Consumption-Savings with Retirement: Identification in Dynamic Discrete-Continuous Choice Models

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August, 2021

Introduction

Motivation

- Agents make complex decisions which involve continuous and discrete decisions: either because of the nature of the goods (notably durables); or because consumption decisions require adjustment or transaction costs (e.g. Chetty and Szeidl, 2007) and therefore are infrequently and discretely adjusted.
- Examples: agents choose
    - which retail stores to visit and how much to buy,
    - which car to buy and how much to drive
    - which investment products to buy and how much to invest,
    - which tasks to perform and how much effort to exert,

Introduction

Two distinct approaches:

- Euler equation approaches assume existence of single composite good which decision-maker can freely and continuously adjust
- Discrete-choice models: Discount factor is typically not identified plus additional restrictive normalizations on utility (see for example Magnac and Thesmar (2002))
    - Few exceptions: see Levy & Schiraldi (2021).

Introduction

We make three main theoretical contributions:

- Solve for the selection issue
- Establish new set of (pointwise) identification results of the discount factor and utility function
- Propose a simple two-step method to estimate (semi)parametrically this class of models

Introduction

We then apply our theoretical results to a lifecycle savings model with retirement

- Use continuous consumption and binary retirement choices in Panel Study of Income Dynamics (PSID)
- Find mean annual discount factor of 0.91, with substantial heterogeneity: while the youngest households have a δ approaching one, it falls to 0.863 by age 85.
- At the mean consumption level, we find a RRA coefficient of approximately 0.5 for retired households and 0.3 for working households.

Model

Setup

We assume an infinite time horizon ($t = 1, 2, \ldots$) and a stationary environment.

- Each period, agent chooses a discrete alternative $j \in \mathcal{J}$ and continuous quantity $q_j \in \mathbb{R}$
- State variables $s_{it}, z_{it}, L_{it}, \zeta_{it}, \varepsilon_{it}$. $\bar{s}_{it} \equiv (s_{it}, z_{it}, L_{it})$.
- $s_{it}, z_{it} \in \mathcal{S}$, and $L_{it} \in \mathbb{R}$ are observed by the researcher
    - s_{it} is payoff-relevant state drawn from finite state space
    - z_{it} instruments for the discrete choice
    - $L_{it} \in \mathbb{R}$ is current wealth
- $\varepsilon_{it} \equiv (\varepsilon_{i1t}, \ldots, \varepsilon_{iJt}) \in \mathbb{R}^J$ represents the vector of individual idiosyncratic random preference shocks
- $\zeta_{it} \in \mathbb{R}$ is an individual level shock to the marginal

Setup: Assumptions

- The instantaneous utilities are given by, for each $j \in J$,

$$\bar{u}_{jt}(s_t, \zeta_t, q_{jt}, z_t, \varepsilon_{jt}) = u_{ijt}(s_t, z_t, \zeta_t, q_{jt}) + \varepsilon_{jt}$$

- $F(\varepsilon_t)$ and $G(\zeta_t)$ are known
- Both shocks are observed by agents before making any decisions
- CI assumption similar to Rust

Selection problem

Challenge in the identification: solve selection problem on unobservable — particularly ζ_t — which may affect the conditional continuous choice.

Assumption

(Unlimited encouragement) There exists k such that for all $j \in \mathcal{J} \setminus \{k\}$:

1. *The marginal utility $\partial u_j / \partial q$ is independent of z for all $(q, \bar{s}, \zeta)$.*
2. *For all (s, L) there exists a sequence $\{z_{j,n}\}$ such that $\lim_{n \to \infty} Pr(d_t = j | s, L, z_{j,n}) = 1$*

Unlimited encouragement

It may be viewed as a strengthening of the common encouragement design of experiments with non-random treatment assignment.

- For example, suppose a consumer is choosing between a local bodega in Manhattan and a Costco in New Jersey (which differ in the marginal utility of their goods)
- We require that traffic may be sufficiently bad that the consumer chooses the Manhattan store with arbitrarily high probability — but note that we do not require traffic conditions that force them to New Jersey (and moreover the marginal utility in New Jersey may always depend on traffic).

