

Perceived Income Risks

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Abstract

State-of-art incomplete-market macro models featuring uninsured idiosyncratic income risks typically use estimated risks from panel data of income realizations. But this practice could run into the problem of unobserved heterogeneity, and cannot perfectly approximate the income shocks from the point of view of the agents. This paper calibrates the income risks in a standard OLG/incomplete-market model using a representative expectational survey, which directly elicits density forecasts of individuals' wage growth. It shows that incorporating a number of salient facts of risk perceptions in the survey helps account for the low liquidity-asset holding of a large fraction of agents in the U.S. economy, i.e. hands-to-mouth consumers, and the wealth inequality in the data. I also extend the model to allow for subjective perceptions of income risks to be different from those that objectively drive income inequality. This extension also serves as an experimental model that breaks down the effects of idiosyncratic income risks on wealth inequality into two channels: ex-ante saving behaviors and the ex-post realized income inequality.

Keywords: Income risks, Incomplete market, Perception, Precautionary saving

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1 Introduction

Income risks matter for both individual behaviors and aggregate outcomes. With identical expected income and homogeneous risk preferences, different degrees of risks lead to different saving/consumption and portfolio choices. This is well understood in models in which either the prudence in the utility function (Kimball (1990), Carroll and Kimball (2001)) or occasionally binding constraint induces precautionary savings or self-insurance. It is widely accepted based on various empirical research that idiosyncratic income risks are at most partially insured (Blundell et al. (2008)), such market incompleteness leads to ex-post wealth inequality¹ and different degrees of marginal propensity to consume (MPC) (Krueger et al. (2016); Carroll et al. (2017)). This also changes the mechanisms via which macroeconomic policies take into effect². Furthermore, the aggregate movements in the degree of idiosyncratic labor risks drive time-varying precautionary saving motives, as another source of business cycle fluctuations.³

The size and the nature of the income risks are one of the central inputs in this class of incomplete-market macroeconomic models. One common practice prevailing in this literature thus far is that economists typically approximate/estimate risks under a specified income process, relying upon the cross-sectional dispersion in income realizations, and then treat the estimates as the true model parameters known by the agents making decisions in the model.⁴

But this estimation practice has limitations. Economists who attempt to approximate the real size and nature of unexpected income shocks and risks as perceived by the agents may very likely face omitted variables/unobserved heterogeneity and model mis-specification as to the heterogeneity in risks. The intuition behind this is simple: some information, either intrinsic heterogeneity of each individual or advance information that enters an agent's information set from time to time and is

¹Aiyagari (1994); Huggett (1996); Carroll and Samwick (1997); Krusell and Smith (1998).

²Krueger et al. (2016), Kaplan et al. (2018), Auclert (2019).

³Challe and Ragot (2016); McKay (2017); Heathcote and Perri (2018); Kaplan and Violante (2018); Den Haan et al. (2018); Bayer et al. (2019); Acharya and Dogra (2020); Ravn and Sterk (2021), Harmenberg and Öberg (2021).

⁴Some recent examples include Krueger et al. (2016), Bayer et al. (2019), Kaplan et al. (2018).

used to forecast her income, is not directly observable by economists. Therefore, what is a shock as approximated by economists may be expected by agents already, and what is considered as the risk is not from the agents' point of view, either.

This paper attempts to address these issues by utilizing the recently available density forecasts of labor income surveyed by New York Fed's Survey of Consumer Expectation (SCE). I first characterize the differences between perceived risks reported in the survey and the standard income risk estimates used to parameterize macroeconomic models. My key finding is that perceived earning risks are lower than standard estimates and model parameters, possibly due to the aforementioned unobserved heterogeneity problem ⁵, or under-perceiving of risks of the agents due to behavioral reasons. This is robust to various income processes used in the literature. Second, I explore a variety of drivers of heterogeneity in risk perceptions, including demographics, recent labor market conditions, and individual and macro experience, as detailed below. Despite these reasonable factors, a dominant share of the heterogeneity can be only attributed to unobserved heterogeneity/information. This suggests the importance of incorporating heterogeneity in income risks beyond a limited number of dimensions, such as education and age, as the standard practice in the literature.

These evidence motivate me to utilize survey-implied risks as truly perceived by agents to calibrate income risks in a standard incomplete market, overlapping-generation, and general equilibrium model to quantify these effects. The objective/benchmark model blends [Huggett \(1996\)](#), the income structure of [Carroll and Samwick \(1997\)](#), and persistent unemployment spells and unemployment benefits, a la [Krueger et al. \(2016\)](#) and [Carroll et al. \(2017\)](#). In comparison with conventional practice, I show how calibrating risks using surveyed perceptions helps explain a number of well-documented discrepancies between standard model prediction and that seen in the data: the concentration of households with little liquid wealth, a large fraction of agents with high MPC ,

⁵The unobserved heterogeneity from the point of view of economists is "superior information" from the point of view of agents.

and more wealth inequality.

On the flip side, there is mounting evidence in macroeconomics that people form expectations in ways deviating from full-information rational expectation (FIRE) ⁶ leading to perennial expectational heterogeneity across agents, it is worth considering the robustness of using survey-implied risk perceptions when generating model implications. As this paper cannot fully separately identify the behavioral bias in risk perceptions, I proceed with an additional experiment model, allowing the risk perceptions (subjective risks) to be different from the underlying income process (objective risks). This experiment model kills two birds with one stone. On one hand, it serves as a robustness check with an alternative model assumption deviating from FIRE. On the other hand, it can be invoked as an intermediary to break down the model implications into two channels: one via ex-anted saving behaviors resulting from risk perceptions and the other via ex-post realized income inequality.

The most important novelty of this paper compared to previous work studying partial insurance with expectational surveys ⁷ is that I particularly use the density survey which contains directly perceived risks. What is special about the density survey is that agents are asked to provide histogram-type forecasts of their wage growth over the next 12 months, together with a set of expectational questions about the macroeconomy. When the individual density forecast is available, a parametric density estimation can be made to obtain the individual-specific subjective distribution. And the second moment, namely the implied variance of the subjective distribution, allow me to directly characterize the perceived risk profile without relying on external estimates from cross-sectional microdata. This provides the first-hand measured perceptions on income risks that are truly relevant to individual decisions.

I established the following novel facts about the perceived income risks.

- People do have reasonable clues about the income risks they are facing, in the sense that

⁶For instance, [Mankiw et al. \(2003\)](#), [Reis \(2006\)](#), [Coibion and Gorodnichenko \(2012\)](#), [Wang \(2021\)](#), although most of these evidence are based on macroeconomic expectations such as inflation.

⁷For instance, [Pistaferri \(2001\)](#), [Kaufmann and Pistaferri \(2009\)](#).

heterogeneity in risk perceptions across different groups are consistent with the between-group differences in income volatility revealed from past data. For instance, younger, low-income, being female, low-education with more volatile income growth also perceive higher risks.

- But huge heterogeneity remains. People with the same observable characteristics still show wide dispersion in risk perceptions.
- State-dependence. Perceived income risks counter-cyclically react to nationwide/regional labor market conditions.
- Extrapolation. People who have experienced earning volatility and unemployment have higher risk perceptions.
- Past-dependence. Higher experienced volatility and experience of negative labor market conditions is associated with higher perceived income risks.
- Decision. Perceived income risks translate into economic decisions in a way consistent with precautionary saving motives. In particular, households with higher income risk perceptions expect a higher growth in expenditure, i.e. lower consumption today versus tomorrow.

These empirical patterns in perceived income risks have clear implications for both the level of optimal savings and cross-sectional wealth inequality via its effects on precautionary saving motives. As to the level, lower perceived risks than standard estimates used to parameterize the model implies lower precautionary savings, while the state-dependence and extrapolation both induce additional precautionary savings, as shown by [Caballero \(1990\)](#). Therefore, the quantitative implications of the survey-implied risk perceptions on the level of savings depend on the counterbalance of the two forces. The effect on wealth inequality is less ambiguous. Allowing for heterogeneity in risk perceptions induce straightforward increase in wealth inequality, simply because different risks induce different optimal savings.

As to the extension of the standard model with different subjective and objective risks, it features a single deviation by introducing an idiosyncratic subjective state that swings between low and high-risk perceptions, the process of which is estimated from the survey data. This assumption easily accommodates heterogeneity, state-dependence, and extrapolation of risk perceptions. In comparison with the objective model, the subjective model adds a state variable to individuals' consumption problems, and its dynamics also drive the distributional evolution of the economy in wealth. I characterize the economy with a stationary equilibrium. I also explore an extension of the subjective model by assuming the risk perception state depends on employment status of the individuals.

With the standard calibration of the model as in the literature, I *expect* to show that incorporating survey-implied risk perceptions separately helps generate sizable additional wealth inequality needed to match the data. It will also shed light on if the subjective income risk profiles on average result in a higher or lower level of aggregate savings in equilibrium. As a methodological contribution, my exercises highlight the usefulness and provide methods dealing with the resulting issues of utilizing expectational surveys such as measurement error and unobserved heterogeneity in heterogeneous-agent macro models, which has become the new paradigm of the macroeconomic analysis.

Related literature

First, this paper closely builds on the literature estimating both cross-sectional and time trends of labor income risks and partial insurance. Early work estimating income risks includes [Gottschalk et al. \(1994\)](#); [Carroll and Samwick \(1997\)](#). Later, the literature explores time-varying patterns of the income risks. For instance, [Meghir and Pistaferri \(2004\)](#) allows for time-varying risks or conditional heteroscedasticity in the traditional permanent-transitory model. [Blundell et al. \(2008\)](#) uses the same specification to income to estimate partial insurance in conjunction with consumption

data. More recently, [Bloom et al. \(2018\)](#) found the risks have declined in recent decades. Moreover, recent evidence relied upon detailed social security records and larger data samples highlight the asymmetry and cyclical behaviors of idiosyncratic earning/income risks ([Storesletten et al., 2004](#); [Guvenen et al., 2014](#); [Arellano et al., 2017](#); [Guvenen et al., 2019](#)). Besides, a separate literature focus on job-separation and unemployment risks([Low et al., 2010](#); [Davis and Von Wachter, 2011](#)). The novelty of this paper lies in the focus on the subjective perceptions of labor risks and how it is correlated with the realized income risks estimated from the income panel.

Second, it is related to an old but recently reviving interest in studying consumption/saving behaviors in models incorporating imperfect expectations and perceptions. For instance, the closest to the current paper, [Pischke \(1995\)](#) explores the implications of the incomplete information about aggregate/individual income innovations by modeling agent's learning about income component as a signal extraction problem. [Wang \(2004\)](#) extends the framework to incorporate precautionary saving motives. In a similar spirit, [Carroll et al. \(2018\)](#) reconciles the low micro-MPC and high macro-MPCs by introducing to the model an information rigidity of households in learning about macro news while being updated about micro news. [Rozsypal and Schlafmann \(2017\)](#) found that households' expectation of income exhibits an over-persistent bias using both expected and realized household income from Michigan household survey. The paper also shows that incorporating such bias affects the aggregate consumption function by distorting the cross-sectional distributions of marginal propensity to consume(MPCs) across the population. [Lian \(2019\)](#) shows that an imperfect perception of wealth accounts for such phenomenon as excess sensitivity to current income and higher MPCs out of wealth than current income and so forth. My paper has a similar flavor to all of these works in that I also explore the behavioral implications of households' perceptual imperfection. But it has important two distinctions. First, this paper focuses on higher moments such as income risks. Second, most of these existing work either considers inattention of shocks or bias introduced by the model parameter, none of these explores the possible misperception of the

nature of income shocks.⁸

Third, empirically, this paper also contributes to the literature studying expectation formation using subjective surveys. There has been a long list of “irrational expectation” theories developed in recent decades on how agents deviate from full-information rationality benchmark, such as sticky expectation, noisy signal extraction, least-square learning, etc. Also, empirical work has been devoted to testing these theories in a comparable manner ([Coibion and Gorodnichenko \(2012\)](#), [Fuhrer \(2018\)](#)). But it is fair to say that thus far, relatively little work has been done on individual variables such as labor income, which may well be more relevant to individual economic decisions. Therefore, understanding expectation formation of the individual variables, in particular, concerning both mean and higher moments, will provide fruitful insights for macroeconomic modeling assumptions.

Fourth, the paper is indirectly related to the research that advocated for eliciting probabilistic questions measuring subjective uncertainty in economic surveys ([Manski \(2004\)](#), [Delavande et al. \(2011\)](#), [Manski \(2018\)](#)). Although the initial suspicion concerning to people’s ability in understanding, using and answering probabilistic questions is understandable, [Bertrand and Mullainathan \(2001\)](#) and other works have shown respondents have the consistent ability and willingness to assign a probability (or “percent chance”) to future events. [Armantier et al. \(2017\)](#) have a thorough discussion on designing, experimenting and implementing the consumer expectation surveys to ensure the quality of the responses. Broadly speaking, the advocates have argued that going beyond the revealed preference approach, availability to survey data provides economists with direct information on agents’ expectations and helps avoids imposing arbitrary assumptions. This insight holds for not only point forecast but also and even more importantly, for uncertainty, because for any economic decision made by a risk-averse agent, not only the expectation but also the perceived risks matter a great deal.

