CSE 130: Programming Languages

Environments & Closures

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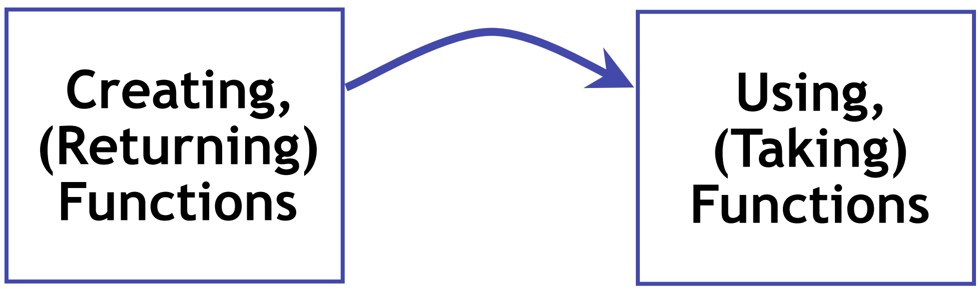
Recap: Functions as “first-class” values

- Arguments, return values, bindings …
- What are the benefits ?

Parameterized, similar functions (e.g. Testers)



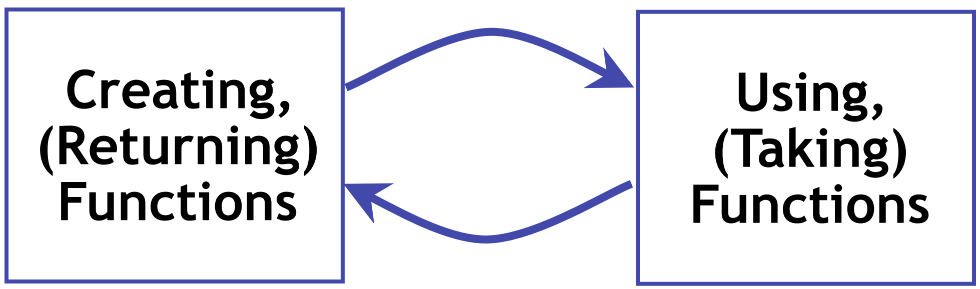
Iterator, Accumul, Reuse computation pattern w/o exposing local info

Functions are "first-class" values

- Arguments, return values, bindings …
- What are the benefits ?

Parameterized,
similar functions
(e.g. Testers)

Iterator, Accumul,
Reuse computation
pattern w/o
exposing local info



Compose Functions:
Flexible way to build
Complex functions
from primitives.

Funcs taking/returning funcs

Higher-order funcs enable modular code

- Each part only needs local information

Data Structure Client
Uses `list`

Data Structure Library
`list`

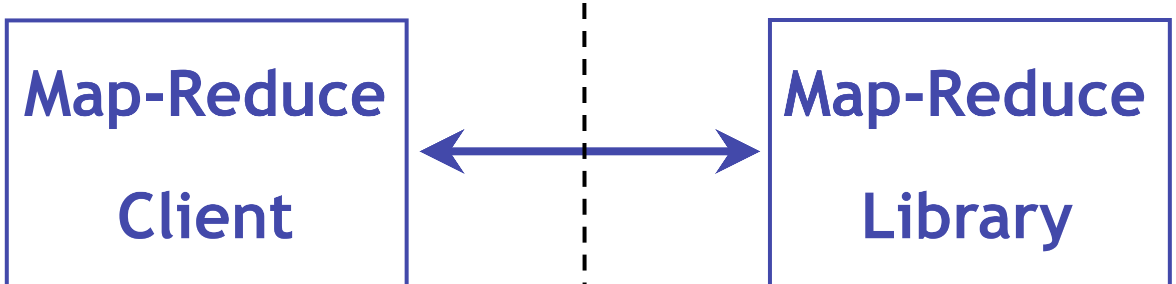
Uses meta-functions:
`map`,`fold`,`filter`
With locally-dependent funs
`(lt h)`, `square` etc.
Without requiring Implement. details of data structure

Provides meta-functions:
`map`,`fold`,`filter`
to traverse, accumulate over lists, trees etc.
Meta-functions don't need client info (tester ? accumulator ?)

