Discrete-Continuous Dynamic Choice Models: Identification and Conditional Choice Probability Estimation

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Introduction

Many economic problems involve joint **discrete** and **continuous dynamic** choices.

→ **Labor force participation** and **Consumption**

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- $\rightarrow$ **Labor force participation** and **Consumption**
- $\rightarrow$ Retirement and Consumption
- $\rightarrow$ Product choice and quantity consumed
- $\rightarrow$ Housing Tenure and Housing size
- $\rightarrow$ Product quality and Sale price
- $\rightarrow$ Pricing scheme and Price level
- $\rightarrow$ Student major choice and effort level

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This paper: addresses both issues altogether and *extends* ***Conditional Choice Probability Estimation*** *insights to* ***Dynamic Discrete-Continuous Choice models***.

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2. Use the identified CCPs and CCCs to identify the **primitives (payoffs, utility) of the model**, which are linked to the optimal choices through *first order conditions*, *Euler equation*, ...

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→ **Two-step estimation** procedure building upon identification.

→ Do not need to solve the model/value function.

→ **Sizeable computation gains**: 50 times faster in 2 period toy model. The **more complex** the model, the **larger the gains**.

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Objective:

Facilitate and **spread the use** of discrete-continuous dynamic models.

Many applications in **labor** (this paper), housing, education, IO, ...

Roadmap

Framework

Identification of the optimal choice policy functions

Dynamic Models and Identification of the Primitives

Estimation

Application

Conclusion

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Given a state x, the agent *simultaneously* makes a discrete choice $d \in \{0, 1\}$ and a continuous choice $c_d \in \mathcal{C}_d$ in order to maximize:

$$\max_{d, c_d} \quad v_d(c_d, \eta, x) + m_d(w, \eta, x) + \epsilon_d$$

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  - Only impacts discrete choice d:

$$\begin{aligned} d = 1 \iff & \max_{c_1} v_1(c_1, \eta, x) + \epsilon_1 \quad > \quad \max_{c_0} v_0(c_0, \eta, x) + \epsilon_0 \\ \iff & \max_{c_1} v_1(c_1, \eta, x) - \max_{c_0} v_0(c_0, \eta, x) \quad > \quad \epsilon_0 - \epsilon_1 \end{aligned}$$

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  - Also impacts discrete choice d

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Example:

- d is labor force participation, $d = 1$ if work, $d = 0$ if unemployed.
- c_d is consumption.
- x can include family background variables, education, work experience, asset, income, etc.
- η are unobserved idiosyncratic taste for consumption shocks.
- ϵ are unobserved idiosyncratic preference for work.

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Assumptions:

- Monotone Choice: the payoff functions are such that the optimal policies functions $c_d^*(\eta, x)$ are $\mathcal{C}^1$ and strictly increasing with respect to η.

$$\frac{\partial^2 v_d(c_d, \eta, x)}{\partial c_d \partial \eta} > 0$$

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- Shock independence: $\eta \perp (x, w)$, $\epsilon \perp (x, w)$, $\eta \perp \epsilon$
- Normalization: $\eta \sim \mathcal{U}(0, 1)$

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Given a state x, and instrument w, the agent *simultaneously* makes a discrete choice $d \in \{0,1\}$ and a continuous choice $c_d \in \mathcal{C}_d$ in order to max:

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→ impacting the **selection** $Pr(D = 0|\eta, W, X)$ (and thus $F_{C_d|d,w,x}(c)$) **but not the continuous choice** $c_d^*(\eta, x)$.

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- Example: w can be the *previous labor force participation*.
  If switching cost in and out of employment, conditional on current d it does not impact the consumption c_d. But it impacts the probability of working.

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→ Triangular structure for the reduced forms:

$$\begin{cases} C_d = & c_d^*(X, \eta) \\ D = & d^*(X, W, \eta, \epsilon) \end{cases}$$

Roadmap

- Framework
- Identification of the optimal choice policy functions
  - Conditional Continuous Choices (CCCs)
  - Conditional Choice Probabilities (CCPs)
- Dynamic Models and Identification of the Primitives
- Estimation
- Application
- Conclusion

Identification

What is observed by the econometrician?

- (C_d, D, W, X) for all individuals.
  Where $c_d = c_0\,(1-d) + c_1 d \rightarrow$ do not observe *both* choices c_0 and c_1.

- **Reduced forms**: joint distributions

$$Pr(D = d|W = w, X = x) \quad \text{and} \quad F_{C_d|D=d,W=w,X=x}(c_d)$$

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Want to **identify** the **optimal choices**:

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Abstract from X without loss of generality in this section.

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Identification: Conditional Continuous Choices (CCCs)

The econometrician observes (C_d, D, W) for all individuals.
i.e. observes the *reduced forms*: $F_{C_d|d,w}(c)$ and $Pr(D = d|W = w)$.

Gives the following system of equations $\forall h$:

$$\begin{cases} h = F_{C_0|D=0,W=0}(c_0(h))Pr(D = 0|W = 0) + F_{C_1|D=1,W=0}(c_1(h))Pr(D = 1|W = 0) \\ h = F_{C_0|D=0,W=1}(c_0(h))Pr(D = 0|W = 1) + F_{C_1|D=1,W=1}(c_1(h))Pr(D = 1|W = 1) \end{cases}$$

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Proof: since $\eta \sim \mathcal{U}(0, 1)$ and $w \perp \eta$, we have, $\forall w, \forall h$:

$$\begin{aligned} h = Pr(\eta \le h) &= Pr(\eta \le h|w) \\ &= Pr(\eta \le h \mid D = 0, w)Pr(D = 0|w) + Pr(\eta \le h \mid D = 1, w)Pr(D = 1|w) \\ &= Pr(C_0 \le c_0^*(h) \mid D = 0, w)Pr(D = 0|w) \\ &\quad + Pr(C_1 \le c_1^*(h) \mid D = 1, w)Pr(D = 1|w) \quad \text{by monotonicity of } c_d^*(h) \\ &= F_{C_0|D=0,w}(c_0^*(h))Pr(D = 0|w) + F_{C_1|D=1,w}(c_1^*(h))Pr(D = 1|w) \end{aligned}$$

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IDENTIFICATION:
The policy functions $c_d^*(h)$ are identified iff there exists a **unique solution** with *increasing* $c_d(h)$ to the system.

$\rightarrow$ Uniqueness requires an *additional assumption* on the *effect of the instrument.*

Identification: Conditional Continuous Choices (CCCs)

Assumption: Relevant instrument

The additive terms of the payoff are such that:

$$m_0(w=0,h) - m_1(w=0,h) \neq m_0(w=1,h) - m_1(w=1,h) \quad \text{except, at most, at a finite set of points.}$$

Equivalently,

$$Pr(D=0|\eta=h, \mathbf{W}=\mathbf{1}) \neq Pr(D=0|\eta=h, \mathbf{W}=\mathbf{0}) \quad \text{except, at most, at a finite set of points.}$$

Full rank

Identification: Conditional Continuous Choices (CCCs)

Theorem: Identification

Under our assumptions, for any reduced form drawn from the model, there exists unique CCCs $c_d^0(h)$ mapping $[0, 1]$ into $\mathcal{C}_d$ which are strictly increasing and solve the system of equation.

This unique solution identifies the optimal CCCs.

▸ Proof

Identification: Conditional Continuous Choices (CCCs)

Data on (C_d, D, W) (and X). Only observes $c_d = c_0(1-d) + c_1 d$. ▸ More
$\rightarrow$ Observed reduced forms: $F_{C_d|d,w}(c)$ and $Pr(D = d|W = w)$.

