The Intergenerational Elasticity of Earnings: Exploring the Mechanisms

Uta Bolt, Eric French, Jamie Hentall Maccuish, and Cormac O'Dea

UCL, IFS, Cambridge, and Yale

August 2021

Why do high income parents have high income children?

Potential explanations: Children of high income families ...

... attain more years of schooling

... have higher cognitive skills

... receive more investments: parental time & school quality

... face different family environment: more educated parents, fewer siblings

Why do high income parents have high income children?

Potential explanations: Children of high income families ...

... attain more years of schooling

... have higher cognitive skills

... receive more investments: parental time & school quality

... face different family environment: more educated parents, fewer siblings

Why do high income parents have high income children?

Potential explanations: Children of high income families ...

... attain more years of schooling

... have higher cognitive skills

... receive more investments: parental time & school quality

... face different family environment: more educated parents, fewer siblings

Why do high income parents have high income children?

Potential explanations: Children of high income families ...

... attain more years of schooling

... have higher cognitive skills

... receive more investments: parental time & school quality

... face different family environment: more educated parents, fewer siblings

Why do high income parents have high income children?

Potential explanations: Children of high income families ...

... attain more years of schooling

... have higher cognitive skills

... receive more investments: parental time & school quality

... face different family environment: more educated parents, fewer siblings

Why do high income parents have high income children?

Potential explanations: Children of high income families ...

... attain more years of schooling

... have higher cognitive skills

... receive more investments: parental time & school quality

... face different family environment: more educated parents, fewer siblings

Why do high income parents have high income children?

What we do in this paper:

$\Rightarrow$ We quantify the importance of these explanations

$\Rightarrow$ We analyse how these different explanations are related

Why do high income parents have high income children?

What we do in this paper:

$\Rightarrow$ We quantify the importance of these explanations

$\Rightarrow$ We analyse how these different explanations are related

Literature

How can we explain intergenerational earnings persistence?

1. Potential mechanisms:
   - Schooling: Carneiro & Heckman (2002), Caucutt & Lochner (2020)
   - Cognition: Dahl & Lochner (2012), Agostinelli & Sorrenti (2018)
   - Parental Investments: Cunha & Heckman (2008), Cunha et al. (2010), Attanasio et al. (2020), Dearden et. al (2002)
   - Family background: Meghir & Palme (2005), Heckman & Karapakula (2019), Bhalotra & Clarke (2020)
2. Decomposition: Blanden, Gregg, Macmillan (2007)
3. Dynamic lifecycle models: Gayle, Golan, Soytas (2018), Lee & Seshadri (2019), Daruich (2020),Bolt et al. (2021)

Literature

How can we explain intergenerational earnings persistence?

1. Potential mechanisms:
   - Schooling: Carneiro & Heckman (2002), Caucutt & Lochner (2020)
   - Cognition: Dahl & Lochner (2012), Agostinelli & Sorrenti (2018)
   - Parental Investments: Cunha & Heckman (2008), Cunha et al. (2010), Attanasio et al. (2020), Dearden et. al (2002)
   - Family background: Meghir & Palme (2005), Heckman & Karapakula (2019), Bhalotra & Clarke (2020)
2. Decomposition: Blanden, Gregg, Macmillan (2007)
3. Dynamic lifecycle models: Gayle, Golan, Soytas (2018), Lee & Seshadri (2019), Daruich (2020),Bolt et al. (2021)

Literature

How can we explain intergenerational earnings persistence?

1. Potential mechanisms:
   - Schooling: Carneiro & Heckman (2002), Caucutt & Lochner (2020)
   - Cognition: Dahl & Lochner (2012), Agostinelli & Sorrenti (2018)
   - Parental Investments: Cunha & Heckman (2008), Cunha et al. (2010), Attanasio et al. (2020), Dearden et. al (2002)
   - Family background: Meghir & Palme (2005), Heckman & Karapakula (2019), Bhalotra & Clarke (2020)
2. Decomposition: Blanden, Gregg, Macmillan (2007)
3. Dynamic lifecycle models: Gayle, Golan, Soytas (2018), Lee & Seshadri (2019), Daruich (2020),Bolt et al. (2021)

Literature

How can we explain intergenerational earnings persistence?

1. Potential mechanisms:
   - Schooling: Carneiro & Heckman (2002), Caucutt & Lochner (2020)
   - Cognition: Dahl & Lochner (2012), Agostinelli & Sorrenti (2018)
   - Parental Investments: Cunha & Heckman (2008), Cunha et al. (2010), Attanasio et al. (2020), Dearden et. al (2002)
   - Family background: Meghir & Palme (2005), Heckman & Karapakula (2019), Bhalotra & Clarke (2020)
2. Decomposition: Blanden, Gregg, Macmillan (2007)
3. Dynamic lifecycle models: Gayle, Golan, Soytas (2018), Lee & Seshadri (2019), Daruich (2020),Bolt et al. (2021)

Our contribution

- Understand how these channels operate and interact
- Mediation analysis - Large number of direct and indirect effects of each channel on lifetime income:
  - School quality → lifetime income
  - School quality → schooling → lifetime income
  - School quality → cognition → schooling → lifetime income
- Solving this would be very difficult with a structural model (just think of the no. of states you'd have to keep track of....)
- Use of data that links early life circumstances to outcomes across the lifecycle for the same sample of individuals

Our contribution

- Understand how these channels operate and interact
- Mediation analysis - Large number of direct and indirect effects of each channel on lifetime income:

  School quality → lifetime income
  School quality → schooling → lifetime income
  School quality → cognition → schooling → lifetime income

- Solving this would be very difficult with a structural model (just think of the no. of states you'd have to keep track of....)
- Use of data that links early life circumstances to outcomes across the lifecycle for the same sample of individuals

Our contribution

- Understand how these channels operate and interact
- Mediation analysis - Large number of direct and indirect effects of each channel on lifetime income:

  School quality → lifetime income
  School quality → schooling → lifetime income
  School quality → cognition → schooling → lifetime income

- Solving this would be very difficult with a structural model (just think of the no. of states you'd have to keep track of....)
- Use of data that links early life circumstances to outcomes across the lifecycle for the same sample of individuals

Our contribution

- Understand how these channels operate and interact
- Mediation analysis - Large number of direct and indirect effects of each channel on lifetime income:

  School quality $\rightarrow$ lifetime income
  School quality $\rightarrow$ schooling $\rightarrow$ lifetime income
  School quality $\rightarrow$ cognition $\rightarrow$ schooling $\rightarrow$ lifetime income
- Solving this would be very difficult with a structural model (just think of the no. of states you'd have to keep track of....)
- Use of data that links early life circumstances to outcomes across the lifecycle for the same sample of individuals

Our contribution

- Understand how these channels operate and interact
- Mediation analysis - Large number of direct and indirect effects of each channel on lifetime income:

  School quality → lifetime income
  School quality → schooling → lifetime income
  School quality → cognition → schooling → lifetime income

- Solving this would be very difficult with a structural model (just think of the no. of states you'd have to keep track of....)
- Use of data that links early life circumstances to outcomes across the lifecycle for the same sample of individuals

Outline

Data - National Child Development Study (NCDS)

- Population born in one week in Britain in 1958
- Followed at ages 0, 7, 11, 16, 23, 26, 33, 37, 42, 49, 55, (60)
- Data on:
  - Parental income
  - Individual's earnings over the lifecycle
  - Potential drivers of the Intergenerational Elasticity of Earnings (IGE)

Data - National Child Development Study (NCDS)

- Population born in one week in Britain in 1958
- Followed at ages 0, 7, 11, 16, 23, 26, 33, 37, 42, 49, 55, (60)
- Data on:
  - Parental income
  - Individual's earnings over the lifecycle
  - Potential drivers of the Intergenerational Elasticity of Earnings (IGE)

Key Facts: Children from high income families ...

