HW1

2022年1月14日 9:17

0 negtive side

For $t \in \mathbb{R}, n \in \mathbb{Z}$: $\quad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}, \quad u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}, \quad r(t) = tu(t), \quad r[n] = nu[n]$

1. For each of the following continuous and discrete examples, determine if the signal is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x(t) = \cos(\pi t) + 2\cos(2t)$

   (b) $x(t) = \sin(\pi t) - 3\cos\left(2\pi t + \frac{\pi}{4}\right)$

   (c) $x(t) = 2\cos(\pi t) + e^{-t}\cos(2\pi t)$

   (d) $x[n] = A\sin\left(\frac{3\pi}{4}n\right)$

   (e) $x[n] = (-1)^n$

   (f) $x[n] = A\cos(3n)$

a) $T_1 = \frac{2\pi}{\pi} = 2 \qquad T_2 = \frac{2\pi}{2} = \pi$

$\frac{T_1}{T_2} = \frac{2}{\pi} \rightarrow$ not rational $\rightarrow \times$ periodic

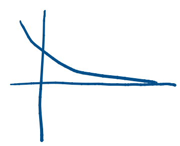
b) $T_1 = \frac{2\pi}{\pi} = 2 \qquad T_2 = \frac{2\pi}{2\pi} = 1$

$\frac{T_1}{T_2} = \frac{2}{1} = 2 \qquad LCM(2, 1) = 2$

periodic $\quad T = 2$

c) $x(t) = 2\cos(\pi t) + e^{-t}\cos(2\pi t)$

e^{-t} decress with time



Then, $e^{-t}\cos(2\pi t)$ is not periodic,

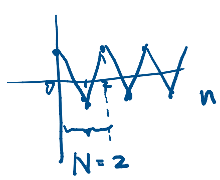
then, whole $x(t)$ is not periodic

d) $N_0 = \frac{2\pi}{3\pi/4} = \frac{2 \cdot 4}{3} = \frac{8}{3} \longrightarrow$ rational

$N = 8$, periodic

e)

n	$x[n] = (-1)^n$
0	1
1	-1
2	1
3	-1
⋮	⋮



peridic

$N = 2$

f) $\frac{2\pi}{3} = \frac{2\pi}{3} \longrightarrow$ not rational $\longrightarrow$ not periodic

2. For each of the following continuous and discrete examples, determine if the signal is bounded. If so, determine the signal's minimal finite upper bound.

(a) $x(t) = e^{-2t}$

(b) $x(t) = Ae^{-1.5|t|}$

(c) $x(t) = u(t-2)u(3-t)t$ such that $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$

(d) $x[n] = a^{0.1n}u[-n], \quad |a| > 1$

(e) $x[n] = a^{-0.2n}$

a) not bound $\lim_{t\to-\infty} x(t) = \infty$

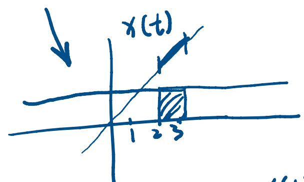
no upper limit

b) $|t| \geq 0$, $x(t)$ have maxium at $t=0$

$M = Ae^{-1.5\cdot 0} = A$ upper limit is A

c)

$x(t) = u(t-2)\cdot u(-t+3)\, t$ $u(-t+3) = u(-(t-3))$



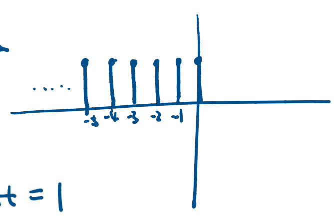
max at $t=3$, $x(t)=3 \Rightarrow$ upper limit

$x(t) = 0$ always

upper limit = 0

d)

$x[n] = a^{0.1n} \cdot \underbrace{u[-n]}$



$n\uparrow,\ x\uparrow$

max: $x[0] = a^0 = 1$, upper limit = 1

e) $x[n] = a^{-0.2n}$, $x[n]$ decrease , as n increase.

