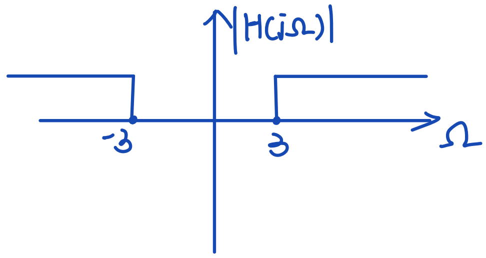
Problem 1

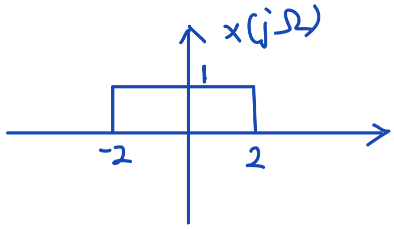
(a) $|H(jw)| = \begin{cases} |e^{-j2w}| & |\Omega| > 3 \\ 0 & \text{elsewhere} \end{cases}$



pass high frequency and stop low frequency

high pass filter

(b)



$y = x(j\Omega) \times H(j\Omega) = 0$

(c) $x(t) = \sin\left(\frac{\pi t}{2}\right) + \cos(5\pi t)$

$X(j\Omega) = \frac{\pi}{j}\left[\delta\left(\Omega - \frac{\pi}{2}\right) - \delta\left(\Omega + \frac{\pi}{2}\right)\right] + \pi\left[\delta(\Omega - 5\pi) + \delta(\Omega + 5\pi)\right]$

$Y(j\Omega) = X(j\Omega) \cdot H(j\Omega)$

$= \pi\left[\delta(\Omega - 5\pi) + \delta(\Omega + 5\pi)\right] e^{-j2\pi}$

$y(t) = \cos(5\pi(t-2))$

Problem 2

$\frac{dy(t)}{dt} + y(t) = x(t)$

apply laplace transform $sY(s) + Y(s) = X(s)$

(a) impulse response $X(s) = 1$

(b) $H(s) = \frac{1}{s+1} \cdot 1$

$h(t) = e^{-t}$

$H(jw) = \frac{1}{1 + jw}$

(c) $w = 0 \quad |H(jw)| = 1$

$w = \infty \quad |H(jw)| = 0$

it is a low pass filter

(d) $h_1(t) = h(t)\cdot\cos(1000t)$

$$h_1(s) = \frac{1}{1+s}\cdot\frac{s}{s^2+10^6}$$

$$= \frac{s}{(1+s)(s^2+10^6)}$$

$$H(j\omega) = \frac{j\omega}{(10^6-\omega^2)(1+j\omega)}$$

$\omega=0$ $\quad |H(j\omega)|=0$

$\omega=\infty$ $\quad |H(j\omega)|=0$

band pass filter

Problem #3

impulse response

$h(n) = 3\delta(n) - 5\delta(n-1) + a\delta(n-2) + b\delta(n-3)$

to have linear shase in its frequency response

$h(n) = \pm h(N-1-n)$, $n=0,1\cdots N-1$

$h(0) = \pm h(3)$

$3 = \pm b$ $\qquad b=\pm 3$

$h(1) = \pm h(2)$

$a = \pm 5$

Problem #4

linear phase $\rightarrow$ symmetircal signal

$a_1 = a_5$ $\qquad a_2 = a_4$

$h(n) = a_1\delta(n+2) + a_2\delta(n+1) + a_3\delta(n) + a_2\delta(n-1) + a_1\delta(n-2)$

$H(e^{j\omega}) = a_1e^{j2\omega} + a_2e^{j\omega} + a_3 + a_2e^{-j\omega} + a_1e^{-j2\omega}$

$H(e^{j\omega}) = a_3 + 2a_1\cos(2\omega) + 2a_2\cos(\omega)$

so if $a_1=a_5$ $\quad a_2=a_4$ then for any a_3, it will have 0 phase