

Theoretical Foundations of Buffer Stock Saving

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Abstract

This paper builds foundations for rigorous and intuitive understanding of ‘buffer stock’ saving models (Bewley (1977)-like models with a wealth target), pairing each theoretical result with quantitative illustrations. After describing conditions under which a consumption function exists, the paper articulates stricter ‘Growth Impatience’ conditions that guarantee alternative forms of stability — either at the population level, or for individual consumers. Together, the numerical tools and analytical results constitute a comprehensive toolkit for understanding buffer stock models.

Keywords Precautionary saving, buffer stock saving, marginal propensity to consume, permanent income hypothesis, income fluctuation problem

JEL codes D81, D91, E21

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The paper's results can be automatically reproduced using the Econ-ARK/HARK toolkit, which can be cited per our references (Carroll, Kaufman, Kazil, Palmer, and White (2018)); for reference to the toolkit itself see Acknowledging Econ-ARK. Thanks to the Consumer Financial Protection Bureau for funding the original creation of the Econ-ARK toolkit; and to the Sloan Foundation for funding Econ-ARK's extensive further development that brought it to the point where it could be used for this project. The toolkit can be cited with its digital object identifier, 10.5281/zenodo.1001067, as is done in the paper's own references as Carroll, Kaufman, Kazil, Palmer, and White (2018). Thanks to Will Du, James Feigenbaum, Joseph Kaboski, Miles Kimball, Qingyin Ma, Misuzu Otsuka, Damiano Sandri, John Stachurski, Adam Szeidl, Alexis Akira Toda, Metin Uyanik, Mateo Velásquez-Giraldo, Weifeng Wu, Jiaxiong Yao, and Xudong Zheng for comments on earlier versions of this paper, John Boyd for help in applying his weighted contraction mapping theorem, Ryoji Hiraguchi for extraordinary mathematical insight that improved the paper greatly, David Zervos for early guidance to the literature, and participants in a seminar at the Johns Hopkins University, a presentation at the 2009 meetings of the Society of Economic Dynamics for their insights, and at a presentation at the Australian National University.

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1 Introduction

In the presence of realistic transitory and permanent shocks to income *a la* Friedman (1957) and Muth (1960), only one further ingredient is required to construct a microeconomically testable model of consumption: A description of preferences. Zeldes (1989) was the first to calibrate a quantitatively plausible example; his paper spawned a literature showing that such models' predictions can match household life cycle data reasonably well, whether or not explicit liquidity constraints are imposed.¹

A connected literature in macroeconomic theory, starting with Bewley (1977), has derived limiting properties of related infinite-horizon problems – but only in models more complex than the case with just shocks and preferences, because standard contraction mapping theorems (beginning with Bellman (1957) and including those building on Stokey et al. (1989)) cannot be applied when utility and/or marginal utility are unbounded. Many proof methods also rule out permanent shocks *a la* Friedman (1957) and Muth (1960) (and Zeldes).²

This paper's first contribution is to articulate conditions under which the infinite-horizon Friedman-Muth(-Zeldes) problem (without complications like a consumption floor or liquidity constraints) defines a contraction mapping whose limit is sensible as the horizon approaches infinity. A 'Finite Value of Autarky Condition' is mostly sufficient (the other requirement, the 'Weak Return Impatience Condition',³ is unlikely to bind). Because the infinite-horizon solution is the limit of finite-horizon recursions, many intermediate results are also useful for solving finite-horizon problems.