Identification: Conditional Continuous Choices (CCCs)

More

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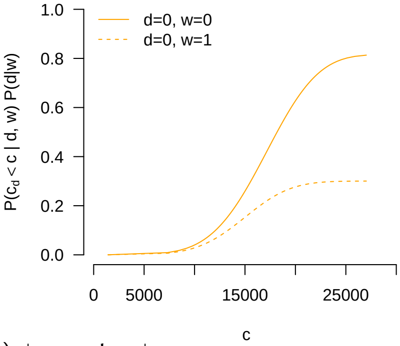
Figure: **Joint distributions $\mathbf{F_{C_d|d,w}(c_d)Pr(d|w)}$**



Assumptions: $c_d^*(\eta) \perp w$ and $w \perp \eta$
$\implies$ Observable differences caused by unobserved:

$$Pr(D = 0|\eta, W = 1) - Pr(D = 0|\eta, W = 0)$$

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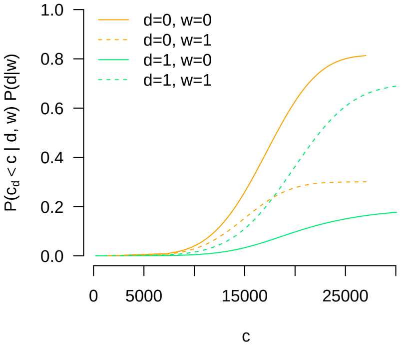
Figure: **Joint distributions** $F_{C_d|d,w}(c_d)Pr(d|w)$



Similarly for the **other choice** $\mathbf{D = 1}$.

Moreover, link between $D = 1$ and $D = 0$: $\mathbf{Pr(D = 0|w) = 1 - Pr(D = 1|w)}$

$\Longrightarrow$ the differences between the conditional distributions of c_d are related.

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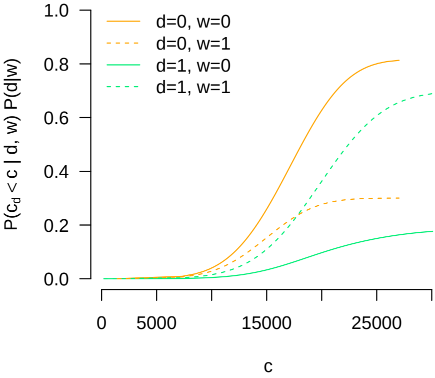


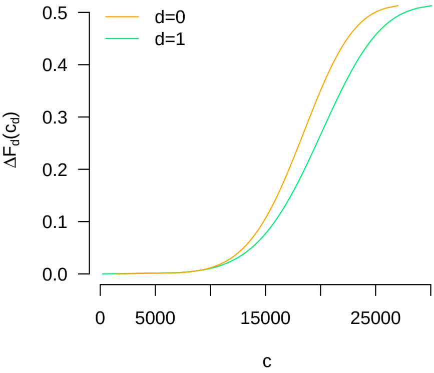
Bayes: $F_{C_1|1,w=1}(c_1(\eta))Pr(1|w=1) - F_{C_1|1,w=0}(c_1(\eta))Pr(1|w=0)$

$$= -\left(F_{C_0|0,w=1}(c_0(\eta))Pr(0|w=1) - F_{C_0|0,w=0}(c_0(\eta))Pr(0|w=0)\right)$$

Identification: Conditional Continuous Choices (CCCs)

Figure: **Difference within joint distributions $\implies$ Identified CCCs**

More



$$\Delta F_{C_0}(c_0(\eta)) = \Delta F_{C_1}(c_1(\eta)) \overset{monot}{\iff} \Delta F_{C_0}(c_0(c_1)) = \Delta F_{C_1}(c_1)$$

where $\Delta F_{C_d}(c_d)$ are observed. The only unknown is the **mapping** $c_0(c_1)$.

Identification: Conditional Continuous Choices (CCCs)

Figure: **<u>Difference</u> within joint distributions $\implies$ Identified CCCs**

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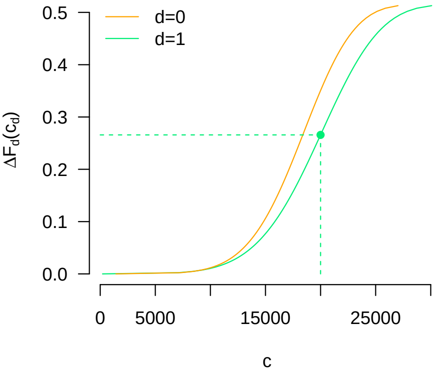

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Figure: Difference within joint distribution $\implies$ **Identified CCCs**

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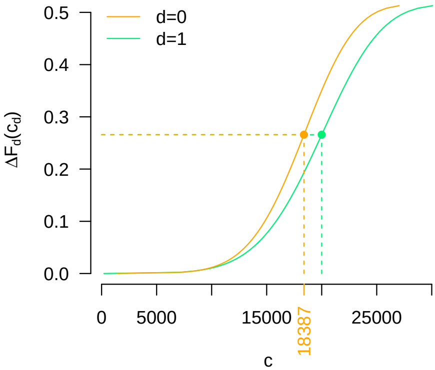

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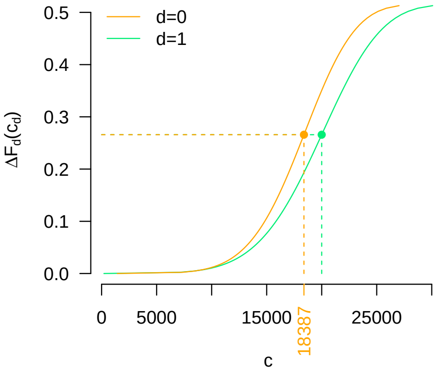
Figure: Difference within joint distribution $\implies$ **Identified CCCs**

More Nonmonot



$\rightarrow$ **Identifies** correspondence between c_0 and c_1: $c_0(c_1)$.
$\rightarrow$ Then use system to find corresponding η:

$$\eta(c_1) = F_{C_0|D=0,w}(\mathbf{c_0}(\mathbf{c_1}))Pr(D=0|w) + F_{C_1|D=1,w}(\mathbf{c_1})Pr(D=1|w) \quad \forall c_1$$

Identification: Conditional Continuous Choices (CCCs)

What is the role of the **relevance assumption**?

Determines the **shape of $\mathbf{\Delta F_{C_d}}$** functions.

- Here *monotone* $\Delta F_{C_d} \iff$ strict relevance: ▸ Full rank

$$Pr(D = 0|\eta, W = 0) < Pr(D = 0|\eta, W = 1) \qquad \forall\eta$$

- General relevance assumption

  $Pr(D = 0|\eta = h, W = 1) \neq Pr(D = 0|\eta = h, W = 0)$ except at a finite set of points.

  $\iff$ **Piecewise monotone** ΔF_{C_d}. ▸ Nonmonot

- If not relevant on a segment: flat ΔF_{C_d}, partial identification.

Roadmap

Framework

Identification of the optimal choice policy functions

- Conditional Continuous Choices (CCCs)
- Conditional Choice Probabilities (CCPs)

Dynamic Models and Identification of the Primitives

Estimation

Application

Conclusion

Identification: Conditional Choice Probabilities (CCPs)

To recover the Conditional Choice Probabilities (CCPs): $Pr(D = d|\eta, W = w)$:

- First, invert the policy function to identify η from the observed c_d^{obs}:

$$\eta = c_d^{*\,-1}(c_d^{obs})$$

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  as if η was **observed** from now on.

- Then using it, recover the conditional choice probabilities $Pr(D = d|\eta, W = w)\ \forall \eta, \forall w$ (at the optimal continuous policy choice) from data (η, D, W).

Roadmap

Dynamic Models

Extension to dynamic: *How do **dynamic** models enter the general framework?*

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- Period utility:

$$u_{dt}(c_t, \eta_t, \tilde{x}_t) + m_{dt}(w_t, \eta_t, \tilde{x}_t) + \epsilon_{dt}$$

subject to: $$a_{t+1} = (1 + r)a_t - c_t + \text{income}_t d_t$$

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- Given states $x_t = (\tilde{x}_t, \text{income}_t, a_t)$, the agent solves:

$$\begin{aligned} \max_{d, c_{dt}} \quad & u_{dt}(c_{dt}, \eta_t, \tilde{x}_t) + m_{dt}(w_t, \eta_t, \tilde{x}_t) + \epsilon_{dt} \\ & + \beta \mathbb{E}_t \left[V_{t+1}(x_{t+1}, w_{t+1} | x_t, d_t, c_t, w_t) \right] \end{aligned}$$

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- And we obtain back general form (where a_t is part of x_t):

$$\max_{d, c_{dt}} \quad \overbrace{u_{dt}(c_{dt}, \eta_t, \tilde{x}_t) + \beta \mathbb{E}_t \left[V_{t+1}(x_{t+1}, w_{t+1} | x_t, d_t, c_t, w_t) \right]}^{\equiv v_{dt}(c_{dt}, \eta_t, x_t, w_t)} + m_{dt}(w_t, \eta_t, \tilde{x}_t) + \epsilon_{dt}$$

Problem: w_t *not excluded* from c_t optimal choice without additional assumptions.

