Problem 1

(a) $E=\lim_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}}|e^{-at}\cdot u(t)|^2\,dt=\frac{1}{2a}$ does not equal to infinity

not power signal

$P=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|e^{-at}\cdot u(t)^2|\,dt=0$

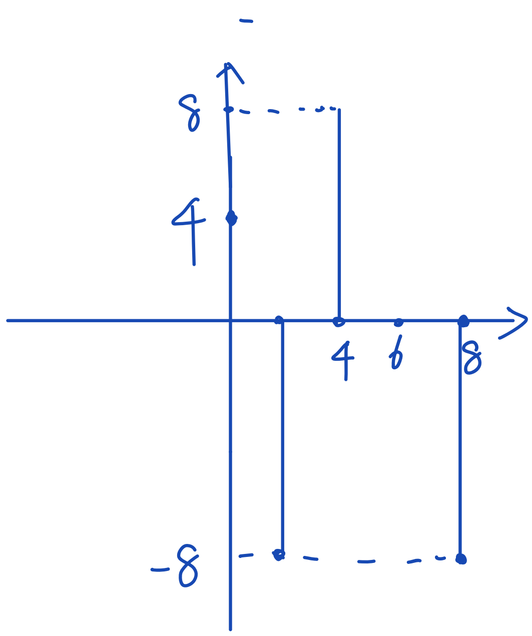
energy signals

(b) $x_2[n]=e^{j\cdot 0.4\pi n}$ $\quad |x_2[n]|=1$

$E=\sum_{n=-\infty}^{\infty}1^2=\infty$

$P=\lim_{n\to\infty}\frac{1}{2N+1}\sum_{-N}^{N}1^2=\lim_{n\to\infty}\frac{2N}{2N+1}=1$ $\quad$ power signal

Problem 2



Problem 3

(a) $y[n]-a\,y[n-1]=x[n]$

$Y(e^{jw})=\sum_{k=0}^{k=M}b_k\cdot e^{-jwk}$

(b) $H(e^{jw}) = \frac{1}{1-a\cdot e^{-jw}}$

if $a<0$ $\quad a\cdot e^{-jw}<0$ $\quad$ low pass filter

if $a>0$ $\quad a\cdot e^{-jw}>0$ $\quad$ high pass filter

Problem 4

(a) $H(e^{jw}) = 1+2e^{-jw}+2e^{-j2w}+e^{-j3w}$

$= e^{-j\frac{3}{2}w}[(e^{j\frac{3}{2}w}+e^{-j\frac{3}{2}w})+(2\cdot e^{j\frac{1}{2}w}+2\cdot e^{-j\frac{1}{2}w})]$

we have $H(e^{jw}) = e^{-j\frac{3}{2}w}\cdot(2\cos(\frac{3}{2}w)+4\cos(\frac{1}{2}w))$

which has linear phase

(b) frequency response is said to be linear phase if the coefficient of the frequency response are symmetrical

$h[n] = h[N-1-n]$

$h[0] = h[4-1] = h[3]$

$h[1] = h[4-1-1] = h[2]$

so it's linear phase

Problem 5.

(a) $j\omega Y(j\omega) + 4Y(j\omega) = X(j\omega)$ $(j\omega+4)Y(j\omega) = X(j\omega)$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{4+j\omega}$$

(b) $\frac{1}{a+j\omega} \Rightarrow e^{-at}u(t)$ $a=4$

$$h(t) = e^{-4t}u(t)$$

(c) when $\omega=0$ $H(j\omega) = \frac{1}{4}$

$\omega=\infty$ $H(j\omega) = 0$

low pass filter

(d) $h_1(t) = h(t) \times u(t)$

$$H_1(j\omega) = \frac{1}{2\cancel{\pi}} H(j\omega) \cdot (\cancel{\pi}\delta(\omega) + \frac{1}{j\omega})$$

$$= \frac{1}{2}\left(H(j\omega) + \frac{H(j\omega)}{j\omega}\right)$$