HW2

2022年1月19日 19:49

HW2 Topics: Integrals, System equations, Causality, Linearity, Time Invariance, Memory/Memoryless Systems

HW2 References: Lectures 4, 5, 6

NOTE: You will notice that some selected problems are given answers through "Show that" statements. Make sure you show all your work clearly for every problem.
Throughout the assignment, $u(t)$ is the unit step function, $r(t) = tu(t)$ and $p(t) = u(t) - u(t-1)$.
HW1 Problems:

1. For each of the following systems $T[\cdot]$, determine if the system is (i) memoryless, (ii) causal, (iii) stable, (iv) linear, (v) time-invariant. Prove your statements using the definitions for these conditions covered in lecture.

   (a) $T[x(t)] = -x(t+4)x(t)$

   (b) $T[x[n]] = x[4n+1]$

   (c) $T[x[n]] = \begin{cases} x[n+1] & n \geq 0 \\ x[n] & n < 0 \end{cases}$

   (d) $T[x(t)] = \int_{-\infty}^{-t} x(\tau)d\tau$

   (e) $T[x(t)] = \int_{-\infty}^{0} \tau^{-2}x(t-\tau)d\tau$

future in time

a) i) $T\{x(1)\} = -x(5)x(1)$ not memoryless

ii) future in time $\longrightarrow$ not causal

iii) $|x(t)| \leq B$, $|-x(t+4)| \leq B$

$T\{x(t)\} \leq B \cdot B \leq B^2$ finite value, stable

iv) $ay(t) = a(-x(t+4)x(t))$ $\overset{?}{\neq}$ not linear

$T[ax(t)] = -ax(t+4)ax(t)$

v) $y(t-t_0) = -x(t-t_0+4)x(t-t_0)$ TI

$T\{x(t-t_0)\} = -x(t-t_0+4)x(t-t_0)$

b) i) $T\{x[n]\} = x[4n+1]$

0 $\quad$ $0+1=1$ future $\quad$ not memoryless

ii) future $\longrightarrow$ not causal

iii) $|x[n]| \leq B \rightarrow |x[4n+1]| \leq B \rightarrow$ stable

iv) $ay[n] = ax[4n+1]$ $\quad$ $ay_1 + by_2 = ax_1[4n+1] + bx_2[4n+1]$ $\}$ equal

iv) $ay[n] = ax[4n+1]$ $\quad ay_1 + by_2 = ax_1[4n+1] + bx_2[4n+1]$

$T\{ax[n]\} = ax[4n+1]$ $\quad T\{ax_1[n]+bx_2[n]\} = ax_1[4n+1] + bx_2[4n+1]$ } equal

linear

v) $y[n-n_0] = x[4(n-n_0)+1]$

$T\{x[n-n_0]\} = x[4n-n_0+1]$ } not equal $\rightarrow$ not TI

c)

i) $T\{x[n]\} = \begin{cases} x[n+1] & n \geq 0 \\ x[n] & n < 0 \end{cases}$ $\quad 1 \rightarrow$ future in time $\quad$ not memoryless

$\uparrow$ 0

ii) future $\rightarrow$ not causal

iii) $|x[n]| \leq B$ $\quad |x[n+1]| = |x[n]| \leq B \rightarrow$ stable

iv) $ay[n] = \begin{cases} ax[n+1] & n \geq 0 \\ ax[n] & n < 0 \end{cases}$ $\quad ay_1[n] + by_2[n] = \begin{cases} ax_1[n+1] + bx_2[n+1] & n \geq 0 \\ ax_1[n] + bx_2[n] & n < 0 \end{cases}$

$T\{ax_1[n] + bx_2[n]\} = \begin{cases} ax_1[n+1] + bx_2[n+1] & n \geq 0 \\ ax_1[n] + bx_2[n] & n < 0 \end{cases}$ equal, linear

v) $y[n-n_0] = \begin{cases} x[n-n_0+1] & n-n_0 \geq 0 \\ x[n-n_0] & n-n_0 < 0 \end{cases}$

$T\{x[n-n_0]\} = \begin{cases} x[n-n_0+1] & n \geq 0 \\ x[n-n_0] & n < 0 \end{cases}$ } not equal, not TI

d) i) $T[x(t)] = \int_{-\infty}^{-t} x(\tau)\, d\tau$ $\quad \rightarrow 5$ (future) $\quad$ not memoryless

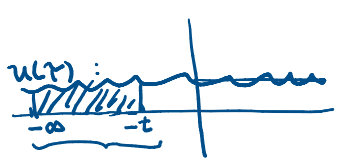
$\uparrow$ -5

ii) future $\longrightarrow$ not causal

iii) $\int_{-\infty}^{-t} x(\tau)\, d\tau$ $\quad x(\tau) \Rightarrow \cos(\tau)$

$\Downarrow$

$\int_{-\infty}^{-t} \cos(\tau)\, d\tau$



area $= \infty$

unstable,

iv) $ay_1(t) = a\int_{-\infty}^{-t} x(\tau)\, d\tau$ $\quad ay_1(t) + by_2(t) = \int_{-\infty}^{-t} ax_1(\tau) + bx_2(\tau)\, d\tau$

