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This year, TU Delft celebrates its 180th anniversary. To mark the occasion, many different activities are planned to highlight our role in the energy transition, also in the educational programme. In the Python course, we're incorporating the energy transition theme in two of this year's assignments, of which this one is the first.

AE1205 Assignment 3: Aviation CO2 Reduction

Following the Paris Agreement, the aviation sector has adopted a set of short-, medium-, and long-term climate goals. Its medium-term goal is to achieve carbon-neutral growth from 2020 on. The long-term goal is to achieve net zero CO_2 emissions by 2050. In this assignment we'll try to assess how achievable these goals are, based on the different available measures. We'll consider four categories of aircraft:

Range	**Percentage of total passenger-kilometres**
<500 km	9%
500-1500 km	24%
1500-3000 km	19%
>3000 km	48%

We'll assume that the relative contribution of different aircraft categories to the total numer of passenger-kilometers remains constant (i.e., the percentages in the above table don't change). The Eurocontrol aviation outlook 2050 gives an overview different ways how aviation can achieve the objective of net zero by 2050. It identifies four key emission reduction measures:

1. Evolutionary improvements to aircraft and engines, making them more efficient.
2. Revolutionary new aircraft technologies, such as the deployment of electric, hybrid-electric and hydrogen-powered aircraft, together with the required infrastructure.
3. More efficient flights, thanks to improved air traffic management.
4. Increasing production and use of Sustainable Aviation Fuels (SAF).

These measures will, in addition to reducing current emissions, also have to compensate for the aviation growth that is expected to continue. In the assignment, we'll start out with the unimpeded emission growth. In successive steps, we'll add the effects of different emission reduction measures. We'll make our calculations based on the following (quite optimistic) assumptions:

- The relative contribution of different aircraft categories to the total numer of passenger-kilometers remains constant.
- Turbofan engines have a continuing improvement of 1% per year (Evolutionary improvement)
- Short-range new-generation aircraft will be able to fly emissionless (100% reduction, fully electric), this percentage drops for higher distance ranges, where electric flight is no longer feasible (benefits come instead from new configurations, SAF, ...) (Revolutionary improvement)
- The lifetime of a plane is assumed to be 20 years only, worldwide!
- From the year of introduction onwards only new-generation planes will be built, i.e. not a single "old-technology" plane.

- The CO_2 reduction figures for the new generation planes are those for the so-called best case.

Modeling exponential growth

Despite the impact of COVID-19 on aviation, the global traffic volume is still expected to grow several percent, each year. So if the number of aircraft in some reference year is given by N_0, and growth rate r is specified as $r = \frac{\text{growth percentage}}{100}$, the number of aircraft in year t, $N(t)$ can be calculated as:

$$N(t) = N_0(1+r)^t$$

where t is the number of years after the reference year.

In the analysis of exponential growth phenomena, it is common to reformulate such growth equations to *base-e*. This can be done by defining the so-called *e-folding time* τ, which is the time it takes to grow by a factor e, given a growth rate r in a period p (p = one year in our case).

$$\tau = \frac{p}{\ln(1+r)} = \frac{1}{\ln(1+r)}$$

When considering that $e^{\ln(x)} == x$, we can prove that:

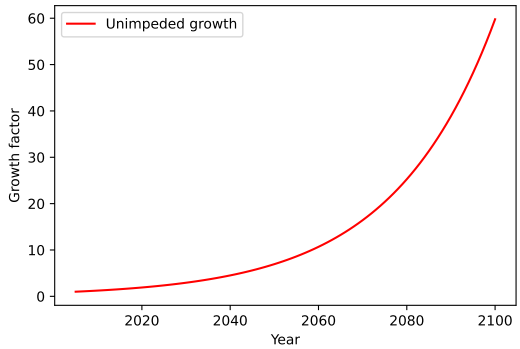
$$(1+r)^t = \left(e^{\ln(1+r)}\right)^t = e^{\ln(1+r)\cdot t}$$

