1 When Is Consumption Growth Declining in m?

Figure 4 depicts the expected consumption growth factor as a strictly declining function of the cash-on-hand ratio. To investigate this, define

$$\Upsilon(m_t) \equiv \Gamma_{t+1} c(\mathcal{R}_{t+1} a(m_t) + \xi_{t+1}) / c(m_t) = \mathbf{c}_{t+1} / \mathbf{c}_t$$

and the proposition in which we are interested is

$$(d/dm_t) \, \mathbb{E}_t [\underbrace{\Upsilon(m_t)}_{\equiv \Upsilon_{t+1}}] < 0$$

or differentiating through the expectations operator, what we want is

$$\mathbb{E}_t \left[\Gamma_{t+1} \left(\frac{\mathbf{c}'(m_{t+1}) \mathcal{R}_{t+1} \mathbf{a}'(m_t) \mathbf{c}(m_t) - \mathbf{c}(m_{t+1}) \mathbf{c}'(m_t)}{\mathbf{c}(m_t)^2} \right) \right] < 0. \tag{1}$$

Henceforth indicating appropriate arguments by the corresponding subscript (e.g. $c'_{t+1} \equiv c'(m_{t+1})$), since $\Gamma_{t+1} \mathcal{R}_{t+1} = R$, the portion of the LHS of equation (1) in brackets can be manipulated to yield

$$c_t \Upsilon'_{t+1} = c'_{t+1} a'_t \mathsf{R} - c'_t \Gamma_{t+1} c_{t+1} / c_t$$

= $c'_{t+1} a'_t \mathsf{R} - c'_t \Upsilon_{t+1}$.

Now differentiate the Euler equation with respect to m_t :

$$1 = \mathsf{R}\beta \, \mathbb{E}_{t}[\boldsymbol{\Upsilon}_{t+1}^{-\rho}]$$

$$0 = \mathbb{E}_{t}[\boldsymbol{\Upsilon}_{t+1}^{-\rho-1} \boldsymbol{\Upsilon}_{t+1}']$$

$$= \mathbb{E}_{t}[\boldsymbol{\Upsilon}_{t+1}^{-\rho-1}] \, \mathbb{E}_{t}[\boldsymbol{\Upsilon}_{t+1}'] + \operatorname{cov}_{t}(\boldsymbol{\Upsilon}_{t+1}^{-\rho-1}, \boldsymbol{\Upsilon}_{t+1}')$$

$$\mathbb{E}_{t}[\boldsymbol{\Upsilon}_{t+1}'] = -\operatorname{cov}_{t}(\boldsymbol{\Upsilon}_{t+1}^{-\rho-1}, \boldsymbol{\Upsilon}_{t+1}') / \, \mathbb{E}_{t}[\boldsymbol{\Upsilon}_{t+1}^{-\rho-1}]$$

$$(2)$$

but since $\Upsilon_{t+1} > 0$ we can see from (2) that (1) is equivalent to

$$cov_t(\mathbf{\Upsilon}_{t+1}^{-\rho-1},\mathbf{\Upsilon}_{t+1}') > 0$$

which, using (2), will be true if

$$\operatorname{cov}_{t}(\boldsymbol{\Upsilon}_{t+1}^{-\rho-1}, \operatorname{c}'_{t+1} \operatorname{a}'_{t} \mathsf{R} - \operatorname{c}'_{t} \boldsymbol{\Upsilon}_{t+1}) > 0$$

which in turn will be true if both

$$cov_t(\Upsilon_{t+1}^{-\rho-1}, c'_{t+1}) > 0$$

and

$$\operatorname{cov}_t(\boldsymbol{\Upsilon}_{t+1}^{-\rho-1},\boldsymbol{\Upsilon}_{t+1})<0.$$

The latter proposition is obviously true under our assumption $\rho > 1$. The former will be true if

$$\operatorname{cov}_{t} ((\Gamma \psi_{t+1} c(m_{t+1}))^{-\rho-1}, c'(m_{t+1})) > 0.$$

The two shocks cause two kinds of variation in m_{t+1} . Variations due to ξ_{t+1} satisfy the proposition, since a higher draw of ξ both reduces $c_{t+1}^{-\rho-1}$ and reduces the marginal

propensity to consume. However, permanent shocks have conflicting effects. On the one hand, a higher draw of ψ_{t+1} will reduce m_{t+1} , thus increasing both $c_{t+1}^{-\rho-1}$ and c_{t+1}' . On the other hand, the $c_{t+1}^{-\rho-1}$ term is multiplied by $\Gamma \psi_{t+1}$, so the effect of a higher ψ_{t+1} could be to decrease the first term in the covariance, leading to a negative covariance with the second term. (Analogously, a lower permanent shock ψ_{t+1} can also lead a negative correlation.)