

**Table 1** Microeconomic Model Calibration

Calibrated Parameters			
Description	Parameter	Value	Source
Permanent Income Growth Factor	$\Phi$	1.03	PSID: Carroll (1992)
Interest Factor	R	1.04	Conventional
Time Preference Factor	$\beta$	0.96	Conventional
Coefficient of Relative Risk Aversion	$\rho$	2	Conventional
Probability of Zero Income	$\wp$	0.005	PSID: Carroll (1992)
Std Dev of Log Permanent Shock	$\sigma_\Psi$	0.1	PSID: Carroll (1992)
Std Dev of Log Transitory Shock	$\sigma_\theta$	0.1	PSID: Carroll (1992)

**Table 2** Model Characteristics Calculated from Parameters

Description	Symbol and Formula	Approximate Calculated Value
Finite Human Wealth Factor	$R^{-1} \equiv \Phi/R$	0.990
PF Value of Autarky Factor	$\sqsupset \equiv \beta\Phi^{1-\rho}$	0.932
Growth Compensated Permanent Shock	$\underline{\psi} \equiv (\mathbb{E}[\Psi^{-1}])^{-1}$	0.990
Uncertainty-Adjusted Growth	$\underline{\Phi} \equiv \Phi\underline{\psi}$	1.020
Utility Compensated Permanent Shock	$\underline{\underline{\psi}} \equiv (\mathbb{E}[\Psi^{1-\rho}])^{1/(1-\rho)}$	0.990
Utility Compensated Growth	$\underline{\underline{\Phi}} \equiv \Phi\underline{\underline{\psi}}$	1.020
Absolute Patience Factor	$\mathfrak{P} \equiv (R\beta)^{1/\rho}$	0.999
Return Patience Factor	$\mathfrak{P}_R \equiv \mathfrak{P}/R$	0.961
Growth Patience Factor	$\mathfrak{P}_\Phi \equiv \mathfrak{P}/\Phi$	0.970
Normalized Growth Patience Factor	$\mathfrak{P}_{\underline{\Phi}} \equiv \mathfrak{P}/\underline{\Phi}$	0.980
Value of Autarky Factor	$\sqsubseteq \equiv \beta\Phi^{1-\rho}\underline{\underline{\psi}}^{1-\rho}$	0.941
Weak Return Impatience Factor	$\wp^{1/\rho}\mathfrak{P} \equiv (\wp\beta R)^{1/\rho}$	0.071

**Table 3** Definitions and Comparisons of Conditions

Perfect Foresight Versions	Uncertainty Versions
Finite Human Wealth Condition ( <b>FHWC</b> )	
$\Phi/R < 1$ The growth factor for permanent income $\Phi$ must be smaller than the discounting factor $R$ for human wealth to be finite.	$\Phi/R < 1$ The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction.
Absolute Impatience Condition ( <b>AIC</b> )	
$\mathbf{P} < 1$ The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time: $\mathbf{c}_{t+1} < \mathbf{c}_t$	$\mathbf{P} < 1$ <i>If wealth is large enough, the expectation of consumption next period will be smaller than this period's consumption:</i> $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$
Return Impatience Conditions	
Return Impatience Condition ( <b>RIC</b> )	Weak <b>RIC</b> ( <b>WRIC</b> )
$\mathbf{P}/R < 1$ The growth factor for consumption $\mathbf{P}$ must be smaller than the discounting factor $R$ , so that the PDV of current and future consumption will be finite: $c'(m) = 1 - \mathbf{P}/R < 1$	$\wp^{1/\rho} \mathbf{P}/R < 1$ If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the <b>RIC</b> . Otherwise, weaker. $c'(m) < 1 - \wp^{1/\rho} \mathbf{P}/R < 1$
Growth Impatience Conditions	
<b>GIC</b>	<b>GIC-Nrm</b>
$\mathbf{P}/\Phi < 1$ For an unconstrained PF consumer, the ratio of $\mathbf{c}$ to $\mathbf{p}$ will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees $\lim_{m_t \uparrow \infty} \mathbb{E}_t[\Psi_{t+1} m_{t+1}/m_t] = \mathbf{P}\Phi$	$\mathbf{P} \mathbb{E}[\Psi^{-1}]/\Phi < 1$ By Jensen's inequality stronger than <b>GIC</b> . Ensures consumers will not expect to accumulate $m$ unboundedly. $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{P}\Phi$
Finite Value of Autarky Conditions	
<b>PF-FVAC</b>	<b>FVAC</b>
$\beta \Phi^{1-\rho} < 1$ equivalently $\mathbf{P} < R^{1/\rho} \Phi^{1-1/\rho}$ The discounted utility of constrained consumers who spend their permanent income each period should be finite.	$\beta \Phi^{1-\rho} \mathbb{E}[\Psi^{1-\rho}] < 1$ By Jensen's inequality, stronger than the <b>PF-FVAC</b> because for $\rho > 1$ and nondegenerate $\Psi$ , $\mathbb{E}[\Psi^{1-\rho}] > 1$ .