“Map-Reduce” et al.

Higher-order funcs enable modular code

- Each part only needs local information



Web Analytics “Queries”
Clustering, Page Rank, etc
as map/reduce + **ops**

Provides: **map, reduce**
to traverse, accumulate
over WWW (“Big Data”)
Distributed across “cloud”

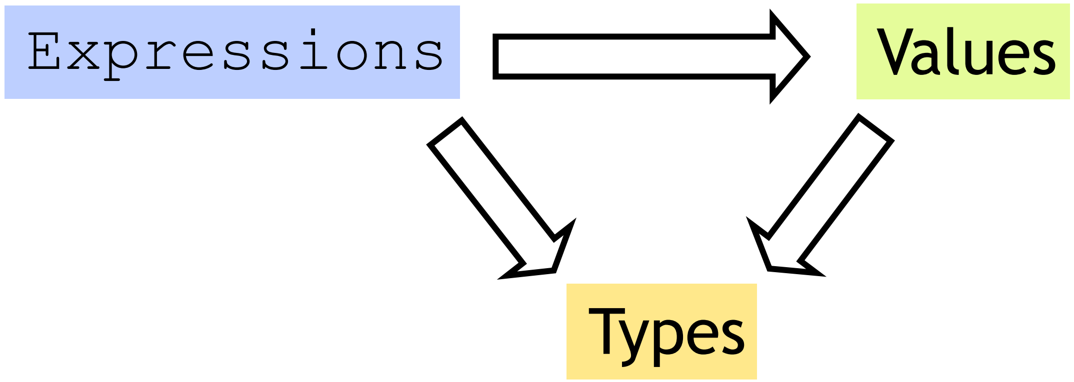
Higher Order Functions

Are Awesome…

Higher Order Functions

..but how do they work

Next: Environments & Functions



Lets start with the humble variable…

Variables and Bindings

Q: How to use variables in ML ?

Q: How to "assign" to a variable ?

```
# let x = 2+2;;
val x : int = 4
```

```
let x = e;;
```

"Bind value of expr *e* to variable x"

Variables and Bindings

```
# let x = 2+2;;
val x : int = 4
# let y = x * x * x;;
val y : int = 64
# let z = [x;y;x+y];;
val z : int list = [4;64;68]
```

Later expressions can **use** `x`

- **Most recent** “bound” value used for evaluation

Sounds like C/Java ?

NO!

Environments (“Phone Book”)

How ML deals with variables

- Variables = “names”
- Values = “phone number”

…	…
x	*4 : int*
y	*64 : int*
z	*[4;64;68] : int list*
x	*8 : int*

Environments and Evaluation

ML begins in a "top-level" environment

- Some names bound (e.g. +,-, print_string...)

```
let x = e;;
```

ML program = Sequence of variable bindings

Program evaluated by evaluating bindings in order

1. Evaluate expr **e** in current env to get value *v* : *t*
2. Extend env to bind **x** to *v* : *t*

(Repeat with next binding)

Environments

"Phone book"

- Variables = "names"
- Values = "phone number"

1. Evaluate:

Find and use most recent value of variable

2. Extend:

Add new binding at end of "phone book"

Example

```
# let x = 2+2;;
val x : int = 4

# let y = x * x * x;;
val y : int = 64

# let z = [x;y;x+y];;
val z : int list = [4;64;68]


# let x = x + x ;;
val x : int = 8
```

…	…

…	…
x	4 : int

…	…
x	4 : int
y	64 : int

…	…
x	4 : int
y	64 : int
z	[4;64;68] : int list

…	…
x	4 : int
y	64 : int
z	[4;64;68] : int list
x	8 : int

New binding!

