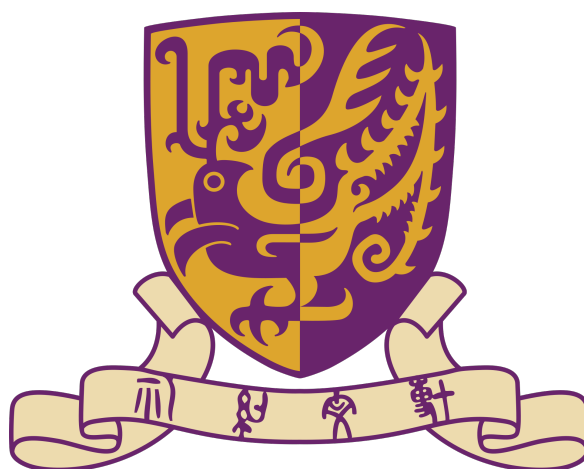


# New Approach of Numerical Relativity

Implementation, tests and applications



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I would like to dedicate this thesis to my loving parents . . .



## **Declaration**

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

Alan Tsz-Lok Lam  
June 2021



## **Acknowledgements**

And I would like to acknowledge ...





## **Abstract**

This is where you write your abstract ...



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