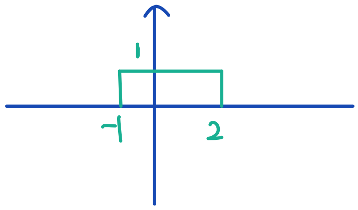
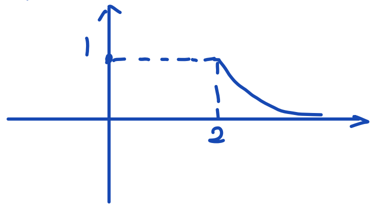
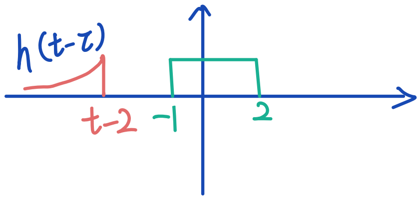
Problem 1. $x(t)$



$h(t)$



$y(t) = x(t) * h(t)$



① $t-2<-1 \quad t<1 \quad y(t)=0$

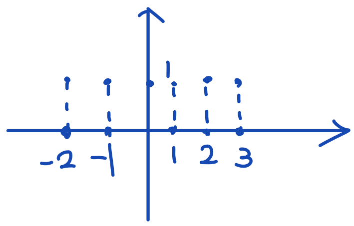
② $-1<t-2<2 \quad y(t) = \int_{-1}^{t-2} e^{-(t-\tau-2)}\, d\tau = 1-e^{1-t}$

$1<t<4$

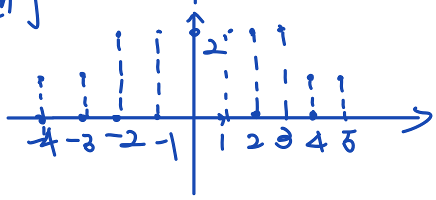
③ $t-2>2 \quad t>4$

$y(t) = \int_{-1}^{2} e^{-(t-\tau-2)}\, d\tau = (e^{3}-1)\cdot e^{1-t}$

Problem 2.a. $x[n]$



$y[n]$

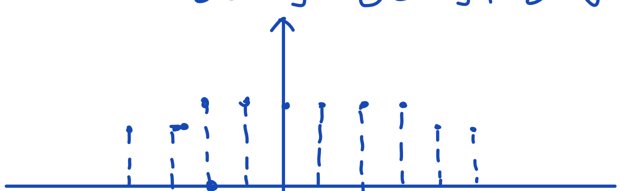


b. if $x[n] = \delta[n]$ $\quad y[n] = h[n]$ $\quad h[n] = \delta[n]+1$

$y[n] = \sum_{-\infty}^{\infty} x[n]\cdot h[n-k]$

$= \sum_{k=-2}^{3} [\delta[n-k]+1]$

$= \delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + 6$



-4 -3 -2 -1 | 1 2 3 4 5

C. No, becaue system T is not linear

$$2y_1[n]+\beta y_2[n]=2x_1[n]+2+\beta x_2[n]+\beta$$

$$T\{x[n]\}=x[n]-1$$

$$T\{2x[n]+\beta x[n]\}=2x[n]+\beta x[n]-2$$

Problem 3.

(a) $y[n]=x[n]\cdot h[n]=\sum_{k=0}^{n} 2^k\cdot\beta^{(n-k)}=\beta^n\cdot\sum_{k=0}^{n}\left(\frac{2}{\beta}\right)^k=\beta^n\cdot\frac{a\left(\frac{a}{b}\right)^n-b}{a-b}\,u[n]$

(c) $y[n]=x[n]\cdot h[n]=\sum_{k=-\infty}^{n-4}\left(-\frac{1}{2}\right)^n\cdot 4^{(n-k)}$