$T\{ax(t)\} = \int_{-\infty}^{-t} ax(\tau)\, d\tau$ $\quad T\{ax_1(t) + bx_2(t)\} = \int_{-\infty}^{-t} ax_1(\tau) + bx_2(\tau)\, d\tau$ } equal, linear

v) $u_1(t-t_0) = \int^{-(t-t_0)} x(\tau)\, d\tau$

v) $y_1(t-t_0) = \int_{-\infty}^{-(t-t_0)} x(\tau)d\tau$

$T\{x(t-t_0)\} = \int_{-\infty}^{-t} x(\tau-t_0)\,d\tau$ $\quad u=\tau-t_0$, $du=d\tau$ $\quad y_1(t-t_0) \neq T\{x(t-t_0)\}$

not TI

$= \int_{-\infty}^{-t-t_0} x(u)du$

e) i) $T[x(t)] = \int_{-\infty}^{0} \tau^{-2} x(t-\tau)\,d\tau$ $\quad t+\infty \to$ future

not memoryless

ii) future $\to$ not causal $\quad x(t-(-\infty)) = x(t+\infty)$

iii) $T[u(t)] = \int_{-\infty}^{0} \tau^{-2} u(t-\tau)\,d\tau$, at 0^- $\tau^{-2} = \infty$

not stable

iv) $ay_1(t) = a\int_{-\infty}^{0} \tau^{-2} x_1(t-\tau)d\tau$ $\quad ay_1(t)+by_2(t) = \int_{-\infty}^{0} a\tau^{-2}x_1(t-\tau) + b\tau^{-2}x_2(t-\tau)\,d\tau$

$T\{ax_1(t)+bx_2(t)\} = \int_{-\infty}^{0} a\tau^{-2}x_1(t-\tau) + b\tau^{-2}x_2(t-\tau)\,d\tau$ $\quad$ same linear.

v) $y(t-t_0) = \int_{-\infty}^{0} \tau^{-2} x(t-t_0-\tau)\,d\tau$

$T\{x(t-t_0)\} = \int_{-\infty}^{0} \tau^{-2} x(t-t_0-\tau)\,d\tau$ $\quad$ same, TI

2. For $T[\cdot]$ in question 1(a), 1(b), and 1(c), determine if the system is invertible. If yes, compute the inverse. If no, demonstrate this fact mathematically.

a) $T\{x(t)\} = -\underbrace{x(t+4)}\,\underbrace{x(t)}$

$x(t+4) = 0$, $x(t) = 0$ $\quad \to$ 2 input refer 1 output

not invertible

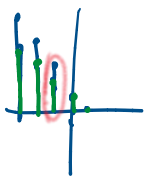
b) $T\{x[n]\} = x[4n+1] = y[n]$

$x[n] = y[\frac{n-1}{4}]$ × $\quad$ not invertible , some values are extracted

$n=0$ $\quad y[-1]$

$x[n] = y[\frac{n-1}{4}]$ × not inver...

c) $T\{x[n]\} = \begin{cases} x[n+1] & n \ge 0 \\ x[n] & n < 0 \end{cases} = y[n]$ $\quad x[n] = \begin{cases} y[n-1] & n \ge 0 \\ y[n] & n < 0 \end{cases}$ $\quad n=0 \;\; y[-1]$, $\; n=-1 \;\; y[-1]$



in T^{-1}, 2 input → 1 output, not invertible

3. For an LTI sytem $T[\cdot]$, an input $x_1(t) = p(t)$ is paired with an output signal $y_1(t) = T[x_1(t)] = p(t) - p(t-1)$. Knowing this, compute $T[x_2(t)]$ when $x_2(t) = p(t) - 2p(t-1)$.

$T\{p(t)\} = p(t) - p(t-1)$ $\quad$ if $T\{x(t)\} = x(t) - x(t-1)$

$\left.\begin{array}{l} T\{x(t-t_0)\} = x(t-t_0) - x(t-1-t_0) \\ y(t-t_0) = x(t-t_0) - x(t-1-t_0) \end{array}\right)$ same

$\left.\begin{array}{l} T\{ax(t) + bx(t)\} = ax(t) - ax(t-1) + bx(t) - bx(t-1) \\ ay(t) + by(t) = a(x(t) - x(t-1)) + b(x(t) - x(t-1)) \end{array}\right)$ Same

therefore $T\{x(t)\} = x(t) - x(t-1)$

$T\{2p(t-1)\} = 2p(t-1) - 2p(t-1-1) = 2p(t-1) - 2p(t-2)$

$$T\{p(t) - 2p(t-1)\} = p(t) - p(t-1) - (2p(t-1) - 2p(t-2))$$

$$= p(t) - p(t-1) - 2p(t-1) + 2p(t-2)$$

$$= p(t) - 3p(t-1) + 2p(t-2)$$