when $n = -\infty$ $x[n] = \infty$, then no upper limite

3. For each of the finite-energy signals, compute the energy of the signal.

(a) $x(t) = 2e^{-\frac{1}{3}|t|}$

(b) $x[n] = u[n-2]u[3-n]n$

3. For each of the finite-energy signals, compute the energy of the signal.

(a) $x(t) = 2e^{-\frac{1}{3}|t|}$

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a) $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

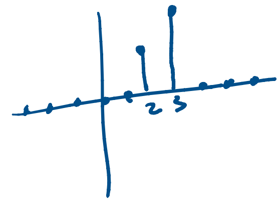
$= \int_{-\infty}^{\infty} \left|2e^{-\frac{1}{3}|t|}\right|^2 dt$

$= 12$

b) $E = \sum_{n=-\infty}^{\infty} \left| u[n-2]u[3-n]\,n \right|^2$

$= 2^2 + 3^2$

$= 13$



$u(-(t+1))$

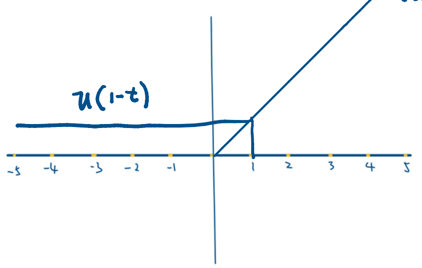
4. Draw a plot of $x(t) = u(1-t) + r(t) - r(t-2) - 2u(t-3)$ for $x \in [-5, 5]$. r is the ramp function $r(t) = tu(t)$.

$-t+1$ $\quad tu(t) \quad (t-2)u(t-2)$

0 negtive side

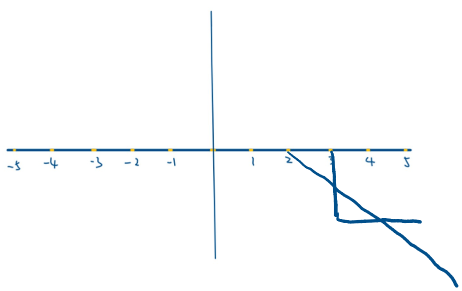
For $t \in \mathbb{R}, n \in \mathbb{Z}$: $\quad u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}, \quad u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}, \quad r(t) = tu(t), \quad r[n] = nu[n]$

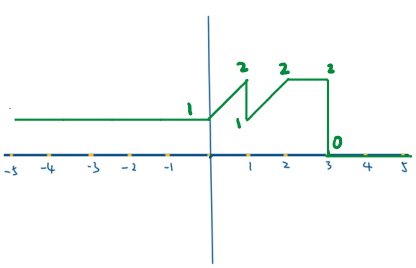
$tu(t)$



$x(t) = u(-t+1) + tu(t)$

$- \left[(t-2)u(t-2) + 2u(t-3)\right]$

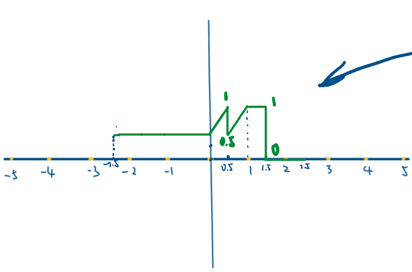




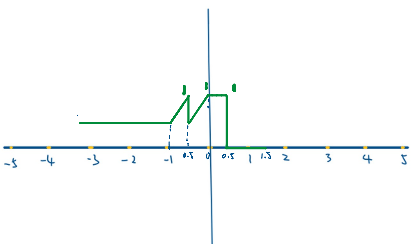
$x(t)$

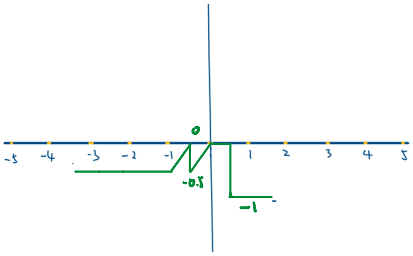
5. Compute and sketch $y(t) = \frac{1}{2}x(2(t+1)) - 1$ where $x(t)$ is defined in problem 4.

