Countdown, Geosodic Expansion, and an Application to Ethical AI

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1 Introduction

The classic *Countdown numbers game* can be viewed as a bounded constraint satisfaction problem (CSP), wherein one attempts to reach a target number T using at most six given numbers with the operations $\{+, -, \times, \div\}$.

In this short paper, we show how the *finite search space* of Countdown naturally embeds into a "meltdown-free" or "geosodic" expansion framework, initially introduced in our companion paper [1], wherein no prior constraints are re-labeled or destroyed ("no meltdown"), and each new "depth" of the tree expands in a carefully structured, pivot-based manner.

Beyond Countdown, we also illustrate how this *geosodic* viewpoint applies to AI ethics decisions: the meltdown-free layering ensures old moral constraints cannot be discarded, yet the system can keep refining its solutions within a bounded or tolerance-driven approach.

2 Preliminaries

2.1 Countdown as a Finite CSP

We briefly recall the *Countdown numbers game*:

- We have six initial numbers, e.g. $n_1, \ldots, n_6$.
- We have a target integer T.
- We may use each number at most once.
- Allowed operations: $\{+, -, \times, \div\}$ (no division by zero).

We can form an expression by combining any two available numbers (or partial expressions) at a time, until at most five operations have been performed. Because there is a finite set of combinations, this is a bounded CSP.

2.2 Geosodic (Meltdown-Free) Expansion

We rely on the meltdown-free tree construction detailed in [1]. In essence:

- At depth $d = 0$, there is one root node.

- Moving from depth d to $d+1$ adds exactly 2^{d+1} nodes: a new pivot (root) plus a perfect right subtree of depth d.
- No node from a previous level is re-labeled or shuffled (no "meltdown").

This yields a perfectly balanced tree at each depth, providing a unique structured form for incremental expansions.

3 Countdown in a Geosodic Subtree: Proof of Concept

Theorem 1 (Countdown Embeds in a Bounded Geosodic Subtree)**.** *Let $n \leq 6$ be the number of initial values and T a target. Then all expressions formed by combining these n values with at most $(n-1)$ operations can be represented in a meltdown-free subtree of depth $d = (n-1)$ of a geosodic expansion.*

Sketch of Proof. For $n = 6$, we have at most five operations. We reserve a geosodic subtree of depth 5. Each node at depth $k \leq 5$ represents a partial expression that has used exactly k operations. The unique meltdown-free expansions guarantee:

- No re-labeling of previous partial expressions.
- Perfectly balanced structure to hold all possible combinations.

Hence, each valid Countdown expression with k steps is assigned to a distinct node in the depth-k layer. By depth 5, we have enumerated all possible outcomes. Full details appear in [1]. □

3.1 Time Complexity and Tolerance Argument

Even though raw CSP search can be exponential, the meltdown-free approach organizes the partial expressions in a layered fashion, preventing excessive redundancy. For well-structured CSPs (like Countdown) or AI decisions with tolerance bounds, this yields a more tractable search in practice.

Definition 2 (Ethical CSP and Tolerance)**.** An *ethical CSP* is one where partial solutions must obey certain fixed moral constraints (no meltdown of previous ethical commitments), and we allow near-solutions if the "cost" is within some tolerance δ.

This parallels the approximate acceptance of near-target expressions in Countdown. As the geosodic structure grows, deeper expansions systematically refine solutions without discarding earlier valid constraints.

4 Conclusion and Future Work

We have shown that the finite search space of Countdown naturally maps into a meltdown-free (geosodic) expansion, ensuring no contradictory re-labeling of partial expressions. This same approach extends to *ethical CSPs* in AI, where tolerances guide incremental moral refinements without discarding established constraints.

Future Work. We foresee further applications of meltdown-free structures to other bounded decision problems, and deeper explorations of moral consistency in AI. In all such settings, the pivot-plus-perfect-subtree pattern provides a canonical way to grow solutions in stable, no-meltdown steps.

References

[1] Your Name and Others. The geosodic tree: Canonical meltdown-free expansions bridging discrete and continuous. arXiv e-prints, 2025. Preprint, available at https://arxiv.org/abs/9999.99999.