**Question 1: Assigned Seating for Harvard Law Students**

Each of the 100 students is assigned a random seat for each of the two courses, independently of their first assignment.

**(a) Exact probability that no one has the same seat for both courses**

For each student, there are 100 seats, and they are assigned a seat for Torts randomly. For Contracts, they must not be assigned the same seat.

* The total number of ways to assign seats for Contracts is 100!100!100!.
* The number of ways to assign students to different seats in Contracts (a derangement) is given by the subfactorial:

!100=100!∑k=0100(−1)kk!!100 = 100! \sum\_{k=0}^{100} \frac{(-1)^k}{k!}!100=100!k=0∑100​k!(−1)k​

* The probability is:

P(no one has the same seat)=!100100!=∑k=0100(−1)kk!P(\text{no one has the same seat}) = \frac{!100}{100!} = \sum\_{k=0}^{100} \frac{(-1)^k}{k!}P(no one has the same seat)=100!!100​=k=0∑100​k!(−1)k​

**(b) Approximate probability that no one has the same seat**

It is well known that the probability of a random permutation being a derangement (no fixed points) approaches:

!nn!≈1e\frac{!n}{n!} \approx \frac{1}{e}n!!n​≈e1​

for large nnn. So, for n=100n = 100n=100:

P(no one has the same seat)≈1e≈0.3679P(\text{no one has the same seat}) \approx \frac{1}{e} \approx 0.3679P(no one has the same seat)≈e1​≈0.3679

**(c) Approximate probability that at least two students have the same seat**

Since the probability that no one has the same seat is 1e\frac{1}{e}e1​, the probability that at least one person does have the same seat is:

1−1e≈1−0.3679=0.63211 - \frac{1}{e} \approx 1 - 0.3679 = 0.63211−e1​≈1−0.3679=0.6321

**Question 2: Airplane Seating Problem**

We have 100 passengers and 100 assigned seats. The first passenger picks a seat randomly. Each subsequent passenger sits in their assigned seat if available or takes a random available seat otherwise.

**Solution:**

The key observation is that the last passenger's fate depends only on whether the first passenger took their seat.

* If the first passenger sits in seat 100, then the last passenger gets their assigned seat.
* If the first passenger sits in a different seat kkk, then at some point a passenger assigned to seat kkk will have to make a random choice among the remaining seats. The last passenger's seat is equally likely to be any of the remaining ones.

This problem has a famous result: The probability that the last passenger gets their assigned seat is **exactly**:

12\frac{1}{2}21​

This result is independent of nnn and follows from symmetry arguments in recursive probability calculations.