## 1 When Is Consumption Growth Declining in m?

Figure 4 depicts the expected consumption growth factor as a strictly declining function of the cash-on-hand ratio. To investigate this, define

$$\Upsilon(m_t) \equiv \Phi_{t+1} c(\mathcal{R}_{t+1} a(m_t) + \boldsymbol{\xi}_{t+1}) / c(m_t) = \mathbf{c}_{t+1} / \mathbf{c}_t$$

and the proposition in which we are interested is

$$(d/dm_t) \mathbb{E}_t[\underbrace{\Upsilon(m_t)}_{\equiv \Upsilon_{t+1}}] < 0$$

or differentiating through the expectations operator, what we want is

$$\mathbb{E}_t \left[ \mathbf{\Phi}_{t+1} \left( \frac{\mathbf{c}'(m_{t+1}) \mathcal{R}_{t+1} \mathbf{a}'(m_t) \mathbf{c}(m_t) - \mathbf{c}(m_{t+1}) \mathbf{c}'(m_t)}{\mathbf{c}(m_t)^2} \right) \right] < 0. \tag{1}$$

Henceforth indicating appropriate arguments by the corresponding subscript (e.g.  $c'_{t+1} \equiv c'(m_{t+1})$ ), since  $\Phi_{t+1}\mathcal{R}_{t+1} = R$ , the portion of the LHS of equation (1) in brackets can be manipulated to yield

$$c_t \Upsilon'_{t+1} = c'_{t+1} a'_t \mathsf{R} - c'_t \Phi_{t+1} c_{t+1} / c_t$$
  
=  $c'_{t+1} a'_t \mathsf{R} - c'_t \Upsilon_{t+1}$ .

Now differentiate the Euler equation with respect to  $m_t$ :

$$1 = \mathsf{R}\beta \, \mathbb{E}_t [\Upsilon_{t+1}^{-\rho}]$$

$$0 = \mathbb{E}_t [\Upsilon_{t+1}^{-\rho-1} \Upsilon_{t+1}']$$

$$= \mathbb{E}_t [\Upsilon_{t+1}^{-\rho-1}] \, \mathbb{E}_t [\Upsilon_{t+1}'] + \operatorname{cov}_t (\Upsilon_{t+1}^{-\rho-1}, \Upsilon_{t+1}')$$

$$\mathbb{E}_t [\Upsilon_{t+1}'] = -\operatorname{cov}_t (\Upsilon_{t+1}^{-\rho-1}, \Upsilon_{t+1}') / \, \mathbb{E}_t [\Upsilon_{t+1}^{-\rho-1}]$$
(2)

but since  $\Upsilon_{t+1} > 0$  we can see from (2) that (1) is equivalent to

$$\operatorname{cov}_t(\mathbf{\Upsilon}_{t+1}^{-\rho-1},\mathbf{\Upsilon}_{t+1}') > 0$$

which, using (2), will be true if

$$\operatorname{cov}_{t}(\boldsymbol{\Upsilon}_{t+1}^{-\rho-1}, \operatorname{c}'_{t+1} \operatorname{a}'_{t} \mathsf{R} - \operatorname{c}'_{t} \boldsymbol{\Upsilon}_{t+1}) > 0$$

which in turn will be true if both

$$cov_t(\Upsilon_{t+1}^{-\rho-1}, c'_{t+1}) > 0$$

and

$$\operatorname{cov}_t(\boldsymbol{\Upsilon}_{t+1}^{-\rho-1},\boldsymbol{\Upsilon}_{t+1})<0.$$

The latter proposition is obviously true under our assumption  $\rho > 1$ . The former will be true if

$$\operatorname{cov}_{t} \left( (\mathbf{\Phi} \Psi_{t+1} c(m_{t+1}))^{-\rho-1}, c'(m_{t+1}) \right) > 0.$$

The two shocks cause two kinds of variation in  $m_{t+1}$ . Variations due to  $\boldsymbol{\xi}_{t+1}$  satisfy the proposition, since a higher draw of  $\boldsymbol{\xi}$  both reduces  $c_{t+1}^{-\rho-1}$  and reduces the marginal

propensity to consume. However, permanent shocks have conflicting effects. On the one hand, a higher draw of  $\Psi_{t+1}$  will reduce  $m_{t+1}$ , thus increasing both  $c_{t+1}^{-\rho-1}$  and  $c_{t+1}'$ . On the other hand, the  $c_{t+1}^{-\rho-1}$  term is multiplied by  $\Phi\Psi_{t+1}$ , so the effect of a higher  $\Psi_{t+1}$  could be to decrease the first term in the covariance, leading to a negative covariance with the second term. (Analogously, a lower permanent shock  $\Psi_{t+1}$  can also lead a negative correlation.)