→ Tutorial 6

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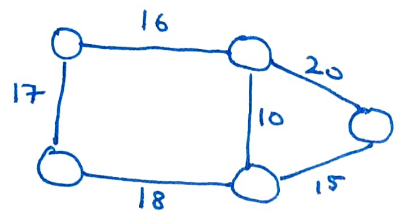
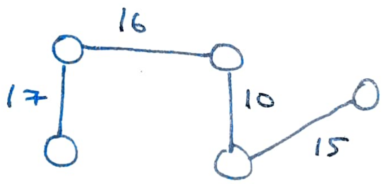
Q1. Minimum Spanning Tree

→ A spanning tree of a undirected graph is a subgraph that is a tree and joined all vertices. One of those tree which has minimum total cost would be its minimum spanning tree.

For eg →



for the above connected, undirected graph minimum cost spanning tree would be.



→ Application of MST

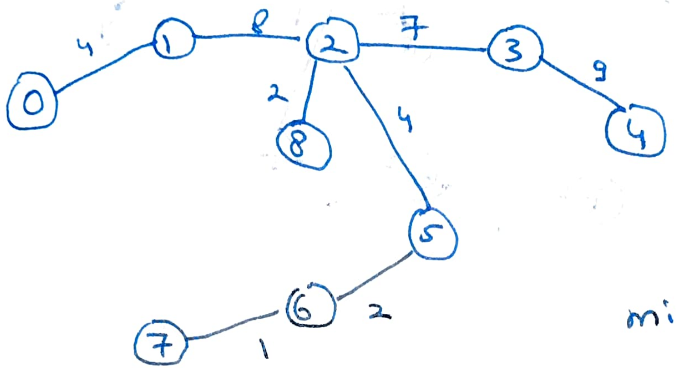
→ minimum spanning Trees have direct applications is the design of network including computer networks, telecommunication network, transportation networks, etc.

Q2.

	Prim's Algorithm	Kruskal's Algorithm.	Dijkstra's Algorithm	Bellman Ford Algo.
T.C	$O(V^2)$	$O(E\log V)$	$O(V+E\log V)$	$O(VE)$
S.C.	$O(V+E)$	$O(\lvert E\rvert+\lvert V\rvert)$	$O(V^2)$	$O(V^2)$

Q.30 Prim's Algo :

0	1	2	3	4	5	6	7	8
$\boxed{0}$	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~
	$\boxed{4}$	$\boxed{8}$	$\boxed{7}$		$\boxed{4}$	~~6~~	8	$\boxed{2}$
				~~10~~		$\boxed{2}$	7	
				$\boxed{9}$			$\boxed{1}$	



min weight = 37

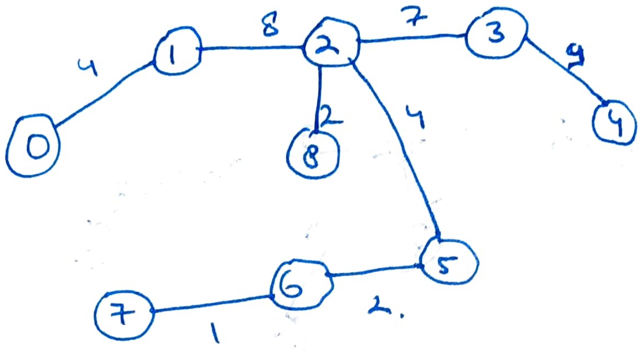
Parent	0	1	2	3	4	5	6	7	8
	-1	~~-1~~	~~-1~~	~~-1~~	-1	~~-1~~	~~-1~~	~~-1~~	~~-1~~
		0	1	2		2		~~0~~	2.
							~~8~~	~~8~~	
					~~8~~		5	6	
					3				

Parent	0	1	2	3	4	5	6	7	8
	-1	0	1	2	3	2	5	6	~~7~~2.

Ans.

Q.3. Kruskal's Algorithm :-

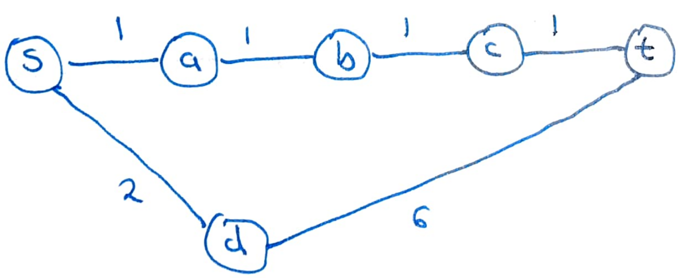
u	v	w	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	✗
7	8	7	✗
2	3	7	✓
1	2	8	✓
0	7	8	✗
3	4	9	✓
5	4	10	✗
1	7	11	✗
3	5	14.	✗



min. weight = 37

Q.4. (i) If 10 units is added to each edge, the overall weight of the path may change.

