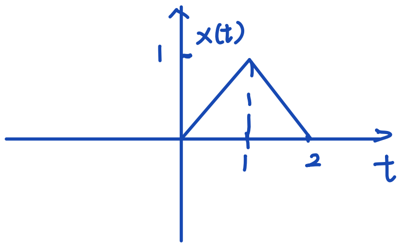
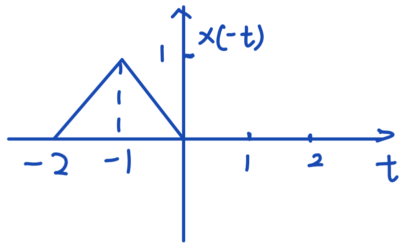
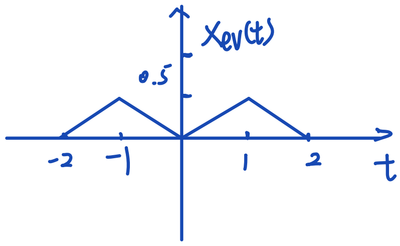
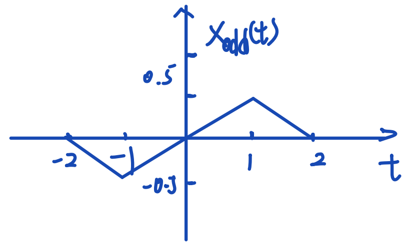
Problem 1.

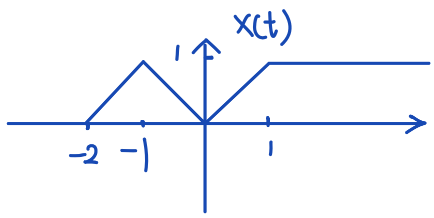


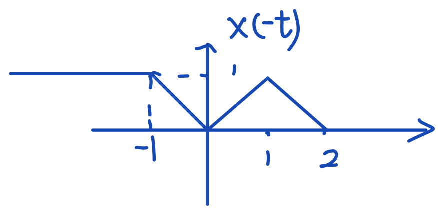


$X_{ev}(t) = \frac{1}{2}(x(t) + x(-t))$ $\quad$ $X_{odd}(t) = \frac{1}{2}(x(t) - x(-t))$

