HW4

2022年2月3日 15:55

Throughout the assignment, $u(t)$ is the unit step function, $r(t) = tu(t)$ and $p(t) = u(t) - u(t-1)$

HW3-b Problems:

Given a system T with an impulse response function $h(t) = p(t+2) - p(t+1) : y(t) = T[x(t)] = (x * h)(t)$, do the following:

(a) Compute and sketch the step response function of the system.

(b) Compute and sketch the system output when given the input $x(t) = p\left(\frac{t}{2}+1\right) - (1-t)\,p(t)$

a) $h(t) = p(t+2) - p(t+1)$

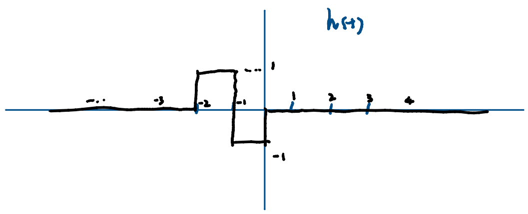
$$= u(t+2) - u(t+2-1) - \left(u(t+1) - u(t+1-1)\right)$$

$$= u(t+2) - u(t+1) - u(t+1) + u(t)$$

$$= u(t+2) - 2u(t+1) + u(t)$$

$$P\left(\frac{\tau}{2}+1\right)$$

$$= \left(\tfrac{1}{2}(\tau+2)\right)$$



$$S(t) = h(t) * u(t)$$

$$= \underbrace{\left(u(t+2) - 2u(t+1) + u(t)\right)}_{x} * \underbrace{u(t)}_{h}$$

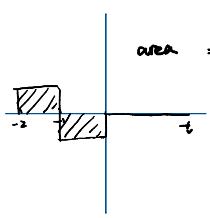
$$= \int_{-\infty}^{\infty} \left(u(\tau+2) - 2u(\tau+1) + u(\tau)\right) u(t-\tau)\, d\tau$$

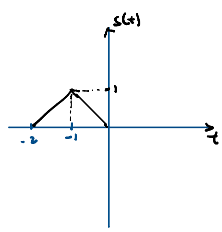
$t+2-2t+2$

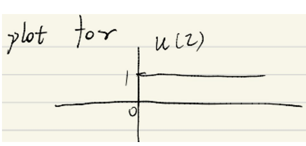
$$= \int_{-\infty}^{t} u(\tau+2)\, d\tau - 2\int_{-\infty}^{t} u(\tau+1)\, d\tau + \int_{-\infty}^{t} u(\tau)\, d\tau$$

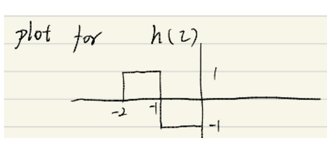
$$= \int_{-2}^{t} 1\, d\tau - 2\int_{-1}^{t} 1\, d\tau + \int_{0}^{t} 1\, d\tau$$

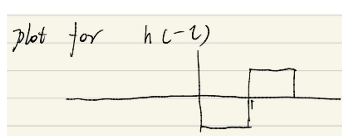
$$S(t) = \begin{cases} 0 & t < -2 \\ t+2 & -2 < t < -1 \\ -t & -1 < t < 0 \\ 0 & 0 < t \end{cases}$$