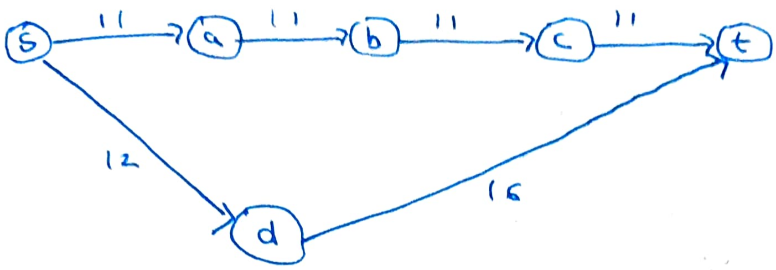
For eg →



shortest path is :- s → a → b → c → t.

weight → 1 + 1 + 1 + 1 = 4.

Now, if 10 unit weight is added to each edge.

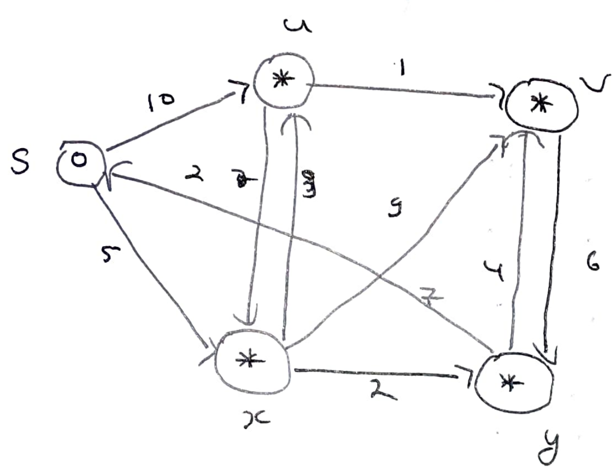


shortest path changed to $s \to d \to t$

weight $\Rightarrow$ 28.

(ii) multiplying the weight of each edge by 10 will have no impact on the shortest path.

Q5 Dijkstra's Algo

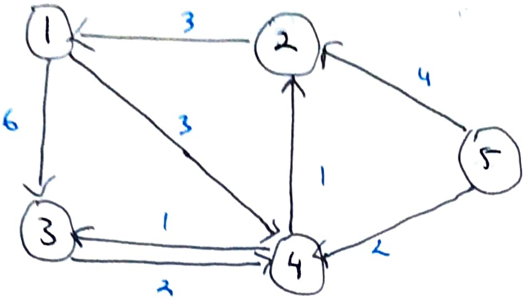


Queue :- ~~s~~ ~~u~~ ~~v~~ ~~x~~ y

Visited :- s u v x y.

s	u	v	x	y
0	$\cancel{\infty}$	∞	$\cancel{\infty}$	$\cancel{\infty}$
0	10	$\cancel{\infty}$	5	∞
0	10	11	5	$\cancel{\infty}$
0	10	11	5	7

Q6. All pair shortest path algorithm → Floyd Marshall.



$$A^0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & \infty & \infty & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$A^1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$A^0[2,3] = \infty$$

$$A^0[2,1] + A^0[1,3] = 3 + 6 = 9.$$

$$9 < \infty$$

Similarly,

$$A^0[2,4] = \infty$$

$$A^0[2,1] + A^0[1,4] = 3 + 3 = 6$$

$$6 < \infty$$

$A^0[2,5] = \infty$

$A^0[2,1] + A^0[1,5] = 3 + \infty$

$$A^2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \boxed{\infty} & \boxed{6} & 3 & \infty \\ 2 & 3 & 0 & \boxed{9} & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & \boxed{1} & 0 & \infty \\ 5 & 7 & 4 & 13 & 2 & 0 \end{array}$$

$A'[1,3] = 6$

$A'[1,2] + A'[2+3] = \infty + 9$

$6 < \infty + 9$

$$A^3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & \boxed{3} & \infty \\ 2 & 3 & 0 & 9 & \boxed{6} & \infty \\ 3 & \infty & \infty & 0 & \boxed{2} & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & 7 & 4 & 13 & \boxed{2} & 0 \end{array}$$

$$A^4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 5 & \infty \\ 2 & 3 & 0 & 7 & 6 & \infty \\ 3 & \infty & 3 & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & 7 & 3 & 3 & 2 & 0 \end{array}$$

$$A^5 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & \infty \\ 2 & 3 & 0 & 7 & 6 & \infty \\ 3 & \infty & 3 & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & 7 & 3 & 3 & 2 & 0 \end{array}$$