Theoretical Foundations of Buffer Stock Saving

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September 12, 2019

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Drawbacks of Numerical Solutions

A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- *Very* Hard To Teach!

I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

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Foundations For Microeconomic Household's Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
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Key Result

Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
  - $\Rightarrow \ \exists$ A Unique Consumption Function $\mathrm{c}()$
- There Is A 'Target' Ratio Of Assets to Permanent Income
  - Requires A Key 'Impatience' Condition To Hold
  - Good News
    - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed

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Limit as horizon T goes to infinity of

$$
\begin{aligned}
{}_{t} &={}_{t} -{}_{t} \\
{}_{t+1} &={}_{t} \\
{}_{t+1} &={}_{t+1} \mathsf{R} \\
{}_{t+1} &={}_{t} \underbrace{\Gamma\psi_{t+1}}_{\equiv\Gamma_{t+1}} \\
{}_{t+1} &={}_{t+1} +_{t+1}\xi_{t+1},
\end{aligned}
$$

$$
\xi_{t+n} = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \theta_{t+n}/(1-\wp) & \text{with probability } (1-\wp) \end{cases} \tag{1}
$$

- $u(\bullet) = \bullet^{1-\rho}/(1-\rho)$; $\mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\xi_{t+n}] = 1 \ \forall \ n > 0$; $\beta < 1, \rho > 1$

Surely This Problem Has Been Solved?

No.

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru Matkowski and Nowak (2011) Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!

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Benchmark: Perfect Foresight Model

Definitions:

Absolute Patience Factor	$Þ$	$=$	$(\mathsf{R}\beta)^{1/\rho}$
Return Patience Factor	$Þ_{\mathsf{R}}$	$=$	$Þ/\mathsf{R}$
Perfect Foresight Growth Patience Factor	$Þ_{\Gamma}$	$=$	$Þ/\Gamma$

Name	Condition	Implication
() Absolute Impatience Condition	$Þ < 1$	$\downarrow$ over time
() Return Impatience Condition	$Þ_{\mathsf{R}} < 1$	$/ \downarrow$ over time
() Growth Impatience Condition	$Þ_{\Gamma} < 1$	$/ \downarrow$ over time

When Does A Useful Limiting Solution Exist?

Finite Human Wealth () condition:

$$\Gamma < R \tag{2}$$

Return Impatience Condition:

$$Þ_{R} < R \tag{3}$$

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What If There Are Liquidity Constraints?

- is *not* necessary for solution to exist
- Other Key Condition For Useful Solution is
  'Perfect Foresight Finite Value of Autarky Condition ()':

$$\beta\Gamma^{1-\rho} < 1 \tag{4}$$

- Without , Constraints Are Irrelevant
  - Because Wealth Always Wants To Rise, So Constraint Never Binds

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Liquidity Constraints and Uncertainty

- Introduce permanent shocks to income
- Finite Value of Autarky Condition Becomes

$$: \quad 0 < \overbrace{\beta\underline{\underline{\Gamma}}^{1-\rho}}^{\equiv \underline{\underline{\beth}} =} < 1 \tag{5}$$

$$0 < \beta < \underline{\underline{\Gamma}}^{\rho-1}.$$

Contraction Mapping Requirements

Finite Value of Autarky Condition: Same As In Liq Constr Problem!

$$: \quad 0 < \overbrace{\beta \underline{\underline{\Gamma}}^{1-\rho}}^{\equiv \underline{\underline{\beth}} =} < 1 \tag{6}$$

$$0 < \beta < \underline{\underline{\Gamma}}^{\rho-1}.$$

'Weak Return Impatience Condition' ()

$$0 \leq \quad \wp^{1/\rho} Þ_{\mathsf{R}} \quad < 1 \tag{7}$$

Requirement For Existence Of A Target

Definitions: 'Uncertainty-Adjusted' Growth:

$$\underline{\Gamma} \equiv \Gamma \underline{\psi} < \Gamma$$

Adjusted Growth Patience Factor:

$$: \ \text{Þ}_{\underline{\Gamma}} = \text{Þ}/\underline{\Gamma} = \mathbb{E}[\text{Þ}/(\Gamma\psi)] \tag{8}$$

Growth Impatience Condition:

$$: \ \text{Þ}_{\underline{\Gamma}} < 1, \tag{9}$$

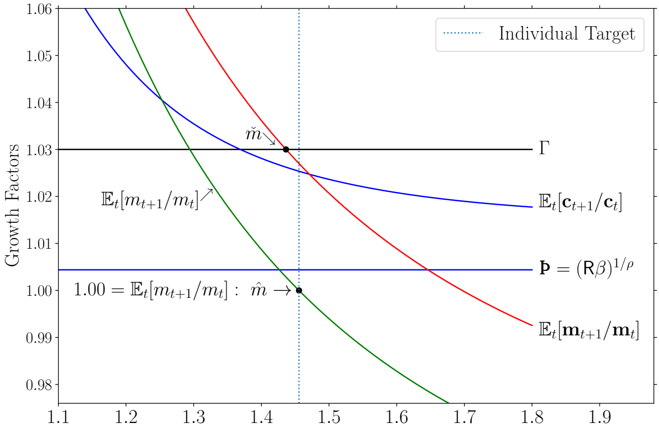
Why? Because it can be shown that

$$\lim_{t\to\infty} \mathbb{E}_t \left[\frac{t+1}{t} \right] \quad = \quad \text{Þ}_{\underline{\Gamma}} \tag{10}$$

