Theoretical Foundations of Buffer Stock Saving

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Econ ARK

A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is Right
- Little Intuition for How Results Might Change With
 - Calibration
 - Structure
- Very Hard To Teach!

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

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The Gap This Paper Fills

Foundations For Microeconomic Household's Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
- (Problem with Liquidity Constraints Is A Limiting Case)

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Key Result

Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
 - \Rightarrow \exists A Unique Consumption Function c(m)
- There Is A 'Target' Ratio Of Assets to Permanent Income
 - Requires A Key 'Impatience' Condition To Hold
 - Good News
 - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed

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Limit as horizon T goes to infinity of

$$a_{t} = m_{t} - c_{t}$$

$$k_{t+1} = a_{t}$$

$$b_{t+1} = k_{t}R$$

$$p_{t+1} = p_{t}\underbrace{\Gamma\psi_{t+1}}_{\equiv \Gamma_{t+1}}$$

$$m_{t+1} = b_{t+1} + p_{t+1}\xi_{t+1},$$
(1)

$$\xi_{t+n} = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \theta_{t+n}/(1-\wp) & \text{with probability } (1-\wp) \end{cases}$$
 (2)

$$\bullet \ u(\bullet) = \bullet^{1-\rho}/(1-\rho); \ \mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\xi_{t+n}] = \underset{\bullet}{1} \ \forall \ n>0; \ \beta<1, \ \rho>1$$

Surely This Problem Has Been Solved?

No

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru ? Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!

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Benchmark: Perfect Foresight Model

Definitions:

| Absolute Patience Factor | Þ | = | $(R\beta)^{1/2}$ |
|--|------------------|---|---------------------|
| Return Patience Factor | \mathbf{p}_R | = | \mathbf{P}/R |
| Perfect Foresight Growth Patience Factor | \mathbf{p}_{L} | = | \mathbf{P}/Γ |

| Name | Condition | | on | Implication | |
|-------------------------------------|------------------|---|----|------------------------------------|--|
| (AIC) Absolute Impatience Condition | Þ | < | 1 | $c \downarrow$ over time | |
| (RIC) Return Impatience Condition | \mathbf{p}_{R} | < | 1 | $c/a \downarrow$ over time | |
| (GIC) Growth Impatience Condition | \mathbf{p}_{L} | < | 1 | $c/\mathbf{p}\downarrow$ over time | |

When Does A Useful Limiting Solution Exist?

Finite Human Wealth (FHWC) condition:

$$\Gamma < R$$
 (3)

Return Impatience Condition:

$$\Phi_{\mathsf{R}} < \mathsf{R} \tag{4}$$

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What If There Are Liquidity Constraints?

- FHWC is not necessary for solution to exist
- Other Key Condition For Useful Solution is 'Perfect Foresight Finite Value of Autarky Condition (PF-FVAC)':

$$\beta \Gamma^{1-\rho} < 1 \tag{5}$$

- Without RIC, Constraints Are Irrelevant
 - Because Wealth Always Wants To Rise, So Constraint Never Binds

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Liquidity Constraints and Uncertainty

- Introduce permanent shocks to income
- Finite Value of Autarky Condition Becomes

$$0 < \overbrace{\beta \underline{\underline{\Gamma}}^{1-\rho}} < 1$$

$$0 < \beta < \underline{\underline{\Gamma}}^{\rho-1}$$
(6)

Contraction Mapping Requirements

Finite Value of Autarky Condition: Same As In Liq Constr Problem!

$$0 < \overbrace{\beta} \underline{\underline{\Gamma}}^{1-\rho} < 1$$

$$0 < \beta < \underline{\underline{\Gamma}}^{\rho-1}$$

$$(7)$$

'Weak Return Impatience Condition' (WRIC)

$$0 \le \wp^{1/\rho} \mathbf{P}_{\mathsf{R}} < 1 \tag{8}$$

Requirement For Existence Of A Target

Definitions: 'Uncertainty-Adjusted' Growth:

$$\underline{\Gamma} \equiv \Gamma \psi < \Gamma \tag{9}$$

Adjusted Growth Patience Factor:

$$\mathbf{P}_{\underline{\Gamma}} = \mathbf{P}/\underline{\Gamma} = \mathbb{E}[\mathbf{P}/(\Gamma\psi)] \tag{10}$$

Growth Impatience Condition:

$$\mathbf{P}_{\underline{\Gamma}} < 1, \tag{11}$$

Why? Because it can be shown that

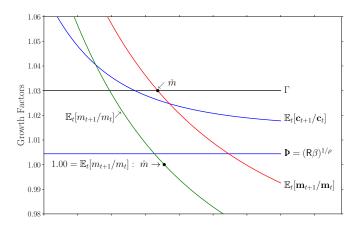
$$\lim_{m_t \to \infty} \mathbb{E}_t \left[\frac{m_{t+1}}{m_t} \right] = \mathbf{P}_{\underline{\Gamma}}$$
 (12)

Five Propositions

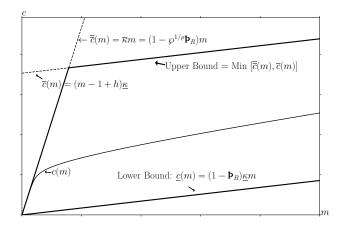
$$\mathbf{0} \ \lim_{m_t \to \infty} \mathbb{E}_t[c_{t+1}/c_t] = \mathbf{P}$$

3 \exists a unique target value of m, called \check{m}

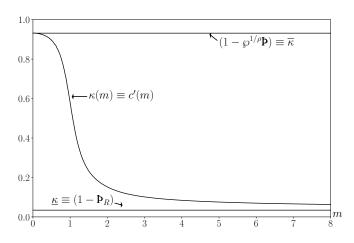
The Target Saving Figure



Bounds On the Consumption Function



The Marginal Propensity to Consume



The Consumption Function and Target Wealth



Convergence To The Invariant Distribution

? Proves Existence of an Invariant Distribution of m, c, a, etc.



Carroll

Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$Y_{t+1}/Y_t = C_{t+1}/C_t = \Gamma$$
 (13)

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving ≈ Liquidity Constraints
- If c(m) is solution for constrained consumer,

$$\lim_{\wp \downarrow 0} c(m;\wp) = \dot{c}(m) \tag{14}$$

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The MPC Out Of Permanent Shocks

https://www.econ2.jhu.edu/people/ccarroll/papers/MPCPerm.pdf

Lots of Recent Papers Trying to Measure the MPCP

Paper Proves:

- MPCP < 1
- But not a lot less:
 - 0.75 to 0.95 (annual rate) for wide range of parameter values

- Defined Conditions Under Which Widely Used Problem Has Solution
 - Finite Value of Autarky Condition Guarantees Contraction (with WRIC)
 - Growth Impatience Condition Prevents $m \to \infty$
- Economy Of Buffer Stock Consumers Exhibits Balanced Growth
 - Even In Absence of General Equilibrium Adj of Interest Rate

Introduction
The Problem
Features Of the Solution
A Small Open Buffer Stock Economy
Conclusions