Hence, using the *e-folding time*, we can rewrite our growth equation to *base-e*:

$$N(t) = N_0 \cdot (1+r)^t = N_0 e^{t/\tau}$$

Step 1: Plot the exponential growth function

Let's assume that our reference year is 2005 (so this is where $t = 0$). Calculate and plot the *emission growth factor* ($N(t)/N_0$) from the reference year until the year 2100. Assume a growth rate of 4.4 % ($r = 0.044$). As shown in the Matplotlib example in lecture 4, you can use a `for`-loop to generate lists with x and y values for your plot. The result should look like this:



In the next steps, the growth factor will not be the same anymore for all aircraft categories. You should therefore calculate each growth factor seperately, and add them together afterwards using the relative contribution percentages ($F_{rel,cat}$) for each aircraft category:

$$\begin{aligned} N(t)/N_0 &= F_{rel,<500km} \cdot N_{<500km}(t) + F_{rel,500-1500km} \cdot N_{500-1500km}(t) + \\ &+ F_{rel,1500-3000km} \cdot N_{1500-3000km}(t) + F_{rel,>3000km} \cdot N_{>3000km}(t) \end{aligned}$$

Step 2: Evolutionary improvements

Today's aircraft are much more efficient than the first jet aircraft from the previous century. Much of this improvement has come from the engines of the aircraft. These improvements can be seen as a continuous improvement over time, and are therefore considered here as 'evolutionary improvements'.

As indicated above, we will model these improvements as an annual efficiency improvement of 1% for turbofan aircraft. Assume that this includes all aircraft categories except the shortest range one.

Compared to the previous step, this means that you will have to account for different growth rates for the different aircraft categories. Calculate the individual emission growth factors for the four aircraft categories, and add them together using the relative contribution of each aircraft category (given by the passenger kilometre percentage). Your code is correct if the resulting growth factor in the year 2100 is 27.1819.

Calculating the impact of revolutionary improvements

Revolutionary improvements, such as electrifying aircraft, apply only to newly built aircraft, from the moment that these new-generation aircraft become available. Technologies like electric, hybrid-electric, and hydrogen-powered aircraft are not available yet, but it is expected that they will become available in the coming decades. It will not be possible to make all categories of aircraft emission-free, so the environmental benefits of newly built aircraft will be different for the different categories. The table below gives our assumptions of year of introduction for next-generation aircraft, and their respective emission reduction in percent.

Aircraft category	**Emission reduction**	**Year of introduction**
<500 km	100%	2030
500-1500 km	80%	2030
1500-3000 km	40%	2035
>3000 km	30%	2040

The emission benefits only count for next-generation aircraft, but of course, the whole fleet of aircraft in a given category won't be replaced in one go. Instead, aircraft that reach the end of their lifetime are replaced, and since all newly-built aircraft are assumed to be next-generation from the year of introduction onwards, the category annual growth will also consist completely of next-generation aircraft from that moment.

This means that in our total number of aircraft for a given year, we need to distinguish between old and new-technology aircraft. In the year that next-gen aircraft are introduced, the number of conventional aircraft, $N_{conv}(t)$, will start reducing, as aircraft that reach the end of their lifetime will be decomissioned, and replaced with next-gen aircraft.

When aircraft are replaced every t_L years, the number of *replaced* conventional aircraft in year t, $N_{replace}(t)$, is equal to the number of newly-built aircraft in year $t - t_L$ (i.e., t_L years earlier):

$$N_{replace}(t) = N_{new}(t - t_L)$$

Here you need to consider that the number of newly built aircraft in a year is not just the growth in that year (N_{grow}), but also the number of decomissioned aircraft that were replaced that year ($N_{replace}$):

$$N_{replace}(t) = N_{new}(t - t_L) = N_{grow}(t - t_L) + N_{replace}(t - t_L)$$

Here, $N_{grow}(t - t_L)$ is the actual growth that year, so the number of aircraft added since the year before:

$$\begin{aligned} N_{grow}(t - t_L) &= N(t - t_L) - N(t - t_L - 1) = N_0 \cdot \left(e^{(t-t_L)/\tau} - e^{(t-t_L-1)/\tau}\right) \\ &= N_0 \cdot \left(1 - e^{-1/\tau}\right) \cdot e^{(t-t_L)/\tau} \end{aligned}$$

