EE 242, Wi 22
Problem Set 1

For $t \in \mathbb{R}, n \in \mathbb{Z}$: $\quad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}, \quad u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}, \quad r(t) = tu(t), \quad r[n] = nu[n]$

1. For each of the following continuous and discrete examples, determine if the signal is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x(t) = \cos(\pi t) + 2\cos(2t)$ (Not periodic, π irrational while 2 is rational. i.e. no $k_1, k_2 \in \mathbb{Z}$ such that $k_1 \frac{1}{2} = k_2 \frac{1}{\pi}$)

   (b) $x(t) = \sin(\pi t) - 3\cos\left(2\pi t + \frac{\pi}{4}\right)$ (Periodic, $T = 2$)

   (c) $x(t) = 2\cos(\pi t) + e^{-t}\cos(2\pi t)$ (Not periodic, $e^{-t} \longrightarrow \nexists T \in \mathbb{R} : x(t) = (t + T) \quad \forall t \in \mathbb{R}$)

   (d) $x[n] = A\sin\left(\frac{3\pi}{4} n\right)$ (Periodic, $N = 8$)

   (e) $x[n] = (-1)^n$ (Periodic, $N = 2$)

   (f) $x[n] = A\cos(3n)$ (Not periodic. no integer multiple of 3 is an integer multiple of 2π, the phase of cos.)

2. For each of the following continuous and discrete examples, determine if the signal is bounded. If so, determine the signal's minimal finite upper bound.

   (a) $x(t) = e^{-2t}$ (Not bounded, $x(t) \to \infty$ as $t \to -\infty$)

   (b) $x(t) = Ae^{-1.5|t|}$ (Bounded, upper bound at A.)

   (c) $x(t) = u(t-2)u(3-t)t \quad$ such that $\quad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$ (Bounded, upper bound of 3 at $t \to 3$)

   (d) $x[n] = a^{0.1n}u[-n]$, $|a| > 1$ (Bounded, maximum of *1* at $n = 0$)

   (e) $x[n] = a^{-0.2n}$ (Not bounded, reaches ∞ as $n \to -\infty$)

3. For each of the finite-energy signals, compute the energy of the signal.

   (a) $x(t) = 2e^{-\frac{1}{3}|t|}$ $\left(E(x) = \int_{t \in \mathbb{R}} x(t)^2 dt = 2\int_{t \in \mathbb{R}^+} 4e^{-\frac{2}{3}t} dt = -12 \left(e^{-\frac{2}{3}t} \right|_0^\infty = -12(0-1) = 12\right)$

   (b) $x[n] = u[n-2]u[3-n]n$ $\left(E(x) = \sum_{n \in \mathbb{Z}} x^2[n] = \left(2^2 + 3^2\right) = 13\right.$