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$\rightarrow$ Need instrument exclusion, also from transitions: $f(x_{t+1}, w_{t+1} | x_t, d_t, c_t, \cancel{w_t})$.

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- Good *example* of instrument: $w_t = d_{t-1}$.

  $\rightarrow$ *Excluded from transition* to w_{t+1} conditional on d_t... since $w_{t+1} = d_t$.
  $\rightarrow$ *Excluded from* c_d conditional on d_t...
  $\rightarrow$ *Relevant for* d_t if **switching costs**.

Dynamic Models

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Dynamic Models

About the **transitions**:

- If $w_t = d_{t-1}$: need **no time-dependence** in η: $\quad \boldsymbol{\eta_t \perp \eta_{t+1}}$.

  Otherwise, instrument independence from shocks is violated: $w_0 = d_{-1} \not\perp \eta_0$.

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  Otherwise, instrument independence from shocks is violated: $w_0 = d_{-1} \not\perp \eta_0$.

Then, how to exploit **observable information about the transitions**?

1. Include **unobserved individual types** (Arcidiacono and Miller, 2011).
2. Find **another instrument**, to identify η_0 in **initial period**.
   Then can use $w_t = d_{t-1}$ for all the other periods by including η_{t-1} in the t state variables: instrument independence conditional on η_{t-1}.
   $\rightarrow$ identify and estimate $f(\eta_{t+1}|\eta_t)$.

Identification of the Primitives

- CCCs and CCPs identified as before (period by period). ▸ More
  Transitions $f_t(x_{t+1}|x_t, d_t, c_t)$ identified directly from the data.

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  - $\rightarrow$ Conditional value (at the optimal CCCs) identified via **FOC with respect to the asset** (Blundell et al., 1997).
  - $\rightarrow$ Differences in additive m_d terms identified via the **CCPs**.

Roadmap

Framework

Identification of the optimal choice policy functions

Dynamic Models and Identification of the Primitives

Estimation

- Optimal choice policy functions
- Structural parameters

Application

Conclusion

Estimation

▸ Application

'Two-step' estimation procedure, à la Hotz and Miller, 1993:

1. Estimate optimal policy functions:

Estimation

Application

'Two-step' estimation procedure, à la Hotz and Miller, 1993:

1. Estimate optimal policy functions:

   - Estimate the CCCs, $c_{dt}(\eta_t, x_t)$, and the CCPs, $Pr(D_t = 1|\eta_t, w_t, x_t)$. Parametric (MLE or GMM) or **Nonparametric**.

Estimation

Application

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2. Recover the payoffs based on the 1st stage policies: naturally linked with structural models, through *first order conditions, Euler equation.*
   $\Longrightarrow$ Do <u>not</u> need to solve the model.

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   $\Longrightarrow$ Do <u>not</u> need to solve the model.

$\rightarrow$ Computation gains: compared with other methods (SMM...).
The **more complex** (covariates, number of periods, ...) the model, the **higher the gains**.

Roadmap

Estimation: CCCs

▸ Fast method ▸ Results

- From data on (D_t, C_{dt}, X_t, W_t), estimate the reduced forms $\hat{F}_{C_{dt}|d_t,x_t,w_t,t}(c_{dt})$ and $Pr(D_t = d|\widehat{X_t = x_t}, W_t = w_t, t)$.
  Nonparametric Kernel, Sieve logistic/probit regressions.

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- From data on (D_t, C_{dt}, X_t, W_t), estimate the reduced forms $\hat{F}_{C_{dt}|d_t,x_t,w_t,t}(c_{dt})$ and $\widehat{Pr(D_t = d|X_t = x_t, W_t = w_t, t)}$.
  Nonparametric Kernel, Sieve logistic/probit regressions.

- Estimate the monotone functions $\hat{c_{dt}}(h, x)$ which solves $\forall h$, $\forall w_t$:

$$
\begin{aligned}
h = & \hat{F}_{C_0|D_t=0,w_t,x_t}(c_{0t}(h, x_t))\widehat{Pr(D_t = 0|w_t, x_t)} \\
& + \quad \hat{F}_{C_1|D_t=1,w_t,x_t}(c_{1t}(h, x))\widehat{Pr(D_t = 1|w_t, x_t)} \\
\equiv & \, g_{w_t,x_t}(c_{0t}(h, x_t), c_{1t}(h, x_t))
\end{aligned}
$$

  We find the CCCs as **strictly monotone functions**, solution to:

$$
\begin{aligned}
\underset{c_{0t}(h,x_t),c_{1t}(h,x_t)}{argmin} \int_0^1 & (h - g_{w_t=0,x_t}(c_{0t}(h, x_t), c_{1t}(h, x_t)))^2 \\
& + (h - g_{w_t=1,x_t}(c_{0t}(h, x_t), c_{1t}(h, x_t)))^2 \, dh
\end{aligned}
$$

  Remark: proceed x_t by x_t and period by period, and solve for the whole functions each time.

Estimation: CCPs

Results

- Recover $\hat{\eta}_t$ from every observed (c_{dt}^{obs}, d_t, x_t) by inverting the CCCs:

$$\hat{\eta} = \widehat{c_{dt}^{-1}}(c_{dt}^{obs}, x_t)$$

Consider $\hat{\eta}$ as *observed* from here onwards.

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Consider $\hat{\eta}$ as *observed* from here onwards.

- Use $\hat{\eta}$ to estimate (nonparametrically or parametrically) the CCPs (at opti c):

$$Pr(D_t \widehat{= d_t | \eta}, w_t, x_t) \quad \forall \eta_t, \ \forall w_t, \ \forall x_t$$

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Estimation: Structural parameters

Take **1st stage** estimated *CCCs*, *CCPs*, and *transitions* as given: ▸ More ▸ Results

1. Pick a set of parameters θ.
   θ determines the period utility functional forms: $u_{dt}()$, $u'_{dt}()$, and $m_{dt}()$.

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1. Pick a set of parameters θ.
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2. **Differences in Euler Equation**.
   Given θ, CCCs, CCPs and transition, estimate both sides of the **Euler equation** for each individuals.
   For the expectation on the RHS, use *forward one period-ahead simulation.*

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5. Inference: *in progress*, tentative by bootstrap.

Roadmap

Application: Female Labor Participation and Consumption

- Dynamic life-cycle model of women Labor Participation and Consumption.
- 1st stage:
  Estimate *distribution* of consumption and work probabilities.
- 2nd stage:
  Recover structural parameters (relative risk aversion, utility cost of work, ...).

Roadmap

Application: Female Labor Participation and Consumption

Working Life: (T periods)
Period utility:

$$u(c_t, d_t, w_t, x_t, \eta_t, \epsilon_t) = \begin{cases} \left(c_t/n_t\right)^{1-\sigma} / (1-\sigma)\ \tilde{\eta}_t^0(\eta_t, couple_t, nchild_t) + \epsilon_{0t} \\ \left(c_t/n_t\right)^{1-\sigma} / (1-\sigma)\ \tilde{\eta}_t^1(\eta_t, couple_t, nchild_t) + \alpha + \omega(1-w_t) + \epsilon_{1t} \end{cases}$$

Where

- t is the **age**. c_t is household consumption. d_t is labor choice. $w_t = d_{t-1}$.
- n_t is an equivalence scale for hh consumption to individual consumption.
- $\tilde{\eta}_t^d \sim \mathcal{LN}(\gamma_d + \gamma_d^c couple_t + \gamma_d^n nchild_t,\ s_d)$.
  So, $\tilde{\eta}$ are just transformations of η: η^{th} quantiles of the lognormal.
- $(\gamma_0, \gamma_1, s_0, s_1)$ measures the effect of the unobserved heterogeneity. Moreover, (γ_d^n, γ_d^c) determines the effect of the family situation, conditional on employment.
- σ is constant relative risk aversion.
- ω is utility cost of searching for a job when previously unemployed.
- α is utility cost of working.

