ME760 Engineering Analysis I

Homework Set 1 due: Wed. Sept. 9, 2020

1. For $\mathbf{a} \neq \mathbf{0}$, what does $\mathbf{a}\bullet\mathbf{b} = \mathbf{a}\bullet\mathbf{c}$ imply about the vectors $\mathbf{b}$ and $\mathbf{c}$? Give a geometric interpretation. Hint: if $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ all begin at the same point, where do the ends of the vectors $\mathbf{b}$ and $\mathbf{c}$ lie?

2. In practical work the following formula is often quite useful.
$$|\mathbf{a} \times \mathbf{b}| = \sqrt{(\mathbf{a}\bullet\mathbf{a})(\mathbf{b}\bullet\mathbf{b}) - (\mathbf{a}\bullet\mathbf{b})^{\mathbf{2}}}$$
Give a proof.

3. Determine the angles in the triangle formed by the three vertices $P_1 = (2, 2, 2)$, $P_2 = (3, 1, 1)$ and $P_3 = (3, 3, 3)$.

4. What is the equation of the plane containing the triangle of the previous problem?

5. Show that the three vectors $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ are coplanar.

6. For general vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ show by expanding in component form
$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (\mathbf{a}\bullet\mathbf{c})\mathbf{b} - (\mathbf{b}\bullet\mathbf{c})\mathbf{a}$$
From this result deduce that the vector product is non-associative.

7. Line L passes through the two points $P_1 = (-2, -2, -2)$ and $P_2 = (-1, -1, -1)$.
   (a) What is the equation of this line?
   (b) What are the coordinates of the two points of intersection of this line with a sphere of radius 2 centered at $(2, 2, 2)$?

8. Two fixed points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$.
   (a) Identify the plane P given by $(\mathbf{a} - \mathbf{b})\bullet\mathbf{r} = (a^2 - b^2)/2$ where a and b are the magnitudes of $\mathbf{a}$ and $\mathbf{b}$.
   (b) Show that the equation
   $$(\mathbf{a} - \mathbf{r})\bullet(\mathbf{b} - \mathbf{r}) = 0$$
   describes a sphere S of radius $R = |\mathbf{a} - \mathbf{b}|/2$. Hint: cast this equation into the standard form $(\mathbf{r} - \mathbf{c})\bullet(\mathbf{r} - \mathbf{c}) = R^2$ where you should find $\mathbf{c} = (\mathbf{a} + \mathbf{b})/2$.
   (c) Show that the intersection of P and S is also the intersection of two spheres centered on A and B each of radius $|\mathbf{a} - \mathbf{b}|/\sqrt{2}$. Hint: Add and subtract the equations for P and S and cast the results into the standard form $|\mathbf{r} - \mathbf{d}|^2 = R^2$.