Problem 1: $a+bj = r(\cos\theta + j\sin\theta) = re^{j\theta} = z$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$$re^{j\theta} = r\cos\theta + jr\sin\theta = a+jb$$

$$a = r\cos\theta \qquad b = r\sin\theta \qquad r = \sqrt{a^2+b^2}$$

Problem 2:

5 : $r = \sqrt{5^2} = 5$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right) = 0$ $\quad 5 = 5\cdot e^{0\cdot j}$

-2 : $r = \sqrt{(-2)^2} = 2$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right) = \pi$ $\quad -2 = 2e^{\pi\cdot j}$

$-3j$: $r = \sqrt{-3^2} = 3$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-3}{0}\right) = -\frac{\pi}{2}$

$-3j = 3e^{j\cdot(-\frac{\pi}{2})}$

$1+j$: $r = \sqrt{1^2+1^2} = \sqrt{2}$ $\quad \theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$1+j = \sqrt{2}\cdot e^{j\cdot\frac{\pi}{4}}$

$\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}}$: $\frac{(\sqrt{2}+j\sqrt{2})(1-j\sqrt{3})}{(1+j\sqrt{3})(1-j\sqrt{3})} = \left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}\right) + \left(\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}\right)j$

$r = \sqrt{\left(\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)\cdot\left(\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}\right)} = 1$ $\quad \theta = \tan^{-1}\left(\frac{\frac{\sqrt{2}-\sqrt{6}}{4}}{\frac{\sqrt{2}+\sqrt{6}}{4}}\right)$

$\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = r^{-j\frac{\pi}{12}}$ $\quad = \tan^{-1}\left(\frac{\sqrt{12}-4}{2}\right) = -\frac{\pi}{12}$

Problem 3

a. $\sum_{k=1}^{100} (-2)^k = \frac{-2-(-2)^{101}}{3}$

b. $\sum_{k=-\infty}^{2} (3)^k = \frac{-3^3}{-2} = \frac{27}{2}$

c. $\sum_{k=1}^{\infty} \left(\frac{1}{3}+j\frac{1}{3}\right)^k = \frac{\frac{1}{3}+j\frac{1}{3}}{1-\frac{1}{3}-j\frac{1}{3}} = \frac{\frac{1}{3}+j\frac{1}{3}}{\frac{2}{3}-j\frac{1}{3}}$

d. $\sum_{k=10}^{\infty} (|a|)^k \quad |a|<1 \quad = \frac{a^{10}}{1-|a|}$

Problem 4

$$|\cos t + j\sin t| = \sqrt{\cos^2 t + \sin^2 t} = 1$$

$$u(t) - u(t-1) = \begin{cases} 1 & 0<t<1 \\ 0 & \text{everywhere else} \end{cases}$$

$E = \int_0^1 |\cos t + j\sin t|^2 = 1$

Problem 5:

(a) $E = \int_{-\infty}^{+\infty} |e^{-at} u(t)|^2 dt$

$= \int_0^{\infty} e^{-2at} dt = \left| \frac{e^{-2at}}{-2a} \right|_0^{\infty} = \frac{1}{2a} < \infty$

energy signal $\quad P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0$

(b) $P = \lim_{P\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |A\cos(w_0 t + \theta)|^2$

$= \frac{A^2}{T} \left| \frac{\sin(w_0 t+\theta)\cdot\cos(w_0 t+\theta)}{2w_0} + \frac{t}{2} \right|_{-\frac{T}{2}}^{\frac{T}{2}} < \infty$

power signal

(c) $E = \int_{-\infty}^{+\infty} |t\cdot u(t)|^2 dt = \int_0^{\infty} t^2 dt = \left| \frac{t^3}{3} \right|_0^{+\infty} = \infty$

$P = \lim_{t\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |t^2| dt = \frac{1}{T} \left| \frac{t^3}{3} \right|_0^{\frac{T}{2}} = \frac{T^2}{24} = \infty$ neither

$\frac{T^3}{8}$

Problem 6.

when $t<3 \quad x(t)=0$

(a) $1-t<3 \quad t>1-3 \quad t>-2$

(b) $\begin{cases} 1-t<3 & t>-2 \\ 2-t<3 & t>-1 \end{cases}$ so $t>-1$.

