

Table 1 Microeconomic Model Calibration

| Calibrated Parameters | | | |
|---------------------------------------|-------------------|-------|----------------------|
| Description | Parameter | Value | Source |
| Permanent Income Growth Factor | Φ | 1.03 | PSID: Carroll (1992) |
| Interest Factor | R | 1.04 | Conventional |
| Time Preference Factor | β | 0.96 | Conventional |
| Coefficient of Relative Risk Aversion | ρ | 2 | Conventional |
| Probability of Zero Income | \wp | 0.005 | PSID: Carroll (1992) |
| Std Dev of Log Permanent Shock | σ_{Ψ} | 0.1 | PSID: Carroll (1992) |
| Std Dev of Log Transitory Shock | σ_{θ} | 0.1 | PSID: Carroll (1992) |

Table 2 Model Characteristics Calculated from Parameters

| Description | Symbol and Formula | Approximate Calculated Value |
|-------------------------------------|--|------------------------------------|
| Finite Human Wealth Factor | $\mathcal{R}^{-1} \equiv \Phi/R$ | 0.990 |
| PF Value of Autarky Factor | $\sqsupset \equiv \beta\Phi^{1-\rho}$ | 0.932 |
| Growth Compensated Permanent Shock | $\underline{\Psi} \equiv (\mathbb{E}[\Psi^{-1}])^{-1}$ | 0.990 |
| Uncertainty-Adjusted Growth | $\underline{\Phi} \equiv \Phi\underline{\Psi}$ | 1.020 |
| Utility Compensated Permanent Shock | $\underline{\underline{\Psi}} \equiv (\mathbb{E}[\Psi^{1-\rho}])^{1/(1-\rho)}$ | 0.990 |
| Utility Compensated Growth | $\underline{\underline{\Phi}} \equiv \Phi\underline{\underline{\Psi}}$ | 1.020 |
| Absolute Patience Factor | $\mathfrak{P} \equiv (R\beta)^{1/\rho}$ | 0.999 |
| Return Patience Factor | $\mathfrak{P}_R \equiv \mathfrak{P}/R$ | 0.961 |
| Growth Patience Factor | $\mathfrak{P}_{\Phi} \equiv \mathfrak{P}/\Phi$ | 0.970 |
| Normalized Growth Patience Factor | $\mathfrak{P}_{\underline{\Phi}} \equiv \mathfrak{P}/\underline{\Phi}$ | 0.980 |
| Value of Autarky Factor | $\sqsubseteq \equiv \beta\Phi^{1-\rho}\underline{\underline{\Psi}}^{1-\rho}$ | 0.941 |
| Weak Return Impatience Factor | $\wp^{1/\rho}\mathfrak{P} \equiv (\wp\beta R)^{1/\rho}$ | 0.071 |

Table 3 Definitions and Comparisons of Conditions

| Perfect Foresight Versions | Uncertainty Versions |
|--|--|
| Finite Human Wealth Condition (FHWC) | |
| $\Phi/R < 1$ The growth factor for permanent income Φ must be smaller than the discounting factor R for human wealth to be finite. | $\Phi/R < 1$ The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction. |
| Absolute Impatience Condition (AIC) | |
| $\mathbf{P} < 1$ The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time: $\mathbf{c}_{t+1} < \mathbf{c}_t$ | $\mathbf{P} < 1$ <i>If wealth is large enough, the expectation of consumption next period will be smaller than this period's consumption:</i> $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$ |
| Return Impatience Conditions | |
| Return Impatience Condition (RIC) | Weak RIC (WRIC) |
| $\mathbf{P}/R < 1$ The growth factor for consumption \mathbf{P} must be smaller than the discounting factor R , so that the PDV of current and future consumption will be finite: $c'(m) = 1 - \mathbf{P}/R < 1$ | $\wp^{1/\rho} \mathbf{P}/R < 1$ If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the RIC . Otherwise, weaker. $c'(m) < 1 - \wp^{1/\rho} \mathbf{P}/R < 1$ |
| Growth Impatience Conditions | |
| GIC | GIC-Nrm |
| $\mathbf{P}/\Phi < 1$ For an unconstrained PF consumer, the ratio of \mathbf{c} to \mathbf{p} will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees $\lim_{m_t \uparrow \infty} \mathbb{E}_t[\Psi_{t+1} m_{t+1}/m_t] = \mathbf{P}_\Phi$ | $\mathbf{P} \mathbb{E}[\Psi^{-1}]/\Phi < 1$ By Jensen's inequality stronger than GIC . Ensures consumers will not expect to accumulate m unboundedly. $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{P}_\Phi$ |
| Finite Value of Autarky Conditions | |
| PF-FVAC | FVAC |
| $\beta \Phi^{1-\rho} < 1$ equivalently $\mathbf{P} < R^{1/\rho} \Phi^{1-1/\rho}$ The discounted utility of constrained consumers who spend their permanent income each period should be finite. | $\beta \Phi^{1-\rho} \mathbb{E}[\Psi^{1-\rho}] < 1$ By Jensen's inequality, stronger than the PF-FVAC because for $\rho > 1$ and nondegenerate Ψ , $\mathbb{E}[\Psi^{1-\rho}] > 1$. |