Lastly, the idea of this paper echoes with an old problem in the consumption insurance liter-

⁸For instance, [Pischke \(1995\)](#) assumes that agents know perfectly about the variance of permanent and transitory income so that they could filter the two components from observable income changes. This paper instead assumes that the agents do not observe the two perfectly.

ature: ‘insurance or information’ (Pistaferri (2001), Kaufmann and Pistaferri (2009), Meghir and Pistaferri (2011), Kaplan and Violante (2010)). In any empirical tests of consumption insurance or consumption response to income, there is always a worry that what is interpreted as the shock has actually already entered the agents’ information set or exactly the opposite. For instance, the notion of excessive sensitivity, namely households consumption highly responsive to anticipated income shock, maybe simply because agents have not incorporated the recently realized shocks that econometricians assume so (Flavin (1988)). Also, recently, in the New York Fed [blog](#), the authors followed a similar approach to decompose the permanent and transitory shocks. My paper shares a similar spirit with these studies in the sense that I try to tackle the identification problem in the same approach: directly using the expectation data and explicitly controlling what are truly conditional expectations of the agents making the decision. This helps economists avoid making assumptions on what is exactly in the agents’ information set. What differentiates my work from other authors is that I focus on higher moments, i.e. income risks and skewness by utilizing the recently available density forecasts of labor income. Previous work only focuses on the sizes of the realized shocks and estimates the variance of the shocks using cross-sectional distribution, while my paper directly studies the individual specific variance of these shocks perceived by different individuals.

2 Theoretical framework

2.1 Income process and risk perceptions

I primarily focus on the wage risk to be consistent with the survey-elicited question. Conditional on employment at the same job and position and same hours of work, the log idiosyncratic earning, basically the wage rate, of an individual i at time t , $w_{i,t}$ consists of a predictable component $z_{i,t}$ and a stochastic component $e_{i,t}$. (Equation 1)

$$w_{i,t} = z_{i,t} + e_{i,t} \quad (1)$$

There is an extensive discussion in the literature about the exact time-series nature of the stochastic component e . For instance, it may consist of a permanent and a transitory component.⁹ Or some literature replaces the permanent component with a persistent component in the form of AR process.¹⁰ The transitory component could be moderately serially correlated following a moving-average (MA) process.¹¹ I first proceed with the generic structure like in Equation 1 without differentiating these various specifications. I defer this discussion to Section 4.2.

Hence, earning growth from t to $t + 1$ consists of predictable changes from $z_{i,t+1}$, and those from realized income shocks.

$$\Delta w_{i,t+1} = \Delta z_{i,t+1} + \Delta e_{i,t} \quad (2)$$

Under the assumption of full-information rational expectation (FIRE), all shocks that have realized till t are observed by the agent at time t . Therefore, the expected volatility under FIRE (with a superscript $*$), or what this paper will refer to as perceived risks (PR), is the expected variance of income growth from t to $t + 1$.

$$Var_t^*(\Delta w_{i,t+1}) = Var_t^*(\Delta e_{i,t+1}) = \sigma_{t+1|t}^2 \quad (3)$$

The predictable changes do not enter PR. Hence, the expected volatility in earning growth is

⁹Abowd and Card (1989),Gottschalk et al. (1994), Carroll and Samwick (1997), Blundell et al. (2008), and Kaplan and Violante (2010).

¹⁰Storesletten et al. (2004) and Guvenen (2009).

¹¹Meghir and Pistaferri (2004).

the *conditional* expected variance of the change in the stochastic component. Notice here that $Var_t^*(\Delta e_{i,t+1})$ crucially depends on the time-series nature of $e_{i,t}$.

The size of the true PR, including the component-specific ones under a specified structure of $e_{i,t}$, is not directly observed by economists. Econometricians usually try to approximate it, relying upon a panel data of earnings. Furthermore, although in theory, the risk as perceived by an FIRE agent could be totally individual-specific, economists can only approximate them at the group level, which we hereby refer to as c , as there are no realizations of risks, but only stochastic outcomes, at individual level. For instance, c could be defined based on age, gender, education or the years of entering job markets.

Specifically, in these estimations, what is used as the proxy of the stochastic component e and later further decomposed into various components, is the regression residual of individual earnings on all the observable characteristics of the individual in a first-step regression. Economists could not perfectly know all the predictable component $z_{i,t}$ from the agent's point of view. They instead include in $\hat{z}_{i,t}$ factors such as age polynomials, gender, education, occupation, etc. Denote the regression-residual or the approximated stochastic component by $\hat{e}_{i,t}$.¹²

Different from the PR by the agent, the cross-sectional variance of the change in residuals within group c , $Var(\Delta \hat{e}_{i,c,t})$, usually referred to as the “income volatility” in the literature,¹³ is an *unconditional* variance.

$$Var(\Delta \hat{e}_{i,c,t}) = \hat{\sigma}_{c,t}^2 + \hat{\sigma}_{c,t+1}^2 - 2Cov^c(\hat{e}_{i,c,t}, \hat{e}_{i,c,t+1}) \quad (4)$$

The distinction between the *conditional* PR by the agent and the *unconditional* volatility approximated by the economists is crucial. There are two important issues around the comparability of the two objects.

¹² $\hat{e}_{i,t} = w_{i,t} - \hat{z}_{i,t}$ ($\hat{z}_{i,t}$ is the approximated counterpart of $z_{i,t}$ from data).

¹³For instance, Gottschalk et al. (1994), Moffitt and Gottschalk (2002), Sabelhaus and Song (2010), Dynan et al. (2012), Bloom et al. (2018).

First, it is very likely that what is controlled for in the first-step income regression, namely $\hat{z}_{i,c,t}$, does not perfectly coincide with what is *predictable* from the point of view of an FIRE¹⁴ agent at time t . The primary reason is that econometricians with the panel data of earnings cannot control for other “unobserved heterogeneity” that is not measured in the data. This is equivalent to the “superior information” problem,¹⁵ which refers to the possibility that agents have advance information or foresight regarding their earning growth that is not available to econometricians. For instance, a worker may already anticipate a recent dispute with her boss may negatively affect her earning next year, but econometricians have no way of knowing this.

Second, the comparison is sensitive to the time-series nature of $e_{i,c,t}$. This is, again, because economists’ estimated volatility is unconditional, while the perception is conditional on the information till time t . To illustrate this point, imagine there is a very persistent component in the income shock, then under the aforementioned process, the estimated income volatility also includes the variance of the realized shock till t , which enters the information set of the agents already. Therefore, even if the econometricians perfectly recover the $e_{i,c,t}$ in the first step regression, any differences in the perceived time-series nature of the $e_{i,c,t}$ by agents and econometricians would lead to differences between PR and income volatility. Therefore, to approximate the true PR from the point of view of agents, economists need to recover a conditional variance using information from the unconditional variance, typically by assuming a time-series structure of the stochastic component e . I return to this discussion in Section 4.2.

To summarize, this paper argues that there are two major reasons why survey-elicited PR has invaluable use and is even preferable to the conventional income risk estimation based on cross-sectional realizations, which is further used to parameterize macro models. First, survey-reported PR is, by construction, conditional on the information set of each agent i , which very likely includes intrinsic heterogeneity specific to the individual or the advance information useful to forecasting

¹⁴In later sections of the paper, I relax the FIRE assumption, which it makes it possible that PR reported in the survey is also subject to incomplete information and behavioral bias of the agents.

¹⁵Pistaferri (2001); Kaufmann and Pistaferri (2009).

her own wage growth¹⁶. Economists who try to approximate the PR cannot do as well as the agents answering the questions, simply because this information is not necessarily available to economists. Second, survey-implied PR provides direct identification of the degree of heterogeneity of income risks across individuals in the economy. This prevents modelers from making possibly imperfect assumptions to estimate group-specific income risks, by grouping individuals by very limited dimension of observable factors, such as education and age.

It is worth pointing out that despite these advantages of the survey-implied PR, they are admittedly subject to the measurement errors and behavioral bias of agents in the real world compared to that is assumed by FIRE. I will explore the robustness of results of the paper with respect to these alternative assumptions in both estimation and models.

3 Data, variables and density estimation

3.1 Data on perceived risks

The data used for this paper is from the core module of Survey of Consumer Expectation(SCE) conducted by the New York Fed, a monthly online survey for a rotating panel of around 1,300 household heads over the period from June 2013 to January 2020, over a total of 80 months. This makes about 95113 household-year observations, among which around 68361 observations provide non-empty answers to the density question on earning growth.

Particularly relevant for my purpose, the questionnaire asks each respondent to fill perceived probabilities of their same-job-hour earning growth to pre-defined non-overlapping bins. The question is framed as “suppose that 12 months from now, you are working in the exact same [“main” if Q11>1] job at the same place you currently work and working the exact same number of hours. In your view, what would you say is the percentage chance that 12 months from now: increased by

¹⁶For the same reason, the literature partial insurance proposes to use expectational surveys, as one resolution of the superior information problem. See [Pistaferri \(2001\)](#), [Kaufmann and Pistaferri \(2009\)](#) and others for examples.

x% or more?”.

As a special feature of the online questionnaire, the survey only moves on to the next question if the probabilities filled in all bins add up to one. This ensures the basic probabilistic consistency of the answers crucial for any further analysis. Besides, the earning growth expectation is formed for exactly the same position, same hours, and the same location. This has two important implications for my analysis. First, these conditions help make sure the comparability of the answers across time and also excludes the potential changes in earnings driven by endogenous labor supply decisions, i.e. working for longer hours. Empirical work estimating income risks are often based on data from received income in which voluntary labor supply changes are inevitably included. Our subjective measure is not subject to this problem and this is a great advantage. Second, the earning expectations and risks measured here are only conditional on non-separation from the current job. It excludes either unemployment, i.e. likely a zero earning, or an upward movement in the job ladder, i.e. a different earning growth rate. Therefore, this only reflects a lower bound for the income risks facing the individuals. I will separately compare unemployment/separation expectations in Section 4.5.

Unemployment and other involuntary job separations are undoubtedly important sources of income risks, but I choose to focus on the same-job/hour earning with the recognition that individuals' income expectations, if any, may be easier to be formed for the current job/hour than when taking into account unemployment risks. Given the focus of this paper being subjective perceptions, this serves as a useful benchmark. What is more assuring is that the bias due to omission of unemployment risk is unambiguous. We could interpret the moments of same-job-hour earning growth as an upper bound for the level of growth rate and a lower bound for the income risk. To put it in another way, the expected earning growth conditional on current employment is higher than the unconditional one, and the conditional earning risk is lower than the unconditional one. At the same time, since SCE separately elicits the perceived probability of losing the current job

for each respondent, I could adjust the measured labor income moments taking into account the unemployment risk.

3.2 Density estimation and variables

With the histogram answers for each individual in hand, I follow [Engelberg et al. \(2009\)](#) to fit each of them with a parametric distribution accordingly for three following cases. In the first case when there are three or more intervals filled with positive probabilities, it was fitted with a generalized beta distribution. In particular, if there is no open-ended bin on the left or right, then two-parameter beta distribution is sufficient. If there is open-ended bin with positive probability on either left or right, since the lower bound or upper bound of the support needs to be determined, a four-parameter beta distribution is estimated. In the second case, in which there are exactly two adjacent intervals with positive probabilities, it is fitted with an isosceles triangular distribution. In the third case, if there is only one positive-probability of interval only, i.e. equal to one, it is fitted with a uniform distribution.

For all the moment's estimates, there are inevitably extreme values. This could be due to the idiosyncratic answers provided by the original respondent, or some non-convergence of the numerical estimation program. Therefore, for each moment of the analysis, I exclude top and bottom 3% observations, leading to a sample size of around 53,180.

I also recognize what is really relevant to many economic decisions such as consumption is real income instead of nominal income. I use the inflation expectation and inflation uncertainty (also estimated from density question) to convert expected nominal earning growth to real growth expectations.

The variance associated with real earning growth, if we treat inflation and nominal earning growth as two independent stochastic variables, is equal to the summed variance of the two. The independence assumption is admittedly an imperfect assumption because of the correlation of wage

growth and inflation at the macro level. In the Appendix, I report results of the paper with alternative assumptions about the correlation between the nominal wage and inflation expectations.

3.3 Labor income data

I use longitudinal data on individual labor earnings from the 2014-2017 and 2018-2020 panels of the Survey of Income and Program Participation (SIPP)¹⁷. Each panel of the SIPP is designed to be a nationally representative sample of the U.S. population and surveys thousands of workers. The interviews are conducted once a year to collect the individual’s monthly earnings and labor market activity¹⁸. On average, each individual is surveyed for 33 months over the multiple waves of the survey.

For the purpose of this paper, there are obvious advantages with using SIPP over another commonly used dataset for income risk estimation, the most notable of which is the Panel Study of Income Dynamics (PSID). SIPP surveys monthly labor outcomes of workers such as earnings, hours of work, and other detailed records of job transitions, while PSID only provides biennial records of labor income for years since 1997. For the overlapping periods between SIPP and SCE, it is possible to make a direct comparison between realized income risks at the annual frequency and the ex-ante perceptions of the earners. This is particularly crucial if income risks are time-varying and dependent upon macroeconomic conditions. Besides, given the surveyed risk perception is conditional on the same job position and hours, income risks explicitly controlling for hours of work and conditional on the same job would make the two more comparable. Given the earning risks expectations regarding the single job for the equal hours of work, I divide the monthly earnings from the *primary job* by the average hours of work for the same job and use it to estimate risks.

I restrict the SIPP sample used for estimating income risks for workers who have been staying

¹⁷Other recent work that estimates income risks using SIPP includes [Bayer et al. \(2019\)](#). Different from this paper, they use quarterly total household income, instead of the monthly job-specific earning of individuals.

¹⁸This causes the “seam” issue well documented in the survey literature([Moore, 2008](#))., which states that cross-wave transitions are systematically larger in magnitudes than within-wave changes. Therefore, I exclude the December-to-January earning growth in estimations to address this issue.

in the same job for the entire surveyed period. In addition, I impose the following criteria. (1) only working-age population between 25-65. (2) only private-sector jobs, excluding workers from government or other public sectors. (3) no days away from work during the reference month without the pay. (4) the same job as the last year. (5) monthly wage rates that are greater than 10 times or smaller than 0.1 times of the average wage are excluded. This leaves me with a monthly panel of 350-1000 individual earners for the sample period 2013m3-2019m12. Appendix A.1 discusses the data selection and summary statistics in greater details.

