

Table 1 Microeconomic Model Calibration

| Calibrated Parameters | | | |
|---------------------------------------|-----------------|-------|----------------------|
| Description | Parameter | Value | Source |
| Permanent Income Growth Factor | Γ | 1.03 | PSID: Carroll (1992) |
| Interest Factor | R | 1.04 | Conventional |
| Time Preference Factor | β | 0.96 | Conventional |
| Coefficient of Relative Risk Aversion | ρ | 2 | Conventional |
| Probability of Zero Income | \wp | 0.005 | PSID: Carroll (1992) |
| Std Dev of Log Permanent Shock | σ_ψ | 0.1 | PSID: Carroll (1992) |
| Std Dev of Log Transitory Shock | σ_θ | 0.1 | PSID: Carroll (1992) |

Table 2 Model Characteristics Calculated from Parameters

| Description | Symbol and Formula | Approximate Calculated Value |
|-------------------------------------|--|------------------------------------|
| Finite Human Wealth Factor | $\mathcal{R}^{-1} \equiv \Gamma/R$ | 0.990 |
| PF Finite Value of Autarky Factor | $\sqsupset \equiv \beta\Gamma^{1-\rho}$ | 0.932 |
| Growth Compensated Permanent Shock | $\underline{\psi} \equiv (\mathbb{E}[\psi^{-1}])^{-1}$ | 0.990 |
| Uncertainty-Adjusted Growth | $\underline{\Gamma} \equiv \Gamma\underline{\psi}$ | 1.020 |
| Utility Compensated Permanent Shock | $\underline{\underline{\psi}} \equiv (\mathbb{E}[\psi^{1-\rho}])^{1/(1-\rho)}$ | 0.990 |
| Utility Compensated Growth | $\underline{\underline{\Gamma}} \equiv \Gamma\underline{\underline{\psi}}$ | 1.020 |
| Absolute Patience Factor | $\mathfrak{P} \equiv (R\beta)^{1/\rho}$ | 0.999 |
| Return Patience Factor | $\mathfrak{P}_R \equiv \mathfrak{P}/R$ | 0.961 |
| Growth Patience Factor | $\mathfrak{P}_\Gamma \equiv \mathfrak{P}/\Gamma$ | 0.970 |
| Normalized Growth Patience Factor | $\mathfrak{P}_{\underline{\Gamma}} \equiv \mathfrak{P}/\underline{\Gamma}$ | 0.980 |
| Finite Value of Autarky Factor | $\sqsubseteq \equiv \beta\Gamma^{1-\rho}\underline{\underline{\psi}}^{1-\rho}$ | 0.941 |
| Weak Impatience Factor | $\wp^{1/\rho}\mathfrak{P} \equiv (\wp\beta R)^{1/\rho}$ | 0.071 |

Table 3 Definitions and Comparisons of Conditions

| Perfect Foresight Versions | Uncertainty Versions |
|--|--|
| Finite Human Wealth Condition (FHC) | |
| $\Gamma/R < 1$ The growth factor for permanent income Γ must be smaller than the discounting factor R for human wealth to be finite. | $\Gamma/R < 1$ The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction. |
| Absolute Impatience Condition (AIC) | |
| $\mathbf{P} < 1$ The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time: $\mathbf{c}_{t+1} < \mathbf{c}_t$ | $\mathbf{P} < 1$ <i>If wealth is large enough, the expectation of consumption next period will be smaller than this period's consumption:</i> $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$ |
| Return Impatience Conditions | |
| Return Impatience Condition (RIC) | Weak RIC (WRIC) |
| $\mathbf{P}/R < 1$ The growth factor for consumption \mathbf{P} must be smaller than the discounting factor R , so that the PDV of current and future consumption will be finite: $c'(m) = 1 - \mathbf{P}/R < 1$ | $\varphi^{1/\rho} \mathbf{P}/R < 1$ If the probability of the zero-income event is $\varphi = 1$ then income is always zero and the condition becomes identical to the RIC. Otherwise, weaker. $c'(m) < 1 - \varphi^{1/\rho} \mathbf{P}/R < 1$ |
| Growth Impatience Conditions | |
| GIC | GIC-Nrm |
| $\mathbf{P}/\Gamma < 1$ For an unconstrained PF consumer, the ratio of \mathbf{c} to \mathbf{p} will fall over time. For constrained, guarantees the constraint eventually binds. Guarantees $\lim_{m_t \uparrow \infty} \mathbb{E}_t[\psi_{t+1} m_{t+1}/m_t] = \mathbf{P}_\Gamma$ | $\mathbf{P} \mathbb{E}[\psi^{-1}]/\Gamma < 1$ By Jensen's inequality stronger than GIC. Ensures consumers will not expect to accumulate m unboundedly. $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{P}_\Gamma$ |
| Finite Value of Autarky Conditions | |
| PF-FVAC | FVAC |
| $\beta \Gamma^{1-\rho} < 1$ equivalently $\mathbf{P} < R^{1/\rho} \Gamma^{1-1/\rho}$ The discounted utility of constrained consumers who spend their permanent income each period should be finite. | $\beta \Gamma^{1-\rho} \mathbb{E}[\psi^{1-\rho}] < 1$ By Jensen's inequality, stronger than the PF-FVAC because for $\rho > 1$ and nondegenerate ψ , $\mathbb{E}[\psi^{1-\rho}] > 1$. |