Environments

1. Evaluate: Use most recent bound value of var
2. Extend: Add new binding at end

How is it different from C/Java's "store" ?

```
# let x = 2+2;;
val x : int = 4

# let f = fun y -> x + y;
val f : int -> int = fn

# let x = x + x ;
val x : int = 8

# f 0;
val it : int = 4
```

...	...
x	*4 : int*

...	...
x	*4 : int*
f	fn <*code*, >: *int->int*

New binding:

- No change or mutation
- Old binding frozen in **f**

Environments

1. Evaluate: Use most recent bound value of var
2. Extend: Add new binding at end

How is it different from C/Java's "store" ?

```
# let x = 2+2;
val : int x = 4

# let f = fun y -> x + y;
val f : int -> int = fn

# let x = x + x ;
val x : int = 8;

# f 0;
val it : int = 4
```

…	…
x	*4 : int*

…	…
x	*4 : int*
f	fn <*code*, >: *int->int*

…	…
x	*4 : int*
f	fn <*code*, >: *int->int*
x	*8 : int*

Environments

1. Evaluate: Use most recent bound value of var
2. Extend: Add new binding at end

How is it different from C/Java's "store" ?

```
# let x = 2+2;
val x : int = 4

# let f = fun y -> x + y;;
val f : int -> int = fn

# let x = x + x ;
val x : int = 8

# f 0;
val it : int = 4
```

Binding used to eval **(f …)**

…	…
x	*4 : int*
f	fn <*code*, >: *int->int*
x	*8 : int*

Binding for subsequent **x**

Cannot change the world

Cannot “assign” to variables

- Can extend the env by adding a fresh binding
- Does not affect previous uses of variable

Environment at fun declaration **frozen inside fun** “value”

- Frozen env used to evaluate application `(f e)`

Q: Why is this a good thing ?

```
# let x = 2+2;;
val x : int = 4
# let f = fun y -> x + y;;
val f : int -> int = fn
# let x = x + x ;;
val x : int = 8;
# f 0;;
val it : int = 4
```

Binding used to eval `(f …)`

…	…
x	*4 : int*
f	fn <*code*, >: *int->int*
x	*8 : int*

Binding for subsequent **`x`**

Cannot change the world

Q: Why is this a good thing ?

A: Function behavior frozen at declaration

Immutability: The Colbert Principle

"A function behaves the same way on Wednesday, as it behaved on Monday, *no matter what happened on Tuesday!*"

Cannot change the world

Q: Why is this a good thing ?

A: Function behavior frozen at declaration

- Nothing entered afterwards affects function
- Same inputs always produce same outputs
  - Localizes debugging
  - Localizes reasoning about the program
  - No “sharing” means no evil aliasing

Examples of no sharing

Remember: No addresses, no sharing.

- Each variable is bound to a "fresh instance" of a value

Tuples, Lists …

- Efficient implementation without sharing ?
  - There is sharing and pointers but hidden from you

- Compiler's job is to optimize code
  - Efficiently implement these "no-sharing" semantics

- Your job is to use the simplified semantics
  - Write correct, cleaner, readable, extendable systems

Q: What is the value of res ?

```
let f = fun x -> 1;;
let f = fun x -> if x<2 then 1 else (x * f(x-1));;
let res = f 5;;
```

(a) 120

(b) 60

(c) 20

(d) 5

(e) 1

Function bindings

Functions are values, can bind using **val**

```
let fname = fun x -> e ;;
```

Problem: Can't define recursive functions !

- `fname` is bound after computing rhs value
- no (or "old") binding for occurences of `fname` inside ***e***

```
let rec fname x = e ;;
```

Occurences of `fname` inside ***e*** bound to "this" definition

```
let rec fac x = if x<=1 then 1 else x*fac (x-1)
```

Q: What is the value of res ?

```
let y = let x = 10 in
          x + x ;;

let res = (x, y);;
```

(a) Unbound Var Error

(b) `(10,20)`

(c) `(10,10)`

(d) Type Error

Local bindings

So far: bindings that remain until a re-binding ("global")

Local, "temporary" variables are useful inside functions

- Avoid repeating computations
- Make functions more readable

```
let x = e1 in
   e2
;;
```

Let-in is an expression!

