Experimental Overview

This describes, at a conceptual level (with appropriate formula for making connections), the integration of the HARK Macro model and the NetLogo market mechanism for determining the price of the risky asset.

There are three primary scripts or blocks of software:

1. The Oversight script. This is python code that controls the overall simulation. It sets up global parameters for each simulation and interacts with the HARK and NetLogo (potentially to be replaced by Zak's C# agent-based market simulation code) code.

2. The HARK software, which solves agent-based macroeconomic calculations. It takes inputs from the Oversight script and returns relevant output based on the price of the risky asset.

3. The NetLogo (wrapped in PyNetLogo) agent-based financial market simulation software. It takes inputs from the Oversight script and returns relevant output – mainly the price of the risky asset after "one day" of trading.

Version 1.0 of The Simulation Algorithm (at a conceptual level)

A. Start – Initialize HARK model

- Create Hark Agents: $HA_i, i = 1, \ldots, n_H$.
  - There are n_H agents.

- Set initial economic condition (wealth, income, health SEB PLEASE REFINE ….): $HEC_{i0}, i = 1, \ldots, n_H$.
  - This is done in the HARK program, these parameters are set and stored inside of the HARK program.
  - The amount of wealth that each agent holds in the Risky Asset is updated at the end of each trading day based on the price of the risky asset (as calculated using the NetLogo code). The updating depends on whether an agent pays attention to the market and wants to change the amount of wealth that they have of the risky asset, or whether they are ignoring the market that day.
  - The rest of these economic condition parameters are 'shocks' that are updated at the beginning of each quarter and then remain constant throughout the rest of the quarter.

- Set risk preferences: $HRP_i, i = 1, \ldots, n_H$.
  - This is something that is set in the HARK program and does these values are not changed throughout the simulation.
- Start with an initial stock price for the risky asset: $RAP_0 = 100$, the Risky Asset Price at time 0.
  - This value of the Risky Asset Price is maintained in the Oversight script.
  - As a convention set the price to 100.
- Use this stock price to determine the initial number of shares of the risky asset held by each agent:
  - $RAS_{i0} = (HEC_{i0}.RAW)/\, RAP_0$.

- $HEC_{i0}.RAW$ is the Risky Asset Wealth held by the i[th] HARK agent at time 0 and RAS_{i0} is the Risky Asset Shares held by the i[th] HARK agent at time 0. This parameter/value is stored in the HARK program (does that work, make sense?).
- RAS_{i0} is a value that could be held in the HARK program (if that makes sense and is easy to update) or it could be tracked in the Oversight script.

B. Update prices for each trading day in each Quarter

- At the beginning of each trading day , $t+1$ (during a quarter), select a random group of HARK agents (with replacement—meaning that the same agent could trade multiple times during a quarter—even back to back days).
  - HA_{i^*}, where i^* indicates that the HARK agent was selected and HA_i, where i indicates that the HARK agent was not selected.
  - Note that time, t, starts at the beginning of the simulation and does not reset each time a quarter starts.

- For the selected group of HARK agents, HA_{i^*}, calculate the desired level of wealth that the agent wants to have allocated to the risky asset based on the previous day's trading price, RAP_{t-1} , then determine the total number of shares that each agent wants to trade, buy or sell.

