Theoretical Foundations of Buffer Stock Saving

August 23, 2021

Christopher D. Carroll[1]

Abstract

This paper builds foundations for rigorous and intuitive understanding of 'buffer stock' saving models (Bewley (1977)-like models that have a wealth target), pairing each theoretical result with quantitative illustrations. After describing conditions under which a consumption function exists, the paper articulates stricter 'Growth Impatience' conditions that guarantee alternative forms of stability — either at the population level, or for individual consumers. Together, the numerical tools and analytical results constitute a comprehensive toolkit for understanding buffer stock models.

Keywords Precautionary saving, buffer stock saving, marginal propensity to consume, permanent income hypothesis, income fluctuation problem

JEL codes D81, D91, E21

Powered by Econ-ARK

Dashboard:	https://econ-ark.org/materials/BufferStockTheory?dashboard
PDF:	https://econ-ark.github.io/BufferStockTheory/BufferStockTheory.pdf
Slides:	https://econ-ark.github.io/BufferStockTheory/BufferStockTheory-Slides.pdf
html:	https://econ-ark.github.io/BufferStockTheory
Appendix:	https://econ-ark.github.io/BufferStockTheory#Appendices
bibtex:	https://econ-ark.github.io/BufferStockTheory/LaTeX/BufferStockTheory-Self.bib
GitHub:	https://github.com/econ-ark/BufferStockTheory

A dashboard allows users to see the consequences of alternative parametric choices in a live interactive framework; a corresponding Jupyter Notebook uses the Econ-ARK/HARK toolkit to produce all of the paper's figures (warning: the notebook may take several minutes to launch).

All figures and numerical results can be automatically reproduced using the Econ-ARK/HARK toolkit, which can be cited per our references (Carroll, Kaufman, Kazil, Palmer, and White (2018)); for reference to the toolkit itself see Acknowleding Econ-ARK. Thanks to the Consumer Financial Protection Bureau for funding the original creation of the Econ-ARK toolkit; and to the Sloan Foundation for funding Econ-ARK's extensive further development that brought it to the point where it could be used for this project. The toolkit can be cited with its digital object identifier, 10.5281/zenodo.1001067, as is done in the paper's own references as Carroll, Kaufman, Kazil, Palmer, and White (2018).

Thanks to Will Du, James Feigenbaum, Joseph Kaboski, Miles Kimball, Qingyin Ma, Misuzu Otsuka, Damiano Sandri, John Stachurski, Adam Szeidl, Alexis Akira Toda, Metin Uyanik, Mateo Velásquez-Giraldo, Weifeng Wu, Jiaxiong Yao, and Xudong Zheng for comments on earlier versions of this paper, John Boyd for help in applying his weighted contraction mapping theorem, Ryoji Hiraguchi for extraordinary mathematical insight that improved the paper greatly, David Zervos for early guidance to the literature, and participants in a seminar at the Johns Hopkins University, a presentation at the 2009 meetings of the Society of Economic Dynamics for their insights, and at a presentation at the Australian National University.

[1]Contact: ccarroll@jhu.edu, Department of Economics, 590 Wyman Hall, Johns Hopkins University, Baltimore, MD 21218, https://www.econ2.jhu.edu/people/ccarroll, and National Bureau of Economic Research.

1 Introduction

In the presence of realistic transitory and permanent shocks to income *a la* Friedman (1957) and Muth (1960), only one further ingredient is required to construct a microeconomically testable model of consumption: A description of preferences. Zeldes (1989) was the first to calibrate a quantitatively plausible example; his paper spawned a literature showing that such models' predictions can match household life cycle data reasonably well, whether or not explicit liquidity constraints are imposed.[1]

References

Abowd, John M., and David Card (1989): "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, 57, 411–445.

Aiyagari, S. Rao (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

Alvarez, Fernando, and Nancy L Stokey (1998): "Dynamic programming with homogeneous functions," *Journal of economic theory*, 82(1), 167–189.

Bellman, Richard (1957): *Dynamic Programming*. Princeton University Press, Princeton, NJ, USA, 1 edn.

Benhabib, Jess, Alberto Bisin, and Shenghao Zhu (2015): "The wealth distribution in Bewley economies with capital income risk," *Journal of Economic Theory*, 159, 489–515, Available at https://www.nber.org/papers/w20157.pdf.

