Table 1 Microeconomic Model Calibration

Calibrated Parameters			
Description	Parameter	Value	Source
Permanent Income Growth Factor	Φ	1.03	PSID: Carroll (1992)
Interest Factor	R	1.04	Conventional
Time Preference Factor	β	0.96	Conventional
Coefficient of Relative Risk Aversion	ρ	2	Conventional
Probability of Zero Income	$\wp$	0.005	PSID: Carroll (1992)
Std Dev of Log Permanent Shock	σ_{Ψ}	0.1	PSID: Carroll (1992)
Std Dev of Log Transitory Shock	σ_{θ}	0.1	PSID: Carroll (1992)

Table 2 Model Characteristics Calculated from Parameters

Description	Symbol and Formula		Approximate Calculated Value
Finite Human Wealth Factor	$\mathcal{R}^{-1}$	$\equiv$ Φ/R	0.990
PF Value of Autarky Factor	$\beth$	$\equiv$ $\beta\Phi^{1-\rho}$	0.932
Growth Compensated Permanent Shock	$\underline{\Psi}$	$\equiv$ $(\mathbb{E}[\Psi^{-1}])^{-1}$	0.990
Uncertainty-Adjusted Growth	$\underline{\Phi}$	$\equiv$ $\Phi\underline{\Psi}$	1.020
Utility Compensated Permanent Shock	$\underline{\underline{\Psi}}$	$\equiv$ $(\mathbb{E}[\Psi^{1-\rho}])^{1/(1-\rho)}$	0.990
Utility Compensated Growth	$\underline{\underline{\Phi}}$	$\equiv$ $\Phi\underline{\underline{\Psi}}$	1.020
Absolute Patience Factor	$\Thorn$	$\equiv$ $(R\beta)^{1/\rho}$	0.999
Return Patience Factor	$\Thorn_{R}$	$\equiv$ $\Thorn/R$	0.961
Growth Patience Factor	$\Thorn_{\Phi}$	$\equiv$ $\Thorn/\Phi$	0.970
Normalized Growth Patience Factor	$\Thorn_{\underline{\Phi}}$	$\equiv$ $\Thorn/\underline{\Phi}$	0.980
Value of Autarky Factor	$\beth$	$\equiv$ $\beta\Phi^{1-\rho}\underline{\underline{\Psi}}^{1-\rho}$	0.941
Weak Return Impatience Factor	$\wp^{1/\rho}\Thorn$	$\equiv$ $(\wp\beta R)^{1/\rho}$	0.071

Table 3 Definitions and Comparisons of Conditions

Perfect Foresight Versions	Uncertainty Versions
Finite Human Wealth Condition (FHWC)	
$\mathbf{\Phi}/\mathsf{R} < 1$	$\mathbf{\Phi}/\mathsf{R} < 1$
The growth factor for permanent income $\mathbf{\Phi}$ must be smaller than the discounting factor R for human wealth to be finite.	The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction.
Absolute Impatience Condition (AIC)	
$\mathbf{Þ} < 1$	$\mathbf{Þ} < 1$
The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time:	*If wealth is large enough*, the *expectation* of consumption next period will be smaller than this period's consumption:
$\mathbf{c}_{t+1} < \mathbf{c}_t$	$\lim_{m_t \to \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$
Return Impatience Conditions	
Return Impatience Condition (RIC)	Weak RIC (WRIC)
$\mathbf{Þ}/\mathsf{R} < 1$	$\wp^{1/\rho}\mathbf{Þ}/\mathsf{R} < 1$
The growth factor for consumption $\mathbf{Þ}$ must be smaller than the discounting factor R, so that the PDV of current and future consumption will be finite:	If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the RIC. Otherwise, weaker.
$c'(m) = 1 - \mathbf{Þ}/\mathsf{R} < 1$	$c'(m) < 1 - \wp^{1/\rho}\mathbf{Þ}/\mathsf{R} < 1$
Growth Impatience Conditions	
GIC	GIC-Nrm
$\mathbf{Þ}/\mathbf{\Phi} < 1$	$\mathbf{Þ}\,\mathbb{E}[\mathbf{\Psi}^{-1}]/\mathbf{\Phi} < 1$
For an unconstrained PF consumer, the ratio of $\mathbf{c}$ to $\mathbf{p}$ will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees $\lim_{m_t \uparrow \infty} \mathbb{E}_t[\mathbf{\Psi}_{t+1}m_{t+1}/m_t] = \mathbf{Þ}_{\mathbf{\Phi}}$	By Jensen's inequality stronger than GIC. Ensures consumers will not expect to accumulate m unboundedly. $\lim_{m_t \to \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{Þ}_{\underline{\mathbf{\Phi}}}$
Finite Value of Autarky Conditions	
PF-FVAC	FVAC
$\beta\mathbf{\Phi}^{1-\rho} < 1$ equivalently $\mathbf{Þ} < \mathsf{R}^{1/\rho}\mathbf{\Phi}^{1-1/\rho}$	$\beta\mathbf{\Phi}^{1-\rho}\,\mathbb{E}[\mathbf{\Psi}^{1-\rho}] < 1$
The discounted utility of constrained consumers who spend their permanent income each period should be finite.	By Jensen's inequality, stronger than the PF-FVAC because for $\rho > 1$ and nondegenerate $\mathbf{\Psi}$, $\mathbb{E}[\mathbf{\Psi}^{1-\rho}] > 1$.