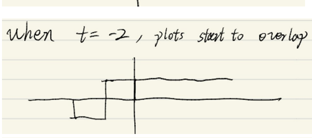


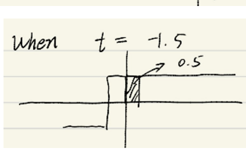


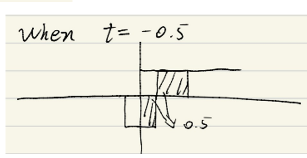


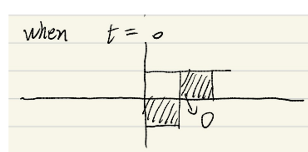


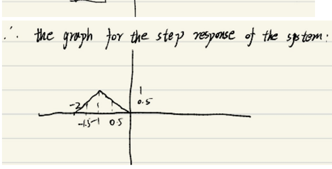












b) $y(t) = T[x(t)] = x(t) * h(t) = x(t) * \left(u(t+2) - 2u(t+1) + u(t) \right)$

$x(t) = u(\frac{t}{2}+1) - u(\frac{t}{2}) - (1-t)\left(u(t) - u(t-1)\right)$

$= u\left(\frac{1}{2}(t+2)\right) - u(t) - u(t) + u(t-1) + tu(t) - t\,u(t-1)$

$= u(t+2) - 2u(t) + tu(t) - t\,u(t-1) + u(t-1)$

$y(t) = \left(u(t+2) + u(t-1) - 2u(t) + tu(t) - t\,u(t-1)\right) * \left(u(t+2) - 2u(t+1) + u(t)\right)$

$= \int_{-\infty}^{\infty} \left(u(\tau+2) + u(\tau-1) - 2u(\tau) + \tau u(\tau) - \tau u(\tau-1)\right) u(t-\tau+2)\, d\tau$

$- 2\int_{-\infty}^{\infty} \left(u(\tau+2) + u(\tau-1) - 2u(\tau) + \tau u(\tau) - \tau u(\tau-1)\right) u(t-\tau+1)\, d\tau$

$+ \int_{-\infty}^{\infty} \left(u(\tau+2) + u(\tau-1) - 2u(\tau) + \tau u(\tau) - \tau u(\tau-1)\right) u(t-\tau)\, d\tau$

$= \int_{-\infty}^{t+2} \left(u(\tau+2) + u(\tau-1) - 2u(\tau) + \tau u(\tau) - \tau u(\tau-1)\right) d\tau$

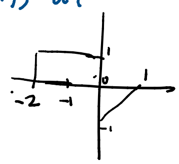
$- 2\int_{-\infty}^{t+1} \left(u(\tau+2) + u(\tau-1) - 2u(\tau) + \tau u(\tau) - \tau u(\tau-1)\right) d\tau$

$+ \int_{-\infty}^{t} \left(u(\tau+2) + u(\tau-1) - 2u(\tau) + \tau u(\tau) - \tau u(\tau-1)\right) d\tau$

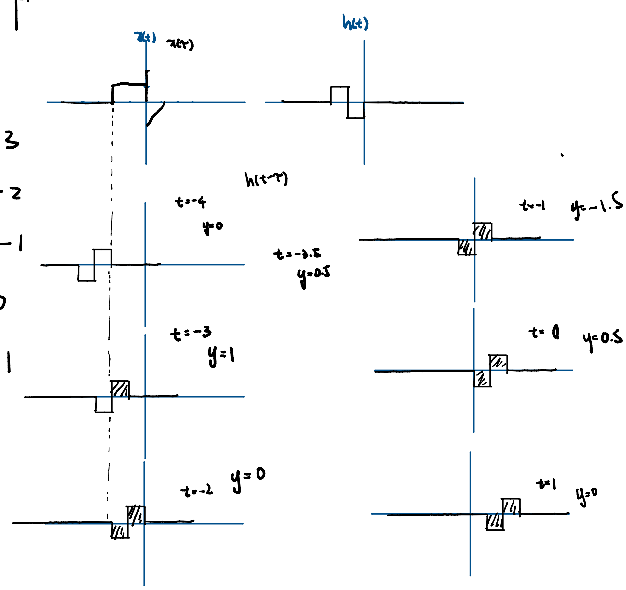
$$y_1 = \begin{cases} 0 & t<-4 \\ t+4 & -4<t<-2 \\ -t+\frac{(t+2)^2}{2} & -2<t<-1 \\ 1.5 & -1<t \end{cases} \checkmark$$