Bewley, Truman (1977): "The Permanent Income Hypothesis: A Theoretical Formulation," *Journal of Economic Theory*, 16, 252–292.

Blanchard, Olivier J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(2), 223–247.

Blundell, Richard, Hamish Low, and Ian Preston (2008): "Decomposing Changes in Income Risk Using Consumption Data," *Manusscript, University College London*.

Boyd, John H. (1990): "Recursive Utility and the Ramsey Problem," *Journal of Economic Theory*, 50(2), 326–345.

Cagetti, Marco (2003): "Wealth Accumulation Over the Life Cycle and Precautionary Savings," *Journal of Business and Economic Statistics*, 21(3), 339–353.

[1] See Carroll (1997) or Gourinchas and Parker (2002) for arguments that models with only 'natural' constraints (see below) match a wide variety of facts; for a model with explicit constraints that produces very similar results, see, e.g., Cagetti (2003).

Carroll, Christopher D. (1992): "The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence," *Brookings Papers on Economic Activity*, 1992(2), 61–156, http://econ.jhu.edu/people/ccarroll/BufferStockBPEA.pdf.

——— (1997): "Buffer Stock Saving and the Life Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, CXII(1), 1–56, http://econ.jhu.edu/people/ccarroll/BSLCPIH.zip.

——— (2006): "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems," *Economics Letters*, 91(3), 312–320, http://econ.jhu.edu/people/ccarroll/EndogenousGridpoints.pdf.

Carroll, Christopher D., Martin Holm, and Miles S. Kimball (2019): "Liquidity Constraints and Precautionary Saving," *Manuscript, Johns Hopkins University*, http://www.econ2.jhu.edu/people/ccarroll/papers/LiqConstr.

Carroll, Christopher D., Alexander M. Kaufman, Jacqueline L. Kazil, Nathan M. Palmer, and Matthew N. White (2018): "The Econ-ARK and HARK: Open Source Tools for Computational Economics," in *Proceedings of the 17th Python in Science Conference*, ed. by Fatih Akici, David Lippa, Dillon Niederhut, and M Pacer, pp. 25 – 30. doi: 10.5281/zenodo.1001067.

Carroll, Christopher D., and Miles S. Kimball (1996): "On the Concavity of the Consumption Function," *Econometrica*, 64(4), 981–992, http://econ.jhu.edu/people/ccarroll/concavity.pdf.

Carroll, Christopher D., and Andrew A. Samwick (1997): "The Nature of Precautionary Wealth," *Journal of Monetary Economics*, 40(1), 41–71.

Carroll, Christopher D., Jiri Slacalek, Kiichi Tokuoka, and Matthew N. White (2017): "The Distribution of Wealth and the Marginal Propensity to Consume," *Quantitative Economics*, 8, 977–1020, At http://econ.jhu.edu/people/ccarroll/papers/cstwMPC.

Chamberlain, Gary, and Charles A. Wilson (2000): "Optimal Intertemporal Consumption Under Uncertainty," *Review of Economic Dynamics*, 3(3), 365–395.

Clarida, Richard H. (1987): "Consumption, Liquidity Constraints, and Asset Accumulation in the Face of Random Fluctuations in Income," *International Economic Review*, XXVIII, 339–351.

Daly, Moira, Dmytro Hryshko, and Iourii Manovskii (2016): "Improving the measurement of earnings dynamics," Discussion paper, National Bureau of Economic Research.

Deaton, Angus S. (1991): "Saving and Liquidity Constraints," *Econometrica*, 59, 1221–1248, http://www.jstor.org/stable/2938366.

Durán, Jorge (2003): "Discounting long run average growth in stochastic dynamic programs," *Economic Theory*, 22(2), 395–413.

Friedman, Milton A. (1957): *A Theory of the Consumption Function*. Princeton University Press.

Gourinchas, Pierre-Olivier, and Jonathan Parker (2002): "Consumption Over the Life Cycle," *Econometrica*, 70(1), 47–89.

Hendricks, Lutz (2001): *Bequests and Retirement Wealth in the United States*. University of Arizona.