Note also that the equation for $N_{replace}(t)$ again contains an $N_{replace}$ term, which can be expressed as:

$$N_{replace}(t - t_L) = N_{new}(t - 2t_L) = N_{grow}(t - 2t_L) + N_{replace}(t - 2t_L)$$

This goes on indefinitely, which means that the equation for $N_{replace}(t)$ can be expressed as the following series:

$$\begin{aligned} N_{replace}(t) &= \sum_{i=1}^{\infty} N_{grow}(t - it_L) \\ &= N_0 \cdot (1 - e^{-1/\tau}) \cdot e^{t/\tau} \cdot \sum_{i=1}^{\infty} e^{-i \cdot t_L/\tau} \end{aligned}$$

Starting from the introduction year of next-gen aircraft, $t_{intro,NG}$, the number of of remaining conventional aircraft in year t_{cur}, $N_{conv}(t_{cur})$ can be calculated by subtracting the number of replaced aircraft for each year since $t_{intro,NG}$:

$$N_{conv}(t_{cur}) = N(t_{intro,NG}) \quad - \sum_{t=t_{intro,NG}}^{t_{cur}} N_{replace}(t)$$

This equation is valid between $t = t_{intro,NG}$ and $t = t_{intro,NG} + t_L$ (after t_L years, all conventional aircraft should be replaced).

The number of next-gen aircraft in year t_{cur} is then:

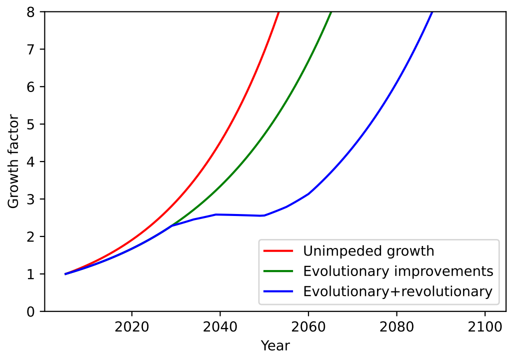
$$N_{NG}(t_{cur}) = N(t_{cur}) - N_{conv}(t_{cur})$$

Step 3: Revolutionary improvements

As described above, for each aircraft category, you need to distinguish between old- and new-technology aircraft starting from the year of introduction of the new-technology aircraft. Implement this through the following steps:

1. Start with the Emission growth calculations from step 2.
2. For each year in your calculations, and for each aircraft category, check if the year exceeds the introduction year of new-technology aircraft for that aircraft type.
3. Starting from this introduction year, start calculating the reduction in numbers of conventional aircraft using the expression with $N_{replace}$ from the previous section. Expand the infinite series from $N_{replace}$ to the fourth order.
4. Calculate the total emission growth for each type as $N_{category}(t)/N_0 = \Big(N_{conv}(t) + (1 - F_{reduction}) \cdot N_{NG}(t)\Big)/N_0$.

The resulting graph should look like this:



As a further check, in year 2050, the values for $N_{new}(t)$ for the four aircraft categories should be:

```
>>>if year == 2050:
       print(Nnew)
[0.6248344258819419, 1.0783840972982002,
 0.7011317165721896, 1.3156487383842765]
```

Step 4: Controlling the variables

As you can see, these already quite optimistic assumptions of emission reduction measures are still not enough to counter the effect of traffic volume growth. Use Python's `input()` function to create a menu that allows the user of your program to specify values for the parameters that affect the effectiveness of the different emission reduction measures (t_L, year of introduction, $F_{reduction}$), and the growth rate r. What values would lead to a halving of the emissions compared to 2005?

*** Optional step 1 ***

Like thinking about ways to reduce the environmental impact of aviation? What other emission-reducing measures could you take into account in your calculation? What would their effect be?

*** Optional step 2 ***

Instead of a plain-text menu with `input()`, you can also create a graphical interface where changing values, sliders, and buttons lead to live changes in your graph. Have a look at Chapter 14 of the reader for tips!