Table 4 Sufficient Conditions for Nondegenerate[‡] Solution

| Consumption Model(s) | Conditions | Comments |
|---|--|--|
| $\bar{c}(m)$: PF Unconstrained $\underline{c}(m) = \underline{\kappa}m$ Section 2.5.3: Section 2.5.3: Eq (21): Eq (22): | RIC, FHW ^o | RIC $\Rightarrow v(m) < \infty$; FHW ^o $\Rightarrow 0 < v(m) $ PF model with no human wealth ($h = 0$) RIC prevents $\bar{c}(m) = \underline{c}(m) = 0$ FHW ^o prevents $\bar{c}(m) = \infty$ PF-FVAC+FHW ^o \Rightarrow RIC GIC+FHW ^o \Rightarrow PF-FVAC |
| $\dot{c}(m)$: PF Constrained Section 2.5.6: Appendix A: Appendix A: | GIC , RIC GIC, RIC GIC, RIC | FHW ^o holds ($\Phi < \mathbf{P} < R \Rightarrow \Phi < R$) $\dot{c}(m) = \bar{c}(m)$ for $m > m_{\#} < 1$ (RIC would yield $m_{\#} = 0$ so $\dot{c}(m) = 0$) $\lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$, $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ kinks where horizon to $b = 0$ changes* $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$ kinks where horizon to $b = 0$ changes* |
| $c(m)$: Friedman/Muth Section 2.10: Section 2.12: Figure 3: Section 2.12.2: Section 2.12.1: Section 3.3: Section 3.3.2: Section 3.3.1: | Section 3.1, Section 3.2 FVAC, WRIC | $\underline{c}(m) < c(m) < \bar{c}(m)$ $\underline{v}(m) < v(m) < \bar{v}(m)$ Sufficient for Contraction WRIC is weaker than RIC FVAC is stronger than PF-FVAC FHW ^o +RIC \Rightarrow GIC, $\lim_{m \rightarrow \infty} \kappa(m) = \underline{\kappa}$ RIC \Rightarrow FHW^o , $\lim_{m \rightarrow \infty} \kappa(m) = 0$ “Buffer Stock Saving” Conditions GIC $\Rightarrow \exists \tilde{m}$ s.t. $0 < \tilde{m} < \infty$ GIC-Nrm $\Rightarrow \exists \hat{m}$ s.t. $0 < \hat{m} < \infty$ |

[‡]For feasible m satisfying $0 < m < \infty$, a nondegenerate limiting consumption function defines a unique optimal value of c satisfying $0 < c(m) < \infty$; a nondegenerate limiting value function defines a corresponding unique value of $-\infty < v(m) < 0$.

^oRIC, FHW^o are necessary as well as sufficient for the perfect foresight case. *That is, the first kink point in $c(m)$ is $m_{\#}$ s.t. for $m < m_{\#}$ the constraint will bind now, while for $m > m_{\#}$ the constraint will bind one period in the future. The second kink point corresponds to the m where the constraint will bind two periods in the future, etc.

**In the Friedman/Muth model, the RIC+FHW^o are sufficient, but *not* necessary for nondegeneracy

Table 5 Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained \dot{c} and unconstrained \bar{c} consumption functions

| Main Condition Subcondition | Math | Outcome, Comments or Results |
|---|---|---|
| GIC and RIC | $1 < \mathbf{P}/\Phi$ $\mathbf{P}/R < 1$ | Constraint never binds for $m \geq 1$ FHWC holds ($R > \Phi$); $\dot{c}(m) = \bar{c}(m)$ for $m \geq 1$ |
| and RIC GIC and RIC | $1 < \mathbf{P}/R$ $\mathbf{P}/\Phi < 1$ $\mathbf{P}/R < 1$ | $\dot{c}(m)$ is degenerate: $\dot{c}(m) = 0$ Constraint binds in finite time $\forall m$ FHWC may or may not hold $\lim_{m \uparrow \infty} \bar{c}(m) - \dot{c}(m) = 0$ $\lim_{m \uparrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ |
| and RIC | $1 < \mathbf{P}/R$ | FHWC $\lim_{m \uparrow \infty} \dot{\kappa}(m) = 0$ |

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and RIC both hold, while the third row indicates that when the **GIC** and the ~~RIC~~ both fail, the consumption function is degenerate; the next row indicates that whenever the **GIC** holds, the constraint will bind in finite time.