Application: Female Labor Participation and Consumption

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subject to the budget constraint:

$$a_{t+1} = (1+r)a_t - c_t + d_t y_t + \text{couple}_t d_t^p y_t^p + T(d_t, x_t)$$

Where

- a_t is the household asset.
- y_t is the woman earnings.
- y_t^p is the partner's earnings. d_t^p indicates if the husband works.
- Missing incomes: estimated with *Heckman Correction* beforehand.
- $T(d_t, x_t)$ are benefits, depending on labor choice and x_t (which includes asset, income, family information).

Application: Female Labor Participation and Consumption

Working Life: (T periods)

Transitions of the state variables, estimated in first stage:

- asset transition given by the budget constraint.

$$a_{t+1} = (1 + r)a_t - c_t + d_t y_t + \text{couple}_t y_t^p + T(d_t, x_t)$$

- $couple_t$ and $educ_t$ fixed.
- $nchild_{t+1} | nchild_t, couple_t, a_t, y_t, y_t^p, educ_t, t$
- y_t evolves through time according to an auto-regressive process:

$$\begin{aligned} y_{t+1} &= (\rho_y^{educ} y_t + \rho_d^{educ} d_t + \rho_{age}^{educ} t) + u_t \\ y_{t+1}^p &= \rho_y^p y_t^p + v_t \end{aligned}$$

$\rightarrow$ **Education** only plays a role in the transitions. It affects c_{dt} through this.

Application: Female Labor Participation and Consumption

Retirement:

- At age $T = 60$, the woman retires. Gets the same utility as when did not work, with $d_t = 0$.
- She lives for another 15 years from her *accumulated assets* and gets a *pension* which is a *proportion of her last income* y_T (and potential husband income). Proportion set to 50% (taux plein).
- No bequest motive.
- Solve retiree problem and get *retirement value*:

$$R(x_T) = R(a_T, y_T, y_T^p, couple_T, nchild_T)$$

Application: Female Labor Participation and Consumption

Parameters:

- Discount future with β. Fixed.
- Set $\gamma_0 = 0$, $s_0 = 0.5$ (normalize the effect of unemployed single with no child).
- Transition function parameters estimated in first stage.
- 9 structural parameters to estimate:

$$\theta = (\sigma, \gamma_0^n, \gamma_0^c, \gamma_1, \gamma_1^n, \gamma_1^c, s_1, \alpha, \omega)$$

Roadmap

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Descriptive statistics

Table: EU-SILC French unbalanced panel, 2004 − 2015, 7391 women

Statistic	N	Mean	St. Dev.	Min	Median	Max
Choices:						
Annual household c (k euros)	21,945	36.58	20.99	3.88	32.54	211.54
$c\|d = 0$	5,330	30.04	19.32	4.02	25.58	204.48
$c\|d = 1$	16,615	38.67	21.07	3.88	34.78	211.54
d	21,945	0.76	0.43	0	1	1
w $= d_{-1}$	21,945	0.76	0.43	0	1	1
$d\|w = 0$	5,354	0.14	0.35	0	0	1
$d\|w = 1$	16,591	0.96	0.20	0	1	1
Covariates:						
Age	21,945	42.37	9.39	26	42	60
Annual Income y (Heckman)	21,945	19.74	5.29	8.10	19.07	43.32
Asset	21,945	108.29	118.55	−32	69.0	528
Nb of children	21,945	1.71	1.09	0	2	4
Couple	21,945	0.75	0.43	0	1	1
Working partner|Couple	16,442	0.93	0.25	0	1	1
Partner's income y^p |Couple	16,442	26.41	13.21	4.02	23.20	147.54
Completed Education	21,945					
$\leq$ Secondary	5,240	0.24	0.43	0	0	1
High School	9,999	0.46	0.50	0	0	1
University	6,706	0.30	0.46	0	0	1
Other:						
Receives Benefits	21,945	0.66	0.47	0	1	1
Benefits|Benefits > 0	14,478	5.16	4.46	0.002	3.60	23.07

c, y, y^p, asset and benefits expressed in real terms (base 2010) and in thousands of euros.

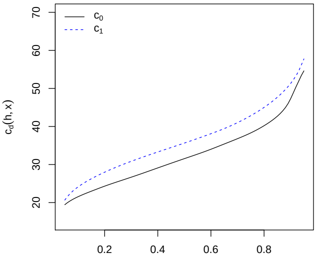
Roadmap

CCCs and CCPs:

1st stage

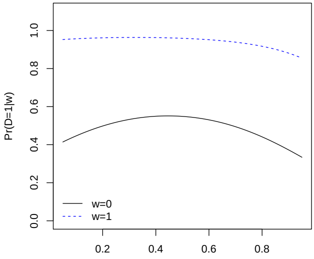
Average evolution of a woman with median characteristics: $26y.o.$ woman, high-school education, with income $17k5$ euros, no asset, in couple, no child, with a partner earning $22k$ euros.

Figure: $\widehat{c_d}(h, x)$



h

Figure: $\widehat{Pr}(D = 1|h, x, w)$



h

Structural parameters

Table: Structural parameter estimates

2nd stage

	Parameter estimates	
	Parameter	Estimate
Discount factor	β	0.98 (fixed)
Relative Risk Aversion	σ	1.63
Effect of η by family...		
... when unemployed:		
$\mathcal{LN}(\gamma_0^c couple + \gamma_0^n nchild,\ s_0)$	γ_0	0 (fixed)
	γ_0^c	-1.80
	γ_0^n	-0.31
	s_0	0.50 (fixed)
... when employed:	γ_1	-1.04
$\mathcal{LN}(\gamma_1 + \gamma_1^c couple + \gamma_1^n nchild,\ s_1)$	γ_1^c	-0.65
	γ_1^n	-0.10
	s_1	0.54
Additive terms:		
Utility cost of working	α	-0.04
Utility cost of search	ω	-2.14

Estimator Comparison

How does my two-step method compare with alternative methods?

Estimator Comparison

How does my two-step method compare with alternative methods?

- Cannot compare in complete model: alternatives take **too long** (months). My method: about 5 hours in total.
- Simplified **Toy model**: ▸ More
  2 periods, binary high/low income, asset and no other covariates.
- Comparison via **Monte Carlo Simulations** of this Toy model.

Estimator Comparison: Toy model Monte Carlo

Table: $T = 2$ periods

	Truth	*Method* DCC	SMM
N		10,000	10,000
σ	1.60	1.6253	1.5924
		(0.0410)	(0.0156)
γ_1	0.00	0.0070	-0.0052
		(0.0238)	(0.0055)
s_1	0.40	0.4078	0.4001
		(0.0228)	(0.0071)
α	-0.50	-0.4727	-0.5023
		(0.0498)	(0.0348)
ω	-1.00	-0.9982	-0.9972
		(0.0581)	(0.0523)
Average Time taken:			
1st stage: CCPs and CCCs		118s	9s
2nd stage: Structural parameters		170s	14328s
Overall		288s	14337s

Other initializations: Number of Monte-Carlo $= 1,000$

$Pr(w_1 = 1) = 0.70$. $y_1 = y_H$ with probability 0.50. $a_1 \sim \mathcal{U}(0, 30)$. $r = 0.05$.

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  $\rightarrow$ *Fixed cost* of computing *1st stage policies.*

  $\rightarrow$ Then estimation of parameters is fast in 2nd stage: *do not compute the value function.*

  Also avoids concerns about kinks in value/discontinuities in optimal policies (Iskhakov et al., 2017).