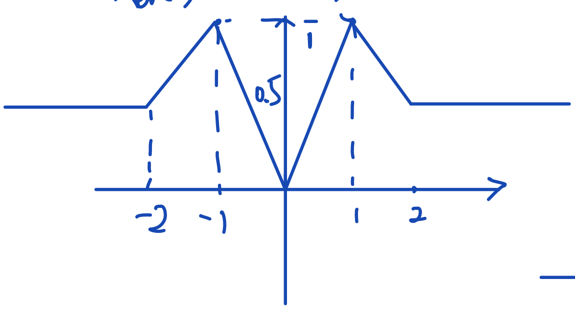


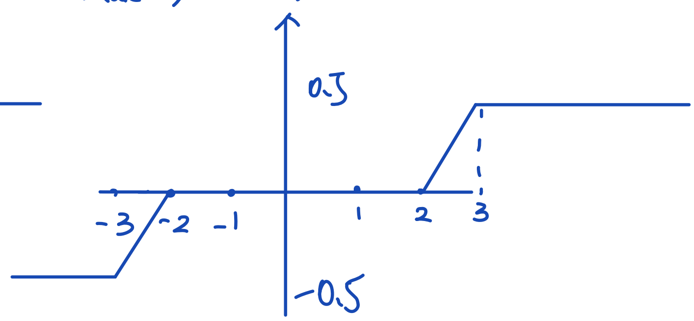


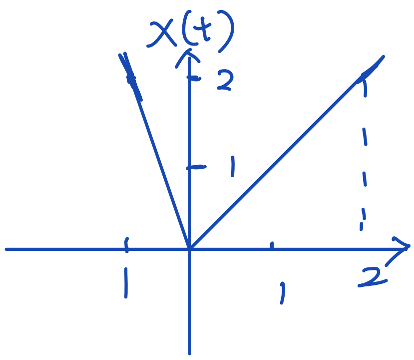


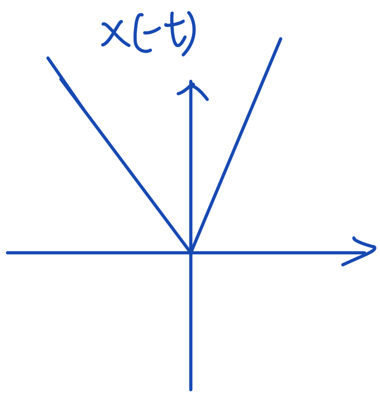


$X_{ev}(t) = \frac{1}{2}(x(t) + x(-t))$ $\quad$ $X_{odd}(t) = \frac{1}{2}(x(t) - x(-t))$

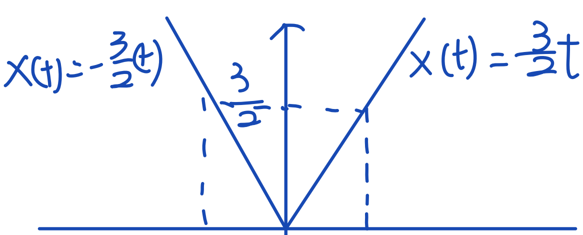


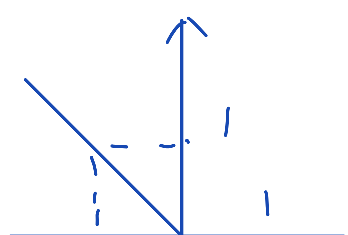


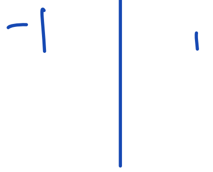


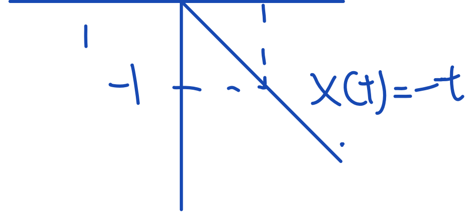


$X_{ev}(t) = \frac{1}{2}(x(t) + x(-t))$ $\quad$ $X_{odd}(t) = \frac{1}{2}(x(t) - x(-t))$ $\quad$ $-2t-$









Problem 2:

(a) $3\cos\left(4t+\frac{\pi}{3}\right) = 3\cdot\left[\cos(4t)\cos\frac{\pi}{3} - \sin(4t)\sin\left(\frac{\pi}{3}\right)\right]$

$= \frac{3}{2}\cos(4t) - \frac{3\sqrt{3}}{2}\sin(4t)$

for $\cos(4t)$ $f_1 = \frac{4}{2\pi}$ $T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$

$\sin(4t)$ $f_2 = \frac{4}{2\pi}$ $T_2 = \frac{\pi}{2}$

LCM is $\frac{\pi}{2}$ $T = \frac{\pi}{2}$.

(b) $e^{j(\pi t-1)} = \cos(\pi t-1) + i\cdot\sin(\pi t-1)$

for $\cos(\pi t-1)$ $f_1 = \frac{\pi}{2\pi} = \frac{1}{2}$ $f = \frac{1}{2}$

$\sin(\pi t-1)$ $f_2 = \frac{\pi}{2\pi} = \frac{1}{2}$ $T = 2$

(c) $\cos\left(2t-\frac{\pi}{3}\right)^2 = \frac{1}{2}\left(1+\cos\left(4t-\frac{\pi}{3}\right)\right)$

$2\pi f = 4$ $f = \frac{2}{\pi}$ $T = \frac{\pi}{2}$

Problem 3:

(a) $y(t) = T\{x(t)\} = x(t-2) + x(2-t)$

- not memoryless. output depends on $x(t-2)$ instead of $x(t)$
- not causal because of $x(2-t)$ time inverse
- stable as a bounded input gives a bounded output
- $T\{\alpha x_1(t) + \beta x_2(t)\} \stackrel{?}{=} \alpha x(t_1-2) + \alpha x(2-t_1) + \beta x(t_1-2)$

$+\beta x(2-t)$

$\alpha y_1(t) = \alpha x(t-2) + \alpha x(2-t_1)$

$\beta y_2(t) = \beta x(t_2-2) + \beta x(2-t_2)$ so linear

$\alpha y_1(t) + \beta y_2(t) = T\{\alpha x_1(t) + \beta x_2(t)\}$.