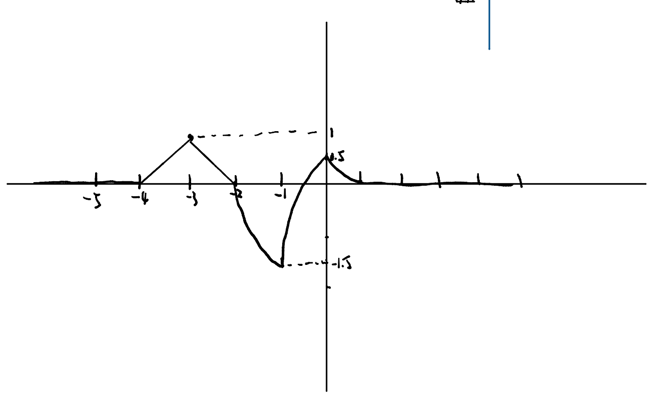
$$y_2 = \begin{cases} 0, & t<-3 \\ 2(t+3) & -3<t<-1 \\ 2\left(1-t+\frac{(t+1)^2}{2}\right) & -1<t<0 \\ 2(1.5) & 0<t \end{cases}$$

$$y_3 = \begin{cases} 0, & t<-2 \\ t+2, & -2<t<0 \\ 2-t+\frac{t^2}{2} & 0<t<1 \\ 1.5 & 1<t \end{cases}$$



$$y(t) = \begin{cases} 0 & t<-4 \\ t+4 & -4<t<-3 \\ t+4-2t-6 = -t-2 & -3<t<-2 \\ \cancel{-t} + \frac{(t+2)^2}{2} - 2t - 6 + \cancel{t} + 2 = -2t + \frac{(t+2)^2}{2} - 4 & -2<t<-1 \\ 1.5 - 2 + 2t - (t+1)^2 + t + 2 = 3t - (t+1)^2 + 1.5 & -1<t<0 \\ 1.5 - 3 + 2 - t + \frac{t^2}{2} = -t + \frac{t^2}{2} + 0.5 & 0<t<1 \\ 0 & 1<t \end{cases}$$





For each of the given step responses $s(t)$ shown below, compute and sketch the corresponding impulse response signal $h(t)$.

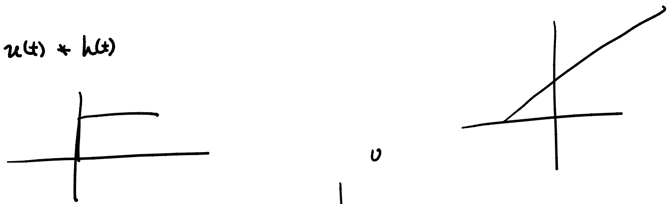
(a) $s(t) = r(t+2) - 2r(t) + 2r(t-1) - r(t-2)$

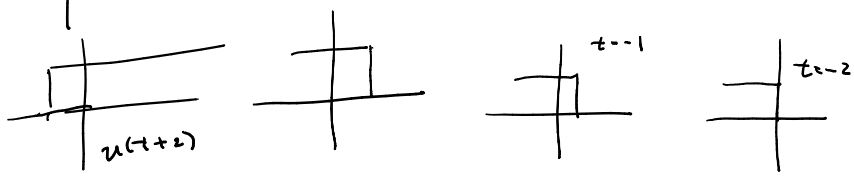
(b) $s[n] = u[n+3] + 2r[n+1] - r[n] - u[n] - 3r[n-1] + 3r[n-3]$