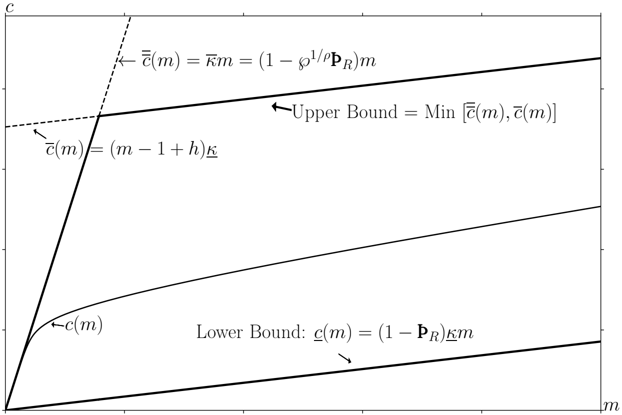
Five Propositions

1. $\lim_{t\rightarrow\infty} \mathbb{E}_t[_{t+1}/_t] = \text{Þ}$
2. $\lim_{t\rightarrow 0} \mathbb{E}_t[_{t+1}/_t] = \infty$
3. $\exists$ a unique target value of , called $\check{}$
4. $\mathbb{E}_t[_{t+1}/_t|_t = \check{m}] = \Gamma - \epsilon$
5. $\left(\frac{d\mathbb{E}_t[_{t+1}/_t]}{d_t}\right) < 0$

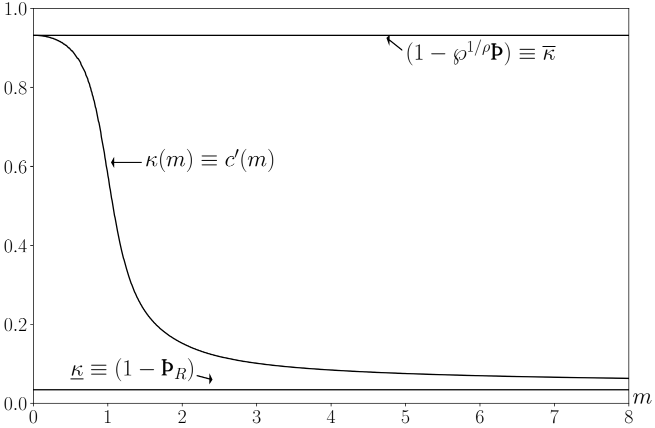
The Target Saving Figure



Bounds On the Consumption Function



The Marginal Propensity to Consume

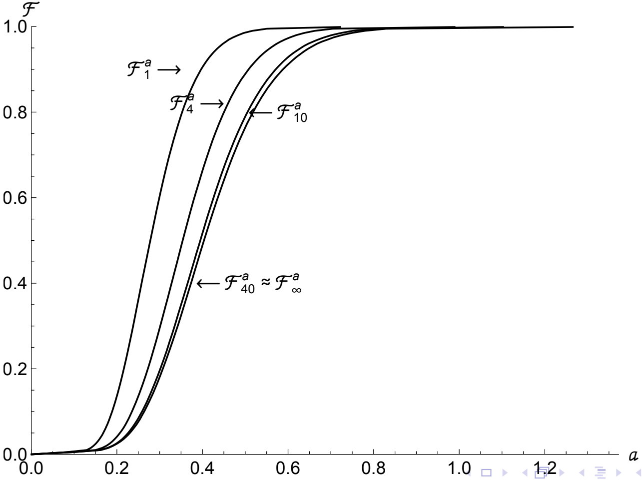


The Consumption Function and Target Wealth

./Figures/cNrmTargetFig.pdf

Convergence To The Invariant Distribution

Szeidl (2013) Proves Existence of an Invariant Distribution of , , , etc.



Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$_{t+1}/_{t} = _{t+1}/_{t} \quad = \quad \Gamma \tag{11}$$

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving $\approx$ Liquidity Constraints
- If $\mathrm{c}()$ is solution for constrained consumer,

$$\lim_{\wp \downarrow 0} \mathrm{c}(;\wp) = \mathrm{c}() \tag{12}$$

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The MPC Out Of Permanent Shocks

https://www.econ2.jhu.edu/people/ccarroll/papers/MPCPerm.pdf

Lots of Recent Papers Trying to Measure the MPCP

Paper Proves:

- MPCP < 1
- But not a lot less:
  - 0.75 to 0.95 (annual rate) for wide range of parameter values

- Defined Conditions Under Which Widely Used Problem Has Solution
  - Finite Value of Autarky Condition Guarantees Contraction (with)
  - Growth Impatience Condition Prevents $m \rightarrow \infty$
- Economy Of Buffer Stock Consumers Exhibits Balanced Growth
  - Even In Absence of General Equilibrium Adj of Interest Rate

Matkowski, Janusz, and Andrzej S. Nowak (2011): "On Discounted Dynamic Programming With Unbounded Returns," *Economic Theory*, 46, 455–474.

Szeidl, Adam (2013): "Stable Invariant Distribution in Buffer-Stock Saving and Stochastic Growth Models," *Manuscript, Central European University*, Available at http://www.personal.ceu.hu/staff/Adam_Szeidl/papers/invariant_revision.pdf.