- $T\{x(t-t_0)\} = x(t-t_0-2) + x(2-t-t_0)$

  $y(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$

  time-variant

$y(t) = T\{x(t)\} = [\cos(3t)]\, x(t)$

- memoryless, output depends on $x(t)$
- causal as it's real time system
- stable as a bounded input gives a bounded output, $\cos(\infty) \leq 1$
- $y(\alpha x_1(t)+\beta x_2(t)) = [\cos(3t)][\alpha x(t) + \beta x(t)]$

  $\alpha y(t_1) = [\cos(3t)]\, \alpha x_1(t)$ $\quad \beta y(t_2) = [\cos(3t)]\, \beta x_2(t)$

  $T\{\alpha x_1(t)+\beta x_2(t)\} = \alpha y_1(t) + \beta y_2(t)$ linear
- $T\{x(t-t_0)\} = [\cos(3t)] \cdot x(t-t_0)$

  $y(t-t_0) = [\cos(3(t-t_0))] \cdot x(t-t_0)$

  time-variant

$y(t) = T\{x(t)\} = \int_{-\infty}^{2t} x(\tau)\, d\tau$

- not memoryless, output does not depend on $x(t)$
- not causal, the upper bound of integral is future time
- not stable because intergral has infinity lowerbound

- $T\{\alpha x_1(t)+\beta x_2(t)\}=\int_{-\infty}^{2t}\alpha x_1(\tau)+\beta x_2(\tau)$

$\alpha y_1(t)=\alpha\int_{-\infty}^{2t}x(\tau)$

$\beta y_2(t)=\beta\int_{-\infty}^{2t}x(\tau)$

$\alpha y_1(t)+\beta y_2(t)=T\{\alpha x_1(t)+\beta x_2(t)\}$ linear

- $T\{x(t-t_0)\}=\int_{-\infty}^{2t}x(\tau-t_0)$

$y(t-t_0)=\int_{-\infty}^{2(t-t_0)}x(\tau-t_0)$ time-variant

$y(t)=T\{x(t)\}=x(\frac{t}{3})$

- not memoryless, output does not depend on $x(t)$
- not causal because slow down.
- stable as a bounded input gives a bounded output
- $T\{\alpha x_1(t)+\beta x_2(t)\}=\alpha x_1(\frac{t}{3})+\beta x_2(\frac{t}{3})$

$\alpha y_1=\alpha x_1(\frac{t}{3})$

$\beta y_2=\beta x_2(\frac{t}{3})$ $\alpha y_1+\beta y_2=T\{\alpha x_1(t)+\beta x_2(t)\}$ linear

$T\{x(t-t_0)\}\ x(\frac{t}{3}-t_0)$

$y(t-t_0)=x(\frac{t-t_0}{3})$ time-variant

Problem 4:

a) $x[n]=x[n+N]=\sin(\frac{6\pi}{7}(n+N)+1)$

$\frac{6\pi}{7}N=2\pi k$

$N=2\pi k\cdot\frac{7}{6\pi}=k\frac{7}{3}$ $k=3$ N is integer

fundemental period is 7

b) $x[n] = x[n+N] = \cos\left(\frac{n+N}{8} - \pi\right) = \cos\left(\frac{n}{8} + \frac{N}{8} - \pi\right)$

$\frac{N}{8} = 2\pi k \quad N = 16\pi k$ not integer

signal is not periodic

c) $x[n] = x[n+N] = \cos\left(\frac{\pi}{8}(n+N)^2\right)$

$= \cos\left(\frac{\pi}{8}n^2 + \frac{\pi}{8}N^2 + \frac{\pi}{4}N \cdot n\right)$