  - **First,** determine the expected return and standard deviation for the risky asset $(\bar{r}_t, sr_t)$ to be used for decision in period $t+1$.
  - To do this the agents use a weighted average of past price movements (the history of the risky asset price process up to the current day, $RAP_0, \ldots, RAP_{t-1}$) and a historic (e.g. S&P 500) average return and standard deviation $(\bar{r}_0, sr_0)$ to calculate the "current" expected return and standard deviation.
  - Formula for $\bar{r}_t = \sum_{l=1}^{t} w_l RAR_l + w_o \bar{r}_0$ and $sr_t = \sqrt{\sum_{l=1}^{t} w_l (RAR_l - \bar{r}_t)^2 + w_o (sr_0)^2}$, where
    - $RAR_l = \frac{RAP_l - RAP_{l-1}}{RAP_{l-1}}$ is the percent return of the risky asset price for period l and where
    - $w_t = (1 - S_t)\frac{exp\{a*t\}}{D_t}$, for $t > 0$ and $w_0 = S_t$ for all t.
    - Further $D_t = \sum_{l=1}^{t} exp\{a * l\}$, and $S_t = exp\{b * t\}$.
    - The parameter $a = -\frac{ln\{p_1\}}{\Delta t_1}$, where $1 > p_1 > 0$ is the proportion (or percentage of decay) for the weights over time Δt_1—with the weight with the largest time (corresponding to the most recent price information) being larger. So $w_t = p_1 w_{t+\Delta t_1}$. This captures the decay of the weights or the influence of the most recent percent returns over time. For example, if $p_1 = 0.1$ and $\Delta t_1 = 120$, then the percent return of the risky asset two quarters ago has 10% of the impact of the percent return of the risky asset today.
    - The parameter $b = \frac{ln\{p_2\}}{\Delta t_2}$, where $1 > p_2 > 0$ is the proportion of the weight that is assigned to the historical mean and standard deviation, or it is the value of w_0, when the amount of data that has been seen is equal to Δt_2. So $w_0 = p_2$, when $t = \Delta t_2$. For example, if $p_2 = 0.1$ and $t = \Delta t_2 = 120$, then $\sum_{l=1}^{120} w_l = 0.9$ and $w_0 = 0.1$.
    - See the accompanying Excel spreadsheet for a couple of numerical examples.

- **Second**, using this $(\bar{r}_t, sr_t)$, calculate the total number of shares of the risky asset that the selected HARK agents want to buy and sell.
  - Have HARK calculate the amount of wealth that each of the selected agents wants to hold in the risky asset (this is an update or new number), or determine $HEC_{i^*t+1}.RAW$.
  - Using the closing price from the previous period, RAP_t, determine the new number of shares that each selected agent wants to hold, or $RAS_{i^*t+1} = (HEC_{i^*t+1}.RAW)/RAP_t$.
  - Aggregate the total number of shares of the risky asset that the selected agents want to buy and the total (aggregate) number of shares of the risky asset that the selected agents want to buy, $BS_{t+1} = \sum(RAS_{i^*t+1} - RAS_{i^*t})^+$, where $(\cdot)^+ = \max(\cdot, 0)$, and the total (aggregate) number of shares of the risky asset that the selected agents want to sell, $SS_{t+1} = \sum(RAS_{i^*t+1} - RAS_{i^*t})^-$, where $(\cdot)^- = \min(\cdot, 0)$.
  - For HARK agents that were not selected leave the number of shares in the risky asset unchanged, or $RAS_{it+1} = RAS_{it}$.

- **Third**, using (BS_{t+1}, SS_{t+1}), as the input to the Buy Broker and Sell broker run a day of NetLogo trading and determine the end of day price of the risky asset, or .
  - There still needs to be some thought as to how (BS_{t+1}, SS_{t+1}) translate into Buy Broker and Sell Broker number of shares in NetLogo.
  - Have HARK calculate the amount of wealth that each of the selected agents wants to hold in the risky asset (this is an update or new number), or RAP_{t+1}.
  - Take the final (the last price from the LMTransaction log file or the equivalent output through PyNetLogo) NetLogo price (which will be close to 400) and divide it by 4, or $NLP_{t+1} = 0.25 * NLP_{t+1}$. Then calculate a percent change, or $NLR_{t+1} = \frac{NLP_{t+1} - 100}{100}$.
  - Adjust the NetLogo return so that it matches historic S&P 500 returns (change the mean and adjust the standard deviation—e.g. assume $(\bar{r}_0 = 0.000628, sr_0 = 0.011988)$ is the S&P 500 mean and standard deviation and$(\bar{r}_{NL} = -0.00052125, sr_{NL} = 0.0068)$ is the NetLogo simulated mean and standard deviation, then $NLR_{t+1} = sr_0 * \left(\frac{NLR_{t+1} - \bar{r}_{NL}}{sr_{NL}}\right) + \bar{r}_0$.
  - Use the NetLogo percent change to calculate the end of day price for the risky asset, or $RAP_{t+1} = RAP_t * (1 + NLR_{t+1})$.

- **Forth**, use the final price to calculate the amount of wealth each (all) HARK agent has allocated to the risky asset, or $HEC_{it+1}.RAW = RAS_{it+1} * RAP_{t+1}$.

- Repeat for each trading day until the end of the quarter.

C. Update the economic conditions for each trader (other than the wealth that they are holding in the risky asset) and then repeat part B.