Table 1 Microeconomic Model Calibration

Calibrated Parameters			
Description	Parameter	Value	Source
Permanent Income Growth Factor	$\boldsymbol{\Phi}$	1.03	PSID: Carroll (1992)
Interest Factor	R	1.04	Conventional
Time Preference Factor	β	0.96	Conventional
Coefficient of Relative Risk Aversion	ρ	2	Conventional
Probability of Zero Income	$\wp$	0.005	PSID: Carroll (1992)
Std Dev of Log Permanent Shock	$\sigma_{\boldsymbol{\Psi}}$	0.1	PSID: Carroll (1992)
Std Dev of Log Transitory Shock	σ_{θ}	0.1	PSID: Carroll (1992)

Table 2 Model Characteristics Calculated from Parameters

Description	Symbol and Formula			Approximate Calculated Value
Finite Human Wealth Factor	$\mathcal{R}^{-1}$	$\equiv$	$\boldsymbol{\Phi}/\mathsf{R}$	0.990
PF Value of Autarky Factor	$\beth$	$\equiv$	$\beta\boldsymbol{\Phi}^{1-\rho}$	0.932
Growth Compensated Permanent Shock	$\underline{\boldsymbol{\Psi}}$	$\equiv$	$(\mathbb{E}[\boldsymbol{\Psi}^{-1}])^{-1}$	0.990
Uncertainty-Adjusted Growth	$\underline{\boldsymbol{\Phi}}$	$\equiv$	$\boldsymbol{\Phi}\underline{\boldsymbol{\Psi}}$	1.020
Utility Compensated Permanent Shock	$\underline{\underline{\boldsymbol{\Psi}}}$	$\equiv$	$(\mathbb{E}[\boldsymbol{\Psi}^{1-\rho}])^{1/(1-\rho)}$	0.990
Utility Compensated Growth	$\underline{\boldsymbol{\Phi}}$	$\equiv$	$\boldsymbol{\Phi}\underline{\underline{\boldsymbol{\Psi}}}$	1.020
Absolute Patience Factor	$Þ$	$\equiv$	$(\mathsf{R}\beta)^{1/\rho}$	0.999
Return Patience Factor	$Þ_{\mathsf{R}}$	$\equiv$	$Þ/\mathsf{R}$	0.961
Growth Patience Factor	$Þ_{\boldsymbol{\Phi}}$	$\equiv$	$Þ/\boldsymbol{\Phi}$	0.970
Normalized Growth Patience Factor	$Þ_{\underline{\boldsymbol{\Phi}}}$	$\equiv$	$Þ/\underline{\boldsymbol{\Phi}}$	0.980
Value of Autarky Factor	$\underline{\underline{\beth}}$	$\equiv$	$\beta\boldsymbol{\Phi}^{1-\rho}\underline{\underline{\boldsymbol{\Psi}}}^{1-\rho}$	0.941
Weak Return Impatience Factor	$\wp^{1/\rho}Þ$	$\equiv$	$(\wp\beta\mathsf{R})^{1/\rho}$	0.071

