

# Theoretical Foundations of Buffer Stock Saving

Chris Carroll

Johns Hopkins University

September 12, 2019

Powered By



# Drawbacks of Numerical Solutions

## A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- *Very Hard To Teach!*

## I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

# Drawbacks of Numerical Solutions

## A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- *Very Hard To Teach!*

## I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

# Drawbacks of Numerical Solutions

## A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- Very Hard To Teach!

## I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

# Drawbacks of Numerical Solutions

## A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- Very Hard To Teach!

## I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

# The Gap This Paper Fills

Foundations For Microeconomic Household's Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
- (Problem with Liquidity Constraints Is A Limiting Case)

# The Gap This Paper Fills

Foundations For Microeconomic Household's Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
- (Problem with Liquidity Constraints Is A Limiting Case)

# Key Result

Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
  - $\Rightarrow \exists$  A Unique Consumption Function  $c()$
- There Is A 'Target' Ratio Of Assets to Permanent Income
  - Requires A Key 'Impatience' Condition To Hold
  - Good News
    - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed



# Key Result

## Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
  - $\Rightarrow \exists$  A Unique Consumption Function  $c()$
- There Is A 'Target' Ratio Of Assets To Permanent Income
  - Requires A Key 'Impatience' Condition To Hold
  - Good News
    - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed

# Key Result

Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
  - $\Rightarrow \exists$  A Unique Consumption Function  $c()$
- There Is A 'Target' Ratio Of Assets To Permanent Income
  - Requires A Key 'Impatience' Condition To Hold
  - Good News
    - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed

Limit as horizon  $T$  goes to infinity of

$$\begin{aligned}
 t &= t - t \\
 t+1 &= t \\
 t+1 &= t+1 \text{ R} \\
 t+1 &= t \underbrace{\Gamma \psi_{t+1}}_{\equiv \Gamma_{t+1}} \\
 t+1 &= t+1 + t+1 \xi_{t+1},
 \end{aligned}$$

$$\xi_{t+n} = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \theta_{t+n}/(1 - \wp) & \text{with probability } (1 - \wp) \end{cases} \quad (1)$$

- $u(\bullet) = \bullet^{1-\rho}/(1 - \rho); \mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\xi_{t+n}] = 1 \forall n > 0; \beta < 1, \rho > 1$

# Surely This Problem Has Been Solved?

No.

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru Matkowski and Nowak (2011) Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!

# Surely This Problem Has Been Solved?

No.

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru Matkowski and Nowak (2011) Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!

# Surely This Problem Has Been Solved?

No.

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru Matkowski and Nowak (2011) Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!

## Benchmark: Perfect Foresight Model

Definitions:

Absolute Patience Factor	$\mathbf{P}$	$=$	$(R\beta)^{1/\rho}$
Return Patience Factor	$\mathbf{P}_R$	$=$	$\mathbf{P}/R$
Perfect Foresight Growth Patience Factor	$\mathbf{P}_\Gamma$	$=$	$\mathbf{P}/\Gamma$

Name	Condition	Implication
( ) Absolute Impatience Condition	$\mathbf{P} < 1$	$\downarrow$ over time
( ) Return Impatience Condition	$\mathbf{P}_R < 1$	$/ \downarrow$ over time
( ) Growth Impatience Condition	$\mathbf{P}_\Gamma < 1$	$/ \downarrow$ over time

# When Does A Useful Limiting Solution Exist?

Finite Human Wealth ( ) condition:

$$\Gamma < R \quad (2)$$

Return Impatience Condition:

$$\mathbb{D}_R < R \quad (3)$$



# When Does A Useful Limiting Solution Exist?

Finite Human Wealth ( ) condition:

$$\Gamma < R \quad (2)$$

Return Impatience Condition:

$$\mathbb{D}_R < R \quad (3)$$

# What If There Are Liquidity Constraints?

- is *not* necessary for solution to exist
- Other Key Condition For Useful Solution is  
'Perfect Foresight Finite Value of Autarky Condition ()':

$$\beta \Gamma^{1-\rho} < 1 \quad (4)$$

- Without , Constraints Are Irrelevant
  - Because Wealth Always Wants To Rise, So Constraint Never Binds

## What If There Are Liquidity Constraints?

- is *not* necessary for solution to exist
- Other Key Condition For Useful Solution is  
'Perfect Foresight Finite Value of Autarky Condition ()':

$$\beta \Gamma^{1-\rho} < 1 \quad (4)$$

- Without , Constraints Are Irrelevant
  - Because Wealth Always Wants To Rise, So Constraint Never Binds



## Contraction Mapping Requirements

Finite Value of Autarky Condition: Same As In Liq Constr Problem!

$$\begin{aligned} & \equiv \underline{\underline{\Gamma}} = \\ & : 0 < \underbrace{\beta \underline{\underline{\Gamma}}^{1-\rho}}_{\underline{\underline{\Gamma}}^{\rho-1}} < 1 \\ & 0 < \beta < \underline{\underline{\Gamma}}^{\rho-1}. \end{aligned} \tag{6}$$