(c) $1-t<3 \quad t>-2$

or

$2-t<3 \quad t>-1$ so $t>-1$

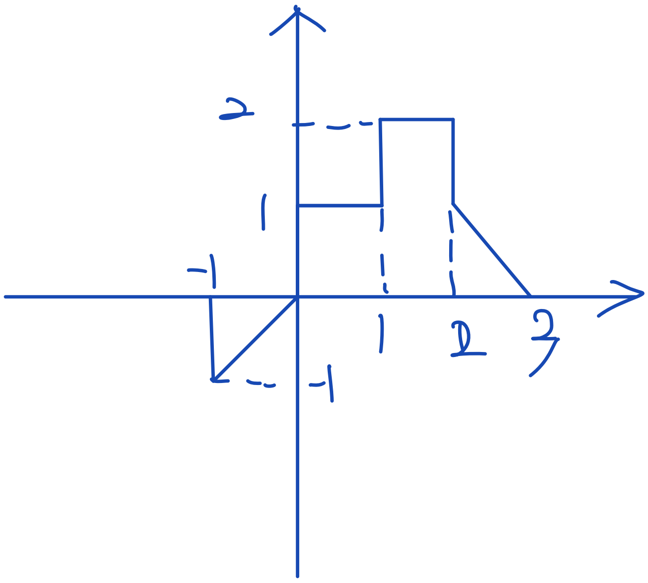
(d) $3t<3 \quad t<1$

(e) $\frac{t}{3}<3 \quad t<9$

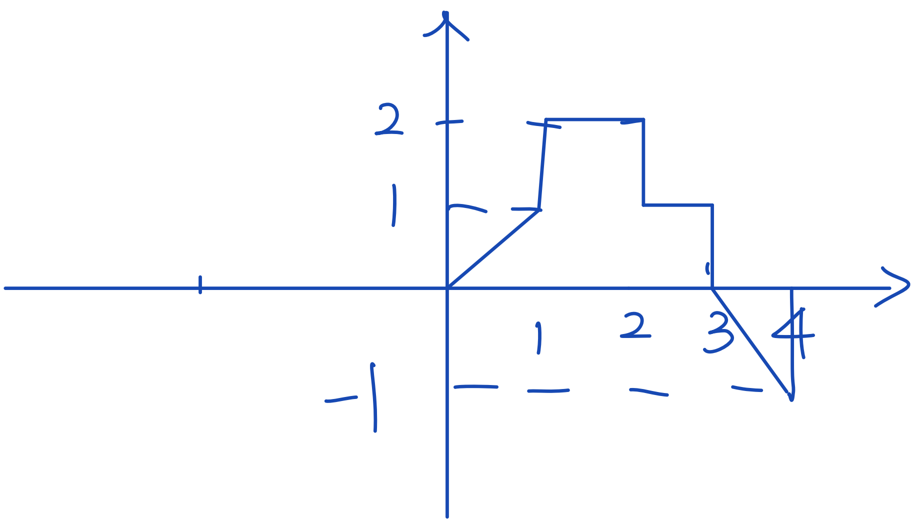
(e)

Problem 7:

(a)



(b)



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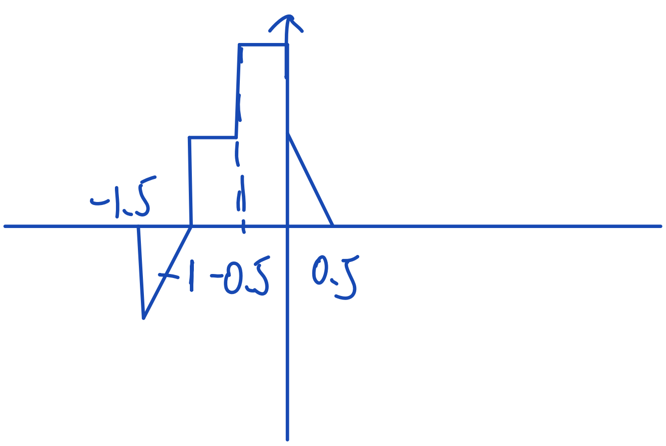