4. Draw a plot of $x(t) = u(1-t) + r(t) - r(t-2) - 2u(t-3)$ for $x \in [-5, 5]$. r is the ramp function $r(t) = tu(t)$.

5. Compute and sketch $y(t) = \frac{1}{2}x(2(t+1)) - 1$ where $x(t)$ is defined in problem 4.

   $y(t) = \frac{1}{2}\left(u(-2t-1) + r(2t+2) - r(2t) - 2u(2t-1)\right) - 1$

6. Draw a plot of $x[n] = u[n+2] + 2u[n+1] - 5u[n] + 2u[n-2]$ for $n \in [-5, 5]$.

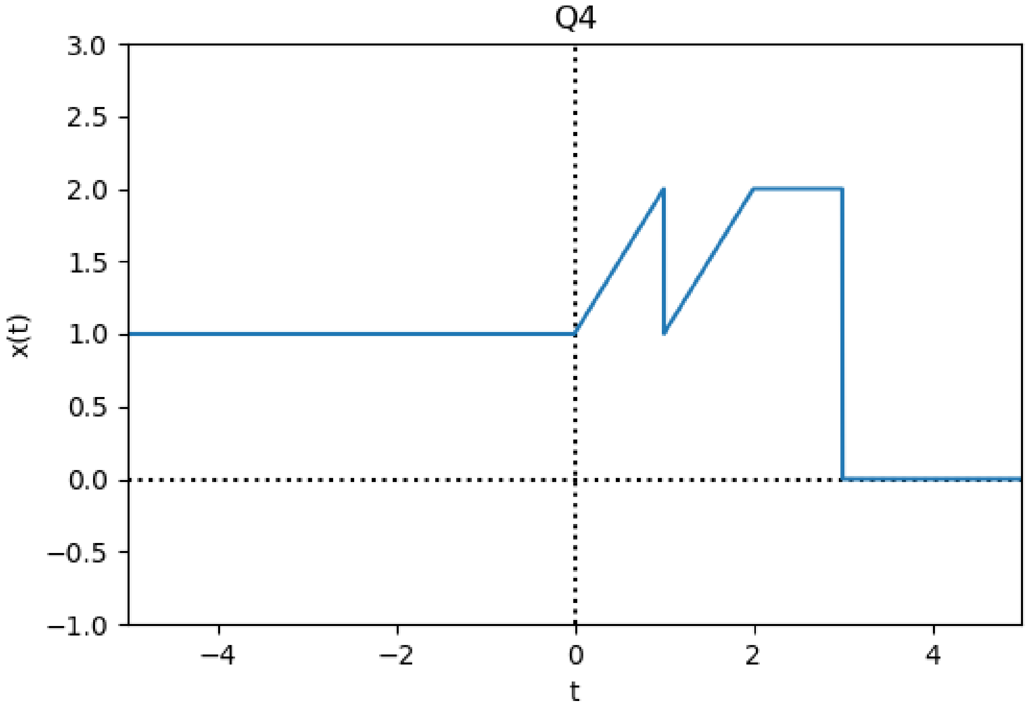

Figure 1: Question 4 Plot

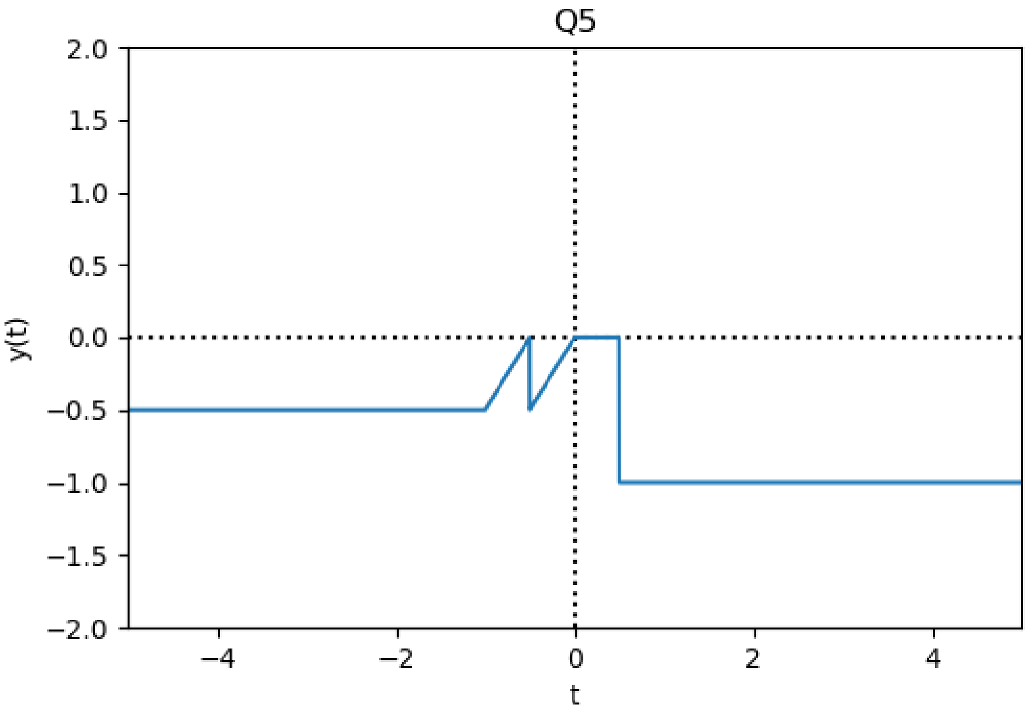

Figure 2: Question 5 Plot

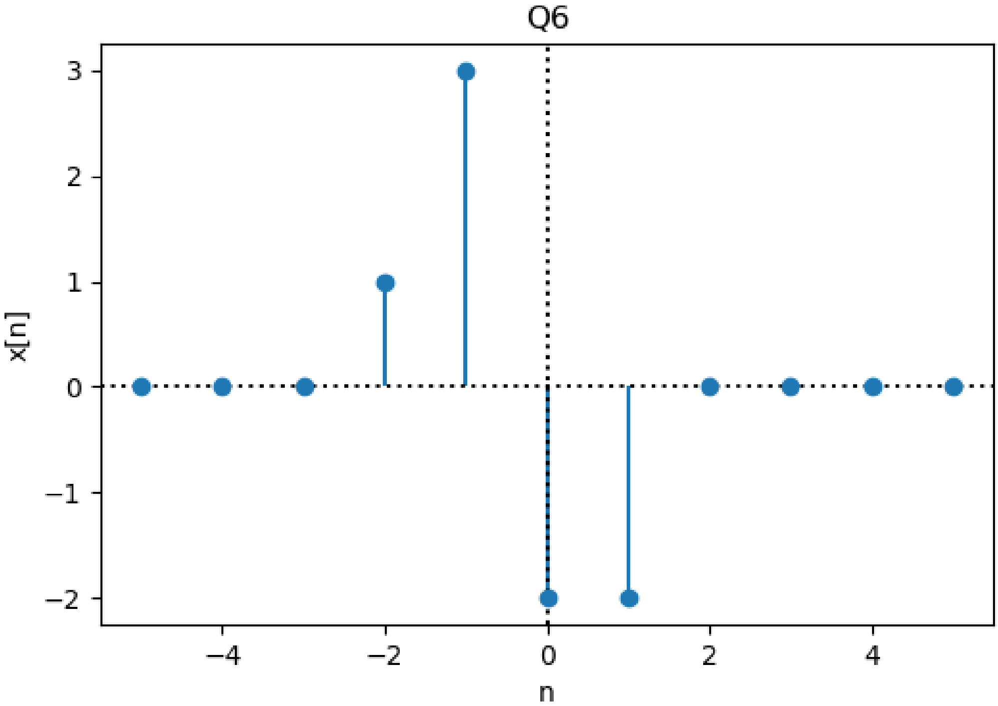

Figure 3: Question 6 Plot

7. Compute and sketch the even and odd components of $x(t)$ as defined in problem 4.

$$x_e(t) = \frac{1}{2}(x(t)+x(-t)) = \frac{1}{2}(u(1-t)+u(t+1)+r(t)+r(-t) \\ -r(t-2)-r(-t-2)-2u(t-3)-2u(t-3)$$

$$x_o(t) = \frac{1}{2}(x(t)-x(-t)) = \frac{1}{2}(u(1-t)-u(t+1)+t-r(t-2)+r(-t-2)-2u(t-3)+2u(t-3))$$

8. Compute and sketch the even and odd components of $y(t)$ as defined in 5.

$$y_e(t) = \frac{1}{2}(y(t)+y(-t)) = \frac{1}{4}(u(-2t-1)+u(2t-1)+r(2t+1)+r(-2t+1) \\ -r(2t-1)-r(-2t-1)-2u(2t-2)-2u(-2t-2))-1$$

$$y_o(t) = \frac{1}{2}(y(t)-y(-t)) = \frac{1}{4}(u(-2t-1)-u(2t-1)+r(2t+1)-r(-2t+1) \\ -r(2t-1)+r(-2t-1)-2u(2t-2)+2u(-2t-2))$$