1. ... grow up in a different family environment: Details
   - More educated parents, less siblings
2. ... receive more investments: Details
   - At home: reading to child, outings with child, interest in child's education
   - At school: student-teacher ratios, PTA, fraction that continues education
3. ... have better outcomes: Details
   - Cognitive skills at age 16: reading score, maths score, teacher-assessed ability
   - Attain more years of schooling

Key Facts: Children from high income families ...

1. ... grow up in a different family environment: Details
   - More educated parents, less siblings
2. ... receive more investments: Details
   - At home: reading to child, outings with child, interest in child's education
   - At school: student-teacher ratios, PTA, fraction that continues education
3. ... have better outcomes: Details
   - Cognitive skills at age 16: reading score, maths score, teacher-assessed ability
   - Attain more years of schooling

Key Facts: Children from high income families ...

1. ... grow up in a different family environment: Details
   - More educated parents, less siblings

2. ... receive more investments: Details
   - At home: reading to child, outings with child, interest in child's education
   - At school: student-teacher ratios, PTA, fraction that continues education

3. ... have better outcomes: Details
   - Cognitive skills at age 16: reading score, maths score, teacher-assessed ability
   - Attain more years of schooling

Key Fact 3:
Child outcomes differ by parental income

	Parental Income Tertile			P-value:
	Bottom	Middle	Top	Bottom vs Top
Cognition				
Reading at age 16	-0.11	0.01	0.10	0.00
Math at age 16	-0.08	-0.02	0.10	0.00
Education				
Age left education	17.9	17.9	18.1	0.02

Key Fact 3: Child outcomes differ by parental income

	Parental Income Tertile			P-value:
	Bottom	Middle	Top	Bottom vs Top
Cognition				
Reading at age 16	-0.11	0.01	0.10	0.00
Math at age 16	-0.08	-0.02	0.10	0.00
Education				
Age left education	17.9	17.9	18.1	0.02

Key Fact 3:
Child outcomes differ by parental income

	Parental Income Tertile			P-value:
	Bottom	Middle	Top	Bottom vs Top
Cognition				
Reading at age 16	-0.11	0.01	0.10	0.00
Math at age 16	-0.08	-0.02	0.10	0.00
Education				
Age left education	17.9	17.9	18.1	0.02

Outline

Summary of our approach

1. Predict latent factor scores for cognition, time investments and school quality
2. Estimate the IGE
3. Decompose IGE into multiple channels, allowing for increasing degrees of mediation

Latent Factors and Measurement Error

- We do not directly observe cognition, time investments, and school quality
- Instead: Multiple noisy measures for each $\omega = C, inv_t, sq_t$

$$\underbrace{Z_{\omega,m}}_{\text{Measure}} = \underbrace{\lambda_{\omega,m}}_{\substack{\text{Loading}\\\text{Parameter}}} \underbrace{\omega}_{\substack{\text{Latent}\\\text{factor}}} + \underbrace{\epsilon_{\omega,m}}_{\substack{\text{Measurement}\\\text{error}}}$$

Latent Factors and Measurement Error

- We do not directly observe cognition, time investments, and school quality
- Instead: Multiple noisy measures for each $\omega = C, inv_t, sq_t$

$$\underbrace{Z_{\omega,m}}_{\text{Measure}} = \underbrace{\lambda_{\omega,m}}_{\text{Loading Parameter}} \underbrace{\omega}_{\text{Latent factor}} + \underbrace{\epsilon_{\omega,m}}_{\text{Measurement error}}$$

Latent Factors and Measurement Error - Details

$$\underbrace{Z_{\omega,m}}_{\text{Measure}} = \underbrace{\lambda_{\omega,m}}_{\substack{\text{Loading}\\\text{Parameter}}} \underbrace{\omega}_{\substack{\text{Latent}\\\text{factor}}} + \underbrace{\epsilon_{\omega,m}}_{\substack{\text{Measurement}\\\text{error}}}$$

- Key estimation steps :
  1. Estimate measurement system parameters $(\lambda_{\omega,m}, Var(\epsilon_{\omega,m}))$ More Details 1 .
  2. Create weighted avg of measures–noisy measure of the latent factor More Details 2 .
  3. Correct for measurement error using errors-in-variables correction:
     - Read appendix of Heckman et al. 2013.
     - Recall OLS measurement error bias.
     - Key trick: We know the variance of measurement error - can be used to correct for measurement error.

Latent Factors and Measurement Error - Details

$$\underbrace{Z_{\omega,m}}_{\text{Measure}} = \underbrace{\lambda_{\omega,m}}_{\substack{\text{Loading}\\ \text{Parameter}}} \underbrace{\omega}_{\substack{\text{Latent}\\ \text{factor}}} + \underbrace{\epsilon_{\omega,m}}_{\substack{\text{Measurement}\\ \text{error}}}$$

- Key estimation steps :
  1. Estimate measurement system parameters $(\lambda_{\omega,m}, Var(\epsilon_{\omega,m}))$ More Details 1 .
  2. Create weighted avg of measures–noisy measure of the latent factor More Details 2.
  3. Correct for measurement error using errors-in-variables correction:
     - Read appendix of Heckman et al. 2013.
     - Recall OLS measurement error bias.
     - Key trick: We know the variance of measurement error - can be used to correct for measurement error.

Signal-to-Noise Ratios

$$Z_{\omega,m} = \lambda_{\omega,m}\omega + \epsilon_{\omega,m}$$

$$s_{\omega,m} = \frac{(\lambda^2_{\omega,m})Var(\omega)}{(\lambda^2_{\omega,m})Var(\omega) + Var(\epsilon_{\omega,m})}$$

Cognition at 16		**Time Inv 16**		**School Quality 16**	
Reading Score	0.56	P:Supportive	0.32	School Type	0.08
Math Score	0.62	M:Interest in ed	0.90	%Cnt School	0.35
Teacher: Math	0.80	F: Interest in ed	0.75	%FT degree	0.82
Teacher: English	0.72			%Passed A-levels	0.93
				%Studying towards A-levels	0.45
				Teacher Student Ratio	0.20

Back

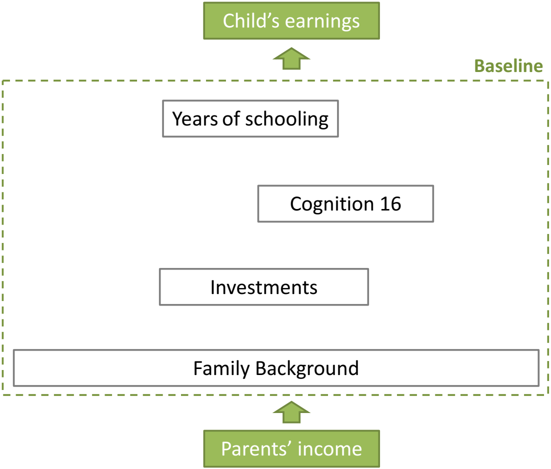
Estimating the IGE

$$\ln Y_i = \rho \ln Y_{Parent,i} + u_i$$

where:

- Y is (demeaned) lifetime earnings
- $Y_{Parent,i}$ (demeaned) lifetime income of parent
- ρ is the Intergenerational Elasticity of Earnings (IGE)