But the main theoretical contribution is to identify, for the infinite-horizon case, conditions under which 'stable' points exist (wealth can be predicted to move toward a 'target'; alternatively, there is a 'balanced growth' equilibrium) either for individual consumers or for the aggregate. The requirement for stability is always that the model's parameters satisfy a 'Growth Impatience Condition' whose details depend on the quantity whose stability is of interest. A model with stable points qualifies as a 'buffer stock' model.⁴

Even without a formal proof of its existence, buffer stock saving has been intuitively understood to underlie central quantitative results in heterogeneous agent macroeconomics; for example, the logic of target saving is central to the claim by Krueger, Mitman, and Perri (2016) in the *Handbook of Macroeconomics* that such models explain why, during the Great Recession, middle-class consumers cut their spending more than the poor or the rich. The theory below provides the rigorous basis for this claim: Learning that the future has become more uncertain does not change the urgent imperatives of

¹See Carroll (1997) or Gourinchas and Parker (2002) for arguments that models with only 'natural' constraints (see below) match a wide variety of facts; for a model with explicit constraints that produces very similar results, see, e.g. Cagetti (2003).

²See the fuller discussion at the end of Section 2.1.

³This is a generalization of a condition in Ma, Stachurski, and Toda (2020).

⁴Buffer stock models are neither a subset nor a superset of Bewley (1977) models. But closed economies in which capital accumulation is limited by declining marginal productivity are always 'buffer stock' economies, because the declining marginal product of capital guarantees that a Growth Impatience Condition will hold in equilibrium (see below). The less obvious applications are to populations (or economies) whose marginal saving behavior does not determine the relevant interest rate, or in which the marginal product of capital does not fall as capital is accumulated (again, see below).

the poor (their high $u'(c)$ means they — optimally — have little room to maneuver). And, increased labor income uncertainty does not much change the behavior of the rich because it poses little risk to their consumption. Only people in the middle have both the motivation and the wiggle-room to respond to uncertainty by reducing their spending.

Analytical derivations required for the proofs also explain many other results familiar from the numerical literature.

The paper begins by describing sufficient conditions for the problem to define a sensible (nondegenerate) limiting consumption function (and explains how the model relates to those previously considered). The conditions are interestingly parallel to those required for the **liquidity constrained perfect foresight model**; that parallel is explored and explained. This analysis establishes limiting properties of the consumption function as resources approach infinity, and as they approach their lower bound; using these limits, the contraction mapping theorem is proven.

The next theoretical contribution demonstrates that a model with an ‘artificial’ liquidity constraint (it prohibits borrowing by consumers who could certainly repay) is a limiting case of the unconstrained model. The analytical appeal of the unconstrained model is that it is both mathematically convenient (e.g., the consumption function is twice continuously differentiable), and arbitrarily close (cf. Section ??) to less tractable models. This congenial environment makes proofs easier (if we define a proposition as holding (in the limit) if it continues to hold as the horizon extends to infinity).

In proving the remaining theorems, the **next section** examines key properties of the model. First, **as cash approaches infinity** the expected growth rate of consumption and the marginal propensity to consume (MPC) converge to their values in the perfect foresight case. Second, **as cash approaches zero** the expected growth rate of consumption approaches infinity, and the MPC approaches a simple analytical limit. Next, the central theorems articulate conditions under which different measures of ‘growth impatience’ imply useful conclusions about points of stability (‘target’ or ‘balanced growth’ points).

The final section elaborates the conditions under which, even with a fixed aggregate interest rate that differs from the time preference rate, a small open economy populated by buffer stock consumers has a balanced growth equilibrium in which growth rates of consumption, income, and wealth match the exogenous growth rate of permanent income (equivalent, here, to productivity growth). In the terms of Schmitt-Grohé and Uribe (2003), buffer stock saving is an appealing method of ‘closing’ a small open economy model, because it requires no ad-hoc assumptions. Not even liquidity constraints.⁵

2 The Problem

2.1 Setup

The infinite horizon solution is the (limiting) first-period solution to a sequence of finite-horizon problems as the horizon (the last period of life) becomes arbitrarily distant.

⁵The paper’s insights are instantiated in the **Econ-ARK** toolkit, whose **buffer stock saving module** flags parametric choices under which a problem is degenerate or under which stable ratios of wealth to income may not exist.

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