Table 3 Definitions and Comparisons of Conditions

Perfect Foresight Versions	Uncertainty Versions
Finite Human Wealth Condition (FHWC)	
$\boldsymbol{\Phi}/\mathsf{R} < 1$ The growth factor for permanent income $\boldsymbol{\Phi}$ must be smaller than the discounting factor R for human wealth to be finite.	$\boldsymbol{\Phi}/\mathsf{R} < 1$ The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction.
Absolute Impatience Condition (AIC)	
$\mathbf{Þ} < 1$ The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time: $\mathbf{c}_{t+1} < \mathbf{c}_t$	$\mathbf{Þ} < 1$ *If wealth is large enough,* the *expectation* of consumption next period will be smaller than this period's consumption: $\lim_{m_t \to \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$
Return Impatience Conditions	
Return Impatience Condition (RIC)	Weak RIC (WRIC)
$\mathbf{Þ}/\mathsf{R} < 1$ The growth factor for consumption $\mathbf{Þ}$ must be smaller than the discounting factor R, so that the PDV of current and future consumption will be finite: $\mathrm{c}'(m) = 1 - \mathbf{Þ}/\mathsf{R} < 1$	$\wp^{1/\rho}\mathbf{Þ}/\mathsf{R} < 1$ If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the RIC. Otherwise, weaker. $\mathrm{c}'(m) < 1 - \wp^{1/\rho}\mathbf{Þ}/\mathsf{R} < 1$
Growth Impatience Conditions	
GIC	GIC-Nrm
$\mathbf{Þ}/\boldsymbol{\Phi} < 1$ For an unconstrained PF consumer, the ratio of $\mathbf{c}$ to $\mathbf{p}$ will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees $\lim_{m_t \uparrow \infty} \mathbb{E}_t[\boldsymbol{\Psi}_{t+1} m_{t+1}/m_t] = \mathbf{Þ}_{\boldsymbol{\Phi}}$	$\mathbf{Þ}\,\mathbb{E}[\boldsymbol{\Psi}^{-1}]/\boldsymbol{\Phi} < 1$ By Jensen's inequality stronger than GIC. Ensures consumers will not expect to accumulate m unboundedly. $\lim_{m_t \to \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{Þ}_{\underline{\boldsymbol{\Phi}}}$
Finite Value of Autarky Conditions	
PF-FVAC	FVAC
$\beta\boldsymbol{\Phi}^{1-\rho} < 1$ equivalently $\mathbf{Þ} < \mathsf{R}^{1/\rho}\boldsymbol{\Phi}^{1-1/\rho}$ The discounted utility of constrained consumers who spend their permanent income each period should be finite.	$\beta\boldsymbol{\Phi}^{1-\rho}\,\mathbb{E}[\boldsymbol{\Psi}^{1-\rho}] < 1$ By Jensen's inequality, stronger than the PF-FVAC because for $\rho > 1$ and nondegenerate $\boldsymbol{\Psi}$, $\mathbb{E}[\boldsymbol{\Psi}^{1-\rho}] > 1$.

Table 4 Sufficient Conditions for Nondegenerate‡ Solution

Consumption Model(s)	Conditions	Comments
$\bar{\mathrm{c}}(m)$: PF Unconstrained	RIC, FHWC°	RIC$\Rightarrow$ $\lvert\mathrm{v}(m)\rvert < \infty$; FHWC$\Rightarrow$ $0 < \lvert\mathrm{v}(m)\rvert$
$\underline{\mathrm{c}}(m) = \underline{\kappa}m$		PF model with no human wealth ($h = 0$)
Section 2.5.3:		RIC prevents $\bar{\mathrm{c}}(m) = \underline{\mathrm{c}}(m) = 0$
Section 2.5.3:		FHWC prevents $\bar{\mathrm{c}}(m) = \infty$
Eq (21):		PF-FVAC+FHWC $\Rightarrow$ RIC
Eq (22):		GIC+FHWC $\Rightarrow$ PF-FVAC
$\grave{\mathrm{c}}(m)$: PF Constrained	~~GIC~~, RIC	FHWC holds ($\mathbf{\Phi} < \text{Þ} < \mathsf{R} \Rightarrow \mathbf{\Phi} < \mathsf{R}$)
Section 2.5.6:		$\grave{\mathrm{c}}(m) = \bar{\mathrm{c}}(m)$ for $m > m_{\#} < 1$
		(~~RIC~~ would yield $m_{\#} = 0$ so $\grave{\mathrm{c}}(m) = 0$)
Appendix E:	GIC,RIC	$\lim_{m\to\infty} \grave{c}(m) = \bar{c}(m), \lim_{m\to\infty} \grave{\boldsymbol{\kappa}}(m) = \underline{\kappa}$
		kinks where horizon to $b = 0$ changes*
Appendix E:	GIC,~~RIC~~	$\lim_{m\to\infty} \grave{\boldsymbol{\kappa}}(m) = 0$
		kinks where horizon to $b = 0$ changes*
$\mathrm{c}(m)$: Friedman/Muth	Section 3.1,	$\underline{\mathrm{c}}(m) < \mathrm{c}(m) < \bar{\mathrm{c}}(m)$
	Section 3.2	$\underline{\mathrm{v}}(m) < \mathrm{v}(m) < \bar{\mathrm{v}}(m)$
Section 2.10:	FVAC, WRIC	Sufficient for Contraction
Section 2.12:		WRIC is weaker than RIC
Figure 3:		FVAC is stronger than PF-FVAC
Section 2.12.2:		~~FHWC~~+RIC $\Rightarrow$GIC, $\lim_{m\to\infty} \boldsymbol{\kappa}(m) = \underline{\kappa}$
Section 2.12.1:		~~RIC~~ $\Rightarrow$~~FHWC~~, $\lim_{m\to\infty} \boldsymbol{\kappa}(m) = 0$
Section 3.3:		"Buffer Stock Saving" Conditions
Section 3.3.2:		GIC $\Rightarrow \exists\ \check{m}$ s.t. $0 < \check{m} < \infty$
Section 3.3.1:		GIC-Nrm $\Rightarrow \exists\ \hat{m}$ s.t. $0 < \hat{m} < \infty$