9. Compute and sketch the even and odd components of $x[n]$ as defined in 6.

$$x_e[n] = \frac{1}{2}(x[n]+x[-n]) = \frac{1}{2}(u[n+2]+u[-n+2]+2u[n+1]+2u[-n+1] \\ -5u[n]-5u[-n]+2u[n-2]+2u[-n-2])$$

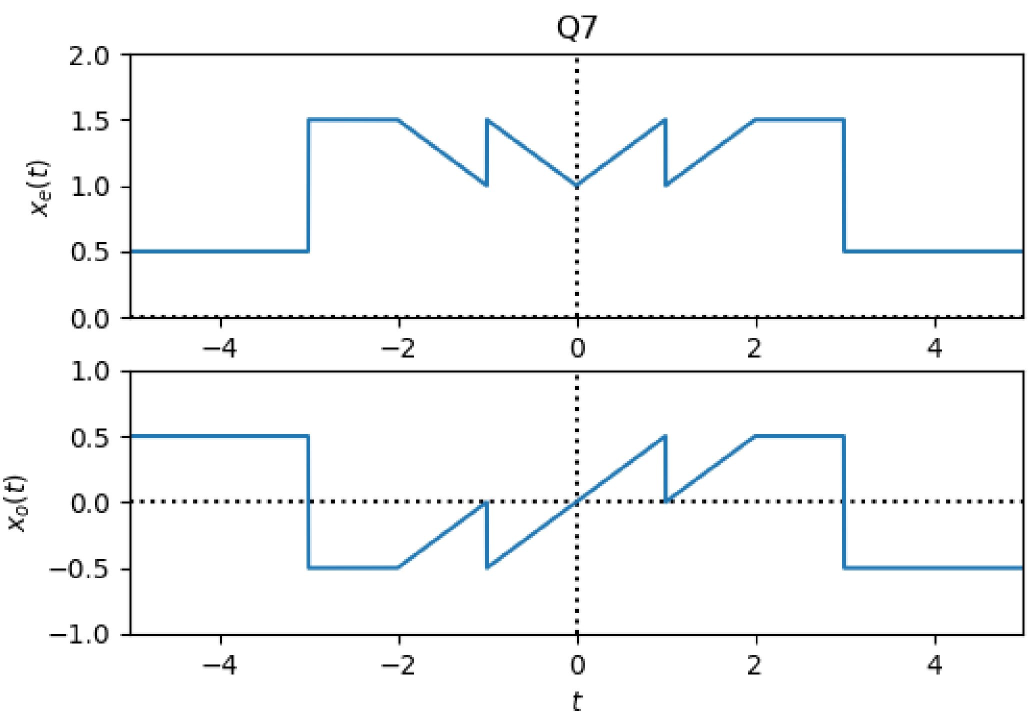

Figure 4: Question 7 Plot

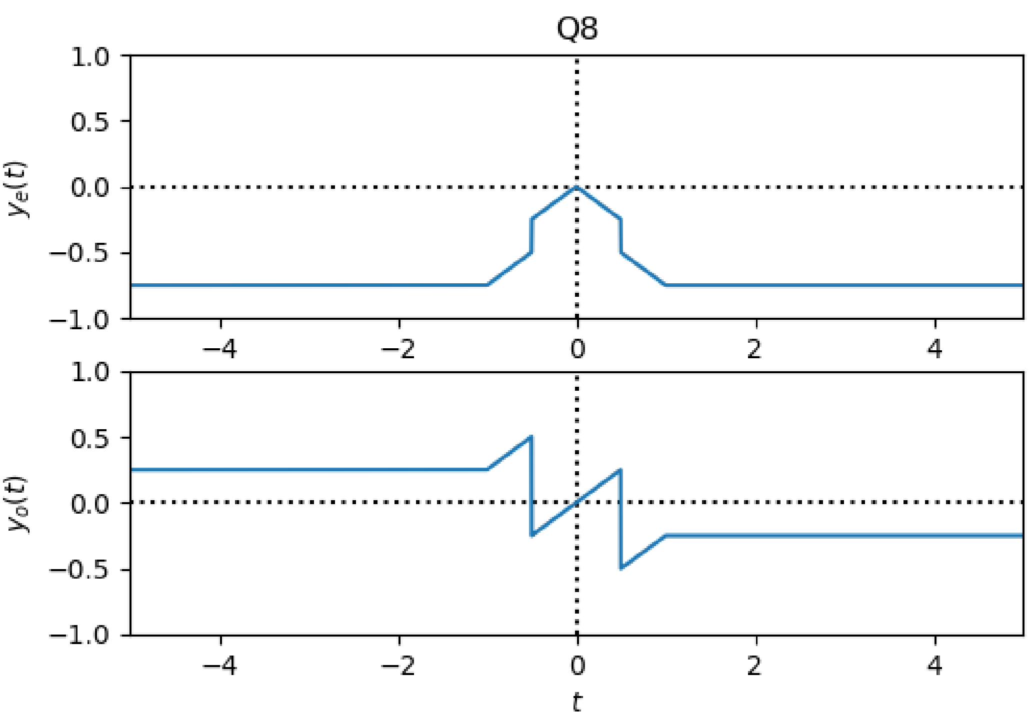