Baseline - Decomposition of IGE



Baseline - Decomposition of IGE

1. Lifetime earnings regression:

$$\ln Y_i = \alpha_S S_i + \alpha_C C_i + \alpha_I \boldsymbol{I}_i + \alpha_F \boldsymbol{F}_i + \alpha_{Y_P} \ln Y_{Parent,i} + u_i^Y$$

$$\boldsymbol{I} = [inv_7, inv_{11}, inv_{16}, sq_7, sq_{11}, sq_{16}]$$
$$\boldsymbol{F} = [ed_m, ed_f, sib]$$

$\Rightarrow$ Can test restrictions, e.g. $\alpha_{sq_7} = \alpha_{sq_{11}} = \alpha_{sq_{16}} = 0$

2. Association between parental income and covariates:

$$C_i = \kappa_C \ln Y_{Parent,i} + v_i^C$$

3. Share of IGE explained by age 16 cognition: $\frac{\alpha_C \kappa_C}{\rho}$

$\Rightarrow$ Only considers direct effect of cognition on lifetime earnings

Baseline - Decomposition of IGE

1. Lifetime earnings regression:

$$\ln Y_i = \alpha_S S_i + \alpha_C C_i + \alpha_I \boldsymbol{I}_i + \alpha_F \boldsymbol{F}_i + \alpha_{Y_P} \ln Y_{Parent,i} + u_i^Y$$

$$\boldsymbol{I} = [inv_7, inv_{11}, inv_{16}, sq_7, sq_{11}, sq_{16}]$$
$$\boldsymbol{F} = [ed_m, ed_f, sib]$$

$\Rightarrow$ Can test restrictions, e.g. $\alpha_{sq_7} = \alpha_{sq_{11}} = \alpha_{sq_{16}} = 0$

2. Association between parental income and covariates:

$$C_i = \kappa_C \ln Y_{Parent,i} + v_i^C$$

3. Share of IGE explained by age 16 cognition: $\frac{\alpha_C \kappa_C}{\rho}$

$\Rightarrow$ Only considers direct effect of cognition on lifetime earnings

Baseline - Decomposition of IGE

1. Lifetime earnings regression:

$$\ln Y_i = \alpha_S S_i + \alpha_C C_i + \alpha_I \boldsymbol{I}_i + \alpha_F \boldsymbol{F}_i + \alpha_{Y_P} \ln Y_{Parent,i} + u_i^Y$$

$$\boldsymbol{I} = [inv_7, inv_{11}, inv_{16}, sq_7, sq_{11}, sq_{16}]$$
$$\boldsymbol{F} = [ed_m, ed_f, sib]$$

$\Rightarrow$ Can test restrictions, e.g. $\alpha_{sq_7} = \alpha_{sq_{11}} = \alpha_{sq_{16}} = 0$

2. Association between parental income and covariates:

$$C_i = \kappa_C \ln Y_{Parent,i} + v_i^C$$

3. Share of IGE explained by age 16 cognition: $\frac{\alpha_C \kappa_C}{\rho}$

$\Rightarrow$ Only considers direct effect of cognition on lifetime earnings

Baseline - Decomposition of IGE

1. Lifetime earnings regression:

$$\ln Y_i = \alpha_S S_i + \alpha_C C_i + \alpha_I \boldsymbol{I}_i + \alpha_F \boldsymbol{F}_i + \alpha_{Y_P} \ln Y_{Parent,i} + u_i^Y$$

$$\begin{aligned} \boldsymbol{I} &= [inv_7, inv_{11}, inv_{16}, sq_7, sq_{11}, sq_{16}] \\ \boldsymbol{F} &= [ed_m, ed_f, sib] \end{aligned}$$

$\Rightarrow$ Can test restrictions, e.g. $\alpha_{sq_7} = \alpha_{sq_{11}} = \alpha_{sq_{16}} = 0$

2. Association between parental income and covariates:

$$C_i = \kappa_C \ln Y_{Parent,i} + v_i^C$$

3. Share of IGE explained by age 16 cognition: $\frac{\alpha_C \kappa_C}{\rho}$

$\Rightarrow$ Only considers direct effect of cognition on lifetime earnings

Baseline - Decomposition of IGE

1. Lifetime earnings regression:

$$\ln Y_i = \alpha_S S_i + \alpha_C C_i + \alpha_I \boldsymbol{I}_i + \alpha_F \boldsymbol{F}_i + \alpha_{Y_P} \ln Y_{Parent,i} + u_i^Y$$

$$\boldsymbol{I} = [inv_7, inv_{11}, inv_{16}, sq_7, sq_{11}, sq_{16}]$$
$$\boldsymbol{F} = [ed_m, ed_f, sib]$$

$\Rightarrow$ Can test restrictions, e.g. $\alpha_{sq_7} = \alpha_{sq_{11}} = \alpha_{sq_{16}} = 0$

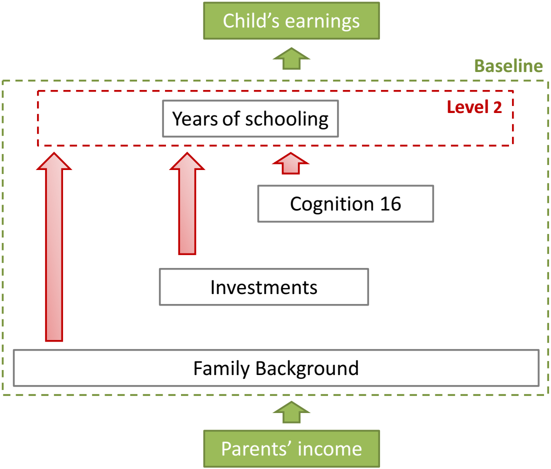
2. Association between parental income and covariates:

$$C_i = \kappa_C \ln Y_{Parent,i} + v_i^C$$

3. Share of IGE explained by age 16 cognition: $\frac{\alpha_C \kappa_C}{\rho}$

$\Rightarrow$ Only considers direct effect of cognition on lifetime earnings

Level 2 - Indirect effects via years of schooling



Level 2 - Indirect effects via years of schooling

Determinants of years of schooling:

$$S_i = \beta_C C_i + \boldsymbol{\beta_I} \boldsymbol{I}_i + \boldsymbol{\beta_F} \boldsymbol{F}_i + \beta_{Y_P} \ln Y_{Parent,i} + u_i^S$$

Share of the IGE explained by age 16 cognition:

$$(\underbrace{\alpha_C \cdot \kappa_C}_{\text{Direct Effect}} + \underbrace{\alpha_S \cdot \beta_C \cdot \kappa_C}_{\text{Indirect Effect via schooling}})/\rho.$$

Level 2 - Indirect effects via years of schooling

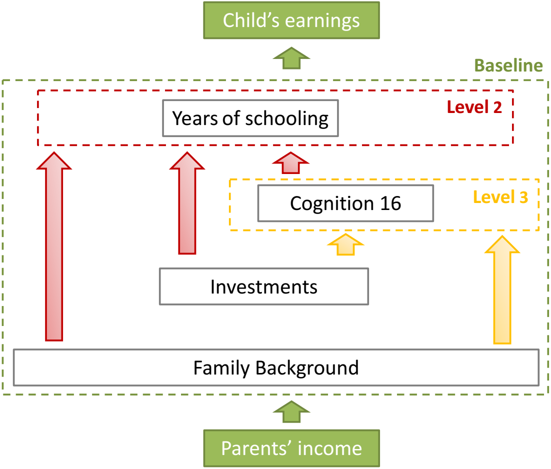
Determinants of years of schooling:

$$S_i \quad = \quad \beta_C C_i + \boldsymbol{\beta_I} \boldsymbol{I}_i + \boldsymbol{\beta_F} \boldsymbol{F}_i + \beta_{Y_P} \ln Y_{Parent,i} + u_i^S$$