Table 4 Sufficient Conditions for Nondegenerate[‡] Solution

Consumption Model(s)	Conditions	Comments
$\bar{c}(m)$: PF Unconstrained $\underline{c}(m) = \underline{\kappa}m$	RIC, FHWC°	RIC$\Rightarrow |v(m)| < \infty$; FHWC$\Rightarrow 0 < |v(m)|$ PF model with no human wealth ($h = 0$)
Section **??**:		RIC prevents $\bar{c}(m) = \underline{c}(m) = 0$
Section **??**:		FHWC prevents $\bar{c}(m) = \infty$
Eq (**??**):		PF-FVAC+FHWC $\Rightarrow$ RIC
Eq (**??**):		GIC+FHWC $\Rightarrow$ PF-FVAC
$\grave{c}(m)$: PF Constrained Section **??**:	~~GIC~~, RIC	FHWC holds ($\boldsymbol{\Phi} < \text{\Thorn} < \mathsf{R} \Rightarrow \boldsymbol{\Phi} < \mathsf{R}$) $\grave{c}(m) = \bar{c}(m)$ for $m > m_{\#} < 1$ (~~RIC~~ would yield $m_{\#} = 0$ so $\grave{c}(m) = 0$)
Appendix **??**:	GIC,RIC	$\lim_{m\to\infty} \grave{c}(m) = \bar{c}(m), \lim_{m\to\infty} \grave{\boldsymbol{\kappa}}(m) = \underline{\kappa}$ kinks where horizon to $b = 0$ changes*
Appendix **??**:	GIC,~~RIC~~	$\lim_{m\to\infty} \grave{\boldsymbol{\kappa}}(m) = 0$ kinks where horizon to $b = 0$ changes*
c(m): Friedman/Muth	Section **??**, Section **??**	$\underline{c}(m) < c(m) < \bar{c}(m)$ $\underline{v}(m) < v(m) < \bar{v}(m)$
Section **??**:	FVAC, WRIC	Sufficient for Contraction
Section **??**:		WRIC is weaker than RIC
Figure **??**:		FVAC is stronger than PF-FVAC
Section **??**:		~~FHWC~~+RIC $\Rightarrow$GIC, $\lim_{m\to\infty} \boldsymbol{\kappa}(m) = \underline{\kappa}$
Section **??**:		~~RIC~~ $\Rightarrow$~~FHWC~~, $\lim_{m\to\infty} \boldsymbol{\kappa}(m) = 0$
Section **??**:		"Buffer Stock Saving" Conditions
Section **??**:		GIC $\Rightarrow \exists \; \check{m}$ s.t. $0 < \check{m} < \infty$
Section **??**:		GIC-Nrm $\Rightarrow \exists \; \hat{m}$ s.t. $0 < \hat{m} < \infty$

[‡]For feasible m satisfying $0 < m < \infty$, a nondegenerate limiting consumption function defines a unique optimal value of c satisfying $0 < c(m) < \infty$; a nondegenerate limiting value function defines a corresponding unique value of $-\infty < v(m) < 0$.

°RIC, FHWC are necessary as well as sufficient for the perfect foresight case. *That is, the first kink point in $c(m)$ is $m_{\#}$ s.t. for $m < m_{\#}$ the constraint will bind now, while for $m > m_{\#}$ the constraint will bind one period in the future. The second kink point corresponds to the m where the constraint will bind two periods in the future, etc.

**In the Friedman/Muth model, the RIC+FHWC are sufficient, but *not* necessary for nondegeneracy

Table 5 Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained $\grave{c}$ and unconstrained $\bar{c}$ consumption functions

Main Condition Subcondition	Math			Outcome, Comments or Results
~~GIC~~		$1 <$	$Þ/\boldsymbol{\Phi}$	Constraint never binds for $m \geq 1$
and RIC	$Þ/R$	< 1		FHWC holds ($R > \boldsymbol{\Phi}$);
				$\grave{c}(m) = \bar{c}(m)$ for $m \geq 1$
and ~~RIC~~		$1 <$	$Þ/R$	$\grave{c}(m)$ is degenerate: $\grave{c}(m) = 0$
GIC	$Þ/\boldsymbol{\Phi}$	< 1		Constraint binds in finite time $\forall\, m$
and RIC	$Þ/R$	< 1		FHWC may or may not hold
				$\lim_{m\uparrow\infty} \bar{c}(m) - \grave{c}(m) = 0$
				$\lim_{m\uparrow\infty} \grave{\boldsymbol{\kappa}}(m) = \underline{\kappa}$
and ~~RIC~~		$1 <$	$Þ/R$	~~FHWC~~
				$\lim_{m\uparrow\infty} \grave{\boldsymbol{\kappa}}(m) = 0$

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and RIC both hold, while the third row indicates that when the GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the GICholds, the constraint will bind in finite time.