$V(t) = \frac{1}{2}x(2t)$



$w(t) = V(t+1) = \frac{1}{2}x(2(t+1))$



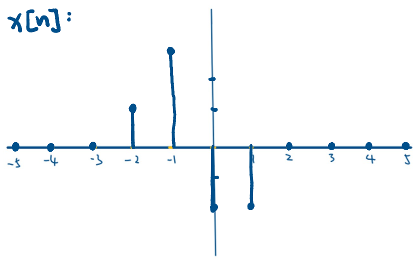


$y = w(t) - 1 = \frac{1}{2}x(2(t+1)) - 1$

$y(t) = \frac{1}{2}\left(u(-2t-1) + r(2t+2) - r(2t) - 2u(2t-1) \right) - 1$

6. Draw a plot of $x[n] = u[n+2] + 2u[n+1] - 5u[n] + 2u[n-2]$ for $n \in [-5, 5]$.

$x[n]$:



7. Compute and sketch the even and odd components of $x(t)$ as defined in problem 4.

$x_e = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$

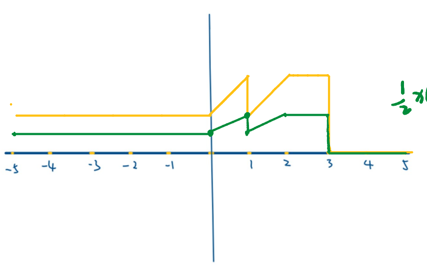
$= \frac{1}{2}u(1-t) + \frac{1}{2}r(t) - \frac{1}{2}r(t-2) - u(t-3)$

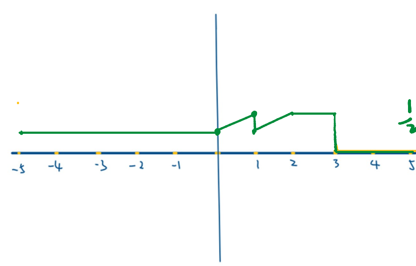
$+ \frac{1}{2}u(1+t) + \frac{1}{2}r(-t) - \frac{1}{2}r(-t-2) - u(-t-3)$

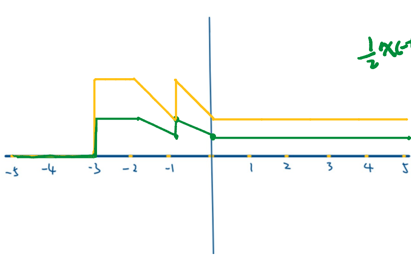
$x_o = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$

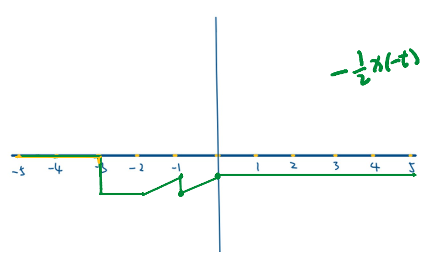
$= \frac{1}{2}u(1-t) + \frac{1}{2}r(t) - \frac{1}{2}r(t-2) - u(t-3)$

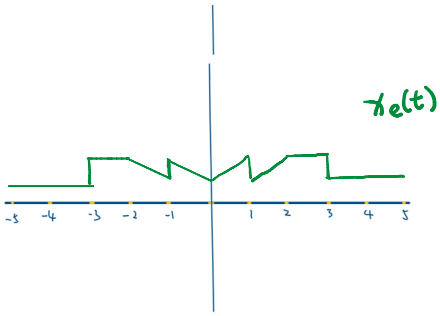
$- \frac{1}{2}u(1+t) - \frac{1}{2}r(-t) + \frac{1}{2}r(-t-2) + u(-t-3)$