‡For feasible m satisfying $0 < m < \infty$, a nondegenerate limiting consumption function defines a unique optimal value of c satisfying $0 < c(m) < \infty$; a nondegenerate limiting value function defines a corresponding unique value of $-\infty < \mathrm{v}(m) < 0$.

°RIC, FHWC are necessary as well as sufficient for the perfect foresight case. *That is, the first kink point in $c(m)$ is $m_{\#}$ s.t. for $m < m_{\#}$ the constraint will bind now, while for $m > m_{\#}$ the constraint will bind one period in the future. The second kink point corresponds to the m where the constraint will bind two periods in the future, etc.

**In the Friedman/Muth model, the RIC+FHWC are sufficient, but *not* necessary for nondegeneracy

Table 5 Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained $\grave{c}$ and unconstrained $\bar{\mathrm{c}}$ consumption functions

Main Condition Subcondition	Math			Outcome, Comments or Results
~~GIC~~		$1 <$	$\mathbf{Þ}/\mathbf{\Phi}$	Constraint never binds for $m \geq 1$
and RIC	$\mathbf{Þ}/\mathsf{R}$	< 1		FHWC holds ($\mathsf{R} > \mathbf{\Phi}$); $\grave{\mathrm{c}}(m) = \bar{\mathrm{c}}(m)$ for $m \geq 1$
and ~~RIC~~		$1 <$	$\mathbf{Þ}/\mathsf{R}$	$\grave{\mathrm{c}}(m)$ is degenerate: $\grave{\mathrm{c}}(m) = 0$
GIC	$\mathbf{Þ}/\mathbf{\Phi}$	< 1		Constraint binds in finite time $\forall\ m$
and RIC	$\mathbf{Þ}/\mathsf{R}$	< 1		FHWC may or may not hold $\lim_{m\uparrow\infty} \bar{\mathrm{c}}(m) - \grave{\mathrm{c}}(m) = 0$ $\lim_{m\uparrow\infty} \grave{\boldsymbol{\kappa}}(m) = \underline{\kappa}$
and ~~RIC~~		$1 <$	$\mathbf{Þ}/\mathsf{R}$	~~FHWC~~ $\lim_{m\uparrow\infty} \grave{\boldsymbol{\kappa}}(m) = 0$

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and RIC both hold, while the third row indicates that when the GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the GIC holds, the constraint will bind in finite time.