Share of the IGE explained by age 16 cognition:

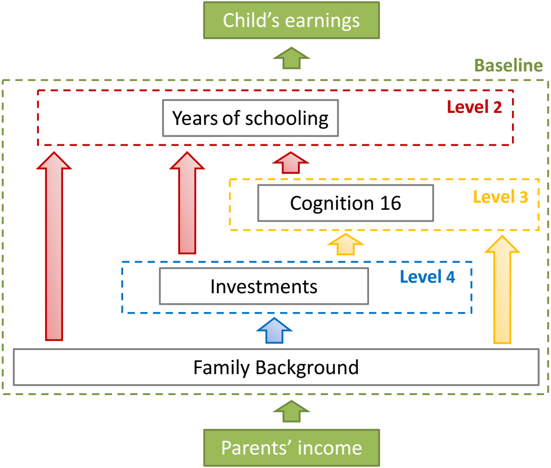
$$(\underbrace{\alpha_C \cdot \kappa_C}_{\text{Direct Effect}} + \underbrace{\alpha_S \cdot \beta_C \cdot \kappa_C}_{\text{Indirect Effect via schooling}})/\rho.$$

Level 3 - Indirect effects via years of schooling



Details

Level 4- Indirect effects via years of schooling



Details

Outline

IGE Estimates

with and without measurement error corrections

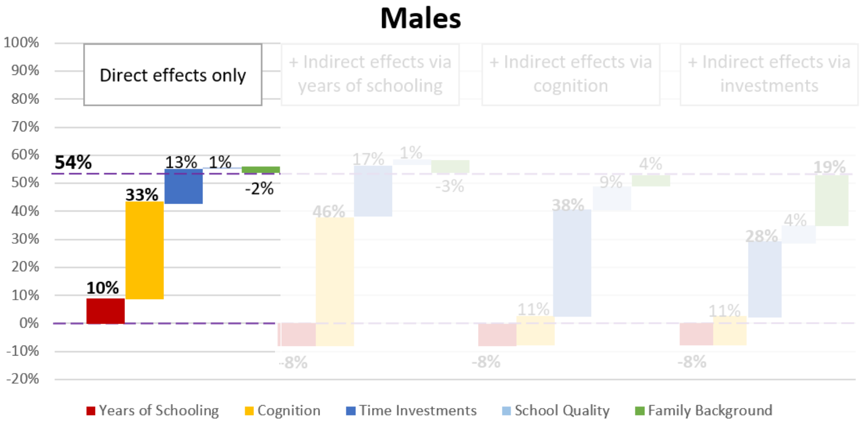
$$\ln Y_i = \rho \ln Y_{Parent,i} + u_i$$

where $\rho =$ Intergenerational Elasticity of Earnings (IGE)

Table: IGE estimates

	Male	Female	Male uncorrected	Female uncorrected
IGE	0.317	0.236	0.155	0.115
	(0.097)	(0.105)	(0.045)	(0.050)
N	1350	1347	1350	1347

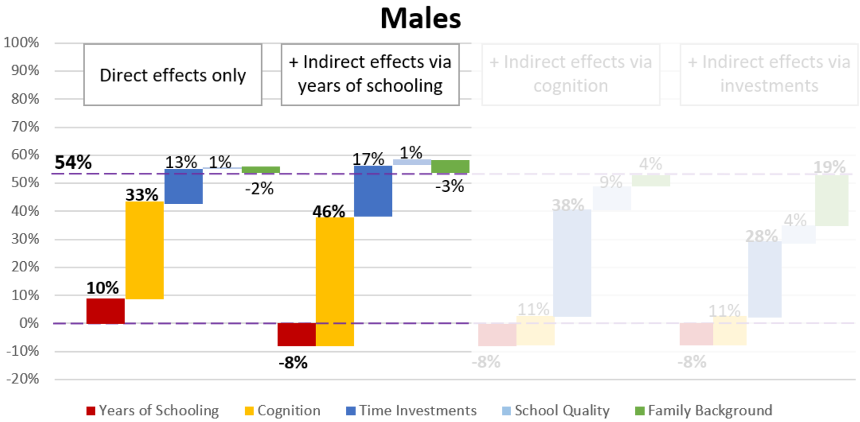
Results: Mediation Analysis - Males



⇒ 54% of IGE is explained by our channels

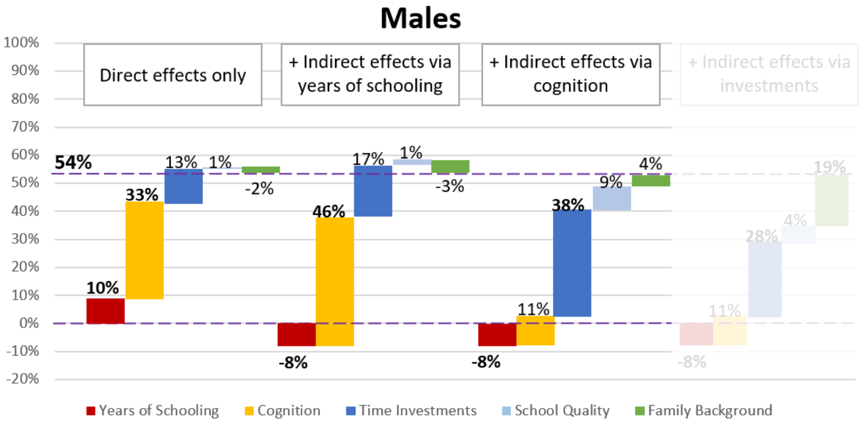
⇒ Cognitive skills and schooling significantly affect IGE

Results: Mediation Analysis - Males



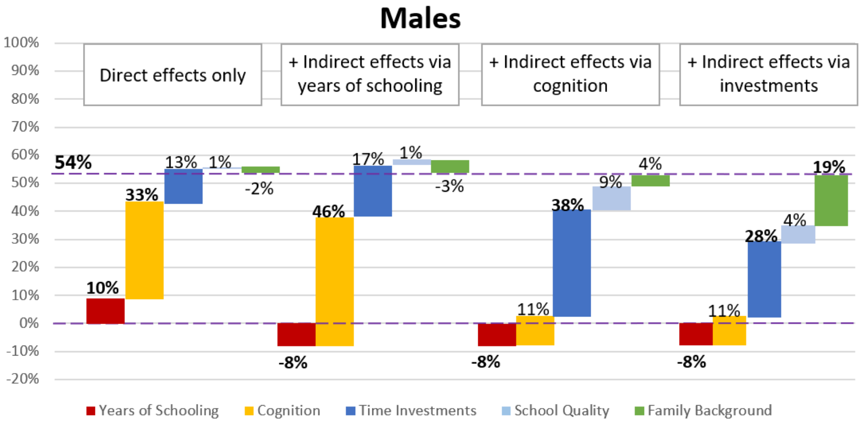
⇒ Effect of schooling is completely mediated by cognitive skills

Results: Mediation Analysis - Males



⇒ Most differences in cognition are explained by differences in time investments and school quality