Figure 5: Question 8 Plot

$$
\begin{aligned}
x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right) = &\frac{1}{2}(u[n+2] - u[-n+2] + 2u[n+1] - 2u[-n+1] \\
&- 5u[n] + 5u[-n] + 2u[n-2] - 2u[-n-2])
\end{aligned}
$$

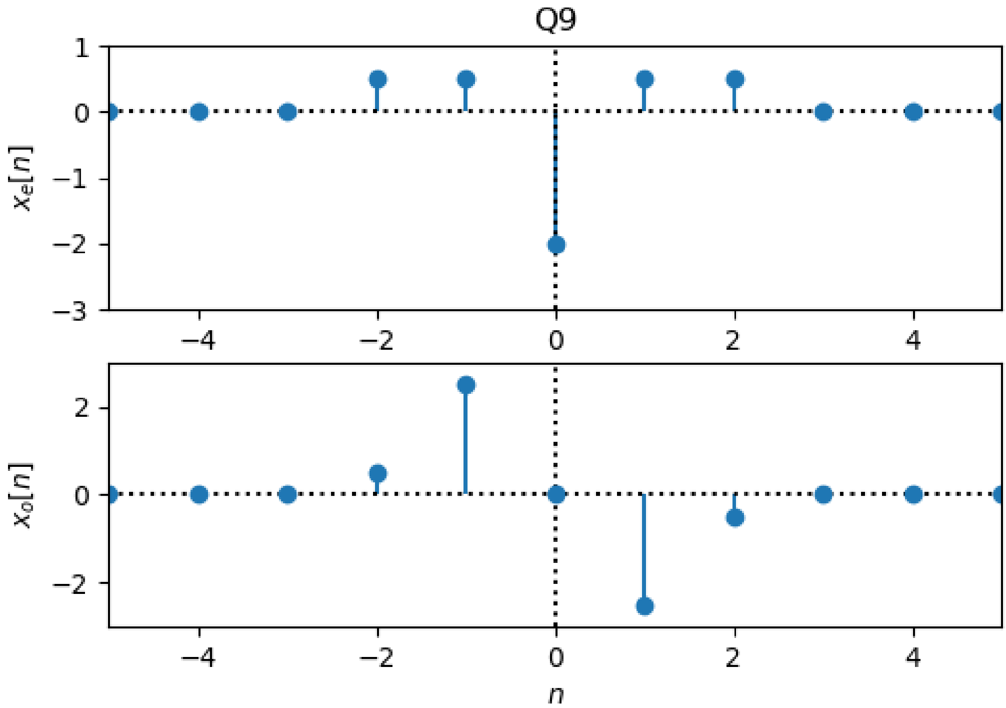

Figure 6: Question 9 Plot