  800 0.50 1.5 1.3792957
## 114  800 0.50 2.0 1.3566995
## 115  800 0.75 1.0 1.6500513
## 116  800 0.75 1.5 1.6381768
## 117  800 0.75 2.0 1.5456057
## 118  800 1.00 1.0 1.9559821
## 119  800 1.00 1.5 1.9433300
## 120  800 1.00 2.0 1.7349365
## 121  900 0.00 1.0 0.3115343
## 122  900 0.00 1.5 0.4367016
## 123  900 0.00 2.0 0.2858076
## 124  900 0.25 1.0 0.9442217
## 125  900 0.25 1.5 0.6907896
## 126  900 0.25 2.0 1.0341143
## 127  900 0.50 1.0 1.2262295
## 128  900 0.50 1.5 1.2989843
## 129  900 0.50 2.0 1.2271353
## 130  900 0.75 1.0 1.5904047
## 131  900 0.75 1.5 1.8411995
## 132  900 0.75 2.0 1.4171470
## 133  900 1.00 1.0 2.1290987
## 134  900 1.00 1.5 1.9064864
## 135  900 1.00 2.0 1.8460896
## 136 1000 0.00 1.0 0.5441020
## 137 1000 0.00 1.5 0.7751388
## 138 1000 0.00 2.0 0.6366835
## 139 1000 0.25 1.0 0.7992841
## 140 1000 0.25 1.5 0.8256762
## 141 1000 0.25 2.0 0.6412979
## 142 1000 0.50 1.0 1.2921433
## 143 1000 0.50 1.5 1.2680054
## 144 1000 0.50 2.0 1.2986345
## 145 1000 0.75 1.0 1.6698362
## 146 1000 0.75 1.5 1.5722088
## 147 1000 0.75 2.0 1.4382501
## 148 1000 1.00 1.0 2.0434242
## 149 1000 1.00 1.5 1.8520927
## 150 1000 1.00 2.0 1.5812816
##
## attr(,"class")
## [1] "Eco"
```

As seen as on the summary part, total execution time of the simulation took 36.35 seconds which also proves that parallel process works well.As a reminder, those simulations ran on MacBook Air 10 with total 8 (4 performance and 4 efficiency) cores. Execution times of the simulation might differ on other computers.

Now, lets use the simulation results and visualize them by using ggplot2 methods.Here, additional `MSE`(Mean Square Error) are calculated for each simulation and saved as `out$average$mse`.

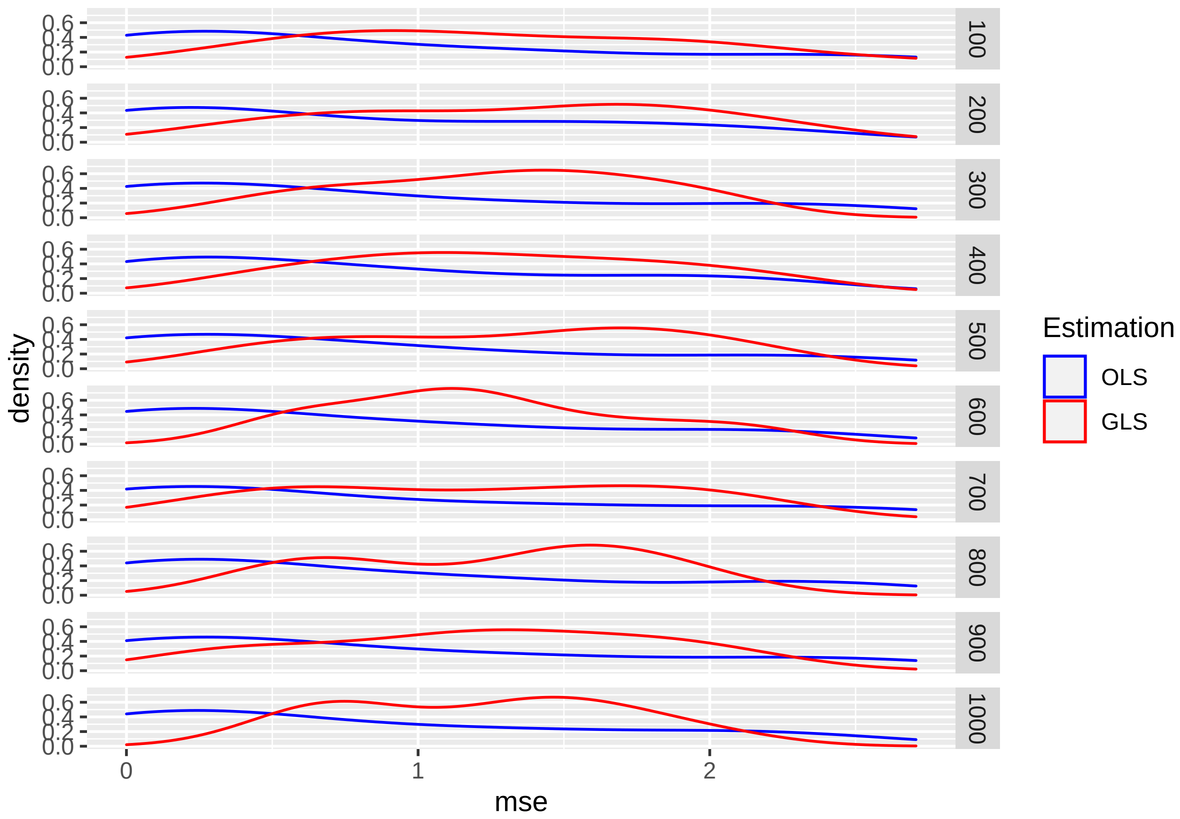
```
gls$average <-  gls$average %>% mutate(mse =(2-avg)^2 )
ols$average <-  ols$average %>% mutate(mse =(2-avg)^2 )

ols$average %>% ggplot(aes(x=avg,y=mu,col="OLS"))+
  facet_grid(n~mean(mse))+geom_line()+
 geom_line(data=gls$average,aes(x=avg,y=mu,col="GLS"))+
  scale_color_manual(name = "Estimation", values = c("OLS" = "blue", "GLS" = "red"))