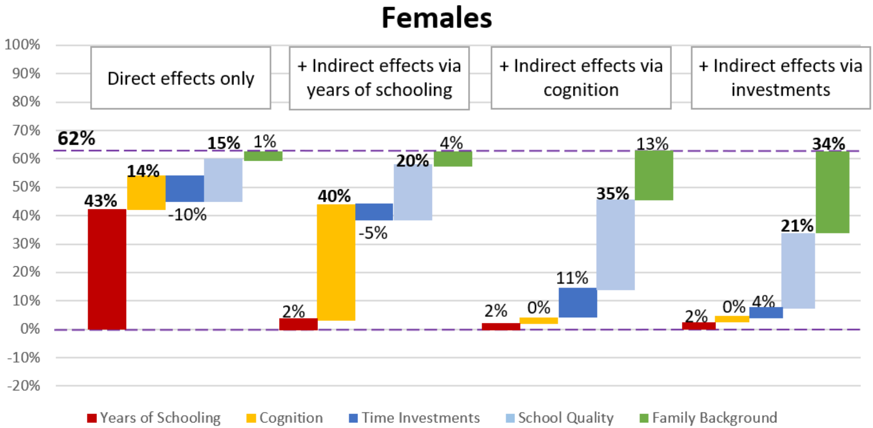
Results: Mediation Analysis - Males



⇒ Family background-related differences explain 19% of IGE.

⇒ Even if we control for family background, the income gradient in investments persists

Results: Mediation Analysis - Females



Key Results - Summary

For both, men and women:

- Years of schooling and cognition explain the large shares of the IGE
- But: Effect of years of schooling is entirely mediated by cognition ...

  ... and cognition is largely mediated by investments

⇒ Differences in investments between rich and poor families really matter for the IGE...

... and not all of them can be explained by family background

Key Results - Summary

For both, men and women:

- Years of schooling and cognition explain the large shares of the IGE
- But: Effect of years of schooling is entirely mediated by cognition ...

  ... and cognition is largely mediated by investments

$\Rightarrow$ Differences in investments between rich and poor families really matter for the IGE...

... and not all of them can be explained by family background

Key Results - Summary

For both, men and women:

- Years of schooling and cognition explain the large shares of the IGE
- But: Effect of years of schooling is entirely mediated by cognition ...

  ... and cognition is largely mediated by investments

$\Rightarrow$ Differences in investments between rich and poor families really matter for the IGE...

... and not all of them can be explained by family background

Key Results - Summary

For both, men and women:

- Years of schooling and cognition explain the large shares of the IGE
- But: Effect of years of schooling is entirely mediated by cognition ...

  ... and cognition is largely mediated by investments

⇒ Differences in investments between rich and poor families really matter for the IGE...

... and not all of them can be explained by family background

Key Results - Summary

For both, men and women:

- Years of schooling and cognition explain the large shares of the IGE
- But: Effect of years of schooling is entirely mediated by cognition ...

  ... and cognition is largely mediated by investments

$\Rightarrow$ Differences in investments between rich and poor families really matter for the IGE...

... and not all of them can be explained by family background

Testing Restrictions

Do time investments, school quality, and family background have direct effects on lifetime earnings?

	Lifetime Earnings		Years of Schooling		Cognition	
	Males	Females	Males	Females	Males	Females
P-values for joint significance:						
Time Investments	0.708	0.842	0.490	0.315	0.096	**0.031**
School Quality	0.501	0.285	0.424	0.183	**0.017**	**0.009**
Family Background	0.291	0.276	0.218	0.408	**0.012**	**0.020**
N	1350	1347	1350	1347	1350	1347

$\Rightarrow$ Only jointly significant for explaining cognition!

Testing Restrictions

Do time investments, school quality, and family background have direct effects on lifetime earnings?

	Lifetime Earnings		**Years of Schooling**		**Cognition**	
	Males	Females	Males	Females	Males	Females
P-values for joint significance:						
Time Investments	0.708	0.842	0.490	0.315	0.096	**0.031**
School Quality	0.501	0.285	0.424	0.183	**0.017**	**0.009**
Family Background	0.291	0.276	0.218	0.408	**0.012**	**0.020**
N	1350	1347	1350	1347	1350	1347

$\Rightarrow$ Only jointly significant for explaining cognition!

Importance of Correcting for Measurement Error

- We combine multiple measures and correct for measurement error in predicted factors.
- What happens if we used a single measure instead?

⇒ under-estimate the importance of cognition by up to 35%

⇒ fraction explained by parental investment attenuated by 45%

	Predicted factor + ME correction				Single measure			
	L1	L2	L3	L4	L1	L2	L3	L4
	Direct	+via years of schooling	+via cognition	+via investments	Direct	+via years of schooling	+via cognition	+via investments
Years of Schooling	**10%**	**-8%**	**-8%**	**-8%**	**18%**	1%	1%	1%
Cognition	**33%**	**46%**	11%	11%	**18%**	**29%**	12%	12%
Investments	14%	19%	**47%**	**33%**	**13%**	**18%**	**29%**	**18%**
Family Background	-2%	-3%	4%	**19%**	2%	3%	9%	**19%**

Importance of Correcting for Measurement Error

- We combine multiple measures and correct for measurement error in predicted factors.
- What happens if we used a single measure instead?

⇒ under-estimate the importance of cognition by up to 35%

⇒ fraction explained by parental investment attenuated by 45%

	Predicted factor + ME correction				Single measure			
	L1	L2	L3	L4	L1	L2	L3	L4
	Direct	+via years of schooling	+via cognition	+via investments	Direct	+via years of schooling	+via cognition	+via investments
Years of Schooling	**10%**	**-8%**	**-8%**	**-8%**	**18%**	1%	1%	1%
Cognition	**33%**	**46%**	11%	11%	**18%**	**29%**	12%	12%
Investments	14%	19%	**47%**	**33%**	**13%**	**18%**	**29%**	**18%**
Family Background	-2%	-3%	4%	**19%**	2%	3%	9%	**19%**

Importance of Correcting for Measurement Error

- We combine multiple measures and correct for measurement error in predicted factors.
- What happens if we used a single measure instead?

⇒ under-estimate the importance of cognition by up to 35%

⇒ fraction explained by parental investment attenuated by 45%

	Predicted factor + ME correction				Single measure			
	L1	L2	L3	L4	L1	L2	L3	L4
	Direct	+via years of schooling	+via cognition	+via investments	Direct	+via years of schooling	+via cognition	+via investments
Years of Schooling	**10%**	**-8%**	**-8%**	**-8%**	**18%**	1%	1%	1%
Cognition	**33%**	**46%**	11%	11%	**18%**	**29%**	12%	12%
Investments	14%	19%	**47%**	**33%**	**13%**	**18%**	**29%**	**18%**
Family Background	-2%	-3%	4%	**19%**	2%	3%	9%	**19%**

Importance of Correcting for Measurement Error

- We combine multiple measures and correct for measurement error in predicted factors.
- What happens if we used a single measure instead?

⇒ under-estimate the importance of cognition by up to 35%

⇒ fraction explained by parental investment attenuated by 45%

	Predicted factor + ME correction				Single measure			
	L1	L2	L3	L4	L1	L2	L3	L4
	Direct	+via years of schooling	+via cognition	+via investments	Direct	+via years of schooling	+via cognition	+via investments
Years of Schooling	**10%**	**-8%**	**-8%**	**-8%**	**18%**	1%	1%	1%
Cognition	**33%**	**46%**	11%	11%	**18%**	**29%**	12%	12%
Investments	14%	19%	**47%**	**33%**	**13%**	**18%**	**29%**	**18%**
Family Background	-2%	-3%	4%	**19%**	2%	3%	9%	**19%**

Robustness

Our results are robust to:

- Accounting for non-cognitive skills see table
- Complementarity between years of schooling and cognition see table
- Including other common family background variables see table

Conclusions

- More than half of the intergenerational elasticity in earnings (IGE) is explained by differences in:
  - years of schooling
  - cognition
  - parental investments
  - family circumstances during childhood.
- Main driver of the IGE
  - Parental investments received early in life
  - ..which leads to higher cognitive development and earnings.