4 Basic facts of perceived income risks

4.1 Observable and unobservable heterogeneity

In both income risk estimation and parameterization of the standard incomplete market macro models, it is common practice to assume idiosyncratic risks differ by certain observable factors such as education, gender, and age, and furthermore, there is no additional within-group heterogeneity in the degree of the risks.¹⁹ This section reports the finding that although the observed heterogeneity in PR across individuals does reflect between-group differences along dimensions economists have commonly assumed, a dominant fraction of the differences in PR is attributed to other unobservable heterogeneity. Furthermore, even in those observable dimensions, the group heterogeneity seen in PR does not coincide with that seen in estimated risks.

Figure 1 plots the group average of perceived risk, approximated wage volatility $Var(\Delta\hat{e})$ as defined in Equation 4, and the estimated risk $Var_t(\Delta\hat{e})$ by age, gender and education. As to the education-risk profile, both wage volatility and approximated risks are higher for more educated workers. This is consistent with the finding of Meghir and Pistaferri (2004) using labor income

¹⁹For instance, Meghir and Pistaferri (2004) found that more educated workers are faced with higher income risks than the less-education ones. In addition, Sabelhaus and Song (2010); Bloom et al. (2018) documented that income risks decrease with age, and vary with the current income level in a non-monotonic U-shape. Cagetti (2003), Blundell et al. (2008), Carroll et al. (2017) allows for heterogeneous risks across different demographic variables in their models.

instead of wage. In contrast, risk perceptions exhibits an opposite pattern with respect to education level, in that less-educated workers perceive wage risks to be higher than more-educated workers. As to the life-cycle pattern of risks, neither wage volatility nor estimated risks shows very clear heterogeneity across different ages.²⁰ In contrast, perceived risks nearly monotonically declines over the life cycle for both males and females. These findings are also confirmed in Table A.1, which reports the group average of PR, wage volatility and estimated risks.

Another salient fact is that PR is always smaller than estimated risks. In particular, both wage volatility and estimated risk of different groups fall in the range of 5-15% per year (in standard deviation), which broadly aligns with the estimates in a large literature and that used in models, as summarized in Table A.3. But the average perceived risks reported in the survey is only around 3-4%, at least half smaller. For instance, a male high school graduate on average perceives annual wage risk to be 4 percentage points in terms of standard deviation, while the income risk implied by wage panel data for the same group is above 9-10 percentage points.

[FIGURE 1 HERE]

Standard models with idiosyncratic income risks do not assume heterogeneity by permanent income in addition to the observed group factors that may affect permanent income, such as education. Is it so in risk perceptions? It turns out that PR does correlate with the realized outcomes of the individuals. For a subsample of around 4000 observations, SCE surveys the annual earning of the respondent along with their risk perceptions. I group individuals into 10 groups based on their reported earning (within the same time) and plot the average risk perceptions against the decile rank in Figure 2. Perceived risks decline as one's earnings increase. This is not exactly consistent with the uptick in income risks for the highest income group, as documented by Bloom et al. (2018) using tax records of income. The most likely explanation is that the small sample I used from SCE

²⁰The homogenous age pattern of wage risks is not necessarily contradictory with the well documented declining pattern estimated using data on household income or total earning²¹. It is likely that the decline of income risks over the life cycle has to do with non-wage risks or better insurance via work arrangements over the life cycle.

does not cover actual top earners. The average annual earning of the top income group is between \$45,000 and \$120,000 in our sample.

[FIGURE 2 HERE]

But despite controlling for all the observable factors of the individuals, there remains a large degree of heterogeneity that seems to be most likely attributable to other unobserved factors. Figure 3 plots the cross-sectional distribution of unexplained residuals of perceived income risks both in nominal and real terms after controlling for observable individual characteristics including age, age polynomial, gender, education, type of work, and time fixed effects, respectively. Controlling for the time fixed effects is important because the focus here is on the idiosyncratic risks perceived by agents.

In both nominal and real terms, the distribution is right-skewed with a long tail. Specifically, most of the workers have perceived a standard deviation of nominal earning growth ranging from zero to 4% wage growth a year). But in the tail, some of the workers perceive risks to be as high as 7 – 8% standard deviation a year. To have a better sense of how large the risk is, consider a median individual in our sample, who has an expected wage growth of 2.4%, and a perceived risk of 1% standard deviation. This implies by no means negligible earning risk. ²²

Besides observable factors controlled in the first-step regression, individual fixed effects are important in explaining the heterogeneity in PRs. In particular, the R^2 of the regression without individual fixed effects is at most 10%, while including fixed effects increases R^2 to 70%. This finding has two implications. First, the role of unobservable heterogeneity seems to suggest that the conventional practice of estimating and modeling income risks differently by demographic groups has limitations. Second, the survey-implied heterogeneity in PR can be directly put into use to model heterogeneous income risks without requiring a strong stance on the explanations of the

²²In the appendix, I also include histograms of expected income growth and subjective skewness, which show intuitive patterns such as nominal rigidity. Besides, about half of the sample exhibits non-zero skewness in their subjective distribution, indicating asymmetric upper/lower tail risks.

source of heterogeneity.

[FIGURE 3 HERE]

4.2 Decomposed risks of different persistence

One crucial aspect of income risks relevant to both economists' estimation and household decisions is its time-series nature. As to the former, perceived income risks by the agents at a particular point of time, say t , are conditional on the information that is available by t . A realized permanent/persistent shock carries information regarding future income growth path, while an entirely transitory shock does not. Therefore, in the two scenarios, agents perceive different degree of income risks. For the latter, permanent income risks affect consumption/savings more substantially than the transitory risks via induced precautionary saving motives, according to the Permanent-Income Hypothesis (PIH).

I follow a large body of literature ²³ to specify the stochastic component $e_{i,c,t}$ to consist of a permanent component p which follows a random walk and a transitory component θ . The shocks to both components are log normally distributed, with mean zero and time-varying risks. This also corresponds to the model specification as in Equation 11.

$$e_{i,c,t} = p_{i,c,t} + \theta_{i,c,t} \quad (5)$$

$$p_{i,c,t} = p_{i,c,t-1} + \psi_{i,c,t}$$

In particular, under such a specification, the observed income volatility defined as in Equation 2, $Var(\Delta \hat{e}_{i,c,t})$ is essentially the sample analogue of the following.

$$Var(\Delta e_{i,c,t}) = \sigma_{\psi,t}^2 + \sigma_{\theta,t-1}^2 + \sigma_{\theta,t}^2 \quad (6)$$

²³Abowd and Card (1989), Gottschalk et al. (1994), Carroll and Samwick (1997), Blundell et al. (2008), etc.

The estimated perceived income risks under full-information rational expectation (FIRE) would be exactly the summation of the variance of the two components $\sigma_{\psi,t}^2 + \sigma_{\theta,t}^2$. The difference between the perceived risks and the income volatility, $\sigma_{\theta,t-1}^2$ is exactly due to the fact that the former is unconditional variance and the latter is conditional on the information available to the agent at the time t .

Using the *GMM* approach in the literature,²⁴ I identified the time-varying variances of the permanent and transitory component of the monthly wage growth, i.e. $\sigma_{\psi,t}^2$ and $\sigma_{\theta,t}^2$, using the *SIPP* data for the same period. Then I convert these monthly risks parameters into annual frequency to be compared to perceived risks about annual risks.²⁵

Figure 4 plots the 1-year-ahead perceived income risks reported in the *SCE* against the estimated *realization* of the total, permanent and transitory risk for the same period. Under correct model-specification and FIRE of the agents, one may expect the perceived risks and expected volatility to be, if not equal, at least close to each other. But the results suggest there is a negligible correlation between the two series.

More importantly, the magnitudes of the perceived risks are significantly lower than the expected income volatility implied by the income risks estimations. For instance, the latter based on the full sample should be 10% in standard deviation a year, while the average earning risk perception in *SCE* is only 2%. The same pattern holds even if we separately estimate income risks for different gender, education and age. (See Table A.2.)

[FIGURE 4 HERE]

What are the reasons behind this substantial differences in magnitudes between risk perceptions and estimated risks based on income data? There are various possible explanations, which may not

²⁴See Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Blundell et al. (2008). Essentially, this approach relies upon moment conditions as that in Equation 6 and the auto-covariance terms of $\Delta e_{i,c,t}$ to pin down the time-varying sizes of σ_{ψ} and σ_{θ} .

²⁵For permanent risks, the annual earning risk is the summation of monthly permanent risks over the next 12 months. The transitory risks of annual earnings, in contrast, is the sample average of monthly risks over the next 12 months.

be mutually exclusive.

The first reason is the unobserved heterogeneity to each individual when economists estimate risks. It is the same as the “superior information”, as recognized in the literature on consumption insurance. When economists estimate income risks, the best we can do is to try to control as many as possible observable factors of individuals that may be anticipated by the agents to approach its true information set at time t . But it is almost surely so that what we treated as unanticipated income shock still contains information available to the agents at time t .

The second reason, however, is that agents are possibly overconfident, i.e. under-perceive income risks for psychological reasons.

The first reason essentially states economists estimating income risks run into an omitted-variable problem, while the second reason states the agents may mis-specifying the data generating process of the income model. Although it is an appealing question to ask, it is hard to differentiate the two possible explanations. The latter part of this paper accommodates both possibilities without taking a strong stance on which explanation is more correct. What matters is the empirical fact that perceived risks reported in the survey seem to be systematically lower than the standard estimates used to calibrate the incomplete market models.

4.3 Counter-cyclicity of perceived risk

Some studies have documented that income risks are counter-cyclical based on cross-sectional income data.²⁶ It is worth inspecting if the subjective income risk profile has a similar pattern. Figure 5 plots the average perceived income risks from SCE against the YoY growth of the average hourly wage across the United States, which shows a clear negative correlation. Table 1 further confirms such a counter-cyclicity by reporting the regression coefficients of different measures of average

²⁶But they differ in exactly which moments of the income are counter-cyclical. For instance, [Storesletten et al. \(2004\)](#) found that variances of income shocks are counter-cyclical, while [Guvenen et al. \(2014\)](#) and [Catherine \(2019\)](#), in contrast, found it to be the left skewness.

risks on the wage rate of different lags. All coefficients are significantly negative.

[FIGURE 5 HERE]

[TABLE 1 HERE]

The pattern can also be seen at the state level. Table 2 reports the regression coefficients of the monthly average perceived risk within each state on the state labor market conditions, measured by either wage growth or the state-level unemployment rate, respectively. It shows that a tighter labor market (higher wage growth or a lower unemployment rate) is associated with lower perceived income risks. Note that our sample stops in June 2019 thus not covering the outbreak of the pandemic in early 2020. The counter-cyclicalities will be very likely more salient if it includes the current period, which was marked by catastrophic labor market deterioration and increase market risks.

[TABLE 2 HERE]

The counter-cyclicalities in subjective risk perceptions seen in the survey may suggest the standard assumption of state-independent symmetry in income shocks is questionable. But it may well be, alternatively, because people's subjective reaction to the positive and negative shocks are asymmetric even if the underlying process being symmetric. The model to be constructed in the theoretical section explores the possible role of both.

4.4 Experiences and perceived risk

[TABLE 3 HERE]

Different generations also have different perceived income risks. Let us explore to what extent the cohort-specific risk perceptions are influenced by the income volatility experienced by that particular cohort. Different cohorts usually have experienced distinct macroeconomic and individual histories.

On one hand, these non-identical experiences could lead to long-lasting differences in realized life-long outcomes. An example is that college graduates graduating during recessions have lower life-long income than others. (Kahn (2010), Oreopoulos et al. (2012), Schwandt and Von Wachter (2019)). On the other hand, experiences may have also shaped people’s expectations directly, leading to behavioral heterogeneity across cohorts (Malmendier and Nagel (2015)). Benefiting from having direct access to the subjective income risk perceptions, I could directly examine the relationship between experiences and perceptions.

Individuals from each cohort are borned in the same year and obtained the same level of their respective highest education. The experienced volatility specific to a certain cohort c at a given time t can be approximated as the average squared residuals from an income regression based on the historical sample only available to the cohort’s life time. This is approximately the unexpected income changes of each person in the sample. I use the labor income panel data from PSID to estimate the income shocks.²⁷ In particular, I first undertake a Mincer-style regression using major demographic variables as regressors, including age, age polynomials, education, gender and time-fixed effect. Then, for each cohort-time sample, the regression mean-squared error (RMSE) is used as the approximate to the cohort/time-specific income volatility.

There are two issues associated with such an approximation of experienced volatility. First, I, as an economist with *PSID* data in my hand, am obviously equipped with a much larger sample than the sample size facing an individual that may have entered her experience. Since larger sample also results in a smaller RMSE, my approximation might be smaller than the real experienced volatility. Second, however, the counteracting effect comes from the superior information problem, i.e. the information set held by earners in the sample contains what is not available to econometricians. Therefore, not all known factors predictable by the individual are used as a regressor. This will bias upward the estimated experienced volatility. Despite these concerns, my method serves as a

²⁷I obtain the labor income records of all household heads between 1970-2017. Farm workers, youth and olds and observations with empty entries of major demographic variables are dropped.

feasible approximation sufficient for my purpose here.

The right figure in Figure 7 plots the (logged) average perceived risk from each cohort c at year t against the (logged) experienced volatility estimated from above. It shows a clear positive correlation between the two, which suggests that cohorts who have experienced higher income volatility also perceived future income to be riskier. The results are reconfirmed in Table 4, for which I run a regression of logged perceived risks of each individual in SCE on the logged experienced volatility specific to her cohort while controlling individuals age, income, educations, etc. What is interesting is that the coefficient of *expvol* declines from 0.73 to 0.41 when controlling the age effect because that variations in experienced volatility are indeed partly from age differences. While controlling more individual factors, the effect of the experienced volatility becomes even stronger. This implies potential heterogeneity as to how experience was translated into perceived risks.