**Table 4** Sufficient Conditions for Nondegenerate<sup>‡</sup> Solution

Consumption Model(s)	Conditions	Comments
$\bar{c}(m)$ : PF Unconstrained $\underline{c}(m) = \underline{\kappa}m$  Section 2.5.3: Section 2.5.3: Eq (21): Eq (22):	RIC, FHW <sup>°</sup>	RIC $\Rightarrow  v(m)  < \infty$ ; FHW $\Rightarrow 0 <  v(m) $ PF model with no human wealth ( $h = 0$ )  RIC prevents $\bar{c}(m) = \underline{c}(m) = 0$ FHW prevents $\bar{c}(m) = \infty$ PF-FVAC + FHW $\Rightarrow$ RIC GIC + FHW $\Rightarrow$ PF-FVAC
$\dot{c}(m)$ : PF Constrained Section 2.5.6:  Appendix A:  Appendix A:	<del>GIC</del> , RIC  GIC, <del>RIC</del>  GIC, <del>RIC</del>	FHW holds ( $\Phi < \mathbf{P} < R \Rightarrow \Phi < R$ ) $\dot{c}(m) = \bar{c}(m)$ for $m > m_{\#} < 1$ ( <del>RIC</del> would yield $m_{\#} = 0$ so $\dot{c}(m) = 0$ ) $\lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$ , $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ kinks where horizon to $b = 0$ changes* $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$ kinks where horizon to $b = 0$ changes*
$c(m)$ : Friedman/Muth  Section 2.10: Section 2.12: Figure 3: Section 2.12.2: Section 2.12.1: Section 3.3: Section 3.3.2: Section 3.3.1:	Section 3.1, Section 3.2  FVAC, WRIC	$\underline{c}(m) < c(m) < \bar{c}(m)$ $\underline{v}(m) < v(m) < \bar{v}(m)$ Sufficient for Contraction WRIC is weaker than RIC FVAC is stronger than PF-FVAC FHW + RIC $\Rightarrow$ GIC, $\lim_{m \rightarrow \infty} \kappa(m) = \underline{\kappa}$ <del>RIC</del> $\Rightarrow$ <del>FHW</del> , $\lim_{m \rightarrow \infty} \kappa(m) = 0$ “Buffer Stock Saving” Conditions GIC $\Rightarrow \exists 0 < \check{m} < \infty$ GIC-Nrm $\Rightarrow \exists 0 < \hat{m} < \infty$

<sup>‡</sup>For feasible  $m$  satisfying  $0 < m < \infty$ , a nondegenerate limiting consumption function defines a unique optimal value of  $c$  satisfying  $0 < c(m) < \infty$ ; a nondegenerate limiting value function defines a corresponding unique value of  $-\infty < v(m) < 0$ .

<sup>°</sup>RIC, FHW are necessary as well as sufficient for the perfect foresight case. \*That is, the first kink point in  $c(m)$  is  $m_{\#}$  s.t. for  $m < m_{\#}$  the constraint will bind now, while for  $m > m_{\#}$  the constraint will bind one period in the future. The second kink point corresponds to the  $m$  where the constraint will bind two periods in the future, etc.

\*\*In the Friedman/Muth model, the RIC + FHW are sufficient, but *not* necessary for nondegeneracy

**Table 5** Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained  $\bar{c}$  and unconstrained  $\bar{c}$  consumption functions

Main Condition Subcondition	Math	Outcome, Comments or Results
<del>GIC</del> and RIC and <del>RIC</del>	$1 < \mathbf{P}/\Phi$ $\mathbf{P}/R < 1$ $1 < \mathbf{P}/R$	Constraint never binds for $m \geq 1$ <b>FHWC</b> holds ( $R > \Phi$ ); $\dot{c}(m) = \bar{c}(m)$ for $m \geq 1$ $\dot{c}(m)$ is degenerate: $\dot{c}(m) = 0$
GIC and RIC	$\mathbf{P}/\Phi < 1$ $\mathbf{P}/R < 1$	Constraint binds in finite time for any $m$ <b>FHWC</b> may or may not hold $\lim_{m \uparrow \infty} \bar{c}(m) - \dot{c}(m) = 0$ $\lim_{m \uparrow \infty} \dot{\kappa}(m) = \underline{\kappa}$
and <del>RIC</del>	$1 < \mathbf{P}/R$	<del>FHWC</del> $\lim_{m \uparrow \infty} \dot{\kappa}(m) = 0$

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and RIC both hold, while the third row indicates that when the GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the GIC holds, the constraint will bind in finite time.