——— (2016): "Wealth Distribution and Bequests," *Lecture Notes, Economics 821, University of North Carolina*.

Hiraguchi, Ryoji (2003): "On the Convergence of Consumption Rules," *Manuscript, Johns Hopkins University*.

Hryshko, Dmytro, and Iourii Manovskii (2020): "How much consumption insurance in the US?," *Manuscript, University of Alerta*.

Jappelli, Tullio, and Luigi Pistaferri (2000): "Intertemporal Choice and Consumption Mobility," *Econometric Society World Congress 2000 Contributed Paper Number 0118*.

Krueger, Dirk, Kurt Mitman, and Fabrizio Perri (2016): "Macroeconomics and Household Heterogeneity," *Handbook of Macroeconomics*, 2, 843–921.

Li, Huiyu, and John Stachurski (2014): "Solving the income fluctuation problem with unbounded rewards," *Journal of Economic Dynamics and Control*, 45, 353–365.

Ma, Qingyin, John Stachurski, and Alexis Akira Toda (2020): "The income fluctuation problem and the evolution of wealth," *Journal of Economic Theory*, 187.

Ma, Qingyin, and Alexis Akira Toda (2020): "A Theory of the Saving Rate of the Rich," .

MaCurdy, Thomas (1982): "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis," *Journal of Econometrics*, 18(1), 83–114.

Martins-da Rocha, V Filipe, and Yiannis Vailakis (2010): "Existence and uniqueness of a fixed point for local contractions," *Econometrica*, 78(3), 1127–1141.

Matkowski, Janusz, and Andrzej S. Nowak (2011): "On Discounted Dynamic Programming With Unbounded Returns," *Economic Theory*, 46, 455–474.

Modigliani, Franco (1966): "The Life Cycle Hypothesis, the Demand for Wealth, and the Supply of Capital," *Social Research*, 33, 160–217.

Muth, John F. (1960): "Optimal Properties of Exponentially Weighted Forecasts," *Journal of the American Statistical Association*, 55(290), 299–306.

Rabault, Guillaume (2002): "When do borrowing constraints bind? Some new results on the income fluctuation problem," *Journal of Economic Dynamics and Control*, 26(2), 217–245.

Ramsey, Frank (1928): "A Mathematical Theory of Saving," *Economic Journal*, 38(152), 543–559.

Riehl, Emily (2017): *Category theory in context*. Courier Dover Publications.

Rincón-Zapatero, Juan Pablo, and Carlos Rodríguez-Palmero (2003): "Existence and uniqueness of solutions to the Bellman equation in the unbounded case," *Econometrica*, 71(5), 1519–1555.

Schechtman, Jack, and Vera Escudero (1977): "Some results on 'An Income Fluctuation Problem'," *Journal of Economic Theory*, 16, 151–166.

Scheinkman, José, and Laurence Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54(1), 23–46.

Schmitt-Grohé, Stephanie, and Martin Uribe (2003): "Closing small open economy models," *Journal of international Economics*, 61(1), 163–185.

Stachurski, John, and Alexis Akira Toda (2019): "An Impossibility Theorem for Wealth in Heterogeneous-Agent Models with Limited Heterogeneity," *Journal of Economic Theory*, 182, 1–24.

Stokey, Nancy L., Robert E. Lucas, and Edward C. Prescott (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron (2004): "Consumption and Risk Sharing Over the Life Cycle," *Journal of Monetary Economics*, 51(3), 609–633.

Szeidl, Adam (2013): "Stable Invariant Distribution in Buffer-Stock Saving and Stochastic Growth Models," *Manuscript, Central European University*, Available at http://www.personal.ceu.hu/staff/Adam_Szeidl/papers/invariant_revision.pdf.

Toche, Patrick (2005): "A Tractable Model of Precautionary Saving in Continuous Time," *Economics Letters*, 87(2), 267–272, http://ideas.repec.org/a/eee/ecolet/v87y2005i2p267-272.html.

Yao, Jiaxiong (2012): "The Theoretical Foundations of Buffer Stock Saving: A Note," *Manuscript, Johns Hopkins University*.

Zeldes, Stephen P. (1989): "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," *Quarterly Journal of Economics*, 104(2), 275–298.