How does experienced income shock per se affect risk perceptions? We can also explore the question by approximating experienced income growth as the growth in unexplained residuals. As shown in the left figure of Figure 7, it turns out that a better past labor market outcome experienced by the cohort is associated with lower risk perceptions. This indicates that it is not just the volatility, but also the change in level of the income, that is asymmetrically extrapolated into their perceptions of risks.

[FIGURE 7 HERE]

4.5 Unemployment risk perceptions

My analysis so far only focuses on the earning risk conditional on employment. But it admittedly only constitutes a lower bound of the labor market risks since major events such as job loss and displacement usually result in more significant changes in labor income and affects household's welfare²⁸. In addition, unemployment risks are usually another central input of the incomplete-

²⁸Low et al. (2010), Davis and Von Wachter (2011).

market macroeconomic models. Similar to the approach with earning risks, the common practice in these models is to model the process of labor market transitions based on externally estimated statistics and assume the agent correctly perceives it within the model. Therefore, by the same token as for the earning risk, this section examines if the survey-reported expectations of job separation align with realized aggregate dynamics revealed from the labor market statistics. Moreover, are perceived unemployment risks extrapolate from recent experiences of aggregate and individual labor market outcomes?

For a fair comparison between perceptions and realizations which are available at different horizons, we cast both probabilities into a continuous-time Poisson rate. Specifically, for the expectation, let the reported probability of separating from the current job in next 12 months be $P_{i,t}(ue_{t+12}|e_t)$, then the corresponding monthly Poisson rate of job-separation $E_{i,t}(s_{t+1})$ is $-\log(1 - P_{i,t}(ue_{t+12}|e_t))/12$ ²⁹. With the realized month-to-month flow rate estimated from CPS $P(ue_{t+1}|e_t)$, the corresponding realized Poisson rate s_{t+1} is $-\log(1 - P(ue_{t+1}|e_t))$.

Figure 6 plots the converted job-separation expectations and realizations against each other. A few important patterns emerge. First, the comparison confirms our earlier findings based on earning risks that individual perceptions did track the aggregate realizations relatively well, assuring us that people's self-reported perceptions are not entirely groundless or naive. But on the other hand, there are systematic differences between perceptions and realizations.

[TABLE 6 HERE]

4.6 Perceived income risk and consumption spending

How individual-specific perceived risks affect household economic spending decisions? One of the key testable predictions is higher perceived risks should induce precautionary saving motive, hence lowering current consumption, or increasing expected consumption growth. SCE directly surveys

²⁹This follows from the following mathematical fact: for a continuous-time Poisson process with an event rate of θ , the arrival probability over a period of Δt units of time is equal to $1 - \exp^{-\theta\Delta t}$.

the self-reported spending plan, i.e. expected spending growth over the next year, which exactly corresponds to the object of our interest ³⁰. Therefore, we can evaluate if higher perceived risks translate affects spending plan consistently with precautionary saving motives.

In general, expected consumption growth with uncertain labor income does not have analytical expression with perceived income risks in it. This is because the optimal consumption paths crucially depends on the income process as well as the nature of this perceived income risks. But under auxiliary assumptions, we could attain a close form expression of expected growth in consumption. Specifically, assume the agent maximizes discounted CRRA utility from consumption with discount rate θ and exogenously given interest factor $1 + r_t$, and the coefficient of relative risk aversion is ρ . Under log normal income process, the expected consumption growth at time t can be approximated as the following when the borrowing constraint is not binding. The expected consumption growth is higher if the borrowing constraint is binding at time t .

$$E_{i,t}(\Delta c_{i,t+1}) \approx \frac{1}{\rho}(r_t - \theta) + \frac{\rho}{2}\sigma_{i,t}^2(c_{i,t+1}) \quad (7)$$

The second term on the right above captures the effect from precautionary saving motive or possibly binding constraint. We could think of both as a consequence of market incompleteness (Parker and Preston, 2005). Regardless of the particular cause of consumption fluctuations, the term increases with the size of expected consumption risks. But we do not directly observe the expected variance of consumption of the individuals. So an additional assumption regarding the degree of insurance of consumption from income risks is necessary to link expected consumption risks to perceived income risks. The scenario of zero insurance or full pass-through, namely $\sigma_{i,t}^2(c_{i,t+1}) = \text{var}_{i,t}(\Delta y_{i,t+1})$, is most likely to happen when the income risks perceived by the agents are permanent. Under partial insurance, the consumption risks anticipated by the agents should be

³⁰Other work that directly examines the impacts of expectations on readiness to spend includes Bachmann et al. (2015) and Coibion et al. (2020). Related to this, there is a recent literature that relies on survey answers to measure marginal propensity to consume, such as Fuster et al. (2020) and Bunn et al. (2018).

smaller than the perceived income risks. Let the partial pass-through parameter being κ , then the relationship between expected spending growth and perceived income risks can be written as the following.

$$E_{i,t}(\Delta c_{i,t+1}) \approx \frac{1}{\rho}(r_t - \theta) + \frac{\rho}{2}\kappa^2 \text{var}_{i,t}(\Delta y_{i,t+1}) \quad (8)$$

Since $\kappa \leq 1$, an OLS estimate coefficient of expected spending growth on perceived income risks reveals a lower bound of the 1/2 of the size of relative risk aversion ρ . Table 5 reports the regression results of planned log spending growth over the next year on real and nominal perceived income risk in the variance terms³¹. Regardless of the specification, the perceived risk is indeed positively correlated with the expected spending growth as the precautionary saving motive would predict. Specifically, after controlling for individual fixed effect, i.e. discount rate, and time fixed effect i.e. interest rate, each unit increase in perceived variance leads to around a 3 percentage points increase in expected spending growth. This implies an estimated risk aversion coefficient in the range of 6-7. Besides, the precautionary saving motives are weaker for real earning risks than the nominal, but the two are not significantly different from each other.

[TABLE 5 HERE]

5 Risk perceptions and wealth inequality

5.1 An overlapping generation model

I set up a standard incomplete market/life-cycle/general-equilibrium model without aggregate risks.

The model structure resembles that of [Huggett \(1996\)](#), although it embeds a more realistic income

³¹One common econometric concern with running regressions of this kind is the measurement error in the regressor, i.e. the perceived risks. In a typical OLS regression in which the regressor has i.i.d. measurement errors, the coefficient estimate for the imperfectly measured regressor has a bias toward zero. For this reason, if I find that expected spending growth is indeed positively correlated with perceived risks, taking into account the bias, it implies that the correlation of the two is greater in size.

risk profile and economic environment a la [Carroll and Samwick \(1997\)](#); [Krueger et al. \(2016\)](#); [Carroll et al. \(2017\)](#).

In each period, a continuum of agents is born. Each agent i lives for L and works for T ($T \leq L$) periods since entering the labor market, during which he/she earns stochastic labor income y_τ at the work-age of τ . After retiring at age of T , the agent lives for another $L - T$ periods of life and receive social security benefits. We assume away aggregate risks in the benchmark model, therefore there is no need to treat calendar time t from working age τ as two separate state variables, hence we suppress time script t . All shocks are idiosyncratic, or to put it differently, specific to the individual i .

5.1.1 Consumer's problem

The consumer chooses the whole future consumption path to maximize expected life-long utility, under a discount factor β and constant survival probability $(1 - D)$.

$$\max \quad \mathbb{E} \left[\sum_{\tau=0}^{\tau=L-1} (1 - D)^\tau \beta^\tau u(c_{i,\tau}) \right] \quad (9)$$

where $c_{i,\tau}$ represents consumption at the work-age of τ . The felicity function $u(c)$ takes a standard CRRA form with relative risk aversion of ρ : $u(c) = \frac{c^{1-\rho}}{1-\rho}$.³²

Denote total cash in hand at the beginning of the period τ as $m_{i,\tau}$, the end-of-period saving in period τ after consumption as $a_{i,\tau}$, the bank balance in period τ as $b_{i,\tau}$. Labor income y_τ is taxed at an income rate of λ and social tax rate λ_{SS} . Also, assume R is the gross real interest factor. The consumer starts with some positive bank balance in the first period of life, b_1 , which may partly come from a lump-sum accidental bequest from the deceased population each period. The household makes consumption and saving decisions subject to the following intertemporal budget constraint.

³²There is are bequest motive and preference-shifter along life cycle, but these features can be easily incorporated.

$$\begin{aligned}
a_{i,\tau} &= m_{i,\tau} - c_{i,\tau} \\
b_{i,\tau+1} &= a_{i,\tau}R \\
m_{i,\tau+1} &= b_{i,\tau+1} + (1 - \lambda)(1 - \lambda_{SS})y_{i,\tau+1} \\
a_{i,\tau} &\geq 0
\end{aligned} \tag{10}$$

In addition, I impose an external zero borrowing constraint. Without the external borrowing constraint, the agent will still self-imposed a lower bound for a_τ to avoid the extremely painful zero consumption next period in the case of the worst draw of income shocks.

5.1.2 Income process

Each agent receives stochastic labor income during working age from $\tau = 0$ to $\tau = T$ and receives social security benefit after retirement. The income processes in both sub-periods can be defined in a generic manner as described below. In particular, it is assumed to follow a slight variant of the standard permanent/transitory income process used in the literature³³ by allowing the possibility of persistent unemployment risks. Specifically, $y_{i,\tau}$ is a multiplication of idiosyncratic labor productivity $n_{i,\tau}$ and the economy-wide wage rate W . The former consists of one permanent component $p_{i,\tau}$ and one potentially persistent or transitory $\xi_{i,\tau}$. The aggregate wage is to be determined by the general equilibrium.³⁴

$$\begin{aligned}
y_{i,\tau} &= n_{i,\tau}W \\
n_{i,\tau} &= p_{i,\tau}\xi_{i,\tau}
\end{aligned} \tag{11}$$

³³Carroll et al. (2017), Kaplan and Violante (2018), etc.

³⁴In the presence of aggregate risk, we need to allow W being time-varying and this also means we need to be explicit about the difference between the calendar year and working age.

During the work, the permanent income component is subject to a mean-one white-noise shock ψ in each period and grows according to a deterministic life-cycle profile governed by G_τ , which usually follows a hump-shape according to existing estimates. (Gourinchas and Parker, 2002)

$$\begin{aligned} p_{i,\tau} &= G_\tau p_{i,\tau-1} \psi_{i,\tau} \\ \log(\psi_{i,\tau}) &\sim N\left(-\frac{\sigma_\psi^2}{2}, \sigma_\psi^2\right) \quad \forall \tau \leq T \end{aligned} \tag{12}$$

The persistent/transitory shock $\xi_{i,\tau}$ takes different values depending on the transitory or persistent state of unemployment following a Markov process.³⁵

$$\begin{aligned} \xi_{i,\tau} &= \begin{cases} \theta_{i,\tau} & \text{if } \nu_{i,\tau} = e \quad \& \quad \tau \leq T \\ \zeta & \text{if } \nu_{i,\tau} = u \quad \& \quad \tau \leq T \\ \mathbb{S} & \text{if } \tau > T \end{cases} \\ \log(\theta_{i,\tau}) &\sim N\left(-\frac{\sigma_\theta^2}{2}, \sigma_\theta^2\right) \end{aligned} \tag{13}$$

where ζ is the replacement ratio of the unemployment insurance and $\theta_{i,\tau}$ is the i.i.d. mean-one white noise shock to the transitory component of the income conditional on staying employed. Notice that the process above also embodies the income process after retirement after $\tau = T$. The agent receives social security with replacement ratio \mathbb{S} and proportional to her permanent income and aggregate wage rate. Therefore, the effective pension benefit received is $\mathbb{S}p_{i,\tau}W$. I assume that the permanent income component just follows the determinist path without additional stochastic shocks.

During work age of any individual i , the transition matrix between unemployment ($\nu_{i,\tau} = u$) and employment ($\nu_{i,\tau} = e$) is the following.

³⁵This formulation follows Krueger et al. (2016).

$$\pi(\nu_{\tau+1}|\nu_{\tau}) = \begin{bmatrix} \mathfrak{U} & 1 - \mathfrak{U} \\ 1 - E & E \end{bmatrix} \quad (14)$$

In general, this assumption implies some degree of the persistence of unemployment risks, but it conveniently nests the special case where the unemployment risk is purely transitory when $\mathfrak{U} = 1 - E$, meaning the probability of unemployment is not dependent on the current status.

Unemployment risks are idiosyncratic, hence by the law of large numbers, the fraction of the population being unemployed and employed at each age, denoted by $\Pi_{\tau}^{\mathfrak{U}}$ and Π_{τ}^E , respectively, are essentially deterministic and does not depend on age.

Notice that in the benchmark model laid out here, I assume the all parameters of income risks σ_{ψ} , σ_{θ} , \mathfrak{U} , and E to be age-invariant (equivalent to time-independent in this setting). By doing this, I avoid making explicit assumptions on the stochastic process of income risks. This is a common practice in the incomplete market macro literature since [Gourinchas and Parker \(2002\)](#) and [Cagetti \(2003\)](#). It is also not fundamentally different from assuming a deterministic age-specific risk profile, as in some variants of the models with the life-cycle component.³⁶ I allow for income risks to be stochastic/state-dependent in one of the extensions of the model discussed later.

5.1.3 Value function and consumption policy

The following value function characterizes the consumer's problem.

$$V_{\tau}(\nu_{i,\tau}, m_{i,\tau}, p_{i,\tau}) = \max_{\{c_{i,\tau}, a_{i,\tau}\}} u(c_{i,\tau}) + (1 - D)\beta \mathbb{E}_{\tau} [V_{\tau+1}(\nu_{i,\tau}, m_{i,\tau+1}, p_{i,\tau+1})] \quad (15)$$

where the three state variables for the agents are current employment status $\nu_{i,\tau}$, total cash in hand $m_{i,\tau}$ and permanent income $p_{i,\tau}$. $\nu_{i,\tau}$ drops from the state variables in the special case of

³⁶See [Carroll et al. \(2017\)](#) and other examples.

purely transitory unemployment shock ($\mathcal{U} = 1 - E$).³⁷

The solution to the problem above is the age-specific optimal consumption policies $c_\tau^*(u_{i,\tau}, m_{i,\tau}, p_{i,\tau})$ and saving policies $a_\tau^*(u_{i,\tau}, m_{i,\tau}, p_{i,\tau})$ both as a function of all state variables.

