

Syntax and Semantics of Nano2

Expression syntax:

$e ::= n \mid x \mid e1 + e2 \mid \text{let } x = e1 \text{ in } e2 \mid \lambda x . e \mid e1 \ e2$

Operational semantics:

[Var] $E, x \Rightarrow E, E[x] \quad \text{if } x \text{ in } \text{dom}(E)$

[Add] $E, n1 + n2 \Rightarrow E, n \quad \text{where } n == n1 + n2$

[Add-L]
$$\frac{E, e1 \Rightarrow E', e1'}{E, e1 + e2 \Rightarrow E', e1' + e2}$$

[Add-R]
$$\frac{E, e2 \Rightarrow E', e2'}{E, n1 + e2 \Rightarrow E', n1 + e2'}$$

[Let] $E, \text{let } x = v \text{ in } e2 \Rightarrow E[x \rightarrow v], e2$

[Let-Def]
$$\frac{E, e1 \Rightarrow E', e1'}{E, \text{let } x = e1 \text{ in } e2 \Rightarrow E', \text{let } x = e1' \text{ in } e2}$$

[Abs] $E, \lambda x . e \Rightarrow E, \langle E, \lambda x . e \rangle$

[App] $E, \langle E1, \lambda x . e \rangle v \Rightarrow E1[x \rightarrow v], e$

[App-L]
$$\frac{E, e1 \Rightarrow E', e1'}{E, e1 \ e2 \Rightarrow E', e1' \ e2}$$

[App-R]
$$\frac{E, e \Rightarrow E', e'}{E, v \ e \Rightarrow E', v \ e'}$$

Syntax of types:

$T ::= \text{Int} \mid T_1 \rightarrow T_2 \mid a$
 $S ::= T \mid \text{forall } a . S$

Typing rules:

[T-Num] $G \vdash n :: \text{Int}$

[T-Add]
$$\frac{G \vdash e_1 :: \text{Int} \quad G \vdash e_2 :: \text{Int}}{G \vdash e_1 + e_2 :: \text{Int}}$$

[T-Var] $G \vdash x :: S \quad \text{if } x:S \text{ in } G$

[T-Abs]
$$\frac{G, x:T_1 \vdash e :: T_2}{G \vdash \lambda x . e :: T_1 \rightarrow T_2}$$

[T-App]
$$\frac{G \vdash e_1 :: T_1 \rightarrow T_2 \quad G \vdash e_2 :: T_1}{G \vdash e_1 e_2 :: T_2}$$

[T-Let]
$$\frac{G \vdash e_1 :: S \quad G, x:S \vdash e_2 :: T}{G \vdash \text{let } x = e_1 \text{ in } e_2 :: T}$$

[T-Inst]
$$\frac{G \vdash e :: \text{forall } a . S}{G \vdash e :: [a / T] S}$$

[T-Gen]
$$\frac{G \vdash e :: S}{G \vdash e :: \text{forall } a . S} \quad \text{if not } (a \text{ in } \text{FTV}(G))$$

Here $n \in \mathbb{N}$ is natural number, $v \in \text{Val}$ is a value, $x \in \text{Var}$ is a variable, $e \in \text{Expr}$ is an expression, $E \in \text{Var} \rightarrow \text{Val}$ is an environment, $a \in \text{TVar}$ is a type variable, $T \in \text{Type}$ is a type, $S \in \text{Poly}$ is a type scheme (a poly-type), $G \in \text{Var} \rightarrow \text{Poly}$ is a type environment (a context).