Tutorial - 4 :-

1. $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a = 3 \quad , b = 2$

$c = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$f(n) = n^2.$

By case 3 :- $f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a = 4, \; b = 2.$

$c = \log_2 4 = 2.$

$n^c = n^2$

$f(n) = n^2.$

By Case 2 : $f(n) = n^c$

$T(n) = \Theta(n^c \log n) = \Theta(n^2 \log n).$

3. $T(n) = T(n/2) + 2^n$

$T(n) = aT(n/b) + f(n)$

$a = 1, \; b = 2$

$c = \log_2 1 = 0$

$n^c = n^0 = 1; \; f(n) = 2^n. \quad ; \quad f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(2^n).$

4. $T(n) = 2^n T(n/2) + n^n$

$T(n) = aT(n/b) + f(n)$

$a = 2^n$, $b = 2$.

$c = \log_2 2^n = n$

$n^c = n^n$, $f(n) = n^n$

By Case : $f(n) = n^c$

$T(n) = \Theta(n^c \log n) \Rightarrow \Theta(n^n \log n)$

5. $T(n) = 16T(n/4) + n$

$T(n) = aT(n/b) + f(n)$

$a = 16$, $b = 4$

$c = \log_4 16 = 2$

$n^c = n^2 \Rightarrow f(n) = n^2$

By Case : $f(n) < n^c$

$T(n) = \Theta(n^c) \Rightarrow T(n) = \Theta(n^2)$.

6. $T(n) = 2T(n/2) + n\log n$

$T(n) = aT(n/b) + f(n)$

$a = 2$, $b = 2$.

$c = \log_2 2 = 1$

$n^c = n$, $f(n) = n\log n$

By Case : $f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n\log n)$.

7. $T(n) = 2T(n/2) + n/\log n$

$T(n) = aT(n/b) + f(n)$

$a = 2, \; b = 2$

$c = \log_2 2 = 1$

$n^c = n^1, \; f(n) = n/\log n$

By Case :- $n^c > f(n)$

$T(n) = \theta(n^c)$

$T(n) = \theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$T(n) = aT(n/b) + f(n)$

$a = 2, \; b = 4$

$c = \log_4 2 = 0.5$

$n^c = n^{0.5}, \; f(n) = n^{0.51}$

By Case :- $f(n) > n^c$

$T(n) = \theta(f(n))$

$T(n) = \theta(n^{0.51})$

9. $T(n) = 0.5T(n/2) + 1/n.$

$T(n) = aT(n/b) + f(n)$

$a = 0.5, \; b = 2$

$c = \log_2 0.5 \Rightarrow -1$

$n^c = n^{-1} \Rightarrow 1/n \Rightarrow f(n) = 1/n$

By Case :- $f(n) = n^c$

$T(n) = \theta(\log n/n).$

10) $T(n) = 16T(n/4) + n!$

$T(n) = aT(n/b) + f(n)$

$a = 16, \ b = 4$

$c = \log_4 16 = 2$

$n^c = n^2, \ f(n) = n!$

By Case :- $f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n!)$

11) $T(n) = 4T(n/2) + \log n$

$T(n) = aT(n/b) + f(n)$

$a = 4, \ b = 2$

$c = \log_2 4 = 2.$

$n^c = n^2 ; \ f(n) = \log n$

By Case :- $f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^2)$

12) $T(n) = \text{sqrt}(n)\, T(n/2) + \log n.$

$T(n) = aT(n/b) + f(n).$

$a = n^{1/2}, \ b = 2., \ f(n) = \log n$

$c = \log_2 n^{1/2} = 1/2 \log n$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^c).$

13. $T(n) = 3T(n/2) + n$

$T(n) = aT(n/b) + f(n)$

$a = 3, \quad b = 2$

$c = \log_2 3 = 1.58$

$n^c = n^{1.58}, \quad f(n) = n$

By Case : $f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^{1.58})$

14. $T(n) = 3T(n/3) + sqrt(n).$

$T(n) = aT(n/b) + f(n)$

$a = 3, \quad b = 3$

$c = \log_3 3 = 1$

$n^c = n^1, \quad f(n) < n^{1/2}.$

By Case : $f(n) < n^c$

$T(n) = \Theta(n).$

15. $T(n) = 4T(n/2) + cn$

$T(n) = aT(n/b) + f(n)$

$a = 4, \quad b = 2$

$c = \log_2 4 = 2.$

$n^c = n^2, \quad f(n) = cn = n$

By Case :

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^2)$

16. $T(n) = 3T(n/4) + n\log n$

$T(n) = aT(n/b) + f(n)$

$a = 3, \quad b = 4$

$c = \log_4 3 = 0.79$

$n^c = n^{0.79}, \quad f(n) = n\log n$

By Case :- $f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n\log n)$

17. $T(n) = 3T(n/3) + n/2$

$T(n) = aT(n/b) + f(n)$

$a = 3, \quad b = 3$

$c = \log_3 3 = 1$

$n^c = n^1, \quad f(n) = n/2$

By case :- $f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n).$

18. $T(n) = 6T(n/3) + n^2\log n$

$T(n) = aT(n/b) + f(n)$

$a = 6, \quad b = 3$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63}, \quad f(n) = n^2\log n$

By Case :- $f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n^2\log n).$

19. $T(n) = 4T(n/2) + n/\log n$

$T(n) = aT(n/b) + f(n)$

$a = 4, b = 2, c = \log_2 4 = 2.$

$n^c = n^2, \quad f(n) = n/\log n$

By case :- $f(n) < n^c$

$T(n) = \theta(n^c) = \theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

$T(n) = aT(n/b) + f(n)$

$a = 64, b = 8, c = \log_8 64 = 2$

$n^c = n^2, \quad f(n) = -n^2 \log n = n^2 \log n^{-1}$

$\Rightarrow n^2 \log 1/n$

By Case :- $f(n) > n^c$

$T(n) = \theta(f(n)) = \theta(n^2 \log 1/n)$

21. $T(n) = 7T(n/3) + n^2$

$T(n) = aT(n/b) + f(n)$

$a = 7, \quad b = 3$

$c = \log_3 7 = 1.77$

$n^c = n^{1.77}, \quad f(n) = n^2$

By Case :- $f(n) > n^c$

$T(n) = \theta(f(n)) = \theta(n^2).$