Conclusions

- More than half of the intergenerational elasticity in earnings (IGE) is explained by differences in:
  - years of schooling
  - cognition
  - parental investments
  - family circumstances during childhood.
- Main driver of the IGE
  - Parental investments received early in life
  - ..which leads to higher cognitive development and earnings.

Robustness Check 1

Accounting for non-cognitive skills

	Males				Females			
	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
Years of Schooling	**0.104**	**-0.078**	**-0.078**	**-0.078**	**0.420**	0.039	0.039	0.039
	[0.031, 0.266]	[-0.274, -0.012]	[-0.274, -0.012]	[-0.274, -0.012]	[0.194, 1.127]	[-0.171, 0.329]	[-0.171, 0.329]	[-0.171, 0.329]
Cognition	**0.338**	**0.474**	0.107	0.107	0.135	**0.394**	0.012	0.012
	[0.181, 0.759]	[0.296, 1.007]	[-0.096, 0.378]	[-0.096, 0.378]	[-0.016, 0.400]	[0.161, 1.071]	[-0.297, 0.212]	[-0.297, 0.212]
Non-cognitive skills	-0.004	-0.005	- 0.046	- 0.046	0.000	0.000	- 0.022	-0.022
	[-0.079, 0.042]	[-0.082, 0.043]	[-0.169, 0.007]	[-0.169, 0.007]	[-0.047, 0.039]	[-0.073, 0.061]	[-0.151, 0.022]	[-0.151, 0.022]
Investments	0.123	0.178	**0.517**	**0.354**	0.033	0.128	**0.444**	0.239
	[-0.133, 0.454]	[-0.063, 0.623]	[0.212, 1.346]	[0.112, 0.974]	[-0.306, 0.351]	[-0.142, 0.525]	[0.158, 1.278]	[-0.038, 0.745]
Family Background	-0.008	-0.018	0.051	0.214	-0.006	0.020	0.108	0.314
	[-0.173, 0.109]	[-0.194, 0.103]	[-0.093, 0.188]	[0.092, 0.558]	[-0.238, 0.215]	[-0.189, 0.303]	[-0.066, 0.504]	[0.089, 0.997]
N	1339	1339	1339	1339	1336	1336	1336	1336

Notes: 95% Confidence intervals in brackets. Coefficients that are significant at the 5% level are **bold**.

Back

Robustness Check 2

Complementarity between schools and cognition

	Males			Females		
	EIV	GMM	GMM	EIV	GMM	GMM
Years of Schooling	**0.093**	**0.165**	**0.162**	**0.425**	**0.452**	**0.487**
	[0.019, 0.228]	[0.073, 0.325]	[0.066, 0.310]	[0.158, 1.337]	[0.231, 1.083]	[0.265, 1.206]
Cognition	**0.333**	**0.368**	**0.365**	0.135	0.094	0.078
	[0.193, 0.729]	[0.173, 0.646]	[0.184, 0.625]	[-0.008, 0.502]	[-0.058, 0.268]	[-0.081, 0.229]
Years of Schooling $\times$ Cognition			-0.016			0.003
			[-0.066, 0.017]			[-0.054, 0.070]
Investments	0.163	0.137	0.122	0.057	0.149	0.122
	[-0.060, 0.456]	[-0.112, 0.428]	[-0.119, 0.392]	[-0.266, 0.437]	[-0.140, 0.554]	[-0.124, 0.513]
Family Background	-0.012	-0.055	-0.053	0.022	0.055	0.102
	[-0.150, 0.112]	[-0.232, 0.074]	[-0.215, 0.077]	[-0.233, 0.302]	[-0.164, 0.297]	[-0.136, 0.374]

Notes: 95% Confidence intervals in brackets. Coefficients that are significant at the 5% level are **bold**.

Back

Robustness Check 3

Including other common family background variables

	Males				Females			
	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
Years of Schooling	**0.095**	-0.096	-0.096	-0.096	**0.423**	0.093	0.093	0.093
Cognition	**0.323**	**0.454**	0.149	0.149	0.129	**0.396**	-0.021	-0.021
Investments	0.134	0.187	**0.469**	0.306	0.049	0.150	**0.449**	0.277
Time Investments	0.132	0.178	**0.388**	**0.281**	-0.093	-0.038	0.116	-0.070
Age 7	0.135	0.156	0.152	0.084	0.149	0.167	0.181	-0.018
Age 11	-0.057	-0.030	0.075	0.066	-0.180	-0.176	-0.133	-0.053
Age 16	0.054	0.052	**0.162**	**0.131**	-0.062	-0.028	0.067	0.002
School Quality	0.002	0.010	0.081	0.024	**0.142**	**0.188**	**0.333**	**0.347**
Age 7	-0.001	-0.001	0.000	0.001	0.047	0.044	0.047	0.062
Age 11	-0.030	-0.028	-0.051	-0.056	0.017	0.019	0.012	0.005
Age 16	0.033	0.038	**0.132**	0.080	0.078	0.125	**0.274**	**0.280**
Family Background	-0.205	-0.197	-0.174	-0.011	-0.264	-0.302	-0.183	-0.012
Mother's education	-0.045	-0.044	-0.021	0.031	-0.027	-0.009	0.043	0.147
Father's education	0.012	0.004	0.032	0.084	0.055	0.066	0.113	0.209
Number of Siblings	0.013	0.012	0.028	0.077	-0.020	-0.021	-0.011	0.004
Stable	-0.145	-0.133	-0.150	-0.103	-0.189	-0.243	-0.178	-0.155
Mum's age	-0.032	-0.028	-0.038	-0.037	-0.099	-0.087	-0.155	-0.213
Dad's age	-0.008	-0.007	-0.025	-0.063	0.017	-0.008	0.004	-0.004
N	1350	1350	1350	1350	1347	1347	1347	1347

Notes: 95% Confidence intervals in brackets. Coefficients that are significant at the 5% level are **bold**.

Back

Mediation Analysis: Share of IGE Explained

	Males				Females			
	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
Years of Schooling	**0.095**	**-0.079**	**-0.079**	**-0.079**	**0.425**	0.024	0.024	0.024
Cognition	**0.327**	**0.456**	0.106	0.106	**0.135**	**0.402**	0.002	0.002
Investments	0.135	0.187	**0.473**	**0.325**	0.050	0.151	**0.463**	0.251
Time Investments	0.127	0.173	**0.384**	**0.284**	-0.100	-0.046	0.114	0.039
... Age 7	0.126	0.147	0.143	0.111	0.143	0.157	**0.176**	0.105
... Age 11	-0.054	-0.027	0.076	0.066	**-0.180**	**-0.175**	-0.133	-0.083
... Age 16	0.056	0.053	**0.166**	**0.108**	-0.062	-0.029	**0.070**	0.016
School Quality	0.008	0.014	0.089	0.041	**0.150**	**0.198**	**0.349**	**0.212**
... Age 7	-0.001	-0.001	0.000	-0.000	0.047	0.044	0.046	0.032
... Age 11	-0.024	-0.023	-0.044	-0.030	0.019	0.022	0.016	-0.010
... Age 16	0.033	0.038	**0.133**	0.072	0.084	**0.132**	**0.287**	**0.191**
Family Background	-0.019	-0.027	0.037	**0.185**	0.006	0.039	0.128	**0.340**
Mother's education	-0.051	-0.049	-0.029	0.020	-0.043	-0.024	0.010	0.104
Father's education	0.016	0.008	0.035	0.084	0.068	0.081	0.126	**0.227**
Number of Siblings	0.016	0.014	0.031	**0.081**	-0.019	-0.019	-0.008	0.009
Total	0.538	0.538	0.538	0.538	0.616	0.616	0.616	0.616
N	1350	1350	1350	1350	1347	1347	1347	1347

