Theoretical Foundations of Buffer Stock Saving

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Abstract

This paper builds foundations for rigorous and intuitive understanding of 'buffer stock' saving models (Bewley (1977)-like models with a wealth target), pairing each theoretical result with quantitative illustrations. After describing conditions under which a consumption function exists, the paper articulates stricter 'Growth Impatience' conditions that guarantee alternative forms of stability — either at the population level, or for individual consumers. Together, the numerical tools and analytical results constitute a comprehensive toolkit for understanding buffer stock models.

Keywords Precautionary saving, buffer stock saving, marginal propensity to consume, permanent income hypothesis, income fluctuation problem

JEL codes D81, D91, E21

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The paper's results can be automatically reproduced using the Econ-ARK/HARK toolkit, which can be cited per our references (Carroll, Kaufman, Kazil, Palmer, and White (2018)); for reference to the toolkit itself see Acknowleding Econ-ARK. Thanks to the Consumer Financial Protection Bureau for funding the original creation of the Econ-ARK toolkit; and to the Sloan Foundation for funding Econ-ARK's extensive further development that brought it to the point where it could be used for this project. The toolkit can be cited with its digital object identifier, 10.5281/zenodo.1001067, as is done in the paper's own references as Carroll, Kaufman, Kazil, Palmer, and White (2018). Thanks to Will Du, James Feigenbaum, Joseph Kaboski, Miles Kimball, Qingyin Ma, Misuzu Otsuka, Damiano Sandri, John Stachurski, Adam Szeidl, Alexis Akira Toda, Metin Uyanik, Mateo Velásquez-Giraldo, Weifeng Wu, Jiaxiong Yao, and Xudong Zheng for comments on earlier versions of this paper, John Boyd for help in applying his weighted contraction mapping theorem, Ryoji Hiraguchi for extraordinary mathematical insight that improved the paper greatly, David Zervos for early guidance to the literature, and participants in a seminar at the Johns Hopkins University, a presentation at the 2009 meetings of the Society of Economic Dynamics for their insights, and at a presentation at the Australian National University.

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1 Introduction

In the presence of realistic transitory and permanent shocks to income *a la* Friedman (1957) and Muth (1960), only one further ingredient is required to construct a microeconomically testable model of consumption: A description of preferences. Zeldes (1989) was the first to calibrate a quantitatively plausible example; his paper spawned a literature showing that such models' predictions can match household life cycle data reasonably well, whether or not explicit liquidity constraints are imposed.[1]

A connected literature in macroeconomic theory, starting with Bewley (1977), has derived limiting properties of related infinite-horizon problems, but only in models more complex than the case with just shocks and preferences. The extra complexity has been imposed because standard contraction mapping theorems (beginning with Bellman (1957) and including those building on Stokey et. al. (1989)) cannot be applied when utility and/or marginal utility are unbounded. Many proof methods also rule out permanent shocks *a la* Friedman (1957), Muth (1960), and Zeldes (1989).[2]

This paper's first technical contribution is to articulate conditions under which the infinite-horizon version of the original Friedman-Muth-Zeldes problem (without complications like a consumption floor or liquidity constraints) defines a contraction mapping whose limiting solution is nondegenerate as the horizon approaches infinity. A 'Finite Value of Autarky Condition' is mostly sufficient (the other imposed condition, the 'Weak Return Impatience Condition' is unlikely to bind).[3] Because the proof constructs the infinite horizon solution as the limit of finite-horizon recursions, many intermediate results are also useful for solving finite-horizon problems.

But the paper's main theoretical contribution is to identify, for the infinite-horizon case, conditions under which 'stable' values of the wealth-to-permanent-income ratio exist, either for individual consumers (a consumer's wealth can be predicted to move toward a 'target' ratio) or for the aggregate (the economy as a whole moves toward a 'balanced growth' equilibrium). The requirement for stability is always that the model's parameters satisfy a 'Growth Impatience Condition' whose details depend on the quantity whose stability is of interest. A model that exhibits stability of either kind qualifies as a 'buffer stock' model.[4]

Even without a formal proof of its existence, buffer stock saving has been intuitively understood to underlie central quantitative results in heterogeneous agent macroeconomics; for example, the logic of target saving is central to the claim by Krueger, Mitman, and Perri (2016) in the *Handbook of Macroeconomics* that such models explain why,

[1] See Carroll (1997) or Gourinchas and Parker (2002) for arguments that models with only 'natural' constraints (see below) match a wide variety of facts; for a model with explicit constraints that produces very similar results, see, e.g., Cagetti (2003).

