

**Table 1** Microeconomic Model Calibration

| Calibrated Parameters                 |                 |       |                      |
|---------------------------------------|-----------------|-------|----------------------|
| Description                           | Parameter       | Value | Source               |
| Permanent Income Growth Factor        | $\Gamma$        | 1.03  | PSID: Carroll (1992) |
| Interest Factor                       | $R$             | 1.04  | Conventional         |
| Time Preference Factor                | $\beta$         | 0.96  | Conventional         |
| Coefficient of Relative Risk Aversion | $\rho$          | 2     | Conventional         |
| Probability of Zero Income            | $\wp$           | 0.005 | PSID: Carroll (1992) |
| Std Dev of Log Permanent Shock        | $\sigma_\psi$   | 0.1   | PSID: Carroll (1992) |
| Std Dev of Log Transitory Shock       | $\sigma_\theta$ | 0.1   | PSID: Carroll (1992) |

**Table 2** Model Characteristics Calculated from Parameters

| Description                         | Symbol and Formula   | Approximate<br>Calculated<br>Value |
|-------------------------------------|--|------------------------------------|
| Finite Human Wealth Factor          | $\mathcal{R}^{-1} \equiv \Gamma/R$   | 0.990                              |
| PF Finite Value of Autarky Factor   | $\sqsupset \equiv \beta\Gamma^{1-\rho}$  | 0.932                              |
| Growth Compensated Permanent Shock  | $\underline{\psi} \equiv (\mathbb{E}[\psi^{-1}])^{-1}$                         | 0.990                              |
| Uncertainty-Adjusted Growth         | $\underline{\Gamma} \equiv \Gamma\underline{\psi}$                             | 1.020                              |
| Utility Compensated Permanent Shock | $\underline{\underline{\psi}} \equiv (\mathbb{E}[\psi^{1-\rho}])^{1/(1-\rho)}$ | 0.990                              |
| Utility Compensated Growth          | $\underline{\underline{\Gamma}} \equiv \Gamma\underline{\underline{\psi}}$     | 1.020                              |
| Absolute Patience Factor            | $\mathfrak{P} \equiv (R\beta)^{1/\rho}$  | 0.999                              |
| Return Patience Factor              | $\mathfrak{P}_R \equiv \mathfrak{P}/R$   | 0.961                              |
| Growth Patience Factor              | $\mathfrak{P}_\Gamma \equiv \mathfrak{P}/\Gamma$                               | 0.970                              |
| Normalized Growth Patience Factor   | $\mathfrak{P}_{\underline{\Gamma}} \equiv \mathfrak{P}/\underline{\Gamma}$     | 0.980                              |
| Finite Value of Autarky Factor      | $\sqsubseteq \equiv \beta\Gamma^{1-\rho}\underline{\underline{\psi}}^{1-\rho}$ | 0.941                              |
| Weak Impatience Factor              | $\wp^{1/\rho}\mathfrak{P} \equiv (\wp\beta R)^{1/\rho}$                        | 0.071                              |