Mediation Analysis: Share of IGE Explained

	Males				Females			
	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
Years of Schooling	**0.095**	**-0.079**	**-0.079**	**-0.079**	**0.425**	0.024	0.024	0.024
Cognition	**0.327**	**0.456**	0.106	0.106	**0.135**	**0.402**	0.002	0.002
Investments	0.135	0.187	**0.473**	**0.325**	0.050	0.151	**0.463**	0.251
Time Investments	0.127	0.173	**0.384**	**0.284**	-0.100	-0.046	0.114	0.039
... Age 7	0.126	0.147	0.143	0.111	0.143	0.157	**0.176**	0.105
... Age 11	-0.054	-0.027	0.076	0.066	**-0.180**	**-0.175**	-0.133	-0.083
... Age 16	0.056	0.053	**0.166**	**0.108**	-0.062	-0.029	**0.070**	0.016
School Quality	0.008	0.014	0.089	0.041	**0.150**	**0.198**	**0.349**	**0.212**
... Age 7	-0.001	-0.001	0.000	-0.000	0.047	0.044	0.046	0.032
... Age 11	-0.024	-0.023	-0.044	-0.030	0.019	0.022	0.016	-0.010
... Age 16	0.033	0.038	**0.133**	0.072	0.084	**0.132**	**0.287**	**0.191**
Family Background	-0.019	-0.027	0.037	**0.185**	0.006	0.039	0.128	**0.340**
Mother's education	-0.051	-0.049	-0.029	0.020	-0.043	-0.024	0.010	0.104
Father's education	0.016	0.008	0.035	0.084	0.068	0.081	0.126	**0.227**
Number of Siblings	0.016	0.014	0.031	**0.081**	-0.019	-0.019	-0.008	0.009
Total	0.538	0.538	0.538	0.538	0.616	0.616	0.616	0.616
N	1350	1350	1350	1350	1347	1347	1347	1347

Mediation Analysis: Share of IGE Explained

	Males				Females			
	Level 1	Level 2	Level 3	Level 4	Level 1	Level 2	Level 3	Level 4
Years of Schooling	**0.095**	**-0.079**	**-0.079**	**-0.079**	**0.425**	0.024	0.024	0.024
Cognition	**0.327**	**0.456**	0.106	0.106	**0.135**	**0.402**	0.002	0.002
Investments	0.135	0.187	**0.473**	**0.325**	0.050	0.151	**0.463**	0.251
Time Investments	0.127	0.173	**0.384**	**0.284**	-0.100	-0.046	0.114	0.039
... Age 7	0.126	0.147	0.143	0.111	0.143	0.157	**0.176**	0.105
... Age 11	-0.054	-0.027	0.076	0.066	**-0.180**	**-0.175**	-0.133	-0.083
... Age 16	0.056	0.053	**0.166**	**0.108**	-0.062	-0.029	**0.070**	0.016
School Quality	0.008	0.014	0.089	0.041	**0.150**	**0.198**	**0.349**	**0.212**
... Age 7	-0.001	-0.001	0.000	-0.000	0.047	0.044	0.046	0.032
... Age 11	-0.024	-0.023	-0.044	-0.030	0.019	0.022	0.016	-0.010
... Age 16	0.033	0.038	**0.133**	0.072	0.084	**0.132**	**0.287**	**0.191**
Family Background	-0.019	-0.027	0.037	**0.185**	0.006	0.039	0.128	**0.340**
Mother's education	-0.051	-0.049	-0.029	0.020	-0.043	-0.024	0.010	0.104
Father's education	0.016	0.008	0.035	0.084	0.068	0.081	0.126	**0.227**
Number of Siblings	0.016	0.014	0.031	**0.081**	-0.019	-0.019	-0.008	0.009
Total	0.538	0.538	0.538	0.538	0.616	0.616	0.616	0.616
N	1350	1350	1350	1350	1347	1347	1347	1347

Back to Level 4

Key Fact 1: Family environment differs by parental income

	Parental Income Tertile:			
	Bottom	Middle	Top	P-val
Family Background				
Number of siblings	2.13	1.93	2.05	0.01
Father's age left school	14.9	14.8	15.2	0.00
Mother's age left school	15.0	15.1	15.3	0.00

Back

Key Fact 2:
Parental investments differ by parental income

	Parental Income Tertile:			
	Bottom	Middle	Top	P-val
Time investment				
% of fathers go on outings w child 7	65.2	72.5	71.5	0.00
% of parents want child to go to uni 11	81.2	82.8	85.2	0.08
% of mothers very interested at age 16	31.5	32.8	35.6	0.19
School quality				
% whose PTA holds meetings 7	56.8	57.6	58.7	0.71
Student-teacher ratio 11	24.8	24.7	24.3	0.06
% from child's class studying for GCEs 16	44.0	44.4	50.5	0.00

Back

Key Fact 2: Parental investments differ by parental income

	Parental Income Tertile:			
	Bottom	Middle	Top	P-val
Time investment				
% of fathers go on outings w child 7	65.2	72.5	71.5	0.00
% of parents want child to go to uni 11	81.2	82.8	85.2	0.08
% of mothers very interested at age 16	31.5	32.8	35.6	0.19
School quality				
% whose PTA holds meetings 7	56.8	57.6	58.7	0.71
Student-teacher ratio 11	24.8	24.7	24.3	0.06
% from child's class studying for GCEs 16	44.0	44.4	50.5	0.00

Back

Key Fact 2: Parental investments differ by parental income

	Parental Income Tertile:			
	Bottom	Middle	Top	P-val
Time investment				
% of fathers go on outings w child 7	65.2	72.5	71.5	0.00
% of parents want child to go to uni 11	81.2	82.8	85.2	0.08
% of mothers very interested at age 16	31.5	32.8	35.6	0.19
School quality				
% whose PTA holds meetings 7	56.8	57.6	58.7	0.71
Student-teacher ratio 11	24.8	24.7	24.3	0.06
% from child's class studying for GCEs 16	44.0	44.4	50.5	0.00

Back

Level 4 - Indirect effects via investments

Determinants of cognition:

$$inv_{16,i} = \boldsymbol{\delta_F F_i} + \delta_{Y_P} \ln Y_{Parent,i} + u_i^{inv_{16}}$$