Lemma 1: ζ and $\{q_j^*(\bar{s}, z, \zeta)\}_{j \in \mathcal{J}}$ are identified

Setup: Value functions

The agent's problem can be written recursively as:

$$V(\bar{s}_t, \zeta_t, \varepsilon_t) = \max_{j_t, q_{jt}} \{u_j(q_{jt}, s_t, \zeta_t, z_t) + \varepsilon_{jt} + \\ + \delta E[V(\bar{s}_{t+1}, L_{t+1}, \zeta_{t+1}, \varepsilon_{t+1}) | j_t, q_{jt}, \bar{s}_t]\}$$

$$\text{s.t.} \quad L_{t+1} = f_j(s_{t+1})(L_t - q_{jt} - \phi_j(s_t, L_t))$$

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$$\text{s.t.} \quad L_{t+1} = f_j(s_{t+1})(L_t - q_{jt} - \phi_j(s_t, L_t))$$

with conditional value function:

$$v_j(\bar{s}_t, \zeta_t) \equiv u_j(q^*_{jt}, \bar{s}_t, \zeta_t) + \delta E\left[V(\bar{s}_{t+1}, \zeta_{t+1}) | j_t, \bar{s}_t, L_t, q^*_{jt}\right]$$

- Note: due to max operator, V not typically concave

Setup: A modified Euler equation

Combining these results, we obtain a modified Euler equation for any alternative j:

$$\frac{\partial u_j(q_t^*, s_t, \zeta_{jt})}{\partial q} = \delta \mathbb{E}\left[f_j(.) \left(\frac{\partial u_j(q_{t+1}^*, s_{t+1}, z_t \zeta_{t+1})}{\partial q} + \frac{\partial \Phi_j(\bar{s}_{t+1}, \zeta_{t+1})}{\partial L_{t+1}} \right) | j_t, \bar{s}_t, q_{jt}^* \right]$$

- If no discrete choice, reduces to familiar Euler equation
- Presence of discrete choice causes two adjustments:
  - Continuation value must account for future marginal utility *and* marginal impact on future choice surplus

Results: Identification

Theorem (1)

$\delta(.)$ and $\partial u(\cdot)/\partial q$ are point-identified.

Level of utility

Assumption (Normalization of utility)

$u_0(0, s, \zeta) = 0$ *for all* $\bar{s}$ *and* ζ

- some degree of normalization is required given that only differences in utilities affect the decision-maker's choices.
- We impose that the utility of consuming $q_0 = 0$ for the reference alternative is normalized to zero across states.

Theorem (4)

Under assumptions above then $(u(\cdot), \delta(\cdot))$ *is point identified.*

Consumption, Savings, and Retirement

Consumption, Savings, and Retirement

The canonical lifecycle consumption model is filled with discrete choices

- We focus on (binary) choice of retirement — i.e. exiting labor force
    - Highly implausible that DM obtains same utility when working and retired

Data

- Primary data source is PSID, 1999-2017.
    - Longitudinal survey of households with bi-annual information about income, employment, consumption, and demographics
    - Greater detail on wealth and consumption added in 1999
- Follow Blundell et al. (2016) to construct measure of consumption.
- Income taxes estimated using NBER Taxsim software
    - Also estimate counterfactual taxes for un-chosen retirement state (incl. social security)
- Returns obtained from Federal Reserve Bank of St. Louis (FRED)

Summary Statistics

	All	Working	Retired	Difference	
Age	47.23	41.84	65.33	-23.49***	-143.58
	(15.85)	(11.60)	(14.74)		
Household Size	2.67	2.84	2.09	0.74***	47.70
	(1.47)	(1.47)	(1.30)		
Years of Education	13.61	13.82	12.89	0.93***	28.27
	(2.63)	(2.50)	(2.93)		
Health Status	2.45	2.29	2.98	-0.69***	-53.67
	(1.05)	(0.97)	(1.14)		
Total Income	49646.97	59272.33	17338.86	41933.46***	96.51
	(49138.39)	(49018.81)	(33155.70)		
Total Wealth	440766.38	461108.95	372485.25	88623.71***	12.40
	(604700.33)	(598687.96)	(619625.53)		
Consumption	25925.37	27287.45	21353.48	5933.97***	29.59
	(19159.19)	(19686.96)	(16466.95)		
Observations	41884	32270	9614	41884	

Estimation

Two-step

- In the first stage, we estimate the policy functions and recover ζ_{it}, and the second stage,
- In the second stage, the marginal utility functions and the discount factor are estimated non-parametrically using a pairwise differencing strategy (Honore and Powell, 2005)

First Stage: discrete choice

- In the first stage, we estimate the policy functions and recover ζ_{it}, and the second stage, we estimate the structural parameters.
- We estimate the unconditional probability of individual i choosing alternative j by SMLE:

$$LL = \sum_{i,j,t} d_{ijt} \log \overline{Pr}_{ijt}(\bar{s}_{it}; \lambda^d) \tag{1}$$

where $\overline{Pr}_{ijt}(\bar{s}_{it}; \lambda^d) = \int Pr_{ijt}(\bar{s}_{it}, \zeta; \lambda^d) g(\zeta) d\zeta$

First Stage: Retired households continuous policy

Estimate continuous policy function and recover ζ_{it}.