Evaluating let-in in env ***E***:

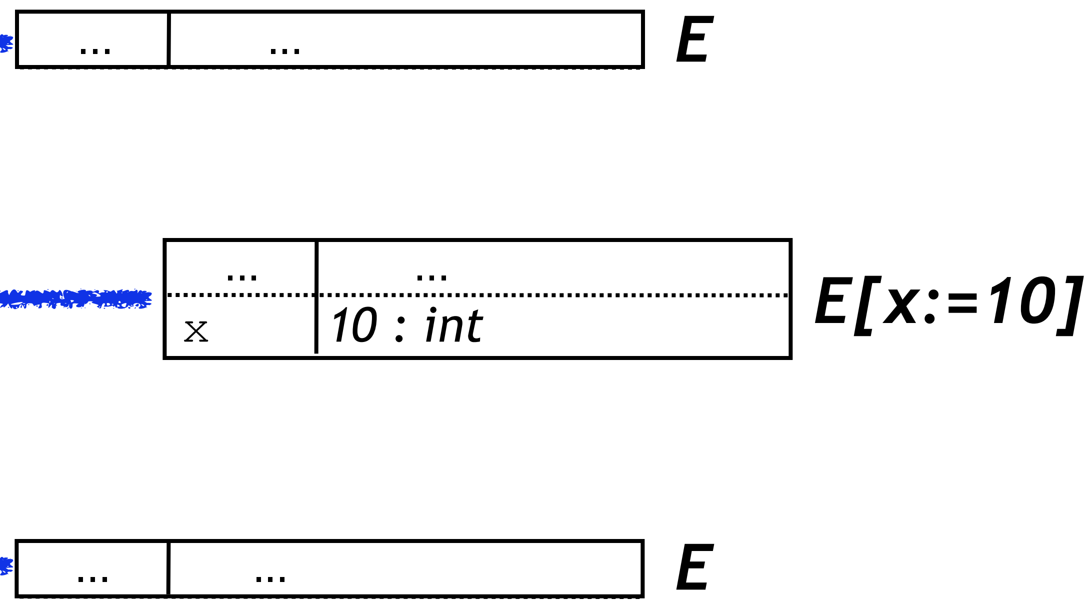
1. Evaluate expr ***e1*** in env ***E*** to get value ***v*** : ***t***
2. Use extended ***E*** [**x** |-> ***v*** : ***t***] (only) to evaluate ***e2***

Local bindings

Evaluating let-in in env ***E***:

1. Evaluate expr ***e1*** in env ***E*** to get value ***v*** : ***t***
2. Use extended ***E*** [**x** |-> ***v*** : ***t***] to evaluate ***e2***

```
let
    x = 5 + 5
in
  x * x
;;
```



Let-in is an expression!

Evaluating let-in in env ***E***:

1. Evaluate expr ***e1*** in env ***E*** to get value ***v*** : ***t***
2. Use extended ***E*** [**x** |-> ***v*** : ***t***] to evaluate ***e2***

```
let y =
  let
     x = 10
  in
     x * x
;;
```