Share of the IGE explained by maternal education:

$$\{ \underbrace{\alpha_{ed_m}}_{\substack{\text{Direct Effect}\\ \text{of mum ed on Earnings}}} + \underbrace{\alpha_S \beta_{ed_m}}_{\substack{\text{Indirect Effect}\\ \text{of mum ed via Schooling}}} + \underbrace{(\alpha_C + \beta_C \alpha_S)\gamma_{ed_m}}_{\substack{\text{Indirect Effect}\\ \text{of mum ed via Cognition}}} +$$

$$\underbrace{[\underbrace{\alpha_{inv_{16}}}_{\substack{\text{Direct Effect of}\\ inv_{16} \text{ on Earnings}}} + \underbrace{\beta_{inv_{16}} \alpha_S}_{\substack{\text{Indirect Effect of}\\ inv_{16} \text{ via schooling}}} + \underbrace{(\underbrace{\alpha_C}_{\substack{\text{Direct Effect of}\\ \text{cognition on Earnings}}} + \underbrace{\beta_C \alpha_S}_{\substack{\text{Indirect Effect of}\\ \text{cognition via schooling}}})\gamma_{inv_{16}}}_{\text{Indirect Effect of } inv_{16} \text{ via cognition}}]}_{\substack{\text{Indirect effect}\\ \text{via } inv_{16}}} \delta_{ed_m, inv_{16}} \} \cdot \kappa_{ed_m} / \rho$$

Back

Level 4 - Indirect effects via investments

Determinants of cognition:

$$inv_{16,i} = \boldsymbol{\delta}_F \boldsymbol{F}_i + \delta_{Y_P} \ln Y_{Parent,i} + u_i^{inv_{16}}$$

Share of the IGE explained by maternal education:

$$\Big\{ \underbrace{\alpha_{ed_m}}_{\substack{\text{Direct Effect}\\ \text{of mum ed on Earnings}}} + \underbrace{\alpha_S \beta_{ed_m}}_{\substack{\text{Indirect Effect}\\ \text{of mum ed via Schooling}}} + \underbrace{(\alpha_C + \beta_C \alpha_S)\gamma_{ed_m}}_{\substack{\text{Indirect Effect}\\ \text{of mum ed via Cognition}}} + \underbrace{\Big[\underbrace{\alpha_{inv_{16}}}_{\substack{\text{Direct Effect of}\\ inv_{16} \text{ on Earnings}}} + \underbrace{\beta_{inv_{16}} \alpha_S}_{\substack{\text{Indirect Effect of}\\ inv_{16} \text{ via schooling}}} + \underbrace{(\underbrace{\alpha_C}_{\substack{\text{Direct Effect of}\\ \text{cognition on Earnings}}} + \underbrace{\beta_C \alpha_S}_{\substack{\text{Indirect Effect of}\\ \text{cognition via schooling}}})\gamma_{inv_{16}}}_{\text{Indirect Effect of } inv_{16} \text{ via cognition}} \Big] \delta_{ed_m, inv_{16}}}_{\substack{\text{Indirect effect}\\ \text{via } inv_{16}}} \Big\} \cdot \kappa_{ed_m} / \rho$$

Back

Level 3- Indirect effects via cognition

Determinants of cognition:

$$C_i = \gamma_I \boldsymbol{I}_i + \gamma_F \boldsymbol{F}_i + \gamma_{Y_P} \ln Y_{Parent,i} + u_i^C$$

Share of the IGE explained by age 16 investments:

$$[\underbrace{\alpha_{inv_{16}}}_{\substack{\text{Direct Effect}\\ \text{of } inv_{16} \text{ on Earnings}}} + \underbrace{\beta_{inv_{16}}\alpha_S}_{\substack{\text{Indirect Effect}\\ \text{of } inv_{16} \text{ via schooling}}} + \underbrace{(\underbrace{\alpha_C}_{\substack{\text{Direct Effect}\\ \text{of cognition on Earnings}}} + \underbrace{\beta_C\alpha_S}_{\substack{\text{Indirect Effect}\\ \text{of cognition via schooling}}})\gamma_{inv_{16}}}_{\text{Indirect Effect of } inv_{16} \text{ via cognition}}] \cdot \kappa_{inv_{16}}/\rho$$

Back

Level 3- Indirect effects via cognition

Determinants of cognition:

$$C_i \quad = \quad \gamma_I \boldsymbol{I}_i + \gamma_F \boldsymbol{F}_i + \gamma_{Y_P} \ln Y_{Parent,i} + u_i^C$$

Share of the IGE explained by age 16 investments:

$$\Big[\underbrace{\alpha_{inv_{16}}}_{\substack{\text{Direct Effect} \\ \text{of } inv_{16} \text{ on Earnings}}} + \underbrace{\beta_{inv_{16}} \alpha_S}_{\substack{\text{Indirect Effect} \\ \text{of } inv_{16} \text{ via schooling}}} + \underbrace{\Big(\underbrace{\alpha_C}_{\substack{\text{Direct Effect} \\ \text{of cognition on Earnings}}} + \underbrace{\beta_C \alpha_S}_{\substack{\text{Indirect Effect} \\ \text{of cognition via schooling}}} \Big) \gamma_{inv_{16}}}_{\text{Indirect Effect of } inv_{16} \text{ via cognition}} \Big] \cdot \kappa_{inv_{16}} / \rho$$

Back

1. Family environment differs by parental income

	Parental Income Tertile			
Variable	Bottom	Middle	Top	P-val
Family Background				
Number of siblings	2.13	1.93	2.05	0.01
Father's age left school	14.9	14.8	15.2	0.00
Mother's age left school	15.0	15.1	15.3	0.00

Back

Determinants of lifetime earnings

$$\ln Y_i = \alpha_S S_i + \alpha_C C_i + \alpha_I \boldsymbol{I}_i + \alpha_F \boldsymbol{F}_i + \alpha_{Y_P} \ln Y_{Parent,i} + u_i^Y$$

$$\boldsymbol{I} = [inv_7, inv_{11}, inv_{16}, sq_7, sq_{11}, sq_{16}]$$
$$\boldsymbol{F} = [ed_m, ed_f, sib]$$

Test potentially relevant restrictions:

- time investments, $\alpha_{inv_7} = \alpha_{inv_{11}} = \alpha_{inv_{16}} = 0$
- school quality, $\alpha_{sq_7} = \alpha_{sq_{11}} = \alpha_{sq_{16}} = 0$
- family background $\alpha_{ed_m} = \alpha_{ed_f} = \alpha_{sib} = 0$
- parental income $\alpha_{Y_P} = 0$

Back

Identification of measurement parameters

$$\boxed{Z_{C,i,m} = \lambda_{C,m}\, C_i + \epsilon_{C,i,m}}$$

Scaling parameter λ:

- Take 3 measures at age 16: Reading score, maths score, teacher rated ability
- We normalized $Var(C) = 1$
- Then: $Cov(Z_{read}, Z_{maths}) = \lambda_{read}\lambda_{maths} Var(C)$
  $Cov(Z_{read}, Z_{teacher}) = \lambda_{read}\lambda_{teacher} Var(C)$
  $Cov(Z_{teacher}, Z_{maths}) = \lambda_{teacher}\lambda_{maths} Var(C)$

$\Rightarrow$ 3 equations in 3 unknowns

Back

Predicting latent factor scores

$$\boxed{Z_{C,i,m} = \lambda_{C,m}\, C_i + \epsilon_{C,i,m}}$$

- We want to predict latent cognition for each individual in our sample
- Easy method: Bartlett scores (Heckman, Pinto, Savelyev, 2013)
- Basic idea: Run GLS of measures on factor loadings for each individual, where weights are inverse of variance of measurement error.

$$\hat{C}_i = (\boldsymbol{\lambda}'\boldsymbol{\Omega}^{-1}\boldsymbol{\lambda})^{-1}\boldsymbol{\lambda}\boldsymbol{\Omega}^{-1}\boldsymbol{Z}_i'$$

where $\boldsymbol{\Omega}$ is a matrix that has the variances of the measurement errors $Var(\epsilon_{C,i,m})$ on the diagonal

Back