Roadmap

Framework

Identification of the optimal choice policy functions

Dynamic Models and Identification of the Primitives

Estimation

Application

Conclusion

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  Nest many models, and allow for unobserved heterogeneity on both choices.

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  - Extension to $J > 2$ alternatives.

Thank you!

Contact: **christophe.bruneel@gmail.com**

Selection on unobservables

Observe d, $c_d|d$ (and x) for every individual $\iff f_{c_d|d,x}(c_d)$ and $Pr(d|x)$.

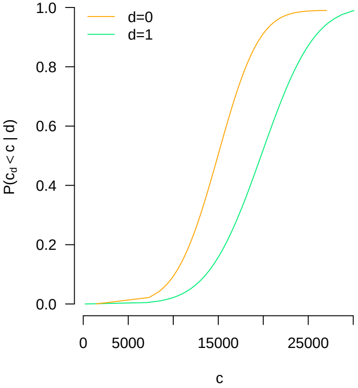

Figure: $F_{c_d|d}(c)$

$Pr(D=1|X)$
0.7003

back

Selection on unobservables

- Cases with reverse interpretation on the selection $Pr(d|\eta)$ can be observationally equivalent... (yield the same observable $F_{c_d|d}(c)$ and $Pr(d)$)

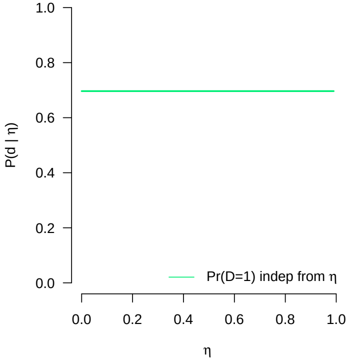

Figure: $Pr(d|\eta)$

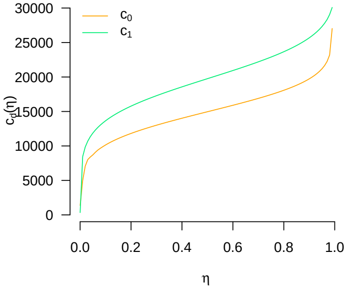

Figure: Corresponding $c_d(h)$

Corresponding unknown $c_d(h)$ is adjusted
$\rightarrow$ yield $F_{c_d|d}(c)$ and $Pr(d)$ $\Longrightarrow$ observationally equivalent to the *true selection.*

back

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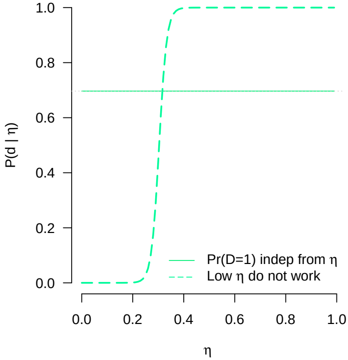

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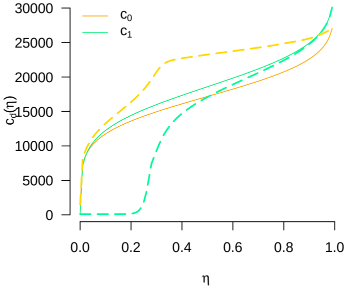

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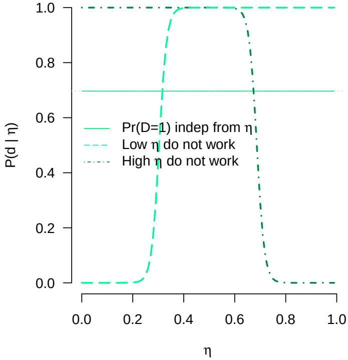

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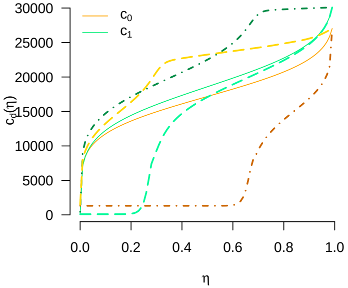

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Relevance

Relevance condition for the instrument w:
relevant as long as for two values of w, e.g. $w = 0$ and $w = 1$, we have:

$$m_0(w=0,\eta,x) - m_1(w=0,\eta,x) \neq m_0(w=1,\eta,x) - m_1(w=1,\eta,x)$$

Because:

$$Pr(D=0|\eta,x,w) = \mathbb{E}_\epsilon\Big[\mathbb{1}\{\overbrace{\max_{c_0} v_0(c_0,\eta,x) - \max_{c_1} v_1(c_1,\eta,x)}^{fixed \perp w|\eta,x} + m_0(1,\eta,x) - m_1(1,\eta,x) > \epsilon_1 - \epsilon_0\}|\,\eta,x,w\Big]$$

Identification: Conditional Continuous Choices (CCCs)

Idea of the proof:

Back

- We observe $F_{c_d|d,w}(c_d)$ and $Pr(d|w)$ $\forall d, w$.

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- Now, we also have a natural relation with the two choices:

$$\begin{aligned} &Pr(d=1|h, w=1) - Pr(d=1|h, w=0) \\ =& -\left(Pr(d=0|h, w=1) - Pr(d=0|h, w=0)\right) \end{aligned}$$

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- Thus, if a given $\tilde{c}_1$ and $\tilde{c}_0$ corresponds to the same h, then:

$$\Delta F_1(\tilde{c}_1) = -\Delta F_0(\tilde{c}_0)$$

  It holds for every h. And under some relevance condition, we can recover the unique mapping between the conditional choices.

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- Once you get it, identify the corresponding h with our Bayesian formula.

Identification: Conditional Continuous Choices (CCCs)

"PROOF":

Back

Denote $p_{d|w} = Pr(d|w)$. We have the system, $\forall h$:

$$\begin{cases} h = F_{c_0|d=0,w=0}(c_0(h))p_{0|0} + F_{c_1|d=1,w=0}(c_1(h))p_{1|0} \\ h = F_{c_0|d=0,w=1}(c_0(h))p_{0|1} + F_{c_1|d=1,w=0}(c_1(h))p_{1|1} \end{cases}$$

Rewrite the system (2) - (1):

$$F_{c_0|d=0,w=1}(c_0(h))p_{0|1} - F_{c_0|d=0,w=0}(c_0(h))p_{0|0} =$$
$$-\left(F_{c_1|d=1,w=1}(c_1(h))p_{1|1} - F_{c_1|d=1,w=0}(c_1(h))p_{1|0} \right)$$
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"Proof": Back

Rewrite the system (2) - (1):

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F_{c_0|d=0,w=1}(c_0(h))p_{0|1} - F_{c_0|d=0,w=0}(c_0(h))p_{0|0} &= \\
-\left(F_{c_1|d=1,w=1}(c_1(h))p_{1|1} - F_{c_1|d=1,w=0}(c_1(h))p_{1|0} \right)& \\
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\end{aligned}
$$

$\rightarrow$ What are the $\Delta F_d()$ functions?

$$
\begin{aligned}
\Delta F_d(c_d(h)) &= F_{c_d|d,w=1}(c_d(h))p_{d|1} - F_{c_d|d,w=0}(c_0(h))p_{d|0} \\
&= \int_0^h \Big(Pr(D=d|\eta=\tilde{h}, w=1) - Pr(D=d|\eta=\tilde{h}, w=0)\Big)\, d\tilde{h}
\end{aligned}
$$

$\implies$ when $Pr(D=d|\eta=h, w=1) = Pr(D=d|\eta=h, w=0)$ the instrument is not 'relevant' at h. And in this case, $\Delta F_d(c)$ presents an inflexion point.