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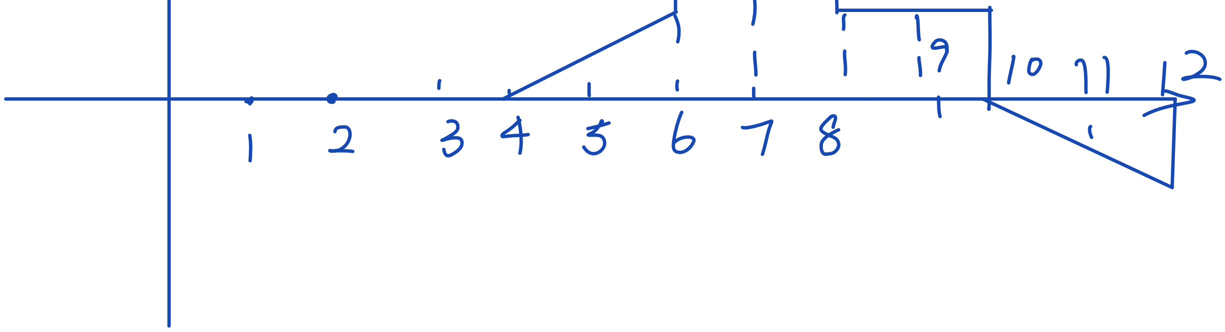
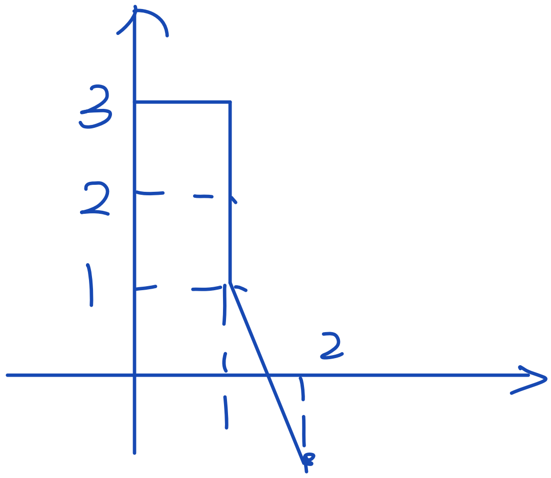


(e)



Problem 8:

(a) $E_\infty = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} u(n) = \frac{4}{3}$

$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \cdot \frac{4}{3} = 0$

(b) $e^{j(\frac{\pi}{2}n + \frac{\pi}{8})} = \cos\left(\frac{\pi}{2}n+\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)$

$E_\infty = \sum_{-\infty}^{\infty} \left|\left(\cos\left(\frac{\pi}{2}n+\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)\right)\right|^2$

$= \sum_{-\infty}^{\infty} \sqrt{\cos^2\left(\frac{\pi}{2}n+\frac{\pi}{8}\right) + \sin^2\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}$

$= \infty$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N} 1 = \frac{2N}{2N+1} = 0$$

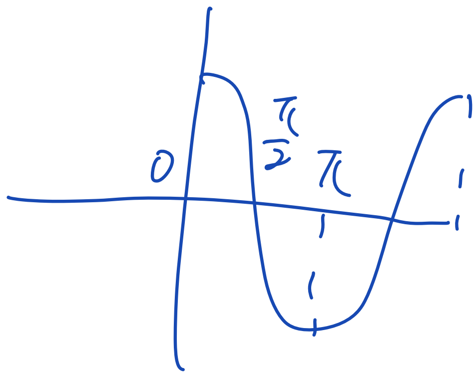
(c) $E_\infty = \sum_{-\infty}^{\infty} \cos(\frac{2\pi}{4} n) = \frac{1}{2}$

$$P_\infty = \lim_{N\to\infty} \frac{E_\infty}{2N+1} = \frac{\infty}{\infty} = \text{undefined}$$

