**Table 1** Microeconomic Model Calibration

| Calibrated Parameters | | | |
|---|---|---|---|
| Description | Parameter | Value | Source |
| Permanent Income Growth Factor | $\Gamma$ | 1.03 | PSID: Carroll (1992) |
| Interest Factor | $\mathsf{R}$ | 1.04 | Conventional |
| Time Preference Factor | $\beta$ | 0.96 | Conventional |
| Coefficient of Relative Risk Aversion | $\rho$ | 2 | Conventional |
| Probability of Zero Income | $\wp$ | 0.005 | PSID: Carroll (1992) |
| Std Dev of Log Permanent Shock | $\sigma_\psi$ | 0.1 | PSID: Carroll (1992) |
| Std Dev of Log Transitory Shock | $\sigma_\theta$ | 0.1 | PSID: Carroll (1992) |

**Table 2** Model Characteristics Calculated from Parameters

| Description | Symbol and Formula | | | Approximate Calculated Value |
|---|---|---|---|---|
| Finite Human Wealth Factor | $\mathcal{R}^{-1}$ | $\equiv$ | $\Gamma/\mathsf{R}$ | 0.990 |
| PF Finite Value of Autarky Factor | $\beth$ | $\equiv$ | $\beta\Gamma^{1-\rho}$ | 0.932 |
| Growth Compensated Permanent Shock | $\underline{\psi}$ | $\equiv$ | $(\mathbb{E}[\psi^{-1}])^{-1}$ | 0.990 |
| Uncertainty-Adjusted Growth | $\underline{\Gamma}$ | $\equiv$ | $\Gamma\underline{\psi}$ | 1.020 |
| Utility Compensated Permanent Shock | $\underline{\underline{\psi}}$ | $\equiv$ | $(\mathbb{E}[\psi^{1-\rho}])^{1/(1-\rho)}$ | 0.990 |
| Utility Compensated Growth | $\underline{\underline{\Gamma}}$ | $\equiv$ | $\Gamma\underline{\underline{\psi}}$ | 1.020 |
| Absolute Patience Factor | $\Þ$ | $\equiv$ | $(\mathsf{R}\beta)^{1/\rho}$ | 0.999 |
| Return Patience Factor | $\Þ_{\mathsf{R}}$ | $\equiv$ | $\Þ/\mathsf{R}$ | 0.961 |
| Growth Patience Factor | $\Þ_{\Gamma}$ | $\equiv$ | $\Þ/\Gamma$ | 0.970 |
| Normalized Growth Patience Factor | $\Þ_{\underline{\Gamma}}$ | $\equiv$ | $\Þ/\underline{\Gamma}$ | 0.980 |
| Finite Value of Autarky Factor | $\underline{\underline{\beth}}$ | $\equiv$ | $\beta\Gamma^{1-\rho}\underline{\underline{\psi}}^{1-\rho}$ | 0.941 |
| Weak Impatience Factor | $\wp^{1/\rho}\Þ$ | $\equiv$ | $(\wp\beta\mathsf{R})^{1/\rho}$ | 0.071 |

**Table 3** Definitions and Comparisons of Conditions

| Perfect Foresight Versions | Uncertainty Versions |
|---|---|
| **Finite Human Wealth Condition (FHWC)** | |
| $\Gamma/\mathsf{R} < 1$<br>The growth factor for permanent income $\Gamma$ must be smaller than the discounting factor $\mathsf{R}$ for human wealth to be finite. | $\Gamma/\mathsf{R} < 1$<br>The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction. |
| **Absolute Impatience Condition (AIC)** | |
| $\Thorn < 1$<br>The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time:<br>$\mathbf{c}_{t+1} < \mathbf{c}_t$ | $\Thorn < 1$<br>*If wealth is large enough,* the *expectation* of consumption next period will be smaller than this period's consumption:<br>$\lim_{m_t \to \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$ |
| **Return Impatience Conditions** | |
| Return Impatience Condition (RIC) | Weak RIC (WRIC) |
| $\Thorn/\mathsf{R} < 1$<br>The growth factor for consumption $\Thorn$ must be smaller than the discounting factor $\mathsf{R}$, so that the PDV of current and future consumption will be finite:<br>$\mathrm{c}'(m) = 1 - \Thorn/\mathsf{R} < 1$ | $\wp^{1/\rho}\Thorn/\mathsf{R} < 1$<br>If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the RIC. Otherwise, weaker.<br>$\mathrm{c}'(m) < 1 - \wp^{1/\rho}\Thorn/\mathsf{R} < 1$ |
| **Growth Impatience Conditions** | |
| GIC | GIC-Nrm |
| $\Thorn/\Gamma < 1$<br>For an unconstrained PF consumer, the ratio of $\mathbf{c}$ to $\mathbf{p}$ will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees $\lim_{m_t \uparrow \infty} \mathbb{E}_t[\psi_{t+1} m_{t+1}/m_t] = \Thorn_\Gamma$ | $\Thorn\, \mathbb{E}[\psi^{-1}]/\Gamma < 1$<br>By Jensen's inequality stronger than GIC. Ensures consumers will not expect to accumulate $m$ unboundedly.<br>$\lim_{m_t \to \infty} \mathbb{E}_t[m_{t+1}/m_t] = \Thorn_{\underline{\Gamma}}$ |
| **Finite Value of Autarky Conditions** | |
| PF-FVAC | FVAC |
| $\beta\Gamma^{1-\rho} < 1$<br>equivalently $\Thorn < \mathsf{R}^{1/\rho}\Gamma^{1-1/\rho}$<br>The discounted utility of constrained consumers who spend their permanent income each period should be finite. | $\beta\Gamma^{1-\rho}\, \mathbb{E}[\psi^{1-\rho}] < 1$<br>By Jensen's inequality, stronger than the PF-FVAC because for $\rho > 1$ and nondegenerate $\psi$, $\mathbb{E}[\psi^{1-\rho}] > 1$. |