"PROOF": ▸ Back

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Since both $c_d(h)$ are increasing wrt h, can *first* focus on finding the increasing mapping $c_1(c_0)$ between the policies. We have:

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$\rightarrow$ *when is the solution unique?*

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Case 1: Monotone $\Delta F_d()$

Back

$$\iff m_0(w=0,h) - m_1(w=0,h) < m_0(w=1,h) - m_1(w=1,h) \quad \forall h$$
$$\iff \quad Pr(D=0|\eta=h, W=0) < Pr(D=0|\eta=h, W=1) \quad \forall h$$

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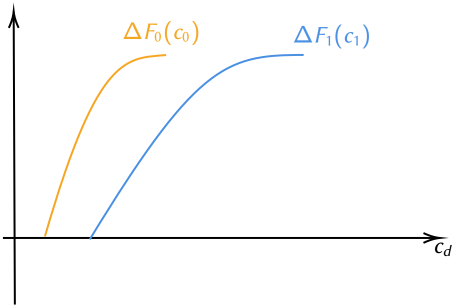

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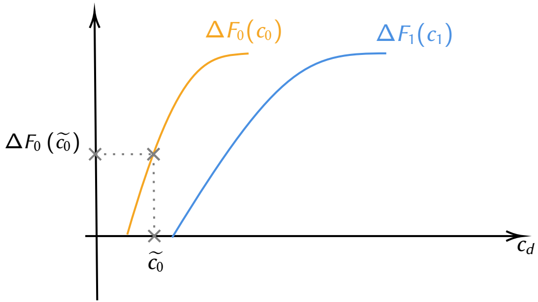

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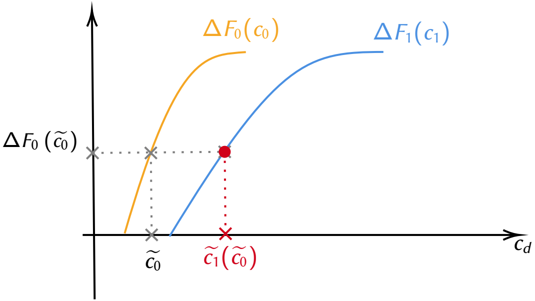

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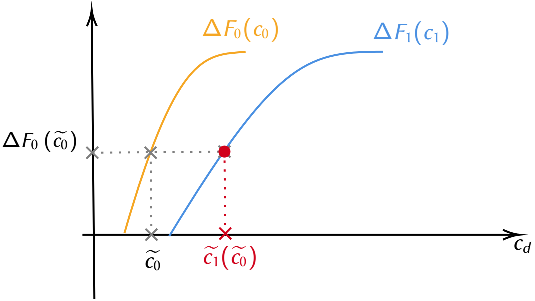
Case 1: Monotone $\Delta F_d()$

Back

$$\iff m_0(w=0,h) - m_1(w=0,h) < m_0(w=1,h) - m_1(w=1,h) \quad \forall h$$

$$\iff \quad Pr(D=0|\eta=h, W=0) < Pr(D=0|\eta=h, W=1) \quad \forall h$$



Point identified: $c_1(c_0) = (\Delta F_1)^{-1}(\Delta F_0(c_0))$

$$\Delta F_0(c_0) \; = \; -\Delta F_1(c_1(c_0)) \qquad \forall c_0$$

Case 2: Piecewise Monotone $\Delta F_d()$

Back

Only a finite number of points h such that:

$$\iff m_0(w=0,h) - m_1(w=0,h) = m_0(w=1,h) - m_1(w=1,h) \quad \forall h$$

$$\iff \quad Pr(D=0|\eta=h, W=0) = Pr(D=0|\eta=h, W=1) \quad \forall h$$

$$\Delta F_0(c_0) \;=\; -\Delta F_1(c_1(c_0)) \qquad \forall c_0$$

Case 2: Piecewise Monotone $\Delta F_d()$

Back

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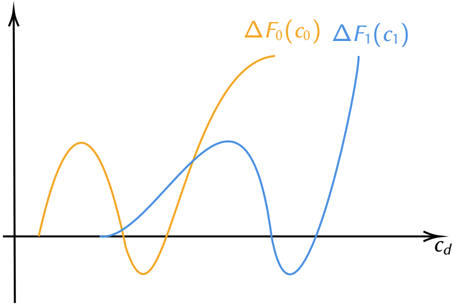

$$\Delta F_0(c_0) \ = \ -\Delta F_1(c_1(c_0)) \qquad \forall c_0$$

Case 2: Piecewise Monotone $\Delta F_d()$

Back

Only a finite number of points h such that:

$$\Longleftrightarrow \ m_0(w=0,h) - m_1(w=0,h) = m_0(w=1,h) - m_1(w=1,h) \quad \forall h$$

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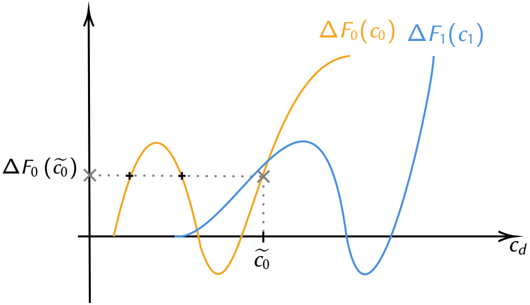

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Back

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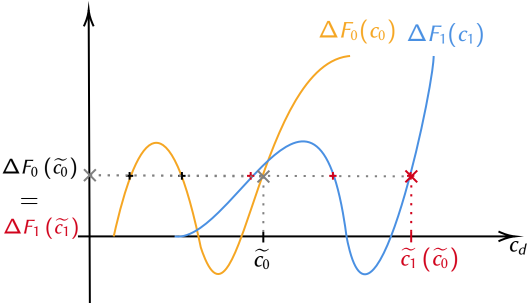

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Back

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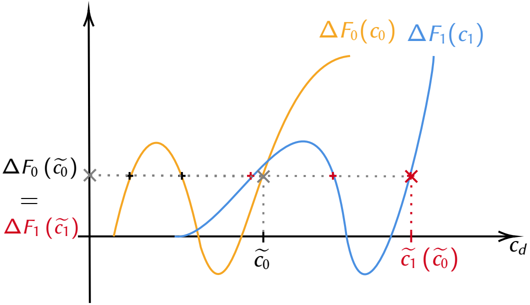

Point identified: piecewise inversion to recover $c_1(c_0)$

$$\Delta F_0(c_0) \;=\; -\Delta F_1(c_1(c_0)) \qquad \forall c_0$$

Case 3: Flat parts $\Delta F_d()$

Back

$\exists$ segment of points h such that: ($\iff$ relevant instrument assumption violated)

$$\iff m_0(w=0,h) - m_1(w=0,h) = m_0(w=1,h) - m_1(w=1,h) \quad \forall h$$
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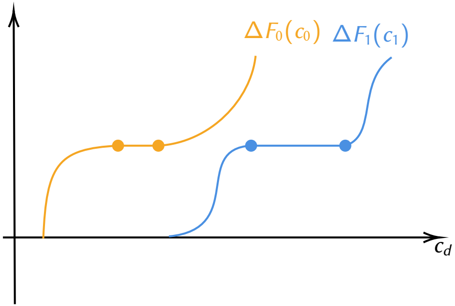

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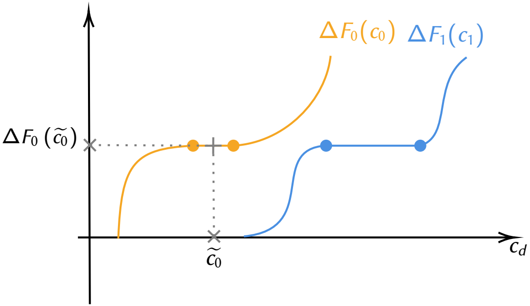

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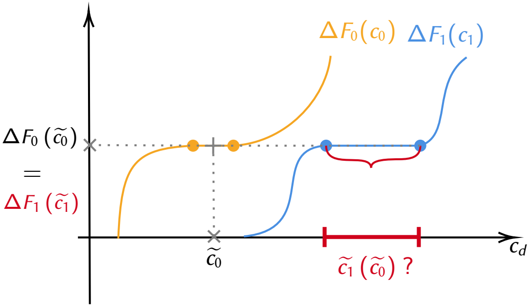

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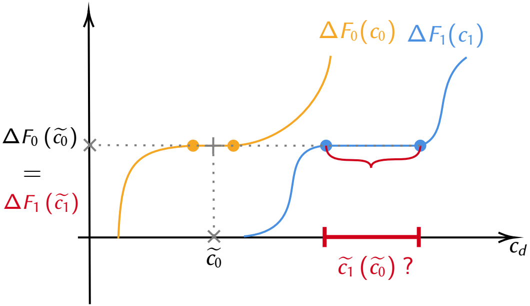