$r(t) = t u(t)$

a) $s(t) = u(t) * h(t)$

$s(t) =$

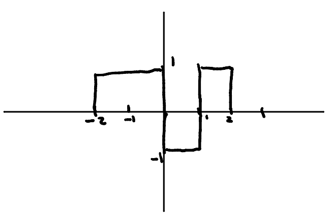




$s(t) = u(t+2) * u(t) - 2u(t) * u(t) + 2u(t-1) * u(t) - u(t-2) * u(t)$

$$= \underbrace{\left(u(t+2) - 2u(t) + 2u(t-1) - u(t-2)\right)}_{h(t)} * u(t)$$

$h(t) = u(t+2) - 2u(t) + 2u(t-1) - u(t-2)$

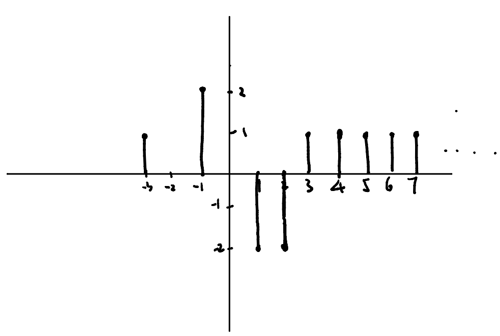


$\delta[n-n_0] * u[n] = u[n-n_0]$

b) $s[n] = \delta[n+3] * u[n] + 2u[n+1] * u[n] - u[n] * u[n] - \delta[n] * u[n] - 3u[n-1] * u[n] + 3u[n-3] * u[n]$

$= (\delta[n+3] + 2u[n+1] - u[n] - \delta[n] - 3u[n-1] + 3u[n-3]) * u[n]$

$h[n] = \delta[n+3] + 2u[n+1] - u[n] - \delta[n] - 3u[n-1] + 3u[n-3]$



Let $y(t) = (x * h)(t)$ with an impulse response $h(t) = te^t u(-t)$. The output $y(t)$ for an input $x(t) = u(t+1) - u(t-2)$ will take the following form:

$$y(t) = \begin{cases} 0 & t \in \{?\} \\ \int_{t_a}^{0} \tau e^{\tau}\, d\tau & t \in \{?\} \\ \int_{t_a}^{t_b} \tau e^{\tau}\, d\tau & t \in \{?\} \end{cases}$$

(a) Solve for t_a, t_b.

(b) Solve for the value and location of $y(t)$'s maximum value, i.e. $\max_{t\in\mathbb{R}}\{y(t)\}$ and $\arg\max_{t\in\mathbb{R}}\{y(t)\}$.

$$y(t) = \underbrace{te^{t}u(-t)}_{x} * \underbrace{(u(t+1) - u(t-2))}_{\text{as } h}$$

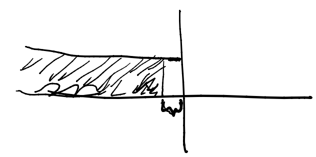
$$y(t) = \int_{-\infty}^{\infty} \tau e^{\tau} u(-\tau)\,(u(t-\tau+1) - u(t-\tau-2))\, d\tau$$

$$= \int_{-\infty}^{0} \tau e^{\tau}\,(u(t-\tau+1)) - u(t-\tau-2))\, d\tau$$

$$= \int_{-\infty}^{0} \tau e^{\tau}\,(u(t-\tau+1))\, d\tau - \int_{-\infty}^{0} \tau e^{\tau}\,(u(t-\tau-2))\, d\tau$$

$$= \int_{-\infty}^{t+1} \tau e^{\tau}\, d\tau - \int_{-\infty}^{t-2} \tau e^{\tau}\, d\tau \qquad \int_{-\infty}^{0} - \int_{-\infty}^{t-2} \tau e^{\tau}\, d\tau \,,$$

$$y(t) \begin{cases} \int_{t-2}^{0} \tau e^{\tau}\, d\tau, & 2 > t > -1 \\ 0, & t > 2 \\ \int_{t-2}^{t+1} \tau e^{\tau}\, d\tau, & t < -1 \end{cases}$$



$t_a = t-2$

$t_b = t+1$

abs max value at $t = -1$ $\quad y(-1) = \int_{-1-2}^{0} \tau e^{\tau}\, d\tau = -0.8009$

0.8009 is abs max

max. is 0., $t > 2$