- Being retire is an absorbing action, the probability of remaining in this absorbing choice is one. It also satisfies the conditional independence assumption as the current choice is a sufficient statistic for the future value of the lagged choice.
- We thus estimate the continuous policy function for retired households by MLE. Specifically, we specify $\ln q^*_{iRt} = \mu(\bar{s}^{\mu}_{iRt}; \lambda^{c1}_R) + \sigma(\bar{s}^{\sigma}_{iRt}; \lambda^{c2}_R) \cdot \zeta_{it}$
- As the policy is invertible, we can retrieve the unobserved ζ for all households that newly choose to retire or who are already retired, i.e. $\hat{\zeta}_{it}(q_{iRt}|d_t = R) = q^{*-1}_{iRt}(q_{iRt}; \hat{\lambda}^c_R)$ where q_{iRt} is the observed quantity consumed.

First Stage: Working households continuous policy

Retrieve the continuous policy function and unobserved ζ_{it} for those households who choose to work.

- As before $\ln q^*_{iWt} = \mu(\bar{s}^{\mu}_{iWt}; \lambda^{c1}_W) + \sigma(\bar{s}^{\sigma}_{iWt}; \lambda^{c2}_W) \cdot \zeta_{it}$.
- To estimate the unknown parameters λ^{c2}_W, we use a GMM estimator where the set of moment conditions on ζ_{it} which is function of the unknown parameters entering the working policy function:

$$\zeta_{it} = q^{*-1}_{iRt}(q_{iRt}; \hat{\lambda}^c_R) \cdot 1_{j=R} + q^{*-1}_{iWt}(q_{iWt}; \lambda^c_W) \cdot 1_{j=W} \qquad (2)$$

Second Stage: Structural Parameters

The estimation follows the proof of Theorem 1.

- Consider 2 households who make different discrete choices (for concreteness, assume $d_{it} = W$), but *would have* chosen the same quantity had both chosen $d_{i't} = R$
- Once we express the marginal utility of choosing W in terms of R, the two equations have the same unknown marginal utility part $\partial u_R(q^*_{iRt}, s_t)/\partial q_{it}$.
  - Note that $ln(Pr_W/Pr_R) = v_W - v_R$
- ζ_{it} is generically different from $\zeta_{i't}$,

Second Stage

we take the difference of the F.O.C.s which determine the continuous choice for each household and we get:

$$\frac{\partial \Psi_W(\bar{s}_{it})}{\partial L} + \zeta_{it} - \zeta_{i't} = \tag{3}$$
$$\delta_{it}\mathbb{E}\Big[(1 + r_W(\bar{s}_{it}))\mathcal{V}(s_{t+1}, L_W, W, \zeta_{it+1}) - (1 + r_R(\bar{s}_{i't}))\mathcal{V}(s_{t+1}, L_R, R, \zeta_{i't+1})\Big]$$

where $\mathcal{V}(s_{t+1}, L_{t+1}, d_t, \zeta_{t+1}) =$ $\mathbb{E}_{\zeta_{t+1}}\left[\frac{\partial \Phi_R(s_{t+1}, L_{t+1}, d_t, \zeta_{t+1})}{\partial L_{t+1}} + \frac{\partial u_R(q^*_{iRt+1}, s_{t+1})}{\partial q_{i,t+1}} + \zeta_{i,t+1}\right]$ is the expected marginal continuation value.