Only Partial identification of non-flat parts

Identification: full rank condition

The rank of $\Pi(c)$ is full for all c:

Back Back2

$$\Pi(c) = \begin{bmatrix} f_{c_0|D=0,W=0}(c_0)Pr(D=0|W=0) & f_{c_1|D=1,W=0}(c_1)Pr(D=1|W=0) \\ f_{c_0|D=0,W=1}(c_0)Pr(D=0|W=1) & f_{c_1|D=1,W=1}(c_1)Pr(D=1|W=1) \end{bmatrix}$$

Which implies a monotone likelihood ratio condition, $\forall c_0, c_1 \in \mathcal{C}_0 \times \mathcal{C}_1$:

$$\frac{f_{c_1|D=1,W=1}(c_1)Pr(D=1|W=1)}{f_{c_0|D=0,W=1}(c_0)Pr(D=0|W=1)} > \frac{f_{c_1|D=1,W=0}(c_1)Pr(D=1|W=0)}{f_{c_0|D=0,W=0}(c_0)Pr(D=0|W=0)}$$

Identification of the Primitives

- CCCs and CCPs identified as before (period by period). ▸ Back
  Transitions $f_t(x_{t+1}|x_t, d_t, c_t)$ identified directly from the data.

Identification of the Primitives

▸ Back

- CCCs and CCPs identified as before (period by period).
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- Use First Order Conditions relating the structure and the optimal choices to identify the structure of the model.

Identification of the Primitives

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Back

- Use First Order Conditions relating the structure and the optimal choices to identify the structure of the model.

$\rightarrow$ Marginal (period) utility identified by Euler Equation at optimal choices

$$u_d'^*(x_t, \eta_t) = \beta \mathbb{E}_t \left[(1+r) \frac{\partial}{\partial c_{dt+1}} u_{d_{t+1}}'^*(x_{t+1}, \eta_{t+1}) \,\middle|\, x_t, c_{dt} = c_{dt}^*(\eta_t, x_t), d_t = d \right]$$

where the marginal utility at optimal choices are

$$u_d'^*(x_t, \eta_t) = \frac{\partial}{\partial c_{dt}} u_d(c_{dt}, \tilde{x}_t, \eta_t)|_{c_{dt} = c_{dt}^*(\eta_t, x_t)}.$$

$\rightarrow$ Escanciano et al., 2015: nonparametrically identified (up to a scale) if stationary period utility: $u_{dt} = u_d \; \forall t$, and if $\partial u_d / \partial c_d > 0 \; \forall c, d$.

$\rightarrow$ Parametric non stationary period utility can also be identified.

Identification of the Primitives

Back

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- Use First Order Conditions relating the structure and the optimal choices to identify the structure of the model.

→ Marginal (period) utility identified by Euler Equation at optimal choices

→ Conditional value (at the optimal $c^*_{dt}(\eta_t, x_t)$) identified via FOC with respect to the asset (Blundell et al., 1997)

$$\forall d : \ \forall a_t \qquad \frac{\partial}{\partial a_t} v^*_{dt}(\tilde{x}_t, a_t, \eta_t) = (1+r)\frac{\partial}{\partial c_{dt}} u'^*_d(\tilde{x}_t, a_t, \eta_t)$$

Identified up to an additive constant of integration, normalized to zero.

Identification of the Primitives

Back

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→ Differences in additive m_d terms identified via the CCPs, e.g. if ϵ_t is extreme-value type 1:

$$Pr(D_t = 0|\eta_t, x_t, w_t) = \frac{1}{1 + exp\left(v^*_{1t}(x_t, \eta_t) - v^*_{0t}(x_t, \eta_t) + m_{1t}(w_t, \eta_t, x_t) - m_{0t}(w_t, \eta_t, x_t)\right)}$$

Estimation: CCCs (faster)

▶ Back ▶ Results

- From data on (D_t, C_{dt}, X_t, W_t), estimate the reduced forms $\hat{F}_{C_{dt}|d_t,x_t,w_t,t}(c_{dt})$ and $\widehat{Pr(D_t = d|X_t = x_t, W_t = w_t, t)}$. Nonparametric Kernel, Sieve logistic/probit regressions. $\rightarrow$ gives estimates of $\widehat{\Delta F}_{C_{dt}|x_t}(c)$.

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Back Results

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- Estimate the **mapping** $\hat{c_0}_t(c_{1t}, x_t)$ using that the solution must satisfy:

$$\widehat{\Delta F}_{C_{0t}|x_t}(c_{0t}(c_{1t}, x_t)) = -\widehat{\Delta F}_{C_{1t}|x_t}(c_{1t}) \qquad \forall c_{1t}$$

So, we solve (x_t by x_t and period by period) for the **complete monotone** **<u>function</u>** $\hat{c_0}_t(c_{1t}, x_t)$:

$$\hat{c_0}_t(c_{1t}, x_t) = \underset{c_{0t}(c_{1t}, x_t)}{argmin} \int_{\mathcal{C}_1} \left(\widehat{\Delta F}_{C_{0t}|x_t}(c_{0t}(c_{1t}, x_t)) + \widehat{\Delta F}_{C_{1t}|x_t}(c_{1t})\right)^2 \text{weight}(c_{1t})\, dc_{1t}.$$

Estimation: CCCs (faster)

Back Results

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- Find the corresponding h to a given $(c_{1t}, \hat{c_0}_t(c_{1t}, x_t), x_t)$:

$$\begin{aligned}\widehat{h}_t(c_{1t}, x_t) &= \widehat{F}_{C_{0t}|D_t=0,w_t,x_t}(\widehat{c}_{0t}(c_{1t}, x_t))\ \widehat{Pr(D_t = 0|w_t, x_t)} \\ &+ \widehat{F}_{C_{1t}|D_t=0,w_t,x_t}(c_{1t})\ \widehat{Pr(D_t = 1|w_t, x_t)}.\end{aligned}$$

Estimation: Structural parameters

Take *CCCs*, *CCPs*, and *transitions* as given:

▸ Back ▸ Application

1. Pick a set of parameters θ. Get the $u_{dt}()$, $u'_{dt}()$, and $m_{dt}()$ given θ.

Estimation: Structural parameters

Take *CCCs*, *CCPs*, and *transitions* as given: ▸ Back ▸ Application

1. Pick a set of parameters θ. Get the $u_{dt}()$, $u'_{dt}()$, and $m_{dt}()$ given θ.

2. Estimate the expectation RHS of the Euler equation:

$$u_d'^*(\tilde{x}_t, a_t, \eta_t, \theta) = \beta \mathbb{E}_t \left[(1+r) u_{d_{t+1}}'^*(\tilde{x}_{t+1}, a_{t+1}, \eta_{t+1}, \theta) \middle| x_t, c_{dt} = c_{dt}^*(\eta_t, x_t), d_t = d \right]$$

   Estimate it directly from the data or by *forward one period-ahead simulation.*

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3. Estimate $v_{dt}(c^*_{dt}(\eta_t, x_t), \eta_t, x_t) \equiv v^*_{dt}(\eta_t, x_t)$ by *simulating forward life-cycles.*

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4. Use $v^*_{dt}(\eta_t, x_t)$ to compute the theoretical probabilities:

   $$Pr(D_t = 0 | \eta_t, x_t, w_t, \theta) =$$

   $$\frac{1}{1 + exp\left(v^*_{1t}(x_t, \eta_t, \theta) + m_{1t}(x_t, w_t, \eta_t, \theta) - (v^*_{0t}(x_t, \eta_t, \theta) + m_0(x_t, w_t, \eta_t, \theta)\right))}$$

   Compare these *theoretical probabilities* with the *estimated CCPs.*

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   Compare these *theoretical probabilities* with the *estimated CCPs.*
5. $\widehat{\theta}$ minimizes an objective made of the differences in probabilities and in the Euler equation.