5.1.4 Technology

The economy has a standard CRS technology that turns the capital and supplied efficient units of labor into aggregate output.

$$Y = ZK^\alpha N^{1-\alpha} \quad (16)$$

The capital depreciates at a rate of δ each period.

The factors of input markets are fully competitive. Euler Theorem implies that the output either becomes labor income or capital income.

5.1.5 Demographics

For simplicity, we assume there is no population growth. With a deterministic life-cycle profile of survival probabilities, there exists a stable age distribution $\{\mu_\tau\}_{\mu=1,2,\dots,L}$ such that $\mu_{\tau+1} = (1 - D)\mu_\tau$ and $\sum_{\tau=1}^L \mu_\tau = 1$. The former condition reflects the probability of survivals at each age and the latter is a normalization that guarantees the fraction of all age groups sum up to 1.³⁸

³⁷Another trick used in the literature to reduce the number of state variables is to normalize the value function by permanent income level p_τ , so that it drops from the state variable. I also use endogenous grid method (EGM) by [Carroll \(2006\)](#). See Appendix for the detailed solution algorithm.

³⁸In a more general setting with a constant population growth rate n and age-specific survival probability $1 - D_\tau$, the condition becomes $\mu_{\tau+1} = \frac{(1-D_{\tau+1})}{1+n} \mu_\tau \quad \forall \tau = 1, 2, \dots, L$, as discussed in [Ríos-Rull \(1996\)](#) and [Huggett \(1996\)](#).

5.1.6 Government

Government runs a balanced budget in each period. Therefore, outlays from unemployment insurances are financed by the income tax that is levied on both labor income and unemployment benefit. Given a replacement ratio ζ , and the proportion of employed population, the corresponding tax rate λ can be easily pinned down based on the equation below.³⁹

$$\lambda [1 - \Pi^U + \zeta \Pi^U] = \zeta \Pi^U \quad (17)$$

Social security tax rate λ_{SS} is also determined in the model depending on the pension replacement ratio \mathbb{S} , the permanent income ratio and the relative population size of the retired and the working age, and the aggregate employment rate.

$$\lambda_{SS} \sum_{\tau=1}^T G_{\tau} (1 - \Pi^U) = \mathbb{S} \sum_{\tau=T+1}^L G_{\tau} \quad (18)$$

5.1.7 Stationary equilibrium

Denote $x = \{m, p, \nu\} \in X$ as the idiosyncratic state of individuals. At any point in time, agents in the economy differ in age τ and their idiosyncratic state x . The former is given by $\{\mu_{\tau}\}_{\mu=1,2,\dots,L}$. For the latter, using $\psi_{\tau}(B)$ to represent the fraction of agents at age τ whose individual states lie in B as a proportion of all age τ agents. (B is essentially a subset of Borel σ -algebra on state space X .) The distribution of age $\tau = 1$ agents depend on the initial condition of labor income outcomes and the size of accidental bequests, if any. For any other age $\tau = 2 \dots L$, the distribution $\phi_{\tau}(B)$ evolves as the following.

³⁹This convenient result crucially depends on the assumption that unemployment insurance benefit is paid proportionally to permanent income.

$$\psi_\tau(B) = \int_{x \in X} P(x, \tau - 1, B) d\psi_{\tau-1} \quad \text{for all } B \in B(X) \quad (19)$$

where $P(x, \tau - 1, B)$ is the probability for an agent to transit to B in the next period, conditional on the individual state x at age $\tau - 1$. It depends on the optimal consumption policy $c^*(x, \tau)$ at age τ and the exogenous transition probabilities of income shocks.

In the absence of the aggregate risk, I focus on the stationary equilibrium of the economy (StE) which consists of consumption and saving policies $c(x, \tau), a(x, \tau)$, constant production factor prices, including real interest rate R and the wage W , the initial wealth of newborn b_1 , unemployment benefit ζ , tax rate λ and the time-invariant distribution $(\psi_1, \psi_2, \dots, \psi_L)$ such that

1. Consumption and saving policies are optimal given the real interest rate R , wage W , the tax rate λ .

$$c(x, \tau) = c^*(x, \tau)$$

$$a(x, \tau) = a^*(x, \tau)$$

2. Distributions $(\psi_1, \psi_2, \dots, \psi_L)$ are consistent with optimizing behaviors of household, as described in Equation 19.

3. The factor markets are clearing.

$$\begin{aligned} \sum_{\tau} \mu_{\tau} \int_X a(x, \tau) d\psi_{\tau} &= K \\ \sum_{\tau=0}^{T-1} \mu_{\tau} \Pi_{\tau}^E &= N \end{aligned} \quad (20)$$

4. Firm optimization under competitive factor markets.

$$W = Z(1 - \alpha)(K/N)^{\alpha}$$

$$R = 1 + Z\alpha(K/N)^{\alpha-1} - \delta$$

5. Initial bank balance equal to accidental bequests.

$$b_1 = \sum_{\tau} \mu_{\tau} D \int_{x \in X} a(x, \tau) R d\psi_{\tau}$$

6. Government budget is balanced as described in Equation 17 and 18.

The economy may potentially arrive at different stationary equilibrium depending on the specific assumptions about objective or subjective models under the configurations.

5.2 Calibration

5.2.1 Calibrating income risks from the survey

The parameters to be estimated from the panel data of risk perceptions from *SCE* are the state-dependent risk profile $\tilde{\Gamma}_l = \{\tilde{\sigma}_{\psi}^l, \tilde{\sigma}_{\theta}^l, \tilde{\mathcal{U}}^l, \tilde{E}^l\}$, $\tilde{\Gamma}_h = \{\tilde{\sigma}_{\psi}^h, \tilde{\sigma}_{\theta}^h, \tilde{\mathcal{U}}^h, \tilde{E}^h\}$ and Ω , the transition matrix between the two states.

Denote the reported risk perception of the individual i at time t in the survey by $\tilde{\Gamma}_{i,t}^s$. It consists of the underlying risk perceptions relevant to individual decisions, or the model counterpart $\tilde{\Gamma}_{i,t}$, and an individual-specific, time-specific and an *i.i.d* shock to the survey responses, respectively. The realization of $\tilde{\Gamma}_{i,t}$ depends on a hidden state $J_{i,t}$ which is non-observable to economists working with the survey data. It takes value of 1 if the individual i is at a high-risk-perception state $\tilde{\Gamma}_{i,t} = \tilde{\Gamma}_h$ and zero if at low-risk-perceptions $\tilde{\Gamma}_{i,t} = \tilde{\Gamma}_l$. The *i.i.d* shock $\epsilon_{i,t}$ is assumed to follow a mean-zero normal distribution with variance σ_{ϵ}^2 .

$$\underbrace{\tilde{\Gamma}_{i,t}^s}_{\text{reported PR}} = \underbrace{\tilde{\Gamma}_l + \mathbb{1}(\underbrace{J_{i,t}}_{\tilde{\Gamma}_{i,t}} = 1)}_{\text{Hidden state}} (\tilde{\Gamma}_h - \tilde{\Gamma}_l) + \xi_t + \eta_i + \epsilon_{i,t}$$

$$\text{Prob}(J_{i,t+1}|J_{i,t}) = \Omega$$

Notice that the individuals do not separately report their perceived risks for the permanent and transitory shocks, but instead the overall expected income volatility. Therefore, I make an auxiliary assumption that the agent adopts a constant ratio of decomposition between permanent and transitory risks, $\kappa = \frac{\tilde{\sigma}_{i,t,\psi}}{\tilde{\sigma}_{i,t,\theta}}$, the value of κ is externally estimated from the realized income data.

In addition, since the surveyed risk perceptions is at the monthly frequency, I estimate the underlying risk parameters for monthly shocks. ⁴⁰

For each individual i , we observe at most 12 observations of their perceived income volatility of the earning growth next year $\tilde{var}_{i,t}$ from t to $t + 12$ and their job-separation and job-finding expectations, respectively. The panel structure allows the individual fixed effect η_i and time-fixed effect ξ_t to be easily identified.

Then the parameters can be estimated with a modified 2-regime Markov switching model a la [Hamilton \(1989\)](#) using the maximum-log-likelihood (MLE). (See the detailed implementation in Appendix A.7). Table ?? reports the baseline estimates of the parameters associated with the 2-state Markov model of subjective perceptions. All parameters are converted from monthly into yearly counterparts to be consistent with the model frequency.

The estimates of subjective profile confirms the key finding we have detailed in the previous section. The estimated staying probabilities at low and high risk perceptions, q and p , are around 0.9, indicating a high degree of persistence in individual risk perceptions. Given these estimated transition probabilities, earning risk perceptions are on average lower than the objective level assumed

⁴⁰ $\tilde{var}_{i,t} = (12\tilde{\sigma}_{i,t,\psi}^2 + 1/12\tilde{\sigma}_{i,t,\theta}^2) \exp^{\xi_t} \exp^{\eta_i} \exp^{\epsilon_{i,t}} \rightarrow \log \tilde{var}_{i,t} = \log(12\tilde{\sigma}_{i,t,\psi}^2 + 1/12\tilde{\sigma}_{i,t,\theta}^2) + \xi_t + \eta_i + \epsilon_{i,t} \rightarrow \log(\tilde{var}_{i,t}) = \log[(12 + \frac{1}{12\kappa^2})\tilde{\sigma}_{i,t,\psi}^2] + \xi_t + \eta_i + \epsilon_{i,t}$.

in the literature.

[TABLE ?? HERE]

5.2.2 Other parameters

Life-cycle The model is set at yearly frequency. The working age spans from 25 years old to 65 years old ($T = 40$) and the agent dies with certainty at age of 85 ($L = 60$). The constant death probability before the terminal age is set to be $D = 0.625\%$.

As to the deterministic permanent income profile over the life-cycle, G_τ , I draw on an age polynomial regressions of the earning growth from SIPP for workers aged between 25-65, controlling for other observable demographic variables such as education, gender, occupation, and time fixed effects, etc. This produces very similar estimation results to that in [Gourinchas and Parker \(2002\)](#), [Cagetti \(2003\)](#) and [Kaplan and Violante \(2014\)](#). The estimated income profile is plotted in Appendix [A.3](#). For the retirement phase, I assume a one-time drop of 20% in permanent income in the age of 66, i.e. $G_{41} = 0.8$, and then the permanent income stays flat till death. This produces an average expected growth rate of permanent income over the entire life-cycle exactly equal to one. This serves as a normalization. Note that although alternative assumptions, such as a more smooth decline of income after retirement, do change the wealth distribution across generations among the retired, they do not change the consumption/saving decisions as such a profile is entirely deterministic.

Initial conditions Assumptions about cross-sectional distribution of the initial permanent productivity and liquid asset holdings matter for the subsequent wealthy inequality. I set the standard deviation of the log-normally distributed initial permanent individual productivity $p_{i,\tau}$ to be 0.6 to match the earning heterogeneity in “usual income” (approximated permanent income) at age 25 from the SCF. Initial liquid assets holdings at $\tau = 0$ is assumed to have a cross-sectional standard-deviation of 0.50.

Income risks Given the critical importance of the income risks assumption in my model, In

addition to my estimates from SIPP, as reported in Table A.2, I thoroughly survey the parameters used in the existing incomplete market macro literature, as summarized in Table A.3 in the Appendix. For comparison, I convert all risks into the annual frequency (although the model is set quarterly). Whenever group-specific risks are assumed, i.e. depending on the education and age, I summarize it as a range. Also, for those models which assume a persistent instead of permanent income risk component, I treat their assumed size of the persistent risks as a lower bound for the permanent risk. (One can think of the permanent income shock as a limiting case of AR(1) shock, with the persistence parameter infinitely close to 1. The effective income risks increase with the persistence of the shock.) For models with income risks dependent on aggregate business cycles a la [Krusell and Smith \(1998\)](#), I compute the steady-state size of idiosyncratic risks using the transition probabilities of the aggregate economy used in the paper.

Despite the disagreements in these estimates or model inputs, the earning risks used in these models are constantly larger than those perceived reported in the survey. Meanwhile, the perceived risk of unemployment is higher in the survey than in these models and the perceived employment probability is lower in the survey than in these models. I use the median values of each parameter in the literature as the objective income risks profile Γ . In particular, in our baseline calculation, I set $\sigma_\psi = 0.15$ and $\sigma_\theta = 0.10$. The yearly probability of staying on unemployment is $\mathfrak{U} = 0.18$ and that of staying employed $E = 0.96$, as used in [Krueger et al. \(2016\)](#).

Technology The annual depreciation rate is set to be $\delta = 2.5\%$. The capital share takes a standard value of $\alpha = 0.36$, for the U.S. economy. Without aggregate shocks, Z is simply a normalizer. Therefore, I set its value such that the aggregate wage rate W is equal to one under a capital/output ratio $K/Y = 3$ at the steady-state level of employment in the model.

Government policies Unemployment insurance replacement ratio is set to be $\mu = 0.15$, as the same as [Krueger et al. \(2016\)](#). The pension income relative to the permanent income is assumed to be $\mathfrak{S} = 60\%$. This, plus the 20% drop in permanent income, gives an effective deterministic income

drop of 48% from the working-age to retirement, which corresponds to an empirical replacement ratio estimated for the U.S. economy. The corresponding tax rates financing the unemployment insurance and social security is determined in the equilibrium within the model.

