Bounus7

2022年2月27日 12:10

Bonus
Due Sunday
@ midnight

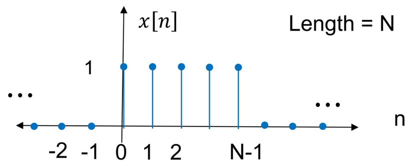
Team exercise

Example: FT of a DT Rectangle

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} e^{-j\omega n} = \sum_{n=0}^{N-1} \left(e^{-j\omega}\right)^n$$

$$= \frac{1-e^{-j\omega N}}{1-e^{-j\omega}} = \frac{e^{-j\omega N/2}}{e^{-j\omega/2}} \frac{\left(e^{j\omega N/2}-e^{-j\omega N/2}\right)}{\left(e^{j\omega/2}-e^{-j\omega/2}\right)} = e^{-j\omega(N-1)/2}\frac{\sin(\omega N/2)}{\sin(\omega/2)} = e^{-j\omega(\frac{2\pi}{\omega}-1)/2}$$



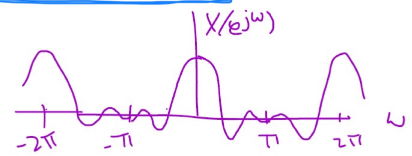
Show math

No sinc

why

If $y[n]$ is an odd length rectangle centered at 0 then: $Y(\omega) = \dfrac{\sin(\omega N/2)}{\sin(\omega/2)}$

N is odd
↓
Lenght



How to plot

1

$$e^{(-j\omega N/2)+j\omega/2}$$

$$e^{\frac{j\omega N}{2}} - e^{\frac{-j\omega N}{2}}$$

set: $a = \frac{\omega N}{2}$

$b = \omega/2$

$= e^{ja} - e^{-ja}$

$= \cos(a) + j\sin(a) - (\cos(-a) + j\sin(-a))$

$= \cos(a) + j\sin(a) - \cos(a) + j\sin(a)$

$= 2j\sin(a)$

$$\frac{\cancel{2j}\sin(a)}{\cancel{2j}\sin(b)} = \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin(\omega/2)} = e^{-j\omega\left(\frac{2\pi}{\omega}-N\right)/2}$$

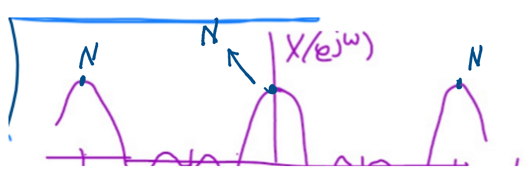
$$= e^{-j\omega(N-1)/2}\frac{\sin(\omega N/2)}{\sin(\omega/2)} = e^{-j\omega(\frac{2\pi}{\omega}-1)/2}$$

$$= e^{-j\omega(N-1)/2}\, e^{-j\omega\left(\frac{2\pi}{\omega}-N\right)/2}$$

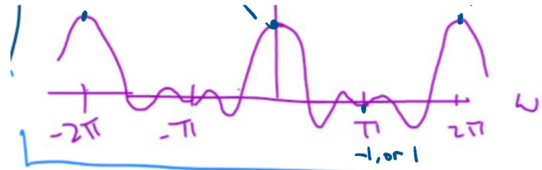
$$= e^{-j\left(\frac{2\pi}{\omega}-1\right)/2}$$

3 → 0
5 → 1
7 → 2
9 → 3

$$Y(\omega) = \frac{\sin(\omega N/2)}{\quad} :$$



$$Y(\omega) = \frac{\sin(\omega N/2)}{\sin(\omega/2)} \;:$$



How to plot 1

mid point:

-1, : $N = 3 + 4k$, k is a constant

1 : $N = 5 + 4k$

number of local min and max: $N-2$

number of zeros: $N-2$