Estimation: Structural parameters

Take *CCCs*, *CCPs*, and *transitions* as given:

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   Compare these *theoretical probabilities* with the *estimated CCPs.*
5. $\widehat{\theta}$ minimizes an objective made of the differences in probabilities and in the Euler equation.
6. Inference: *in progress*, tentative by bootstrap.

Estimator comparison

Comparison of estimators performances:

Back

- Dynamic life-cycle toy model of Labor Participation and Consumption.
- Use *Monte Carlo simulations*.
- Compare my method with other (MLE, SMM).
  Measure statistical and computational efficiency differences.

Toy model: Labor Participation and Consumption

Back

Working Life: (T periods)
Period utility:

$$u(c_t, d_t, w_t, x_t, \eta_t, \epsilon_t) = \begin{cases} c_t^{1-\sigma}/(1-\sigma)\ \tilde{\eta}_t^0(\eta_t, \gamma_0, s_0) + \epsilon_{0t} \\ c_t^{1-\sigma}/(1-\sigma)\ \tilde{\eta}_t^1(\eta_t, \gamma_1, s_1) + \alpha + \omega(1-w_t) + \epsilon_{1t} \end{cases}$$

Where

- t is the age. c_t is individual consumption. d_t is labor choice.
- $\tilde{\eta}_t^d \sim \mathcal{LN}(\gamma_d,\ s_d)$.
  So, $\tilde{\eta}$ are just transformations of η: η^{th} quantiles of the lognormal.
- $(\gamma_0, \gamma_1, s_0, s_1)$ measures the effect of the unobserved heterogeneity.
- σ is risk aversion/ intertemporal elasticity of substitution.
- ω is utility cost of searching for a job when previously unemployed.
- α is utility cost of working.

Toy model: Labor Participation and Consumption

Back

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subject to the budget constraint:

$$a_{t+1} = (1+r)a_t - c_t + d_t y_t + (1-d_t)b_t$$

Where

- a_t is the household asset.
- y_t is the woman earnings. y_t takes two values: $y_L = 10$ and $y_H = 20$. With the following **transitions** (estimated in first stage):

$$Pr(y_{t+1} = y_H | d_t, y_t) = \Pi(d_t, y_t) = \begin{pmatrix} \pi_{0L} & \pi_{0H} \\ \pi_{1L} & \pi_{1H} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.4 \\ 0.6 & 0.8 \end{pmatrix}$$

  where $\pi_{1L} > \pi_{0L}$ and $\pi_{1H} > \pi_{0H}$. And $\pi_{1H} > \pi_{1L}$, and $\pi_{0H} > \pi_{0L}$.
- b_t are benefits for unemployed. We fix them to $b_t = 5$ for all simulation.

Toy model: Labor Participation and Consumption

Back

Retirement: $(T+1)$

- At age $T+1$, the woman retires. Gets the same utility as when did not work, with $d_t = 0$.
- She lives for *one more period* and *dies* in $T+2$. Without besquest motive.
  $\implies a_{T+2} = 0$.
  $\implies$ she consumes everything: $c_{T+1} = (1+r)a_{T+1} + pension(y_T)$.
- Where $pension(y_T)$ is the retirement pension, function of the last income. Set to $0.5y_T$ (taux plein).

Toy model: Labor Participation and Consumption

Back

Parameters:

- Discount future with β. Fixed.
- Set $\gamma_0 = 0$, $s_0 = 0.25$ (normalization).
- Transition function parameters estimated in first stage: $y_{t+1} \sim y_t, d_t$
- 5 structural parameters to estimate:

$$\theta = (\sigma, \gamma_1, s_1, \alpha, \omega)$$

$\rightarrow$ (α, ω) do not impact the marginal utility and consumption.

Simulation

Table: $T = 2$ periods ▸ Back

	Truth	*Method* *DCC*	*SMM*
N		10,000	10,000
σ	1.60	1.6253	1.5924
		(0.0410)	(0.0156)
γ_1	0.00	0.0070	-0.0052
		(0.0298)	(0.0105)
s_1	0.40	0.4078	0.4001
		(0.0228)	(0.0090)
α	-0.50	-0.4727	-0.5023
		(0.0498)	(0.0348)
ω	-1.00	-0.9982	-0.9972
		(0.0581)	(0.0523)
Average Time taken:			
1st stage: CCPs and CCCs		118s	9s
2nd stage: Structural parameters		170s	14328s
Overall		288s	14337s

Other initializations: Number of Monte-Carlo $= 1,000$

$Pr(w_1 = 1) = 0.70$. $y_1 = y_H$ with probability 0.50. $a_1 \sim \mathcal{U}(0, 30)$. $r = 0.05$.

Simulation

Table: $T = 1 \implies$ **closed form policies solution and likelihood**

Back

	Truth	*Method* *DCC* 1,000	*DCC* 10,000	*MLE* 1,000	*MLE* 10,000	*SMM* 1,000	*SMM* 10,000
σ	1.60	1.5806	1.5782	1.6042	1.5992	1.6135	1.5970
		(0.1759)	(0.0827)	(0.0444)	(0.0137)	(0.0560)	(0.0211)
γ_1	0.00	0.0071	0.0040	-0.0061	0.0007	-0.0269	-0.0009
		(0.0714)	(0.0286)	(0.0205)	(0.0072)	(0.0213)	(0.0078)
s_1	0.40	0.4246	0.4043	0.4005	0.4001	0.3926	0.3857
		(0.0747)	(0.0366)	(0.0187)	(0.0060)	(0.0245)	(0.0073)
α	-0.50	-0.4782	-0.5092	-0.4928	-0.5000	-0.4986	-0.4850
		(0.3266)	(0.1016)	(0.0852)	(0.0268)	(0.0989)	(0.0401)
ω	-1.00	-1.0689	-1.0044	-1.0115	-0.9931	-1.0308	-1.0029
		(0.1715)	(0.0484)	(0.1577)	(0.0441)	(0.2919)	(0.0665)
Avg Time taken:		16s	32s	1s	9s	16s	50s

Other initializations: Number of Monte-Carlo $= 1,000$.

$Pr(w_1 = 1) = 0.7$. $y_1 = y_H$ with $Pr(y = y_H) = 1$. $r = 0.05$. $a_1 = 12.5$ for everyone here.

Simulation

Comparison:

Back

1. Statistical Efficiency

Simulation

Comparison:

Back

1. Statistical Efficiency
   $\rightarrow$ **Consistent** estimators.

Simulation

Comparison:

Back

1. Statistical Efficiency
   - $\rightarrow$ **Consistent** estimators.
   - $\rightarrow$ *Efficiency Loss* due to first stage consistent estimation.

Simulation

Comparison:

Back

1. Statistical Efficiency
   - $\rightarrow$ **Consistent** estimators.
   - $\rightarrow$ *Efficiency Loss* due to first stage consistent estimation.
   - $\rightarrow$ SMM with *Endogenous Grid Method more efficient* when both methods built upon the same reduced forms $F_{C_{dt}|d_t,w_t,x_t}(c_t)$ and $Pr(d_t|w_t,x_t)$. But...

Simulation

Comparison:

Back

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   - → **Consistent** estimators.
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2. Computational Efficiency

Simulation

Comparison:

Back

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   - $\rightarrow$ EGM very long even with very simple $T = 2$ periods model.

Simulation

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2. Computational Efficiency
   - $\rightarrow$ EGM very long even with very simple $T = 2$ periods model.
   - $\rightarrow$ My method: about 50 times faster here.
     Fixed cost of computing *1st stage policies*. Then estimation of parameters is fast in the second stage: *do not compute the value function.*

Simulation

Comparison: ▸ Back

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   - → **Consistent** estimators.
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   - → The **more complex** the models (number of periods, number of covariates), the **faster** my method relative to the others.

Simulation

Comparison: ▸ Back

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   - → **Consistent** estimators.
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   - → The **more complex** the models (number of periods, number of covariates), the **faster** my method relative to the others.
   - → Minimal cost of including covariates (as long as data is good enough to estimate correct distributions), or test several specifications.