**Table 3** Definitions and Comparisons of Conditions

| Perfect Foresight Versions  | Uncertainty Versions  |
|---|---|
| Finite Human Wealth Condition ( <b>FHWC</b> )   |   |
| $\Gamma/R < 1$<br>The growth factor for permanent income $\Gamma$ must be smaller than the discounting factor $R$ for human wealth to be finite.  | $\Gamma/R < 1$<br>The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction.  |
| Absolute Impatience Condition ( <b>AIC</b> )  |   |
| $\mathbf{P} < 1$<br>The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time:<br>$\mathbf{c}_{t+1} < \mathbf{c}_t$  | $\mathbf{P} < 1$<br><i>If wealth is large enough, the expectation of consumption next period will be smaller than this period's consumption:</i><br>$\lim_{m_t \rightarrow \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$                               |
| Return Impatience Conditions  |   |
| Return Impatience Condition ( <b>RIC</b> )  | Weak <b>RIC</b> ( <b>WRIC</b> )   |
| $\mathbf{P}/R < 1$<br>The growth factor for consumption $\mathbf{P}$ must be smaller than the discounting factor $R$ , so that the PDV of current and future consumption will be finite:<br>$c'(m) = 1 - \mathbf{P}/R < 1$  | $\wp^{1/\rho} \mathbf{P}/R < 1$<br>If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the <b>RIC</b> . Otherwise, weaker.<br>$c'(m) < 1 - \wp^{1/\rho} \mathbf{P}/R < 1$                |
| Growth Impatience Conditions  |   |
| <b>GIC</b>  | <b>GIC-Nrm</b>  |
| $\mathbf{P}/\Gamma < 1$<br>For an unconstrained PF consumer, the ratio of $\mathbf{c}$ to $\mathbf{p}$ will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees<br>$\lim_{m_t \uparrow \infty} \mathbb{E}_t[\psi_{t+1} m_{t+1}/m_t] = \mathbf{P}\Gamma$ | $\mathbf{P} \mathbb{E}[\psi^{-1}]/\Gamma < 1$<br>By Jensen's inequality stronger than <b>GIC</b> . Ensures consumers will not expect to accumulate $m$ unboundedly.<br>$\lim_{m_t \rightarrow \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{P}\underline{\Gamma}$ |
| Finite Value of Autarky Conditions  |   |
| <b>PF-FVAC</b>  | <b>FVAC</b>   |
| $\beta \Gamma^{1-\rho} < 1$<br>equivalently $\mathbf{P} < R^{1/\rho} \Gamma^{1-1/\rho}$<br>The discounted utility of constrained consumers who spend their permanent income each period should be finite.   | $\beta \Gamma^{1-\rho} \mathbb{E}[\psi^{1-\rho}] < 1$<br>By Jensen's inequality, stronger than the <b>PF-FVAC</b> because for $\rho > 1$ and nondegenerate $\psi$ , $\mathbb{E}[\psi^{1-\rho}] > 1$ .   |

**Table 4** Sufficient Conditions for Nondegenerate<sup>‡</sup> Solution

| Consumption Model(s)  | Conditions  | Comments  |
|---|---|---|
| $\bar{c}(m)$ : PF Unconstrained<br>$\underline{c}(m) = \underline{\kappa}m$<br><br>Section 2.5.3:<br>Section 2.5.3:<br>Eq (22):<br>Eq (23):                         | <b>RIC, FHWC</b> <sup>°</sup>   | <b>RIC</b> $\Rightarrow  v(m)  < \infty$ ; <b>FHWC</b> $\Rightarrow 0 <  v(m) $<br>PF model with no human wealth ( $h = 0$ )<br><br><b>RIC</b> prevents $\bar{c}(m) = \underline{c}(m) = 0$<br><b>FHWC</b> prevents $\bar{c}(m) = \infty$<br><b>PF-FVAC+FHWC</b> $\Rightarrow$ <b>RIC</b><br><b>GIC+FHWC</b> $\Rightarrow$ <b>PF-FVAC</b>   |
| $\dot{c}(m)$ : PF Constrained<br>Section 2.5.6:<br><br>Appendix A:<br><br>Appendix A:   | <del><b>GIC</b></del> , <b>RIC</b><br><br><b>GIC, RIC</b><br><br><b>GIC, <del>RIC</del></b> | <b>FHWC</b> holds ( $\Gamma < \mathbf{D} < \mathbf{R} \Rightarrow \Gamma < \mathbf{R}$ )<br>$\dot{c}(m) = \bar{c}(m)$ for $m > m_{\#} < 1$<br>( <del><b>RIC</b></del> would yield $m_{\#} = 0$ so $\dot{c}(m) = 0$ )<br>$\lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$ , $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$<br>kinks where horizon to $b = 0$ changes*<br>$\lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$<br>kinks where horizon to $b = 0$ changes*   |
| $c(m)$ : Friedman/Muth<br><br>Section 2.10:<br>Section 2.12:<br>Figure 3:<br>Section 2.12.2:<br>Section 2.12.1:<br>Section 3.3:<br>Section 3.3.2:<br>Section 3.3.1: | Section 3.1,<br>Section 3.2<br><b>FVAC, WRIC</b>  | $\underline{c}(m) < c(m) < \bar{c}(m)$<br>$\underline{v}(m) < v(m) < \bar{v}(m)$<br>Sufficient for Contraction<br><b>WRIC</b> is weaker than <b>RIC</b><br><b>FVAC</b> is stronger than <b>PF-FVAC</b><br><del><b>FHWC</b></del> + <b>RIC</b> $\Rightarrow$ <b>GIC</b> , $\lim_{m \rightarrow \infty} \kappa(m) = \underline{\kappa}$<br><del><b>RIC</b></del> $\Rightarrow$ <del><b>FHWC</b></del> , $\lim_{m \rightarrow \infty} \kappa(m) = 0$<br>“Buffer Stock Saving” Conditions<br><b>GIC</b> $\Rightarrow \exists 0 < \check{m} < \infty$<br><b>GIC-Nrm</b> $\Rightarrow \exists 0 < \hat{m} < \infty$ |