…	…

…	…
x	*10 : int*

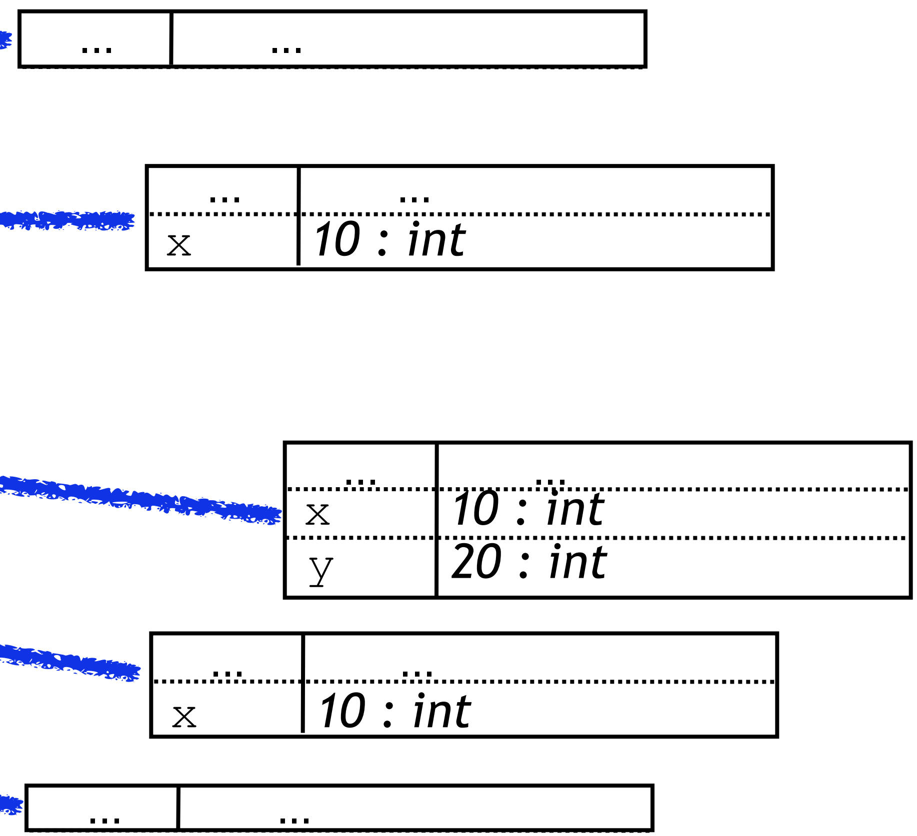
…	…
y	*100 : int*

Nested bindings

Evaluating let-in in env ***E***:

1. Evaluate expr ***e1*** in env ***E*** to get value ***v*** : ***t***
2. Use extended ***E*** [**x** |-> ***v*** : ***t***] to evaluate ***e2***

```
let
   x = 10
in
   (let
      y = 20
    in
      x * y)
   + x
;;
```



Nested bindings

```
let
  x = 10
in
  let
    y = 20
  in
    x * y
;;
```

BAD Formatting

```
let x = 10 in
let y = 20 in
  x * y
;;
```

GOOD Formatting

Example

```
let rec filter f xs =
  match xs with
  | []     -> []
  | x::xs' -> let ys  = if f x then [x] else [] in
              let ys' = filter f xs              in
              ys @ ys'
```

Recap 1: Variables are names for values

- Environment: dictionary/phonebook
- Most recent binding used
- **Entries never change**
- New entries added

Recap 2: Big Exprs With Local Bindings

- **`let-in`** expression
- Variable "in-scope" **`in`**-expression
- Outside, variable not "in-scope"

Recap 3: Env Frozen at Func Definition

- Re-binding vars cannot change function
- Indentical I/O behavior at every call
- Predictable code, localized debugging

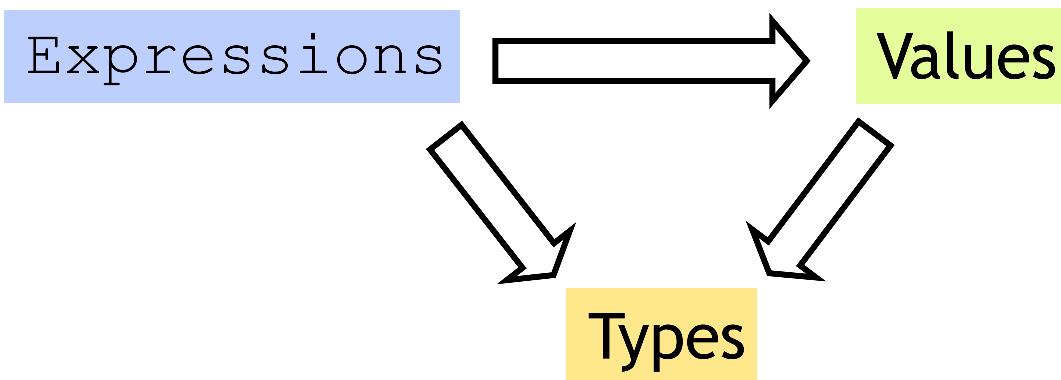
Static/Lexical Scoping

- For each occurrence of a variable,
  A unique place where variable was defined!
  - Most recent binding in environment