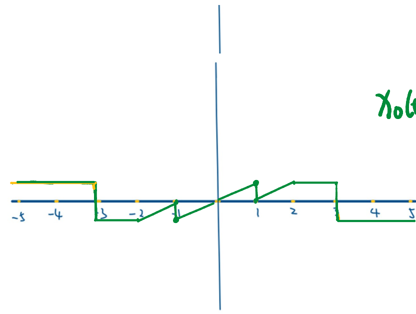












8. Compute and sketch the even and odd components of $y(t)$ as defined in 5.

$$y(t) = \frac{1}{2}x(2(t+1)) - 1$$

$$y_e(t) = \frac{1}{2}y(t) + \frac{1}{2}y(-t)$$

$$= \frac{1}{4}x(2(t+1)) + \frac{1}{4}x(2(-t+1)) - 1$$

$$= \frac{1}{4}\Big(1 - 2(t+1)) + r(2(t+1)) - r(2(t+1)-2) - 2u((2t+1)-3)\Big)$$

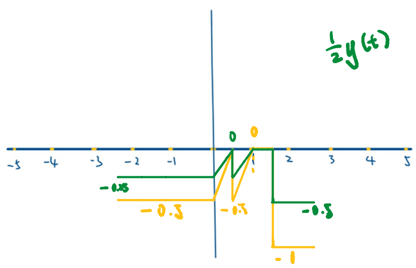
$$+ \frac{1}{4}\Big(1 - 2(-t+1)) + r(2(t+1)) - r(2(-t+1)-2) - 2u(2(-t+1)-3)\Big) - 1$$

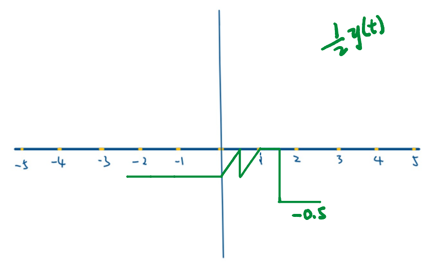
$$y_o = \frac{1}{2}y(t) - \frac{1}{2}y(-t)$$

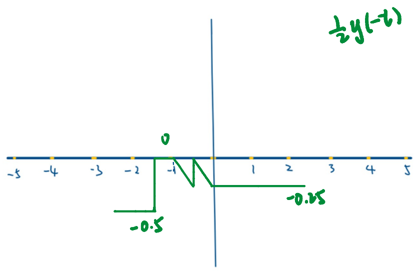
$$= \frac{1}{4}x(2(t+1)) - \frac{1}{4}x(2(-t+1))$$

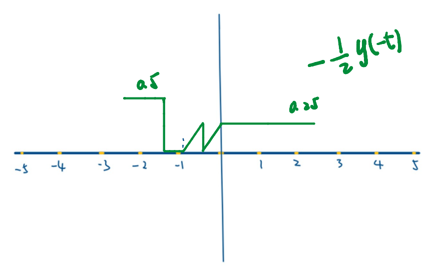
$$= \frac{1}{4}\Big(1 - 2(t+1)) + r(2(t+1)) - r(2(t+1)-2) - 2u((2t+1)-3)\Big)$$

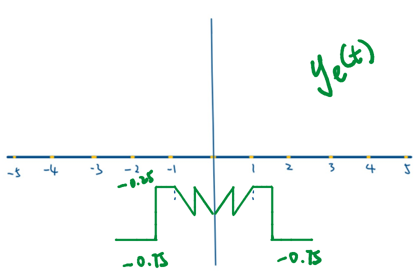
$$- \frac{1}{4}\Big(1 - 2(-t+1)) + r(2(t+1)) - r(2(-t+1)-2) - 2u(2(-t+1)-3)\Big)$$

