

ME760 Engineering Analysis I

Homework Set 6

due: Monday Nov. 16, 2020

1. Under what conditions is $(ax + by)dx + (kx + ly)dy$ an exact differential? Here a , b , k and l are constants. Solve the exact equation.
2. Show that $x^a y^b$ is an integrating factor for the following equation and then find its solution.

$$(b+1)x \frac{dy}{dx} + (a+1)y = 0.$$

3. Find the solution of (a) $y'(x) = x(4 - y)$. (b) Find the solution of $y'(x) + 6x^2y = e^{-2x^3}/x^2$ subject to the condition $y(1) = 0$.
4. Show if the follow two sets of functions are linearly independent on the positive x -axis: (a) $[\cos x, \sin x, \sin 2x]$ and (b) $[e^x \cos x, e^x \sin x, e^x]$
5. Solve the o.d.e. $y'' + 4y' + 3y = 65 \cos 2x$ by (a) the method of undetermined coefficients, and (b) by the variation of parameters method.
6. What is the solution to the initial value problem $d^4y/dx^4 - y = 0$ with $y(0) = -1$, $y'(0) = 7$, $y''(0) = -1$, and $y'''(0) = 7$.
7. Consider the following pair of coupled equations.

$$\mathbf{y}'(t) = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \mathbf{y}(t) + \begin{pmatrix} -t^2 + 6t \\ -t^2 + t - 1 \end{pmatrix}.$$

Find the general solution $\mathbf{y}(t)$ by the methods of (a) undetermined coefficients, (b) variation of parameters, and (c) diagonalization with a similarity transformation.