Second Stage

- If we remove the expectation and add the the expectational error term η_{it}, we can write the equation above as:

$$\Upsilon_{ii't} = \delta_i X_{ii't} + \eta_{ii't} + \delta_i \left((1 + r_{d_{it}}(\bar{s}_{it})) \frac{\partial u_R(q^*_{iRt+1}, s_{it+1})}{\partial q_{Rt+1}} - (1 + r_{d_{i't}}(\bar{s}_{i't})) \frac{\partial u_R(q^*_{i'Rt+1}, s_{i't+1})}{\partial q_{Rt+1}} \right) \tag{4}$$

  - $\Upsilon_{ii't} \equiv \frac{\partial \Psi_{d_{it}}(\bar{s}_{it})}{\partial L_t} + \zeta_{it} - \zeta_{i't}$
  - $X_{ii't} \equiv \left((1 + r_{d_{it}}) \frac{\partial \Phi_{iR}(s_{it+1}, L_{it+1}, d_{it}, \zeta_{it+1})}{\partial L_{it+1}} - (1 + r_{d_{i't}}) \frac{\partial \Phi_{i'R}(s_{i't+1}, L_{i't+1}, d_{i't}, \zeta_{i't+1})}{\partial L_{i't+1}} \right)$
  - the discount factors and the marginal utility are the only unknown objects.

Second Stage: Pairwise differencing

- in practice to write the equation above we need to match two individuals with the same unknown marginal utility at time t
- two individuals to have the same (unknown) marginal utility is that they have the same pay-off relevant state variables and the same optimal quantity chosen.
- We match on the discrete pay-off relevant states (family size and age) and a continuous choice q_t
- we then use a Gaussian kernel to assign weights to each potential match on the basis of differences in their optimal consumption conditioned to household's discrete choice

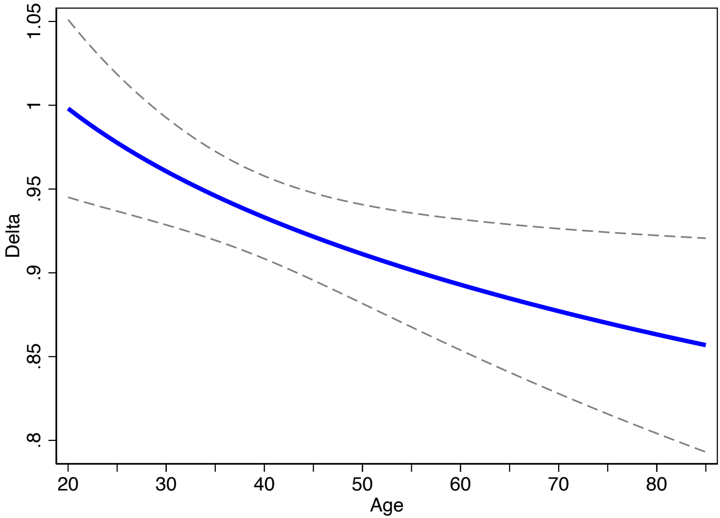
Second Stage: Continuous Choice

- We model the marginal utility as a linear spline for each state
- The equation (4) can be estimated as a WLS/weighted non-linear LS

Results

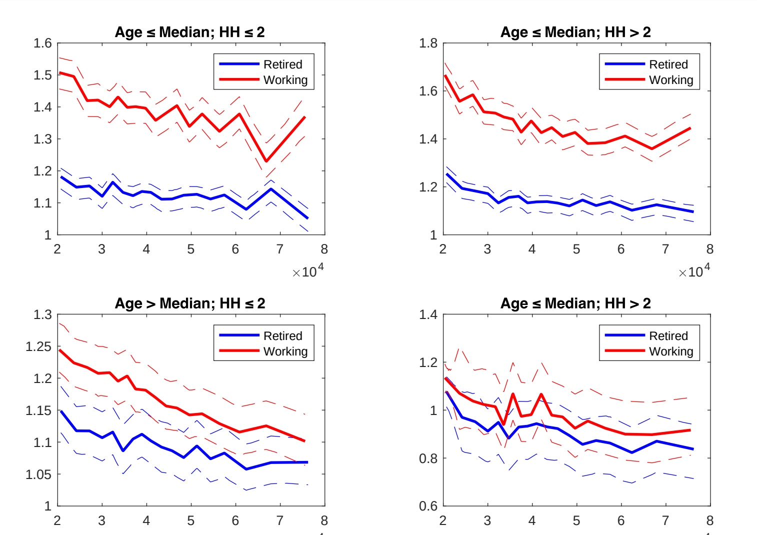
Discount Factors

Figure 1: Estimated Discount Factors



Marginal Utilities

Figure 2: Estimated Marginal Utilities



RRA

- We can use these estimates to calculate a coefficient of relative risk aversion.
- At the mean consumption level, we find a RRA coefficient of approximately 0.5 for retired households and 0.3 for working households.

Conclusion

Conclusion

- We show conditions under which δ and utility are identified
- We use a two-step semi-parametric estimator to estimate a model of lifecycle consumption/retirement
- Heterogeneity in discount factor based on age