Preference The discount factor is set to be $\beta = 0.98$, the average value estimated in the models with heterogeneous time preferences such as [Carroll et al. \(2017\)](#), [Krueger et al. \(2016\)](#). The coefficient of relative risk aversion $\rho = 2.0$, which is common in this literature.

Table 6 summarizes the parameters used in this calibration.

[TABLE 6 HERE]

6 Model implications (preliminary)

6.1 Baseline model results

I first examine the wealthy inequality generated from the benchmark objective model with the income risks calibrated following the existing literature such as [Krueger et al. \(2016\)](#); [Carroll et al. \(2017\)](#). In particular, I use the standard parameterization on permanent and transitory risks at the annual frequency being $\sigma_\psi = 0.10$ and $\sigma_\theta = 0.15$, and the unemployment risks $U2U = 0.18$ and $E2E = 0.96$. The upper panel of Figure 8 reproduces the well-known result⁴¹ that a standard incomplete market model, without additional heterogeneity such as that in time discount rates, imply less (either in partial equilibrium or general equilibrium) wealth inequality than that seen in the data.⁴²

[FIGURE 8 HERE]

The life-cycle profile of wealth is another dimension of the model implications that can be compared to its counterparts seen in the data. The bottom panel of Figure 8 plots the hump-

⁴¹For a thorough survey on this topic, see [Guisen \(2011\)](#), [De Nardi \(2015\)](#), and [Kaplan and Violante \(2018\)](#).

⁴²I use 2016 vintage of the SCF.

shaped average wealth over the life cycle implied by the model against the median net wealth between the age of 25 to 85 from SCF. There are two divergences between the model and the data. First, compared to the data, the model implies a more rapid build-up of buffer-stock wealth before the middle age. This is so because of the precautionary saving motives in the presence of income risks of various kinds in this model.

The second divergence has to do with the high wealth of the old seen in the data, in comparison with the sharp decline in wealth toward zero in the model, as the latter dictates it is optimal to consume all wealth in the end of the life. Such a divergent pattern is usually accounted for by specifically taking into account the bequest motives in the literature.⁴³ As the focus of this paper is on labor income risks of the work-age, I choose not to model such a mechanism.

The difference between the partial equilibrium and general equilibrium in terms of wealth inequality is rather small. Instead, allowing for the endogenous determination of real interest rate in the asset markets clearing as well as the resource constraint imposed by government budget balancing induces a steeper build-up of the buffer stock savings by the young. This is partly due to a higher real return to savings is higher in GE than PE.

6.2 Model results with survey-implied risks

I incorporate the two salient facts as revealed in perceived income risks and explore their implications, respectively. First, an average lower income risk, possibly due to superior information by the agents. Call it the *LPR* (lower perceived risks) model. Second, the heterogeneity in perceived risks. Call it a *HPR* (heterogeneous perceived risks) model. I examine their implications separately compared to the benchmark model.

⁴³De Nardi (2004).

6.2.1 LPR

For LPR calibration, I keep everything the same as baseline calibration above, except for setting the permanent and transitory risks to be smaller based on an upper bound of total perceived risk of 0.04, i.e. $\sigma_\psi = 0.03$ and $\sigma_\theta = 0.02$.

Figure 9 confirms the two straightforward implications of a smaller size of risks. First, a lower PR induce less precautionary saving motive and reduces buffer-stock savings of all working agents, as indicated by a lower line of the wealth before retirement than the benchmark model. This also means that there will be a leftward shift in wealth distribution of the entire population, therefore increasing the proportion of the agents that are close to zero wealth.

Second, if it does reflect an objectively lower size of the idiosyncratic risks than the usual estimation in the benchmark model, a lower PR unambiguously leads to *less* wealth inequality than in the benchmark model, as shown in the Lorenz curve in Figure 9. This means that the simple LPR extension cannot help explain an additional degree of wealth inequality as seen in the data.

[FIGURE 9 HERE]

A lower wealth inequality could be either due to an objectively higher income inequality, which is exogenous to the agents in the model, or partly endogenous responses via consumption/saving decisions. In order to separate the two channels, I solve the model above by separately parameterizing the risks as perceived by the agents (subjective) and the level of risks that determine the realized income distributions (objective). In particular, I set the perceived risks that affect agents' consumption/saving decisions to be the low values as reported in the survey, like in LPR, while keeping the objective risk parameters the same as the baseline model. Call this model SLPR (subjective LPR model).

SLPR can be thought of as a thought experiment I invoked to disentangle the effects via choices and realizations. It can also be a model which accommodates more general possibilities of discrepan-

cies between perceptions and the objective model counterpart, which deviates from the assumption of rational expectations. Appendix A.8 discusses in greater detail on how the consumer problem, the dynamics and stationary distribution of the economy change within a model allowing for subjective risk perceptions and the objective risks to be different.

The difference between SLPR and the baseline model reflects the changes solely attributable to responses in saving behaviors under a lower risk perceptions, and the difference between SLPR and LPR corresponds to income inequality. As shown in Figure 9, the income inequality channel contributes to approximately 80% of the decreases in inequality and the rest is attributable to saving behavior changes. In this class of models, both permanent and transitory income shocks are at most partially insured via self-insurance. This explains why saving behaviors have a smaller effect in driving the distribution than income inequality.

6.2.2 HPR

Compared to the baseline model, HPR model assumes heterogeneous idiosyncratic risks due to stochastic transitions between different levels of risks, as seen from the survey data. This is to model the state-dependence in risk perceptions of individuals, depending on the information set available to agents unobserved by economists. It is also consistent with the observation that individuals in SCE does change their reported risk perceptions from time to time. These changes are possibly due to purely idiosyncratic reasons which economists have no way of knowing, even controlling time individual effects. Such an assumption of stochastic/state-dependent risk assumes that the heterogeneity in risks is not due to ax-ante time-invariant heterogeneity in the level of risks facing everyone. In one of the latter extension, I also allow for ex ante/constant heterogeneity in risks, as estimated from individual fixed effects in PR.

Stochastic risks results in more wealth inequality compared to the baseline model. This is so simply because different risks induce different precautionary saving motives, therefore, different

buffer-stock savings. In addition, the transitions across states with different risks at individual levels also generate more income inequality, separate from the saving behaviors.

As in LPR, I also consider an intermediate model experiment, allowing for the divergence between the perceived risks and that objective risks that drive income inequality. In particular, I assume that in SHPR, agents choose saving/consumption believing the stochastic nature of income risks while the realized income inequality is determined by homogenous degree of income risks. Call it SHPR (subjective HPR) model. The difference between HPR and SHPR speaks to the response in saving behaviors in response to heterogeneity in risks due to the state-dependence. The difference between baseline and SHPR corresponds to the additional inequality that comes from stochastic income risks. Alternatively, this model can be thought of as one where agents do not recognize the heterogeneity in income risks.

6.3 Extensions

6.3.1 Ex-ante heterogeneity in risks

HPR model incorporates heterogeneity in risks via an assumption of stochastic risks at the individual level. But as shown in Section 4.1 and the estimation from a survey in Section 5.2.1, a large degree of heterogeneity in PR is attributable to individual fixed effects, which might reflect the true ex-ante heterogeneity in income risks facing different individuals. Since there are no individual realizations of risks, traditional risk estimation based on cross-sectional income cannot recover the heterogeneity in risks unless grouping individuals by certain group characteristics. Survey-reported PRs, in contrast, allow for direct identification of the cross-sectional heterogeneity across individuals. The advantage of this approach is that modelers can be agnostic about the true reasons for the heterogeneity, and it avoids making arbitrary model specifications as to the heterogeneity in income risks.

Ex-ante heterogeneity in risks in addition to the stochastic transitions unambiguously con-

tributes to more wealth inequality. Again, the effect consists of changes due to saving behaviors via heterogeneous precautionary saving motives, and the changes due to various degrees of income risks.

6.3.2 Unemployment risks

The bulk of the results so far has narrowly focused on only calibrating perceived wage risks using the survey. A natural extension is to incorporate the survey-informed unemployment risks, i.e, the heterogeneity and state-dependence in perceived job loss and job finding probabilities, in the same model. A standard incomplete market model with unemployment spells typically parameterizes the model with one homogenous pair of $U2U$ and $E2E$ probabilities. But this may mask the unobserved heterogeneity among agents and their true perceived unemployment risks given the information they have about their own idiosyncratic circumstances (Mueller and Spinnewijn (2021)).

As detailed in Section 5.2.1, I recover two unobserved states and their transition probabilities by jointly accounting for the changes in reported wage risk, separation and finding expectations. I also further recover the ex-ante differences in risk perceptions using the same approach above.

The resulting model embodies both ex-ante and ex-post heterogeneity in wage risks as well as unemployment risks. By the same token, I introduce the experiment model allowing agents to make saving decisions with such heterogeneity in both wage risks and unemployment risks to be perceived, but I keep the underlying income inequality to be driven by the homogenous degree of income inequality.

6.3.3 Job-switching wage loss and risks

Persistent unemployment spells not only cause income reductions due to the forgone wage otherwise to be earned in employment, but also induces persistent income loss even after reemployment to a new job. It is worth extending the model above to consider the latter channel explicitly. Table A.4

summarizes a large body of micro empirical literature that has found evidence for such an additional channel via which income risks matter for income and wealth inequality. Most of the summarized estimates focus on mass layoffs, but it also highlights the job-displacement costs following purely idiosyncratic unemployment spells followed by job-switching. Some recent job search and match models ⁴⁴ rationalizes such a persistent wage loss associated with unemployment spells with a job-ladder model with on-the-job search. Some other literature attributes the wage loss to employee-employer-specific human capital. And some papers emphasize the human capital decay specific to the individuals.

Regardless of the mechanism, I simply assume in the extended model that there is a one-time wage loss associated with each transition from unemployment to employment. I parameterize it using the median estimate in the literature of a 10% wage loss that permanently affects the wage earned at the same job for each employment spell.

A closely related yet separate effect associated with the unemployment-employment transition is the job-switching wage risk. Each time the worker is reemployed by a new job or new employer, there are possibly good or bad wage rates drawn stochastically specific to this new job. This is separate from the source of income risks modeled so far. [Low et al. \(2010\)](#) emphasizes the importance of separately identifying the conditional wage risk and the job-switching risk, using 1993 SIPP data. Their estimation suggests a standard deviation of the latter component of 20%, well above the upper bound of the wage risks assumed in all the configurations above.

Introducing these two channels to the baseline model would undoubtedly increase income inequality and induce additional precautionary saving motives. It will increase the model-generated wealth inequality.

⁴⁴Such as [Low et al. \(2010\)](#), [Lachowska et al. \(2020\)](#), etc.

7 Conclusion

Incomplete-market macroeconomic models that admit uninsured idiosyncratic income risks have become the new paradigm of macroeconomic analysis in the past decade.

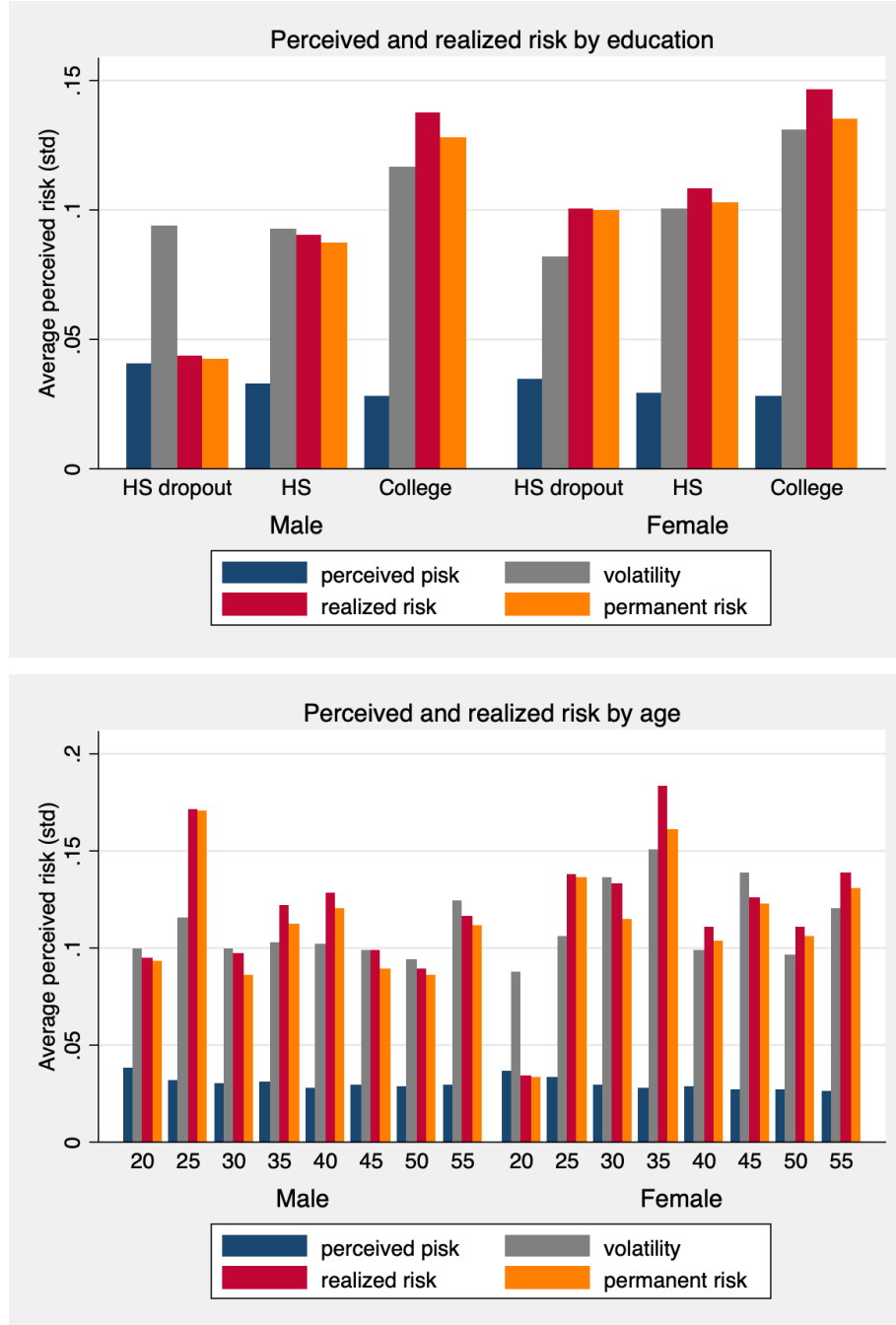
Utilizing the recently available large-scale survey data that elicits density forecasts of wage growth, I incorporate salient empirical patterns of income risk perceptions such as heterogeneity, extrapolation and state-dependence in these models. The survey evidence indicates the possible “unobserved heterogeneity” or the “superior information” problem documented in the literature, confirming an upward bias in the assumed size of income risks in these models compared to what people report in surveys. Incorporating the survey-implied heterogeneity and lower perceived risks helps partly explain the low liquid asset holdings of a large fraction of households, the presence of many hands-to-mouth agents and an additional fraction of wealthy inequality.