**Table 4** Conditions for Nondegenerate[‡] Solution

| Consumption Model(s) | Conditions | Comments |
|---|---|---|
| $\bar{\mathrm{c}}(m)$: PF Unconstrained | RIC, FHWC° | RIC⇒ $\lvert \mathrm{v}(m) \rvert < \infty$; FHWC⇒ $0 < \lvert \mathrm{v}(m) \rvert$ |
| $\underline{\mathrm{c}}(m) = \underline{\kappa} m$ | | PF model with no human wealth ($h = 0$) |
| Section 2.4.2: | | RICprevents $\bar{\mathrm{c}}(m) = \underline{\mathrm{c}}(m) = 0$ |
| Section 2.4.2: | | FHWCprevents $\bar{\mathrm{c}}(m) = \infty$ |
| Eq (26): | | PF-FVAC+FHWC ⇒ RIC |
| Eq (27): | | GIC+FHWC ⇒ PF-FVAC |
| $\grave{\mathrm{c}}(m)$: PF Constrained | ~~GIC~~, RIC | FHWC holds ($\Gamma < \mathbf{\mathfrak{P}} < \mathsf{R} \Rightarrow \Gamma < \mathsf{R}$) |
| Section 2.4.3: | | $\grave{\mathrm{c}}(m) = \bar{\mathrm{c}}(m)$ for $m > m_{\#} < 1$ |
| | | (~~RIC~~ would yield $m_{\#} = 0$ so $\grave{\mathrm{c}}(m) = 0$) |
| Appendix A: | GIC,RIC | $\lim_{m\to\infty} \grave{c}(m) = \bar{c}(m), \lim_{m\to\infty} \grave{\boldsymbol{\kappa}}(m) = \underline{\kappa}$ |
| | | kinks where horizon to $b = 0$ changes* |
| Appendix A: | GIC,~~RIC~~ | $\lim_{m\to\infty} \grave{\boldsymbol{\kappa}}(m) = 0$ |
| | | kinks where horizon to $b = 0$ changes* |
| $\mathrm{c}(m)$: Friedman/Muth | Section 3.1, | $\underline{\mathrm{c}}(m) < \mathrm{c}(m) < \bar{\mathrm{c}}(m)$ |
| | Section 3.2 | $\underline{\mathrm{v}}(m) < \mathrm{v}(m) < \bar{\mathrm{v}}(m)$ |
| Section 2.9: | FVAC, WRIC | Sufficient for Contraction |
| Section 2.11.1: | | WRICis weaker than RIC |
| Figure 3: | | FVACis stronger than PF-FVAC |
| Section 2.11.3: | | ~~FHWC~~+RIC ⇒GIC, $\lim_{m\to\infty} \boldsymbol{\kappa}(m) = \underline{\kappa}$ |
| Section 2.11.2: | | ~~RIC~~ ⇒~~FHWC~~, $\lim_{m\to\infty} \boldsymbol{\kappa}(m) = 0$ |
| Section 3.3: | | "Buffer Stock Saving" Conditions |
| Section 3.3.2: | | GIC ⇒ $\exists 0 < \check{m} < \infty$ |
| Section 3.3.1: | | GIC-Nrm ⇒ $\exists 0 < \hat{m} < \infty$ |

[‡]For feasible $m$ satisfying $0 < m < \infty$, a nondegenerate limiting consumption function defines a unique optimal value of $c$ satisfying $0 < c(m) < \infty$; a nondegenerate limiting value function defines a corresponding unique value of $-\infty < \mathrm{v}(m) < 0$ . °RIC, FHWCare necessary as well as sufficient for the perfect foresight case. *That is, the first kink point in $c(m)$ is $m_{\#}$ s.t. for $m < m_{\#}$ the constraint will bind now, while for $m > m_{\#}$ the constraint will bind one period in the future. The second kink point corresponds to the $m$ where the constraint will bind two periods in the future, etc.**In the Friedman/Muth model, the RIC+FHWC are sufficient, but *not* necessary for nondegeneracy

**Table 5** Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained $\grave{c}$ and unconstrained $\bar{c}$ consumption functions

| Main Condition<br>Subcondition | Math | Outcome, Comments or Results |
|---|---|---|
| ~~GIC~~ | $1 < \Phi/\Gamma$ | Constraint never binds for $m \geq 1$ |
| and RIC | $\Phi/R < 1$ | FHWC holds ($R > \Gamma$); $\grave{c}(m) = \bar{c}(m)$ for $m \geq 1$ |
| and ~~RIC~~ | $1 < \Phi/R$ | $\grave{c}(m)$ is degenerate: $\grave{c}(m) = 0$ |
| GIC | $\Phi/\Gamma < 1$ | Constraint binds in finite time for any $m$ |
| and RIC | $\Phi/R < 1$ | FHWC may or may not hold |
| | | $\lim_{m\uparrow\infty} \bar{c}(m) - \grave{c}(m) = 0$ |
| | | $\lim_{m\uparrow\infty} \grave{\kappa}(m) = \underline{\kappa}$ |
| and ~~RIC~~ | $1 < \Phi/R$ | ~~FHWC~~ |
| | | $\lim_{m\uparrow\infty} \grave{\kappa}(m) = 0$ |

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and RIC both hold, while the third row indicates that when the GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the GICholds, the constraint will bind in finite time.