'Weak Return Impatience Condition' ()

$$0 \leq \beta^{1/\rho} \underline{\underline{\Gamma}} < 1 \tag{7}$$

## Requirement For Existence Of A Target

Definitions: 'Uncertainty-Adjusted' Growth:

$$\underline{\Gamma} \equiv \Gamma \psi < \Gamma$$

Adjusted Growth Patience Factor:

$$: \mathbf{P}_{\underline{\Gamma}} = \mathbf{P} / \underline{\Gamma} = \mathbb{E}[\mathbf{P} / (\Gamma \psi)] \quad (8)$$

Growth Impatience Condition:

$$: \mathbf{P}_{\underline{\Gamma}} < 1, \quad (9)$$

Why? Because it can be shown that

$$\lim_{t \rightarrow \infty} \mathbb{E}_t \left[ \frac{t+1}{t} \right] = \mathbf{P}_{\underline{\Gamma}} \quad (10)$$

# Five Propositions

- 1  $\lim_{t \rightarrow \infty} \mathbb{E}_t[t+1/t] = \mathfrak{P}$
- 2  $\lim_{t \rightarrow 0} \mathbb{E}_t[t+1/t] = \infty$
- 3  $\exists$  a unique target value of , called  $\checkmark$
- 4  $\mathbb{E}_t[t+1/t | t = \checkmark] = \Gamma - \epsilon$
- 5  $\left( \frac{d\mathbb{E}_t[t+1/t]}{d_t} \right) < 0$

# The Target Saving Figure





# Bounds On the Consumption Function



# The Marginal Propensity to Consume

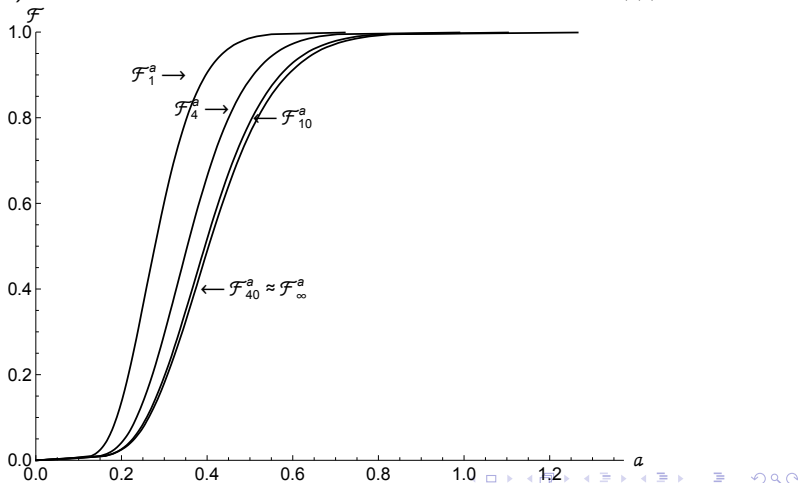


# The Consumption Function and Target Wealth

`./Figures/cNrmTargetFig.pdf`

# Convergence To The Invariant Distribution

Szeidl (2013) Proves Existence of an Invariant Distribution of  $\mathcal{F}$ ,  $a$ , etc.



## Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$x_{t+1}/x_t = y_{t+1}/y_t = \Gamma \quad (11)$$

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving  $\approx$  Liquidity Constraints
- If  $c()$  is solution for constrained consumer,

$$\lim_{\phi \downarrow 0} c(\phi) = c() \quad (12)$$

## Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$x_{t+1}/x_t = y_{t+1}/y_t = \Gamma \quad (11)$$

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving  $\approx$  Liquidity Constraints
- If  $c()$  is solution for constrained consumer,

$$\lim_{\phi \downarrow 0} c(\phi) = c() \quad (12)$$

## Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$x_{t+1}/x_t = y_{t+1}/y_t = \Gamma \quad (11)$$

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving  $\approx$  Liquidity Constraints
- If  $c()$  is solution for constrained consumer,

$$\lim_{\varphi \downarrow 0} c(\varphi) = c() \quad (12)$$

# The MPC Out Of Permanent Shocks

<https://www.econ2.jhu.edu/people/ccarroll1/papers/MPCPerm.pdf>

Lots of Recent Papers Trying to Measure the MPCP

Paper Proves:

- $MPCP < 1$
- But not a lot less:
  - 0.75 to 0.95 (annual rate) for wide range of parameter values



- Defined Conditions Under Which Widely Used Problem Has Solution
  - Finite Value of Autarky Condition Guarantees Contraction (with )
  - Growth Impatience Condition Prevents  $m \rightarrow \infty$
- Economy Of Buffer Stock Consumers Exhibits Balanced Growth
  - Even In Absence of General Equilibrium Adj of Interest Rate

MATKOWSKI, JANUSZ, AND ANDRZEJ S. NOWAK (2011): "On Discounted Dynamic Programming With Unbounded Returns," *Economic Theory*, 46, 455–474.

SZEIDL, ADAM (2013): "Stable Invariant Distribution in Buffer-Stock Saving and Stochastic Growth Models," *Manuscript, Central European University*, Available at [http://www.personal.ceu.hu/staff/Adam\\_Szeidl/papers/invariant\\_revision.pdf](http://www.personal.ceu.hu/staff/Adam_Szeidl/papers/invariant_revision.pdf).