- Static/Lexical: Determined from program text
  - Without executing the program

- Very useful for readability, debugging:
  - Don't have to figure out "where" a variable got assigned
  - Unique, statically known definition for each occurrence

Next: Functions



Q: What's the value of a function ?

Immutability: The Colbert Principle

"A function behaves the same way on Wednesday, as it behaved on Monday, *no matter what happened on Tuesday!*"

Functions

Expressions

Two ways of writing function expressions:

1. Anonymous functions:

Parameter (formal)

Body Expr

```
let fname = fun x -> e
```

2. Named functions:

Parameter (formal)

Body Expr

```
let fname x = e
```

Function Application `Expressions`

Application: fancy word for "call"

`(e1 e2)`

- Function value ***e1***
- Argument ***e2***
- "apply" argument ***e2*** to function value ***e1***

Functions

Type

The type of any function is:

- *T1* : the type of the "input"
- *T2* : the type of the "output"

T1 -> T2

```
let fname = fun x -> e
```

T1 -> T2

```
let fname x = e
```

T1 -> T2

Functions

Type

The type of any function is:

- *T1* : the type of the "input"
- *T2* : the type of the "output"

T1->T2

T1, *T2* can be any types, including functions!

Whats an example of ?

- *int -> int*
- *int * int -> bool*
- *(int -> int) -> (int -> int)*

Type of function application

Application: fancy word for "call"

`(e1 e2)`

- "apply" argument ***e2*** to function value ***e1***

$$\frac{\texttt{e1} : T1 \to T2 \qquad \texttt{e2} : T1}{\texttt{(e1 e2)} : T2}$$

- Argument must have same type as "input" *T1*
- Result has the same type as "output" *T2*

Functions

Values

Two questions about function values:

What is the value:

1. … of a function ? `fun x -> e`

2. … of a function “application” (call) ? `(e1 e2)`

Values of function = "Closure"

Two questions about function values:

What is the value:

1. … of a function ? `fun x -> e`

Closure =

Code of Fun. (formal ***x*** + body ***e***)

\+ Environment at Fun. Definition

Values of function = "Closure"

Two questions about function values:

What is the value:

1. ... of a function ? `fun x -> e`

Closure =

Code of Fun. (formal ***x*** + body ***e***)

\+ Environment at Fun. Definition

Closure

Q: Which vars in env. of f ?

```
let x   = 2 + 2 ;;
let f y = x + y ;;
let z   = x + 1 ;;
```

(a) `x`

(b) `y`

(c) `x y`

(d) `x y z`

(e) None

Values of functions: Closures

- Function value = **"Closure"**
  - <code + environment at definition>

- Body not evaluated until application
  - But type-checking when function is defined

```
# let x = 2+2;;
val x : int = 4
# let f = fun y -> x + y;;
val f : int -> int = fn
# let x = x + x;;
val x : int = 8
# f 0;;
val it : int = 4
```

Binding used to eval `(f …)`

x	*4 : int*
f	fn <*code*, >: *int->int*
x	*8 : int*

Binding for subsequent `x`

Q: Vars in closure-env of f ?

```
let a = 20;;

let f x =
   let y   = x + 1 in
   let g z = y + z in
     a + (g x)
;;
```

(a) a y

(b) a

(c) y

(d) z

(e) y z

Free vs. Bound Variables

```
let a = 20;;

let f x =
   let y = 1 in
   let g z = y + z in
      a + (g x)
;;

f 0;;
```

```
(e1 e2)
```

Environment frozen with function

Used to evaluate fun application

Which vars needed in frozen env?