As an additional exploration, I also extend the benchmark model to allow for subjective perceived risks to be different from the objective ones that drive the realized dispersion of the shocks, possibly due to behavioral bias. With such an extension of the model, I can explicitly break down the aggregate effects of idiosyncratic income risks into two components: one via ex-ante choices, i.e. the saving behaviors according to the perceived risks, and the other via ex post outcomes, i.e. the realized income inequality.

This paper also presents a demonstration of the rich possibility of incorporating survey data reflecting real-time heterogeneity in expectations/perceptions in heterogeneous-agent models. In a world with increasingly rich survey data that directly measures expectations, economists are no longer forced to make stringent assumption of rational expectations. Directly using survey-implied heterogeneity helps match empirical patterns of the macroeconomy.

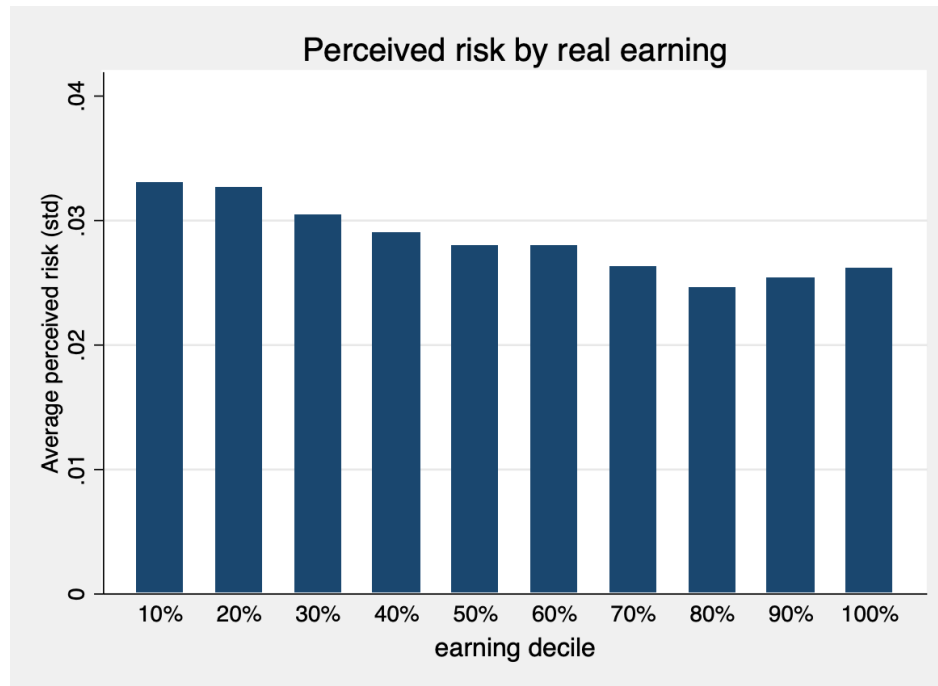
Tables and Figures

Figure 1: Perceived Risks, Wage Volatility and Estimated Wage Risks by Observable Factors



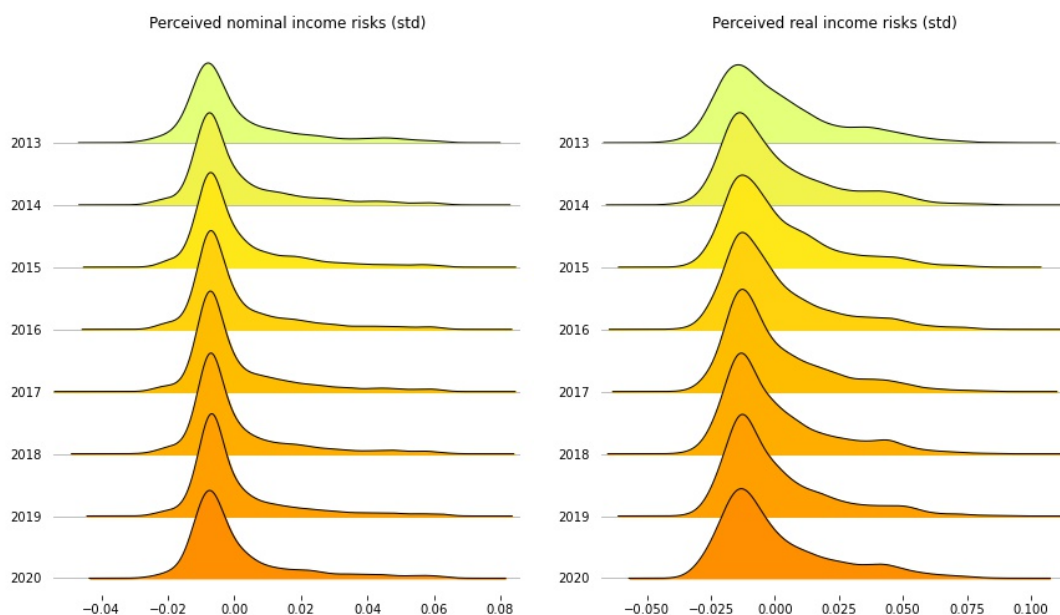
Note: this figure plots perceived risk (from SCE), average estimated wage volatility, approximated wage risk, and permanent risk (from SIPP) of each education-gender (upper panel) or age-gender (bottom panel) group. The volatility is approximated by the within-group cross-sectional standard deviation of log changes in unexplained wage residuals, as defined in Equation 4. The estimated risk is based on the process specified in Equation 6.

Figure 2: Perceived Wage Risks by Earning Decile

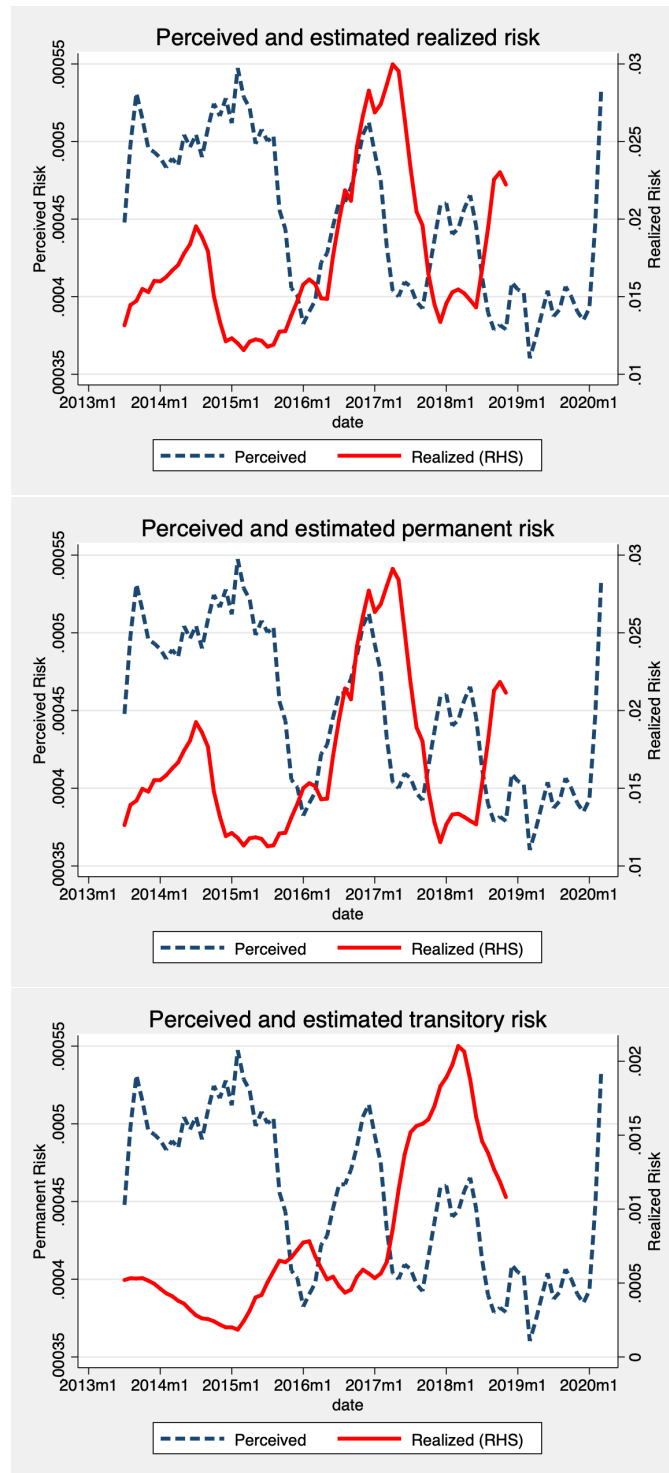


Note: this figure plots average perceived income risks by the decile of annual earning of the same individual.

Figure 3: Dispersion in Unexplained Perceived Income Risks

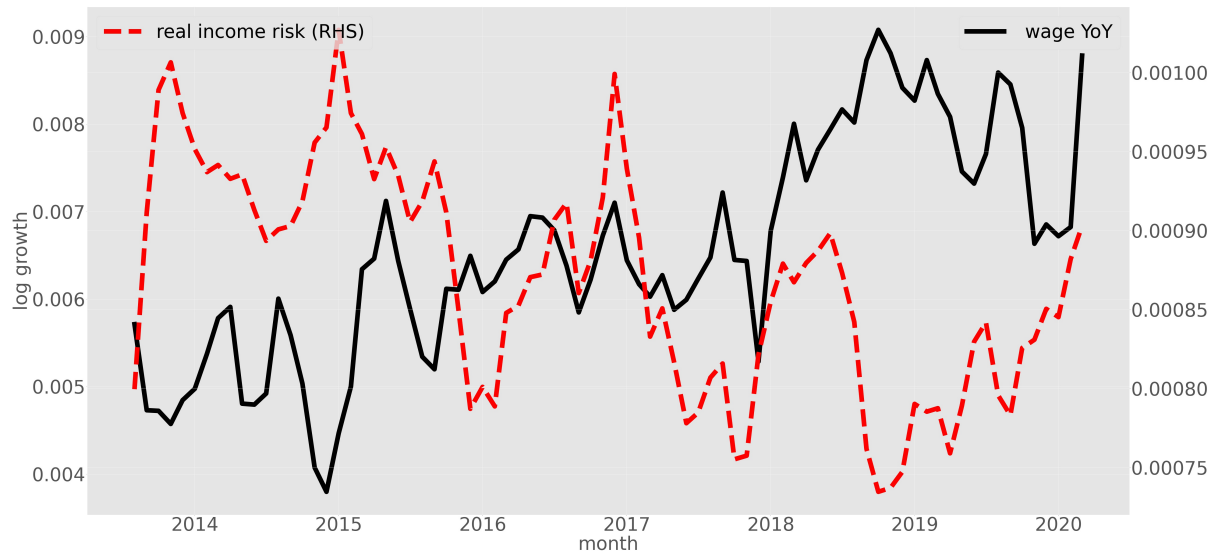


Note: this figure plots the distributions of residuals of the perceived standard deviation of 1-year-ahead earning growth in nominal (left) and real terms (right) after controlling age, age polynomial, gender, education, type of work arrangement, and time fixed effects. The real risk is the sum of the perceived risk of nominal income and inflation uncertainty.

Figure 4: Perceived and Realized Risks

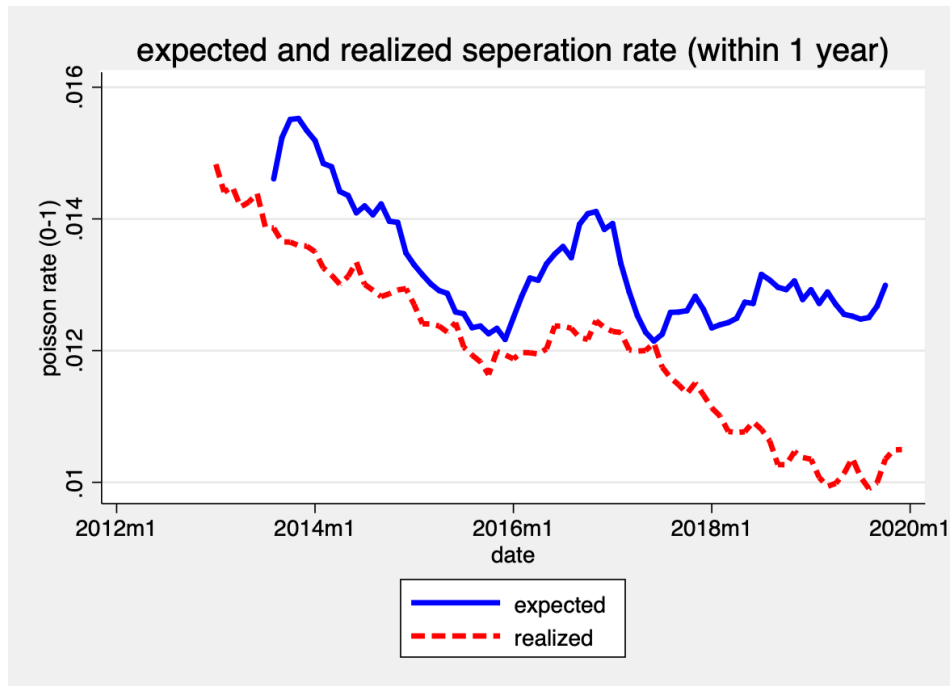
Note: this figure plots median 1-year-ahead perceived income risks in the whole SCE sample against the estimated realized risks, permanent, and transitory risks over the *same* period. Both series are regarding the real wage. The realized risks are first estimated monthly from SIPP and then aggregated into annual frequency. Specifically, the permanent risks are the sum of monthly permanent risks and the annual transitory risks are the simple average over the corresponding 12 months.