<sup>‡</sup>For feasible  $m$  satisfying  $0 < m < \infty$ , a nondegenerate limiting consumption function defines a unique optimal value of  $c$  satisfying  $0 < c(m) < \infty$ ; a nondegenerate limiting value function defines a corresponding unique value of  $-\infty < v(m) < 0$ . <sup>°</sup>**RIC**, **FHWC** are necessary as well as sufficient for the perfect foresight case. <sup>\*</sup>That is, the first kink point in  $c(m)$  is  $m_{\#}$  s.t. for  $m < m_{\#}$  the constraint will bind now, while for  $m > m_{\#}$  the constraint will bind one period in the future. The second kink point corresponds to the  $m$  where the constraint will bind two periods in the future, etc. <sup>\*\*</sup>In the Friedman/Muth model, the **RIC+FHWC** are sufficient, but *not* necessary for nondegeneracy

**Table 5** Taxonomy of Perfect Foresight Liquidity Constrained Outcomes

For constrained  $\dot{c}$  and unconstrained  $\bar{c}$  consumption functions

| Main Condition<br>Subcondition                  | Math  | Outcome, Comments or Results  |
|---|---|---|
| <del>GIC</del><br>and RIC<br>and <del>RIC</del> | $1 < \mathbf{P}/\Gamma$<br>$\mathbf{P}/R < 1$<br>$1 < \mathbf{P}/R$ | Constraint never binds for $m \geq 1$<br><b>FHWC</b> holds ( $R > \Gamma$ ); $\dot{c}(m) = \bar{c}(m)$ for $m \geq 1$<br>$\dot{c}(m)$ is degenerate: $\dot{c}(m) = 0$                                       |
| <b>GIC</b><br>and RIC<br>and <del>RIC</del>     | $\mathbf{P}/\Gamma < 1$<br>$\mathbf{P}/R < 1$<br>$1 < \mathbf{P}/R$ | Constraint binds in finite time for any $m$<br><b>FHWC</b> may or may not hold<br>$\lim_{m \uparrow \infty} \bar{c}(m) - \dot{c}(m) = 0$<br>$\lim_{m \uparrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ |
| and <del>RIC</del>                              |   | <del><b>FHWC</b></del><br>$\lim_{m \uparrow \infty} \dot{\kappa}(m) = 0$  |

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and **RIC** both hold, while the third row indicates that when the **GIC** and the **RIC** both fail, the consumption function is degenerate; the next row indicates that whenever the **GIC** holds, the constraint will bind in finite time.