[2] See the fuller discussion at the end of Section 2.1.

[3] This is a generalization of a condition in Ma, Stachurski, and Toda (2020).

[4] Such models are neither a subset nor a superset of Bewley (1977) models. But closed economies in which capital results from saving and has declining marginal productivity are always 'buffer stock' economies under some definition of that term, because capital accumulation causes interest rates to fall, which guarantees that a Growth Impatience Condition will hold in equilibrium (see below). The more interesting applications are to populations (or economies) whose marginal saving behavior does not determine the relevant interest rate, or in which the marginal product of capital does not fall as capital is accumulated (again, see below).

during the Great Recession, middle-class consumers cut their spending more than the poor or the rich. The theory below provides the rigorous basis for this claim: Learning that the future has become more uncertain does not change the urgent imperatives of the poor (their high $\mathrm{u}'(c)$ means they — optimally — have little room to maneuver). And, increased labor income uncertainty does not much change the behavior of the rich because it poses little risk to their consumption. Only people in the middle have both the motivation and the wiggle-room to respond by reducing their spending.

Analytical derivations for the proofs also provide foundations for many other results familiar from the numerical literature.

The paper's first part begins by defining sufficient conditions for the problem to define a useful (nondegenerate) limiting consumption function (and explains how the model relates to those previously considered in the literature). The conditions are interestingly parallel to those required for the liquidity constrained perfect foresight model; that parallel is explored and explained. This analysis establishes limiting properties of the consumption function as resources approach infinity, and as they approach their lower bound; using these limits, the contraction mapping theorem is proven.

The next theoretical contribution is to show that a corresponding model with an 'artificial' liquidity constraint (a model that prohibits borrowing, even by consumers who could certainly repay) is a limiting case of the model without constraints. The analytical appeal of the unconstrained model is that it is both mathematically convenient (e.g., the consumption function is twice continuously differentiable), and arbitraily close (cf. Section **??**) to less tractable models that have elsewhere been tackled with less convenient methods. This treatment models a strategy of proving interesting propositions in a congenial environment, and then appealing to a limiting argument to define the analogous proposition as holding (in the limit) in a more unwieldy environment.

In proving the remaining theorems, the next section examines the key properties of the model. First, as cash approaches infinity the expected growth rate of consumption and the marginal propensity to consume (MPC) converge to their values in the perfect foresight case. Second, as cash approaches zero the expected growth rate of consumption approaches infinity, and the MPC approaches a simple analytical limit. Next, the central theorems articulate conditions under which different measures of 'growth impatience' imply useful conclusions about points of stability ('target' or 'balanced growth' points).

The final section elaborates the conditions under which, even with a fixed aggregate interest rate that differs from the time preference rate, a small open economy populated by buffer stock consumers has a balanced growth equilibrium in which growth rates of consumption, income, and wealth match the exogenous growth rate of permanent income (equivalent, here, to productivity growth). In the terms of Schmitt-Grohé and Uribe (2003), buffer stock saving is an appealing method of 'closing' a small open economy model, because it requires no ad-hoc assumptions. Not even liquidity constraints.[5]

[5]The paper's insights are instantiated in the Econ-ARK toolkit, whose buffer stock saving module flags parametric choices under which a problem is degenerate or under which stable ratios of wealth to income may not exist.

2 The Problem

2.1 Setup

The infinite horizon solution is the (limiting) first-period solution to a sequence of finite-horizon problems as the horizon (the last period of life) becomes arbitrarily distant.

That is, for the value function, fixing a terminal date T, we are interested in the term $\mathbf{v}_{T-n}$ in the sequence of value functions $\{\mathbf{v}_T, \mathbf{v}_{T-1}, \ldots, \mathbf{v}_{T-n}\}$. We will say that the problem has a 'nondegenerate' infinite horizon solution if, corresponding to that $\mathbf{v}$, as $n \uparrow \infty$ there is a limiting consumption function $\mathrm{c}(m) = \lim_{n\uparrow\infty} \mathrm{c}_{T-n}$ which is neither $\mathrm{c}(m) = 0$ everywhere (for all m) nor $\mathrm{c}(m) = \infty$ everywhere.

Concretely, a consumer born n periods before date T solves the problem

$$\mathbf{v}_{T-n} = \max\ \mathbb{E}_t \left[\sum_{i=0}^{n} \beta^i \mathrm{u}(\mathbf{c}_{t+i}) \right]$$

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