# Theoretical Foundations of Buffer Stock Saving

Chris Carroll

Johns Hopkins University

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Econ ARK

#### A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is Right
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- Very Hard To Teach!

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

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Foundations For Microeconomic Household's Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
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## Key Result

Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
  - $\Rightarrow \exists$  A Unique Consumption Function c(m)
- There Is A 'Target' Ratio Of Assets to Permanent Income
  - Requires A Key 'Impatience' Condition To Hold
  - Good News
    - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed

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Limit as horizon T goes to infinity of

$$a_{t} = m_{t} - c_{t}$$

$$k_{t+1} = a_{t}$$

$$b_{t+1} = k_{t}R$$

$$p_{t+1} = p_{t}\underbrace{\Gamma\psi_{t+1}}_{\equiv \Gamma_{t+1}}$$

$$m_{t+1} = b_{t+1} + p_{t+1}\xi_{t+1},$$
(1)

$$\xi_{t+n} = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \theta_{t+n}/(1-\wp) & \text{with probability } (1-\wp) \end{cases}$$
 (2)

$$\bullet \ u(\bullet) = \bullet^{1-\rho}/(1-\rho); \ \mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\xi_{t+n}] = \underset{\bullet}{1} \ \forall \ n>0; \ \beta<1, \ \rho>1$$

## Surely This Problem Has Been Solved?

#### No

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru Matkowski and Nowak (2011) Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

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# Benchmark: Perfect Foresight Model

#### Definitions:

Absolute Patience Factor	Þ	=	$(R\beta)^{1/2}$
Return Patience Factor	$\mathbf{p}_R$	=	$\mathbf{P}/R$
Perfect Foresight Growth Patience Factor	$\mathbf{p}_{L}$	=	$\mathbf{P}/\Gamma$

Name	Condition		on	Implication	
(AIC) Absolute Impatience Condition	Þ	<	1	$c \downarrow$ over time	
(RIC) Return Impatience Condition	$\mathbf{p}_{R}$	<	1	$c/a \downarrow$ over time	
(GIC) Growth Impatience Condition	$\mathbf{p}_{L}$	<	1	$c/\mathbf{p}\downarrow$ over time	

# When Does A Useful Limiting Solution Exist?

Finite Human Wealth (FHWC) condition:

$$\Gamma < R$$
 (3)

Return Impatience Condition:

$$\Phi_{\mathsf{R}} < \mathsf{R} \tag{4}$$

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# What If There Are Liquidity Constraints?

- FHWC is not necessary for solution to exist
- Other Key Condition For Useful Solution is 'Perfect Foresight Finite Value of Autarky Condition (PF-FVAC)':

$$\beta \Gamma^{1-\rho} < 1 \tag{5}$$

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# Liquidity Constraints and Uncertainty

- Introduce permanent shocks to income
- Finite Value of Autarky Condition Becomes

$$0 < \overbrace{\beta \underline{\underline{\Gamma}}^{1-\rho}} < 1$$

$$0 < \beta < \underline{\underline{\Gamma}}^{\rho-1}$$
(6)

## Contraction Mapping Requirements

Finite Value of Autarky Condition: Same As In Liq Constr Problem!

$$0 < \overbrace{\beta} \underline{\underline{\Gamma}}^{1-\rho} < 1$$

$$0 < \beta < \underline{\underline{\Gamma}}^{\rho-1}$$

$$(7)$$

'Weak Return Impatience Condition' (WRIC)

$$0 \le \wp^{1/\rho} \mathbf{P}_{\mathsf{R}} < 1 \tag{8}$$

# Requirement For Existence Of A Target

Definitions: 'Uncertainty-Adjusted' Growth:

$$\underline{\Gamma} \equiv \Gamma \psi < \Gamma \tag{9}$$

Adjusted Growth Patience Factor:

$$\mathbf{P}_{\underline{\Gamma}} = \mathbf{P}/\underline{\Gamma} = \mathbb{E}[\mathbf{P}/(\Gamma\psi)] \tag{10}$$

Growth Impatience Condition:

$$\mathbf{P}_{\underline{\Gamma}} < 1, \tag{11}$$

Why? Because it can be shown that

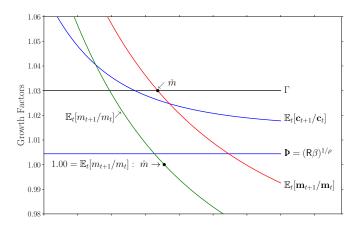
$$\lim_{m_t \to \infty} \mathbb{E}_t \left[ \frac{m_{t+1}}{m_t} \right] = \mathbf{P}_{\underline{\Gamma}}$$
 (12)

# Five Propositions

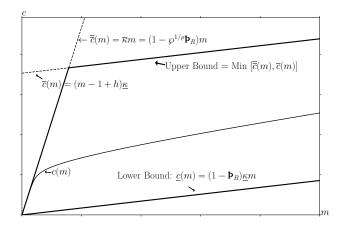
$$\mathbf{0} \ \lim_{m_t \to \infty} \mathbb{E}_t[c_{t+1}/c_t] = \mathbf{P}$$

**3**  $\exists$  a unique target value of m, called  $\check{m}$ 

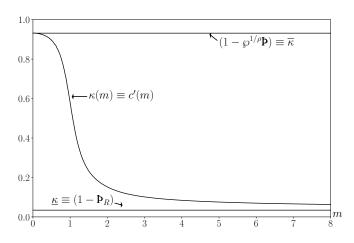
# The Target Saving Figure



## Bounds On the Consumption Function



# The Marginal Propensity to Consume



# The Consumption Function and Target Wealth



### Convergence To The Invariant Distribution

Szeidl (2013) Proves Existence of an Invariant Distribution of m, c, a, etc.



Carroll

## Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$Y_{t+1}/Y_t = C_{t+1}/C_t = \Gamma$$
 (13)

Fisherian Separation Fails, Even Without Liquidity Constraints!

### Insight:

- Precautionary Saving ≈ Liquidity Constraints
- If c(m) is solution for constrained consumer,

$$\lim_{\wp \downarrow 0} c(m;\wp) = \dot{c}(m) \tag{14}$$

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### The MPC Out Of Permanent Shocks

https://www.econ2.jhu.edu/people/ccarroll/papers/MPCPerm.pdf

Lots of Recent Papers Trying to Measure the MPCP

#### Paper Proves:

- MPCP < 1</li>
- But not a lot less:
  - 0.75 to 0.95 (annual rate) for wide range of parameter values

- Defined Conditions Under Which Widely Used Problem Has Solution
  - Finite Value of Autarky Condition Guarantees Contraction (with WRIC)
  - Growth Impatience Condition Prevents  $m \to \infty$
- Economy Of Buffer Stock Consumers Exhibits Balanced Growth
  - Even In Absence of General Equilibrium Adj of Interest Rate

Introduction
The Problem
Features Of the Solution
A Small Open Buffer Stock Economy
Conclusions

MATKOWSKI, JANUSZ, AND ANDRZEJ S. NOWAK (2011): "On Discounted Dynamic Programming With Unbounded Returns," Economic Theory, 46, 455–474.

SZEIDL, ADAM (2013): "Stable Invariant Distribution in Buffer-Stock Saving and Stochastic Growth Models," Manuscript, Central European University, Available at http://www.personal.ceu.hu/staff/Adam\_Szeidl/papers/invariant\_revision.pdf.