$n-2\leq 4 \quad n\leq 6$

$$y[n]=\sum_{k=4}^{\infty}\left(-\frac{1}{2}\right)^k\cdot 4^{n-k}$$

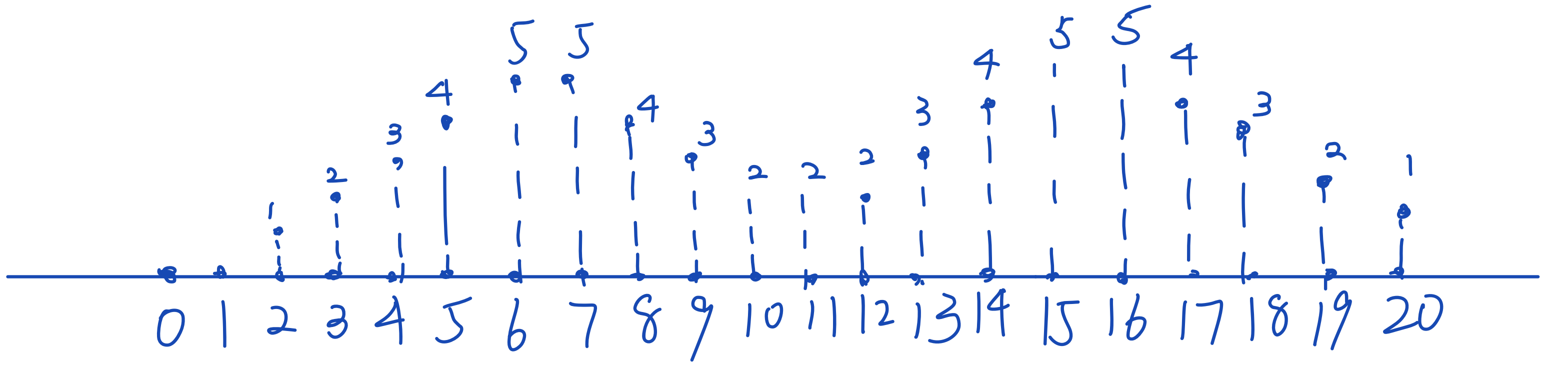
$$=4^n\cdot\sum_{k=4}^{\infty}\left(-\frac{1}{8}\right)^k=\frac{4^n}{4608}$$

$n>6$

$$y[n]=\sum_{k=n-2}^{\infty}\left(-\frac{1}{8}\right)^k=4^n\cdot\left[\frac{\left(-\frac{1}{8}\right)^{n-2}}{1+\frac{1}{8}}\right]$$

(d) $y[n]=\sum_{k=0}^{\infty}x[k]h[n-k]$

$$y[n]=x[0]h[0]+x[1]h[n-1]+x[2]h[n-2]$$
$$+x[3]h[n-3]+x[4]h[n-4]$$



Problem 4

(a) Causal $h(t)=0$ for all $t<0$

$\int_2^\infty e^{-4\tau}\,d\tau = \frac{e^{-8}}{4}$ stable

(b) not causal $h(t) \neq 0$ for $t > 3$

$\int_{-\infty}^{3} e^{-6\tau} d\tau = \infty$ unstable

(c) not causal $h(t) \neq 0$ for $t < 50$

$\int_{-\infty}^{3} e^{-2\tau} d\tau = \infty$ unstable

(d) not causal $h(t) \neq 0$ for $t > -1$

$\int_{-\infty}^{-1} e^{2\tau} d\tau = \frac{e^{-2}}{2}$ stable

(e) not causal $h(t) \neq 0$

$\int_{-\infty}^{\infty} e^{-6|\tau|} d\tau = \frac{1}{3}$ stable

(f) causal $h(t) = 0$ for $t < 0$

$\int_{0}^{\infty} (\tau \cdot e^{-\tau}) d\tau = 1$ stable

(g) causal $h(t) = 0$ for $t < 0$

$\int_{0}^{\infty} (2 \cdot e^{-x} - e^{\frac{x-100}{100}}) dx = -\infty$ unstable

Problem 5

(a) causal. no shifting

$\int_{0}^{\infty} (\frac{1}{5})^k dk = \infty$ unstable

(b) not causal since right shifting

$\int_{-2}^{\infty} 0.8^k dk = 7$ stable

(c) not causal since $h[-2]$ depends on $u[2]$ which is future time

$\int_{-\infty}^{0} (\frac{1}{2})^k dk = \infty$ unstable

(d) not causal since $h[-1]$ depends on $u[4]$ which is future time

$\int_{-\infty}^{3} 5^{-n} dk = \frac{125}{\ln(5)}$ stable

(e) causal, only right shift

$\int_{1}^{\infty} (1.01)^k dk = \infty$ unstable

(f) not causal since $h[-1]$ depends on $u[2]$ which is future time

$\int_{0}^{\infty} (-\frac{1}{2})^k dk + \int_{-\infty}^{1} (1.01)^k dk = -\frac{1}{\ln 2} + 101$

stable

(g) causal, only right shift

$\int_{1}^{\infty} n = \infty$ unstabl