```



Also density graph of `MSE` for `GLS` and ‘OLS *beta*2 coefficients.

```
ggplot(ols$average,aes(x=mse,col="OLS"))+facet_grid(n~.)+geom_density()+
  geom_density(data=gls$average,aes(x=avg,col="GLS"))+
  scale_color_manual(name = "Estimation", values = c("OLS" = "blue", "GLS" = "red"))
```


density
0.6
0.4
0.2
0.0
100
200
300
400
500
600
700
800
900
1000
0
1
2
mse
Estimation
OLS
GLS

4 Conclusion

The above section illustrates the power of our implemented model and gives the fairly easy to use tool, that still allows for a variety of different specifications in terms of used parameters, data generation processes and summary functions. Researchers, who use Monte Carlo studys on a regular basis, may save a lot of time using a tool like this in the long run.

By nature, there may be cases, where our implementation doesnt satisfy the needs of the user to the fullest, but for a wide variety of examples we showed, that it worked well and served the goal that we aimed for. Our functional programming approach allows for easy and flexible adjustments in case the use of our functions should be expanded, f.e. if a grid of more than 3 (or 4?) parameters is needed.

Theoretically, this work could be implemented as an R package to share it with the R community. But since the `MonteCarlo()` function of the `vigniette` package already provides a well working alternative to our project besides some minor differences, there is currently no need in doing that.

5 References

6 Contributions

Eidesstattliche Versicherung

Ich versichere an Eides statt durch meine Unterschrift, dass ich die vorstehende Arbeit selbständig und ohne fremde Hilfe angefertigt und alle Stellen, die ich wörtlich oder annähernd wörtlich aus Veröffentlichungen entnommen habe, als solche kenntlich gemacht habe, mich auch keiner anderen als der angegebenen Literatur oder sonstiger Hilfsmittel bedient habe. Die Arbeit hat in dieser oder ähnlicher Form noch keiner anderen Prüfungsbehörde vorgelegen.

Essen, den ____________________ ______________________________

Alexander Langnau, Öcal Kaptan, Sunyoung Ji