Free vs. Bound Variables

```
let a = 20;;

let f x =
  let y = 1 in
  let g z = y + z in
    a + (g x)
;;

f 0;;
```

Inside a function:

A "bound" occurrence:

1. Formal variable
2. Variable bound in **`let-in`**

`x`, **`y`**, **`z`** are "bound" inside **`f`**

A "free" occurrence:

- **Non-bound** occurrence

`a` is "free" inside **`f`**

Frozen Environment

needed for values of free vars

Q: Which vars are free in f ?

```
let a = 20;;

let f x =
  let a    = 1 in
    let g z = a + z in
      a + (g x)
;;
```

(a) a

(b) x

(c) y

(d) z

(e) None

Free vs. Bound Variables

```
let a = 20;;

let f x =
  let a = 1 in
  let g z = a + z in
    a + (g x)
  ;;

f 0;
```

Inside a function:

A "bound" occurrence:

1. Formal variable
2. Variable bound in `let-in-end`

`x`, `a`, `z` are "bound" inside `f`

A "free" occurrence:

Not bound occurrence

nothing is "free" inside `f`

Where do bound-vars values come from?

```
let a = 20;;

let f x =
   let a = 1 in
   let g z = a + z in
      a + (g x)
   ;;

f 0;
```

Bound values determined when function is evaluated ("called")

- Arguments
- Local variable bindings

Values of function application

Two questions about function values:

What is the value:

1. … of a function ? `fun x -> e`

2. … of a function “application” (call) ? `(e1 e2)`

“apply” the argument *e2* to the (function) *e1*

Values of function application

Value of a function "application" (call) `(e1 e2)`

1. Find **closure** of ***e1***
2. Execute body of **closure** with param ***e2***

Free values found in **closure-environment**

Bound values by executing **closure-body**

Values of function application

Value of a function "application" (call) `(e1 e2)`

1. Evaluate ***e1*** in current-env to get (**closure**)
   = **code** (formal ***x*** + body ***e***)   **+ env *E***

2. Evaluate ***e2*** in current-env to get (argument) ***v2***

3. Evaluate body ***e*** in env ***E*** extended with ***x*** := ***v2***

Q: What is the value of `res` ?

```
let x   = 1;;
let y   = 10;;
let f y = x + y;;
let x   = 2;;
let y   = 3;;
let res = f (x + y);;
```

(a) 4 (b) 5 (c) 6 (d) 11 (e) 12

Q: What is the value of `res` ?

```
let x   = 1;;
let y   = 10;;
let f y = x + y;;
let x   = 2;;
let y   = 3;;
let res = f (x + y);;
```

```
f |-> formal:= y
      body  := x + y
      env   := [x|->1]
x |-> 2
y |-> 3

x + y ====> 5
```

Application: `f (x + y)`

Eval `body` in `env` extended with `formal|->` 5

Eval `x+y` in `[x|->1, y|->5]` `====>` 6

Example

```
let x = 1;;
let f y =
  let x = 2 in
  fun z -> x + y + z
;;
let x = 100;;
let g = f 4;;
let y = 100;;
(g 1);;
```

Q: Closure value of g?

```
formal  z
body    x + y + z
env     [x|->2, y|->4]
```

Eval body in env extended with formal|-> 1

Eval x+y+z in [x|->2, y|->4,z|->1] ====> 7

Q: What is the value of `res` ?

```
let f g =
  let x = 0 in
  g 2
;;

let x = 100;;

let h y = x + y;;

let res = f h;;
```

(a) Syntax Error

(b) 102

(c) Type Error

(d) 2

(e) 100

Example 3

```
let f g =
   let x = 0 in
   g 2
;;

let x = 100;;

let h y = x + y;;

f h;;
```

Static/Lexical Scoping

- For each occurrence of a variable,
  - Unique place in program text where variable defined
  - Most recent binding in environment
- Static/Lexical: Determined from the program text
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- Very useful for readability, debugging:
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