

ME760 Engineering Analysis I

Homework Set 5

due: Monday Nov. 2, 2020

1. Verify that the contour integral $\int_C [2xy^2 dx + 2x^2y dy + dz]$ is independent of the path. Evaluate this integral between the points $(0, 0, 0)$ and (a, b, c) .
2. Given the parametric form of a cone $\mathbf{r}(u, v) = [u \cos v, u \sin v, cu]$ (a) find an explicit representation of the form $z = f(x, y)$, (b) find and identify the *parameter curves* defined as $u = \text{const}$ and $v = \text{const}$, and (c) find a normal vector \mathbf{N} to the conical surface.
3. In class we discussed surface integrals without regard to orientation. By reparameterizing the surface integral could be written as

$$I = \iint_S G(\mathbf{r}) dS = \iint_R G(r(u, v)) |\mathbf{N}(u, v)| du dv$$

(a) Consider the case with $G = z$ and the surface S is the hemisphere $x^2 + y^2 + z^2 = 9$ with $z \geq 0$. Use polar coordinates and evaluate the right-hand side of the above result. (b) The surface S is also given explicitly by $z = f(x, y) = \sqrt{9 - x^2 - y^2}$. For such cases the surface integral can be rewritten as

$$\iint_S G(\mathbf{r}) dA = \iint_{R^*} G(x, y, f(x, y)) \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dx dy.$$

Evaluate the right-hand side of this result.

4. Evaluate $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}} dA$ using the divergence (Gauss') theorem when (a) $\mathbf{F} = [x^3, y^3, z^3]$ and the surface S is the sphere $x^2 + y^2 + z^2 = 9$, and (b) when $\mathbf{F} = [9x, y \cosh^2 x, -z \sinh^2 x]$ and S is the ellipsoid $4x^2 + y^2 + 9z^2 = 36$.
5. Consider the vector function $\mathbf{F} = [e^z, e^z \sin y, e^z \cos y]$ and the surface $S : z = y^2, 0 \leq x \leq 4, 0 \leq y \leq 2$. Stoke's theorem states that

$$\iint_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}} dS = \oint_C \mathbf{F} \cdot d\mathbf{r}.$$

- (a) Evaluate the left-hand side of this result, and (b) evaluate the right-hand side.