$\frac{\pi}{8}N^2 + \frac{\pi}{4}N \cdot n = 2\pi k$ when n is 0

$\frac{\pi}{8}N^2 = 2\pi k \quad N^2 = 16k$ try $N=4$

$2\pi + \pi n = 2\pi k$

$n = 2k - 2$

fundemental period $N=4$

d) $x[n] = x[n+N] = \cos\left(\frac{\pi}{2}(n+N)\right)\cos\left(\frac{\pi}{4}(n+N)\right)$

① $\frac{\pi}{2}N = 2\pi k \quad N = 4k \qquad k=1 \quad LCM = 8$

② $\frac{\pi}{4}N = 2\pi k \quad N = 8k$

fundemental period is 8

e) $x[n] = x[n+\pi] = 2\cos\left(\frac{\pi}{4}(n+N)\right) + \sin\left(\frac{\pi}{8}(n+N)\right)$

$-2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$

$\frac{\pi}{4}N = 2\pi k \quad N = 8k \qquad LCM = 16$

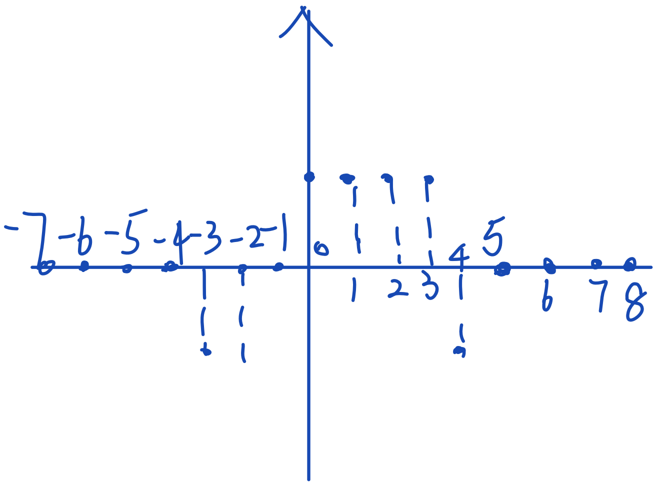
$\frac{\pi}{8}N = 2\pi k \quad N = 16k$

$\frac{\pi}{2}N = 2\pi k \quad N = 4k$

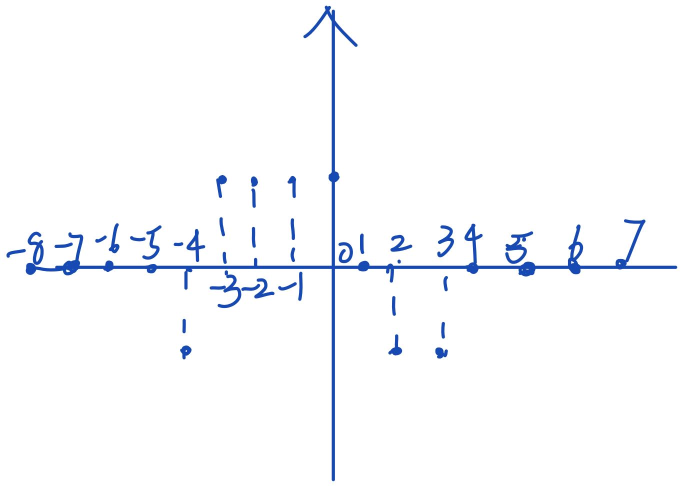
fundemental period is 16

Problem 5

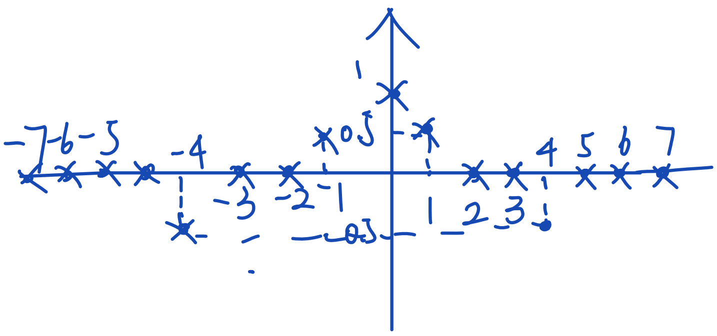
a. $x[n]$



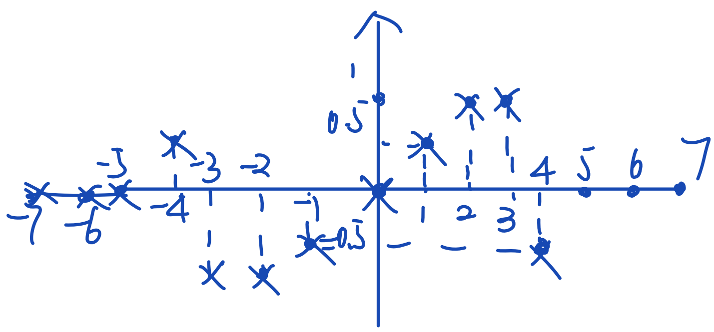
$x[-n]$



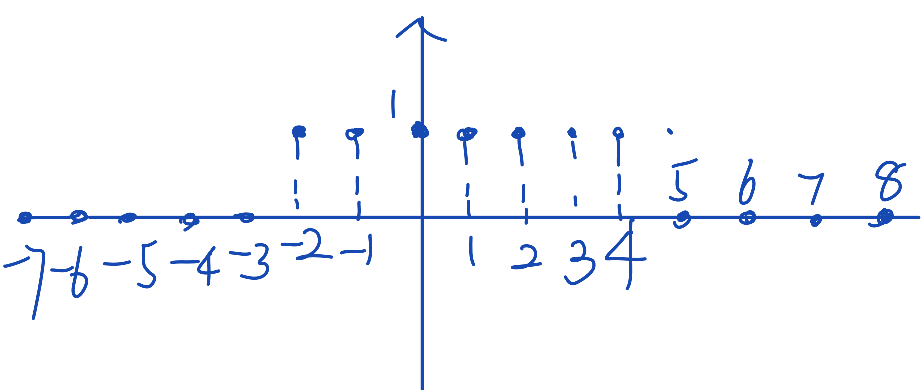
$$x_{even}[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$$



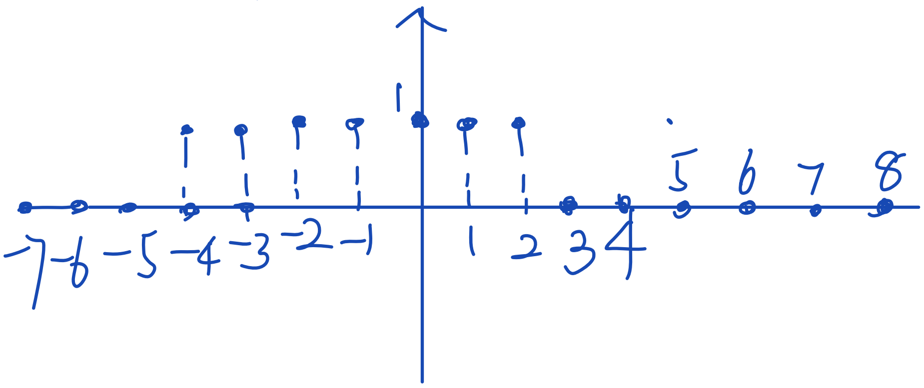
$$x_{odd}[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$$