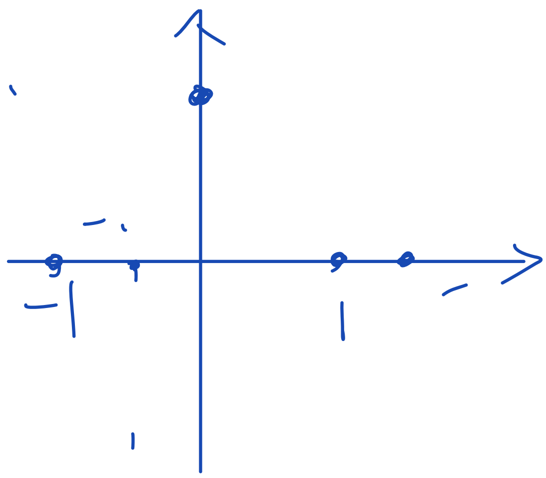
Problem 9: $x(t) \Rightarrow x(2t) \Rightarrow x(2t-4) \Rightarrow x(-2t-4)$

$x(t) \Rightarrow x(-t) \Rightarrow x(-t+2 \Rightarrow x(-2t+2)$

Problem 10:



a.



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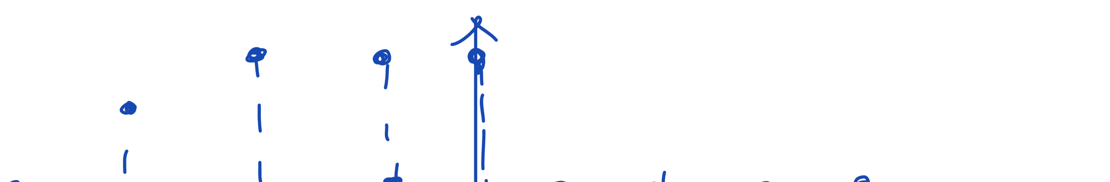
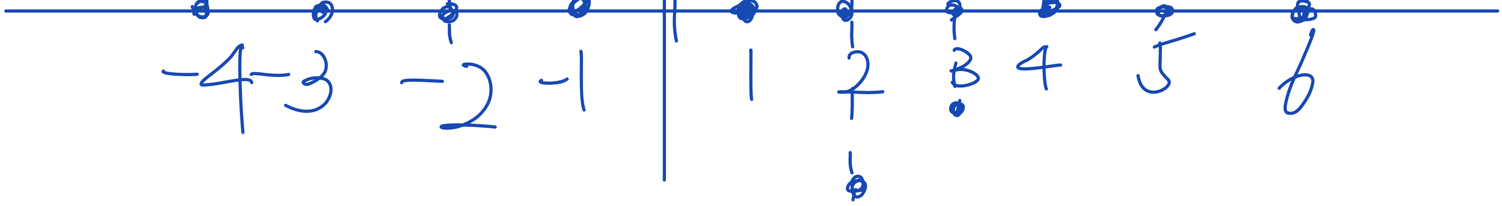
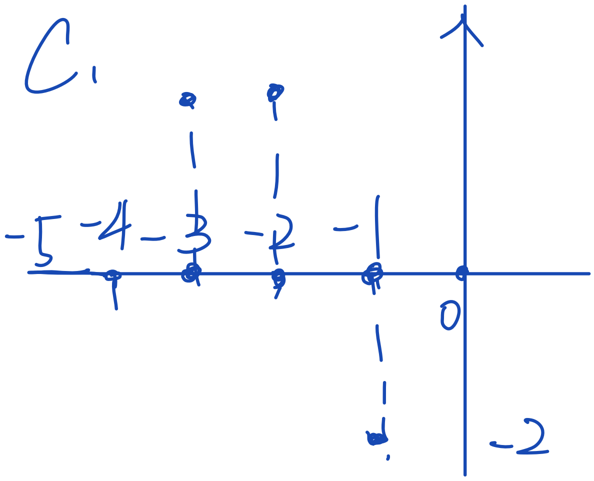




c.



d.