Table 4 Sufficient Conditions for Nondegenerate[‡] Solution

| Consumption Model(s) | Conditions | Comments |
|--|---|---|
| $\bar{c}(m)$: PF Unconstrained $\underline{c}(m) = \underline{\kappa}m$ Section 2.4.2: Section 2.4.2: Eq (26): Eq (27): | RIC, FHWC [°] | RIC $\Rightarrow v(m) < \infty$; FHWC $\Rightarrow 0 < v(m) $ PF model with no human wealth ($h = 0$) RIC prevents $\bar{c}(m) = \underline{c}(m) = 0$ FHWC prevents $\bar{c}(m) = \infty$ PF-FVAC+FHWC \Rightarrow RIC GIC+FHWC \Rightarrow PF-FVAC |
| $\dot{c}(m)$: PF Constrained Section 2.4.3: Appendix A: Appendix A: | $\mathcal{GH}\mathcal{C}$, RIC GIC, RIC GIC, $\mathcal{RH}\mathcal{C}$ | FHWC holds ($\Gamma < \mathbf{D} < \mathbf{R} \Rightarrow \Gamma < \mathbf{R}$) $\dot{c}(m) = \bar{c}(m)$ for $m > m_{\#} < 1$ ($\mathcal{RH}\mathcal{C}$ would yield $m_{\#} = 0$ so $\dot{c}(m) = 0$) $\lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$, $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ kinks where horizon to $b = 0$ changes* $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$ kinks where horizon to $b = 0$ changes* |
| $c(m)$: Friedman/Muth Section 2.9: Section 2.11.1: Figure 3: Section 2.11.3: Section 2.11.2: Section 3.3: Section 3.3.2: Section 3.3.1: | Section 3.1, Section 3.2 FVAC, WRIC | $\underline{c}(m) < c(m) < \bar{c}(m)$ $\underline{v}(m) < v(m) < \bar{v}(m)$ Sufficient for Contraction WRIC is weaker than RIC FVAC is stronger than PF-FVAC $\mathcal{FHWC} + \text{RIC} \Rightarrow \text{GIC}$, $\lim_{m \rightarrow \infty} \kappa(m) = \underline{\kappa}$ $\mathcal{RH}\mathcal{C} \Rightarrow \mathcal{FHWC}$, $\lim_{m \rightarrow \infty} \kappa(m) = 0$ “Buffer Stock Saving” Conditions GIC $\Rightarrow \exists 0 < \check{m} < \infty$ GIC-Nrm $\Rightarrow \exists 0 < \hat{m} < \infty$ |

[‡]For feasible m satisfying $0 < m < \infty$, a nondegenerate limiting consumption function defines a unique optimal value of c satisfying $0 < c(m) < \infty$; a nondegenerate limiting value function defines a corresponding unique value of $-\infty < v(m) < 0$. [°]RIC, FHWC are necessary as well as sufficient for the perfect foresight case. ^{*}That is, the first kink point in $c(m)$ is $m_{\#}$ s.t. for $m < m_{\#}$ the constraint will bind now, while for $m > m_{\#}$ the constraint will bind one period in the future. The second kink point corresponds to the m where the constraint will bind two periods in the future, etc. ^{**}In the Friedman/Muth model, the RIC+FHWC are sufficient, but *not* necessary for nondegeneracy

Table 5 Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained \bar{c} and unconstrained \bar{c} consumption functions

| Main Condition Subcondition | Math | Outcome, Comments or Results |
|---|---|--|
| GIC and RIC and RIC | $1 < \mathbf{P}/\Gamma$ $\mathbf{P}/R < 1$ $1 < \mathbf{P}/R$ | Constraint never binds for $m \geq 1$ FHWC holds ($R > \Gamma$); $\dot{c}(m) = \bar{c}(m)$ for $m \geq 1$ $\dot{c}(m)$ is degenerate: $\dot{c}(m) = 0$ |
| GIC and RIC | $\mathbf{P}/\Gamma < 1$ $\mathbf{P}/R < 1$ | Constraint binds in finite time for any m FHWC may or may not hold $\lim_{m \uparrow \infty} \bar{c}(m) - \dot{c}(m) = 0$ $\lim_{m \uparrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ |
| and RIC | $1 < \mathbf{P}/R$ | FHWC $\lim_{m \uparrow \infty} \dot{\kappa}(m) = 0$ |

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~GIC~~ and RIC both hold, while the third row indicates that when the GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the GIC holds, the constraint will bind in finite time.