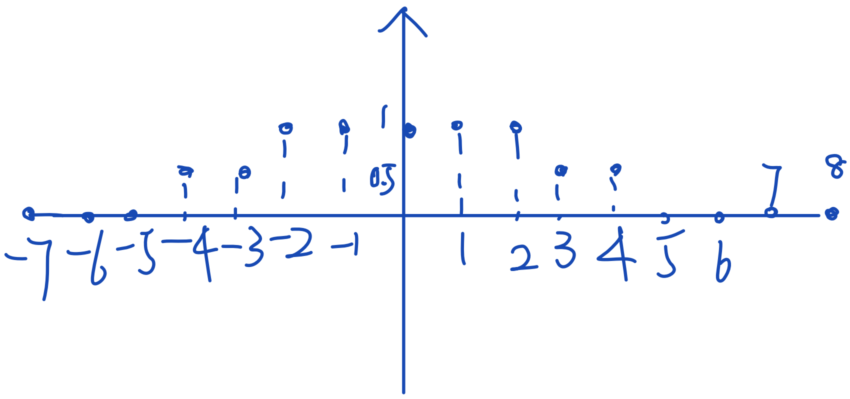
b. $x[n]$



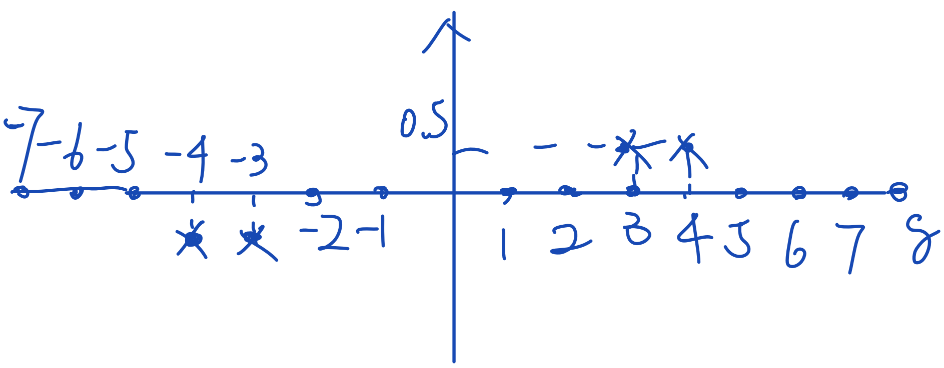
$x[-n]$



$$X_{even}[n] = \frac{1}{2}(x[n]+x[-n])$$



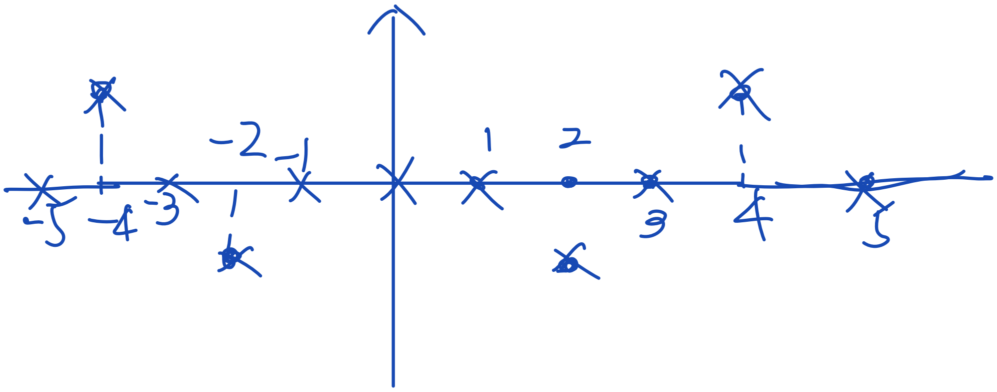
$$X_{odd}[n] = \frac{1}{2}(x[n]-x[-n])$$



c. $x[n] = \cos(\frac{\pi}{2}n)(u[n]-u[n-5])$

$x[-n] = \cos(\frac{\pi}{2}n)(u[-n]-u[-n-5]$

$X_{even} = \frac{1}{2}\cos(\frac{\pi}{2}n)(u[n]-u[n-5]+u[-n]-u[-n-5]$



$$x_{odd} = \frac{1}{2}\cos\left(\frac{\pi}{2}n\right)\left(u[n] - u[n-5] - u[-n] + u[-n-5]\right)$$