ME760 Engineering Analysis I

Homework Set 4 due: Monday, Oct. 19, 2020

1. Estimate an upper bound for the spectral radius of the following matrix. Compare this bound to the actual spectral radius. Show your work.

$$\mathbf{A} = \begin{pmatrix} 7 & 0 & 3 \\ 2 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

2. Given the curve C: $\mathbf{r}(u) = \mathbf{i}\cos u + \mathbf{j}2\sin u$, find

   (a) a tangent vector $\mathbf{r}'(u)$ and the corresponding unit vector $\hat{\mathbf{r}}'(u)$,

   (b) $\mathbf{r}'$ and $\hat{\mathbf{r}}'$ at the point P: $(1/2, \sqrt{3}, 0)$, and

   (c) the equation of the line through P that is tangent to the curve. Sketch the curve and the tangent.

3. Find the length of the circular helix $\mathbf{r}(u) = \mathbf{i}a\cos u + \mathbf{j}a\sin u + \mathbf{k}u$ from $(a, 0, 0)$ to $(a, 0, 2\pi)$.

4. Sketch $\mathbf{r}(t) = \mathbf{i}(R\sin\omega t + \omega Rt) + \mathbf{j}(R\cos\omega t + R)$ taking $R = 1$ and $\omega = 1$. This curve is called a cycloid and is the path of a point on the rim of a wheel of radius R that rolls without slipping along the x-axis. Find the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ at the maximum and minimum y-values of the curve.

5. The flow of heat in a temperature field takes place in the direction of the maximum decrease of temperature. For the temperature field $T(x, y, z) = z/(x^2 + y^2)$ find this direction and magnitude of the heat flow in general and explicitly at the point $(0, 1, 2)$.

6. Find the unit normal (a) to the surface $ax + by + cz + d = 0$ at any point P, and (b) to the surface $x^2 + y^2 + z^2 = 26$ at the point $(1, 4, 3)$.

7. Find the divergence of $(-\mathbf{i}y + \mathbf{j}x)/(x^2 + y^2)$.

8. Prove that $\nabla \cdot (\nabla \times \mathbf{v}) = 0$.