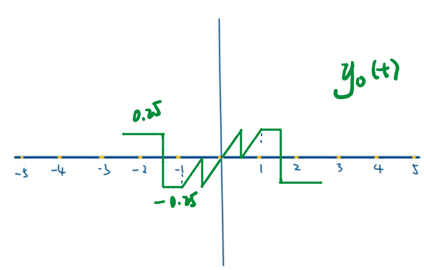






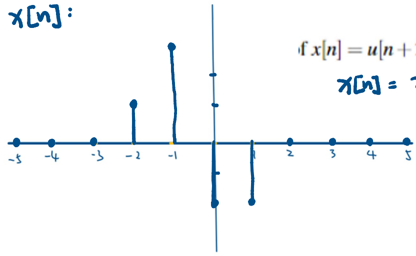






9. Compute and sketch the even and odd components of $x[n]$ as defined in 6.

$x[n]$:



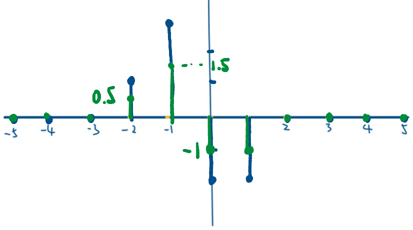
$x[n] = u[n+2] + 2u[n+1] - 5u[n] + 2u[n-2]$

$x[n] = x_e[n] + x_o[n]$

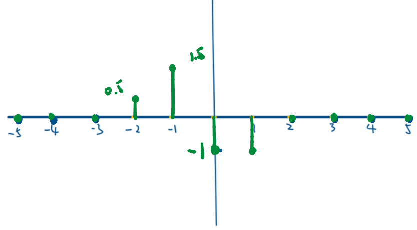
$$x_e[n] = \frac{1}{2}x[n] + \frac{1}{2}x[-n]$$
$$= \frac{1}{2}\left(u[n+2] + 2u[n+1] - 5u[n] + 2u[n-2]\right)$$
$$+ \frac{1}{2}\left(u[-n+2] + 2u[-n+1] - 5u[-n] + 2u[-n-2]\right)$$

$$x_o[n] = \frac{1}{2}x[n] - \frac{1}{2}x[-n]$$
$$= \frac{1}{2}\left(u[n+2] + 2u[n+1] - 5u[n] + 2u[n-2]\right)$$
$$- \frac{1}{2}\left(u[-n+2] + 2u[-n+1] - 5u[-n] + 2u[-n-2]\right)$$

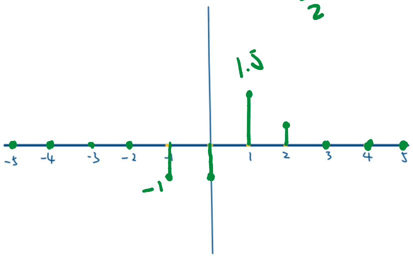
$\frac{1}{2}x[n]$



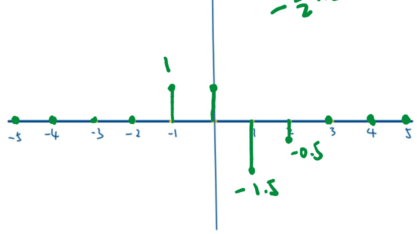
$\frac{1}{2}x[n]$



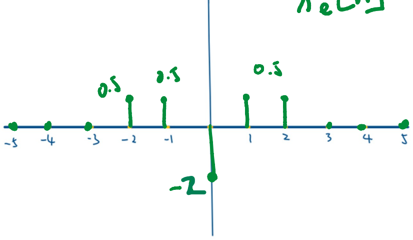
$\frac{1}{2}x[-n]$



$-\frac{1}{2}x[-n]$



$x_e[n]$



$x_o[n]$