HW3a_Q

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EE 242, Win 2022
Homework 3a

HW3 Topics: convolution, LTI systems, impulse response, step response
HW3 References: Lectures 7-10

NOTE: You will notice that some selected problems are given answers through "Show that" statements. Make sure
you show all your work clearly for every problem.
Throughout the assignment, $u(t)$ is the unit step function, $r(t) = tu(t)$ and $p(t) = u(t) - u(t-1)$

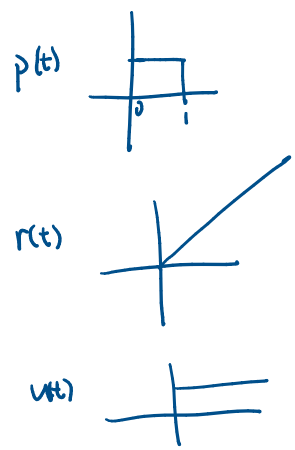
HW3 Problems:

For each of the following LTI systems, compute the system's impulse response function h(t) such that:

$$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

If the system is not LTI, show that this is the case.

a) $y(t) = u(t)\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|} x(t-k)$
b) $y(t) = \int_0^t \tau^a x(t-\tau)d\tau$
c) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau)d\tau$
d) $y[n] = -\frac{1}{8}x[n-3] - \frac{1}{4}x[n-2] - \frac{1}{2}x[n-1]$



a) $a y_1(t) = a\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|} x_1(t-k)$

$a y_1(t) + b y_2(t) = a\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|} x_1(t-k) + b\sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|} x_2(t-k)$

$= \sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}\left(a x_1(t-k) + b x_2(t-k)\right)$

$T\{a x(t)\} = \sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|}\left(a x_1(t-k) + b x_2(t-k)\right)$

same, linear

$y(t-t_0) = \sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|} x(t-t_0-k)$

$T\{x(t-t_0)\} = \sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{|k|} x(t-k-t_0)$

same TI

Then : LTI system

$$\boxed{h(t) = \sum_{k=-1}^{1}\left(\frac{1}{2}\right)^{k}\delta(t-k)}$$

b)

$y(t-t_0) = \int_0^{t-t_0}\tau^a x(t-\tau-t_0)\,d\tau$

$T\{x(t-t_0)\} = \int_0^{t}\tau^a x(t-(\tau-t_0))\,d\tau$ $\boxed{u=\tau-t_0 \\ \tau = u+t_0}$

not same
not TI

$$T\{x(t-t_0)\} = \int_0 \tau^a\, x(t-(\tau-t_0))\, d\tau \qquad \boxed{\begin{matrix} u = \tau - t_0 \\ \tau = u + t_0 \end{matrix}}$$

$$= \int_{-t_0}^{t-t_0} (u+t_0)^a\, x(t-u)\, du$$

$\times$ TI $\rightarrow$ $\times$ LTI

c) $y(t-t_0) = \int_{-\infty}^{t-t_0} e^{-(t-\tau-t_0)}\, x(\tau)\, d\tau$

$$T\{x(t-t_0)\} = \int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau - t_0)\, d\tau \qquad \boxed{\begin{matrix} u = \tau - t_0 \\ \tau = u + t_0 \end{matrix}}$$

$$= \int_{-\infty}^{t-t_0} e^{-(t-u-t_0)}\, x(u)\, d\tau$$

$\rightarrow$ same TI

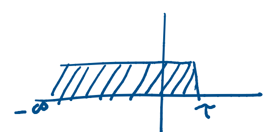
$$a y_1(t) + b y_2(t) = a \int_{-\infty}^{t} e^{(t-\tau)} x_1(\tau)\, d\tau + b \int_{-\infty}^{t} e^{(t-\tau)} x_2(\tau)\, d\tau = \int_{-\infty}^{t} e^{-(t-\tau)} (a x_1(\tau) + b x_2(\tau))\, d\tau$$

$$T\{a x_1(t) + b x_2(t)\} = \int_{-\infty}^{t} e^{-(t-\tau)} (a x_1(\tau) + b x_2(\tau))\, d\tau$$

same Linear

LTI

LTI $\int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau)\, d\tau = \int_{-\infty}^{\infty} \underbrace{x(\tau)}_{x\tau}\, \underbrace{u(t-\tau)\, e^{-(t-\tau)}}_{h(t-\tau)}\, d\tau$



$u(-\tau+t) = u(t-\tau)$

$$\boxed{h(t) = u(t)\, e^{-(t)}}$$

d) $y[n-n_0] = -\frac{1}{8} x[n-n_0-3] - \frac{1}{4} x[n-n_0-2] - \frac{1}{2} x[n-n_0-1]$

$T\{x[n-n_0]\} = -\frac{1}{8} x[n-n_0-3] - \frac{1}{4} x[n-n_0-2] - \frac{1}{2} x[n-n_0-1]$

same, TI

$$a y_1[n] + b y_2[n] = a\left(-\tfrac{1}{8} x_1[n-3] - \tfrac{1}{4} x_1[n-2] - \tfrac{1}{2} x_1[n-1]\right) + b\left(-\tfrac{1}{8} x_2[n-3] - \tfrac{1}{4} x_2[n-2] - \tfrac{1}{2} x_2[n-1]\right)$$

$$T\{a x_1[n] + b x_2[n]\} = -\tfrac{1}{8}(a x_1[n-3] + b x_2[n-3]) - \tfrac{1}{4}(a x_1[n-2] + b x_2[n-2]) - \tfrac{1}{2}(a x_1[n-1] + b x_2[n-1])$$

$$= a\left(-\tfrac{1}{8} x_1[n-3] - \tfrac{1}{4} x_1[n-2] - \tfrac{1}{2} x_1[n-1]\right) + b\left(-\tfrac{1}{8} x_2[n-3] - \tfrac{1}{4} x_2[n-2] - \tfrac{1}{2} x_2[n-1]\right)$$

same, linear

LTI system :

$$h = -\frac{1}{8} \delta[n-3] - \frac{1}{4} \delta[n-2] - \frac{1}{2} \delta[n-1]$$