HW3a_Q

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EE 242, Win 2022
Homework 3a

HW3 Topics: convolution, LTI systems, impulse response, step response
HW3 References: Lectures 7-10

NOTE: You will notice that some selected problems are given answers through "Show that" statements. Make sure
you show all your work clearly for every problem.
Throughout the assignment, $u(t)$ is the unit step function, $r(t) = tu(t)$ and $p(t) = u(t) - u(t-1)$

HW3 Problems:

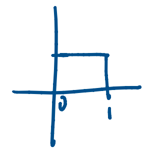
For each of the following LTI systems, compute the system's impulse response function h(t) such that:

$$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

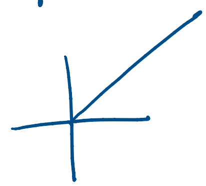
If the system is not LTI, show that this is the case.

a) $y(t) = u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}x(t-k)$
b) $y(t) = \int_0^t \tau^a x(t-\tau)d\tau$
c) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau)d\tau$
d) $y[n] = -\frac{1}{8}x[n-3] - \frac{1}{4}x[n-2] - \frac{1}{2}x[n-1]$

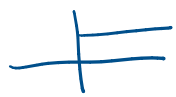
p(t)



r(t)



u(t)



a) $ay_1(t) = a\,u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}x_1(t-k)$

$ay_1(t) + by_2(t) = a\,u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}x_1(t-k) + b\,u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}x_2(t-k)$

$= u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}\left(ax_1(t-k) + bx_2(t-k)\right)$

same, linear

$T\{ax_1(t)\} = u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}\left(ax_1(t-k) + bx_2(t-k)\right)$

$y(t-t_0) = u(t-t_0)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}x(t-t_0-k)$

$T\{x(t-t_0)\} = u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}x(t-k-t_0)$

× same
× TI

not TI → not LTI,

b)

$y(t-t_0) = \int_0^{t-t_0}\tau^a x(t-\tau-t_0)\,d\tau$

$T\{x(t-t_0)\} = \int_0^{t}\tau^a x(t-(\tau-t_0))\,d\tau$

not same
not TI

× TI → × LTI

c) $y(t-t_0) = \int_{-\infty}^{t-t_0} e^{-(t-\tau-t_0)}x(\tau)\,d\tau$

$T\{x(t-t_0)\} = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau-t_0)\,d\tau$

$u = \tau - t_0$
$\tau = u + t_0$

$$T\{x(t-t_0)\} = \int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau - t_0)\, d\tau \qquad \boxed{\begin{aligned} u &= \tau - t_0 \\ \tau &= u + t_0 \end{aligned}}$$

$$= \int_{-\infty}^{t-t_0} e^{-(t-u-t_0)}\, x(u)\, d\tau$$

same TI

$$a y_1(t) + b y_2(t) = a\int_{-\infty}^{t} e(t-\tau)\, x_1(\tau)\, d\tau + b\int_{-\infty}^{t} e(t-\tau)\, x_2(\tau)\, d\tau = \int_{-\infty}^{t} e^{-(t-\tau)} (a x_1(\tau) + b x_2(\tau))\, d\tau$$

$$T\{a x_1(t) + b x_2(t)\} = \int_{-\infty}^{t} e^{-(t-\tau)} (a x_1(\tau) + b x_2(\tau))\, d\tau$$

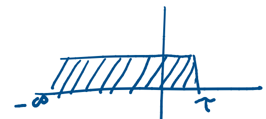
same linear

LTI

LTI

$$\int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau)\, d\tau = \int_{-\infty}^{\infty} \underbrace{x(\tau)}_{x\tau}\, \underbrace{u(t-\tau)\, e^{-(t-\tau)}}_{h(t-\tau)}\, d\tau$$

$$\boxed{h(t) = u(t)\, e^{-(t)}}$$



$u(-\tau+t) = u(t-\tau)$

d)

$$y[n-n_0] = -\frac{1}{8}x[n-n_0-3] - \frac{1}{4}x[n-n_0-2] - \frac{1}{2}x[n-n_0-1]$$

$$T\{x[n-n_0]\} = -\frac{1}{8}x[n-n_0-3] - \frac{1}{4}x[n-n_0-2] - \frac{1}{2}x[n-n_0-1]$$

same, TI

$$a y_1[n] + b y_2[n] = a\left(-\frac{1}{8}x_1[n-3] - \frac{1}{4}x_1[n-2] - \frac{1}{2}x_1[n-1]\right) + b\left(-\frac{1}{8}x_2[n-3] - \frac{1}{4}x_2[n-2] - \frac{1}{2}x_2[n-1]\right)$$

$$T\{a x_1[n] + b x_2[n]\} = -\frac{1}{8}(a x_1[n-3] + b x_2[n-3]) - \frac{1}{4}(a x_1[n-2] + b x_2[n-2]) - \frac{1}{2}(a x_1[n-1] + b x_2[n-1])$$

$$= a\left(-\frac{1}{8}x_1[n-3] - \frac{1}{4}x_1[n-2] - \frac{1}{2}x_1[n-1]\right) + b\left(-\frac{1}{8}x_2[n-3] - \frac{1}{4}x_2[n-2] - \frac{1}{2}x_2[n-1]\right)$$

same, linear

LTI system :

$$h = -\frac{1}{8}\delta[n-3] - \frac{1}{4}\delta[n-2] - \frac{1}{2}\delta[n-1]$$