

# 1 When Is Consumption Growth Declining in $m$ ?

Figure ?? depicts the expected consumption growth factor as a strictly declining function of the cash-on-hand ratio. To investigate this, define

$$\Upsilon(m_t) \equiv \Phi_{t+1}c(\mathcal{R}_{t+1}a(m_t) + \xi_{t+1})/c(m_t) = c_{t+1}/c_t$$

and the proposition in which we are interested is

$$(d/dm_t) \mathbb{E}_t[\underbrace{\Upsilon(m_t)}_{\equiv \Upsilon_{t+1}}] < 0$$

or differentiating through the expectations operator, what we want is

$$\mathbb{E}_t \left[ \Phi_{t+1} \left( \frac{c'(m_{t+1})\mathcal{R}_{t+1}a'(m_t)c(m_t) - c(m_{t+1})c'(m_t)}{c(m_t)^2} \right) \right] < 0. \quad (1)$$

Henceforth indicating appropriate arguments by the corresponding subscript (e.g.  $c'_{t+1} \equiv c'(m_{t+1})$ ), since  $\Phi_{t+1}\mathcal{R}_{t+1} = R$ , the portion of the LHS of equation (1) in brackets can be manipulated to yield

$$\begin{aligned} c_t \Upsilon'_{t+1} &= c'_{t+1}a'_t R - c'_t \Phi_{t+1}c_{t+1}/c_t \\ &= c'_{t+1}a'_t R - c'_t \Upsilon_{t+1}. \end{aligned}$$

Now differentiate the Euler equation with respect to  $m_t$ :

$$\begin{aligned} 1 &= R\beta \mathbb{E}_t[\Upsilon_{t+1}^{-\rho}] \\ 0 &= \mathbb{E}_t[\Upsilon_{t+1}^{-\rho-1} \Upsilon'_{t+1}] \\ &= \mathbb{E}_t[\Upsilon_{t+1}^{-\rho-1}] \mathbb{E}_t[\Upsilon'_{t+1}] + \text{cov}_t(\Upsilon_{t+1}^{-\rho-1}, \Upsilon'_{t+1}) \\ \mathbb{E}_t[\Upsilon'_{t+1}] &= -\text{cov}_t(\Upsilon_{t+1}^{-\rho-1}, \Upsilon'_{t+1}) / \mathbb{E}_t[\Upsilon_{t+1}^{-\rho-1}] \end{aligned} \quad (2)$$

but since  $\Upsilon_{t+1} > 0$  we can see from (2) that (1) is equivalent to

$$\text{cov}_t(\Upsilon_{t+1}^{-\rho-1}, \Upsilon'_{t+1}) > 0$$

which, using (2), will be true if

$$\text{cov}_t(\Upsilon_{t+1}^{-\rho-1}, c'_{t+1}a'_t R - c'_t \Upsilon_{t+1}) > 0$$

which in turn will be true if both

$$\text{cov}_t(\Upsilon_{t+1}^{-\rho-1}, c'_{t+1}) > 0$$

and

$$\text{cov}_t(\Upsilon_{t+1}^{-\rho-1}, \Upsilon_{t+1}) < 0.$$

The latter proposition is obviously true under our assumption  $\rho > 1$ . The former will be true if

$$\text{cov}_t((\Phi \Psi_{t+1}c(m_{t+1}))^{-\rho-1}, c'(m_{t+1})) > 0.$$

The two shocks cause two kinds of variation in  $m_{t+1}$ . Variations due to  $\xi_{t+1}$  satisfy the proposition, since a higher draw of  $\xi$  both reduces  $c_{t+1}^{-\rho-1}$  and reduces the marginal

propensity to consume. However, permanent shocks have conflicting effects. On the one hand, a higher draw of  $\Psi_{t+1}$  will reduce  $m_{t+1}$ , thus increasing both  $c_{t+1}^{-\rho-1}$  and  $c'_{t+1}$ . On the other hand, the  $c_{t+1}^{-\rho-1}$  term is multiplied by  $\Phi\Psi_{t+1}$ , so the effect of a higher  $\Psi_{t+1}$  could be to decrease the first term in the covariance, leading to a negative covariance with the second term. (Analogously, a lower permanent shock  $\Psi_{t+1}$  can also lead a negative correlation.)