Figure 5: Recent Labor Market Conditions and Perceived Risks

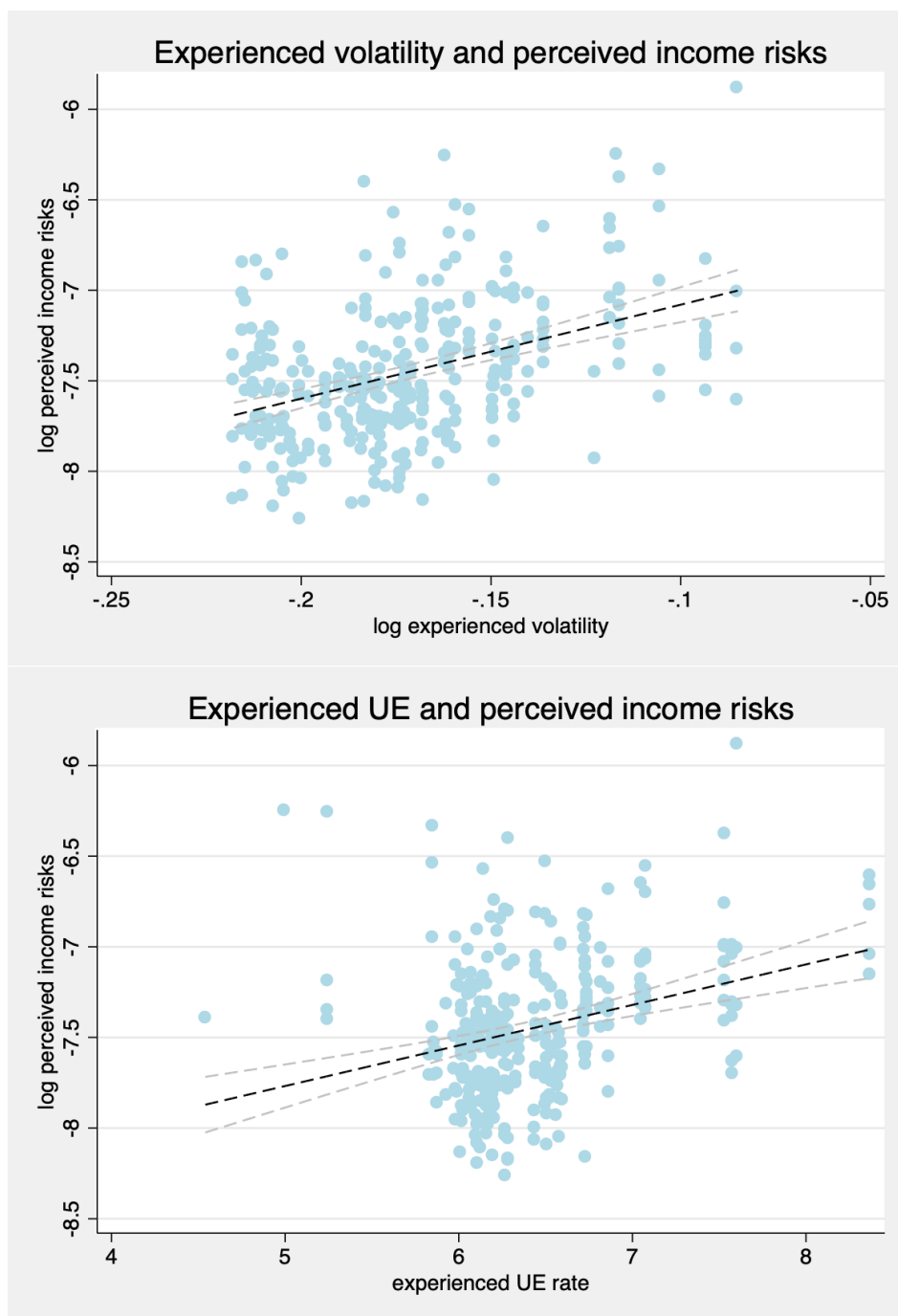


Note: recent labor market outcome is measured by hourly wage growth (YoY). The 3-month moving average is plotted for both series.

Figure 6: Expected and Realized Job-separation Rate

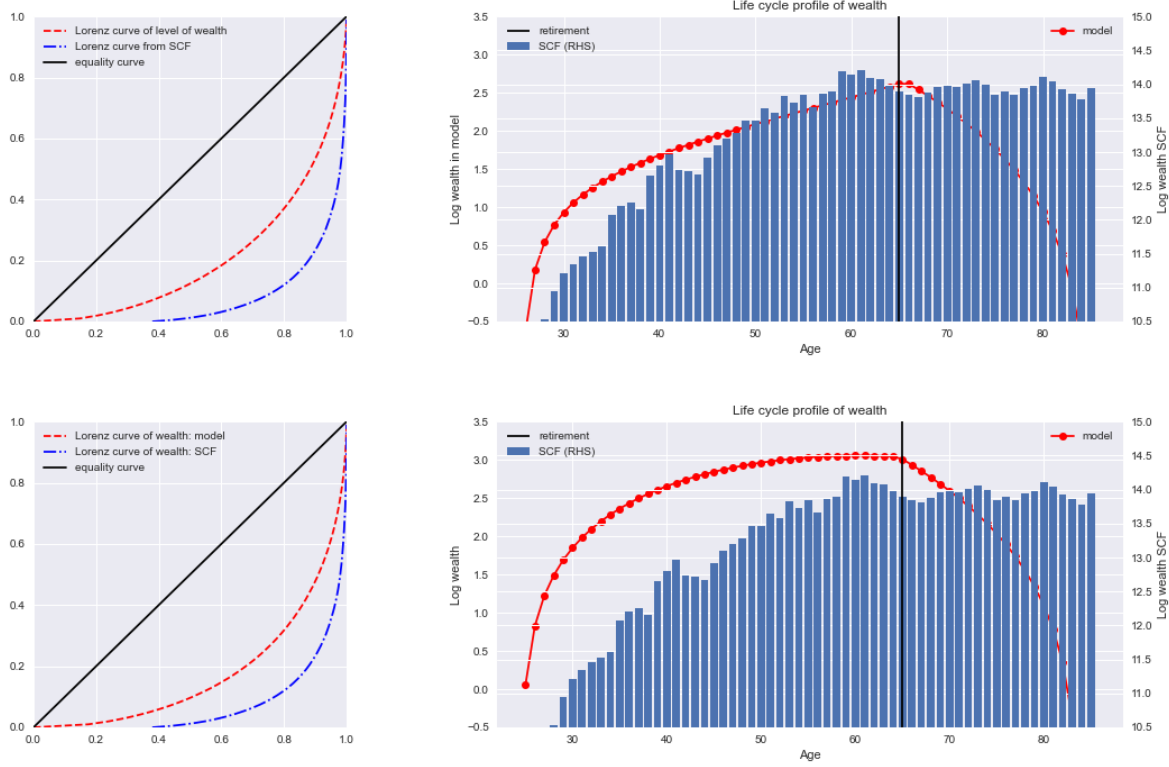


Note: realized job separation rate is computed from CPS. Both are expressed as Poisson arrival rates in continuous time with one month as the unit of time. The 5-month moving average expected rate is plotted.

Figure 7: Experience and Perceived Income Risk

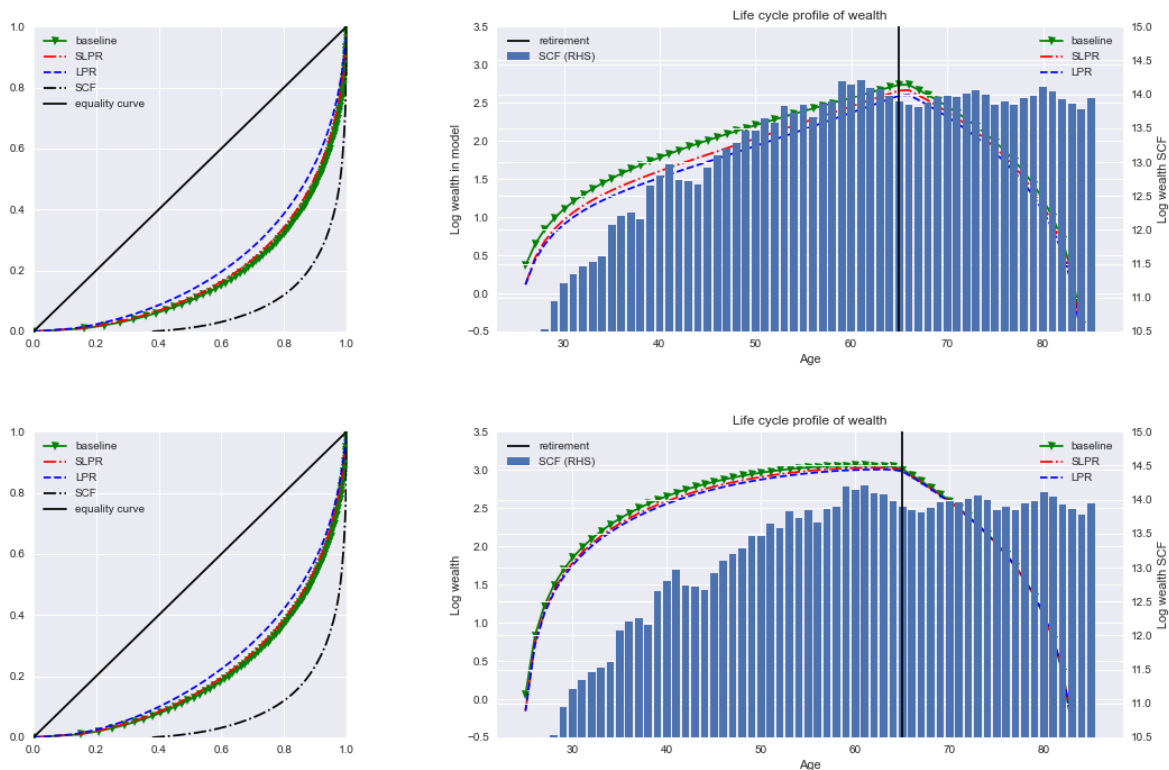
Note: the experienced income volatility is the cross-sectional variance of log change in income residuals estimated using a sub sample restricted to the lifetime of a particular group. For instance, the life experience of a 25-year old till 2015 spans from 1990-2015. The perceived income risk is the average across all individuals from the cohort in that year. Cohorts are time/year-of-birth specific and all cohort sized 30 or smaller are excluded.

Figure 8: Wealth inequality in Partial and General Equilibrium: baseline model



Note: the panel shows, under the baseline objective model, the Lorenz curve of households wealth (left) and the model-generated life-cycle profile of log average wealth compared to that seen in the 2016 vintage of Survey of Consumer Finance (SCF) (right) in the partial equilibrium (upper panel) and in general equilibrium (bottom panel).

Figure 9: Wealth inequality in Partial and General Equilibrium: under Different Model Assumptions



Note: the panel shows, under different assumptions, the Lorenz curve of households wealth (left) and the model-generated life-cycle profile of log average wealth compared to that seen in the 2016 vintage of Survey of Consumer Finance (SCF) (right) in the partial equilibrium (upper panel) and in general equilibrium (bottom panel).

Table 1: Current Labor Market Conditions and Perceived Income Risks

	mean:var	mean:iqr	mean:rvar	median:var	median:iqr	median:rvar
0	-0.28**	-0.42***	-0.48***	-0.16	-0.16	-0.53***
1	-0.44***	-0.54***	-0.51***	-0.02	-0.02	-0.53***
2	-0.39***	-0.44***	-0.43***	-0.05	0.0	-0.45***
3	-0.44***	-0.47***	-0.41***	-0.09	-0.06	-0.5***
4	-0.29**	-0.38***	-0.32***	-0.19	-0.14	-0.5***

*** $p < 0.001$, ** $p < 0.01$ and * $p < 0.05$.

This table reports correlation coefficients between different perceived income moments (inc for nominal and rinc for real) at time t and the quarterly growth rate in hourly earning at $t, t - 1, \dots, t - k$.

Table 2: Average Perceived Risks and Local Labor Market Conditions

	(1) log perceived risk	(2) log perceived risk	(3) log perceived iqr	(4) log perceived iqr
Wage Growth (Median)	-0.05*** (0.01)		-0.03*** (0.01)	
UE (Median)		0.04* (0.02)		0.04*** (0.01)
Observations	3589	3589	3596	3596
R-squared	0.021	0.019	0.025	0.027

*** p<0.001, ** p<0.01 and * p<0.05.

This table reports regression coefficient of the average perceived income risk of each state in different times on current labor market indicators, i.e. wage growth and unemployment rate. Monthly state wage series is from Local Area Unemployment Statistics (LAUS) of BLS. Quarterly state unemployment rate is from Quarterly Census of Employment and Wage (QCEW) of BLS.

Table 3: Extrapolation from Recent Experience

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
income shock squared	0.0225*** (0.00562)	0.0222*** (0.00570)	0.0217*** (0.00562)	0.0207*** (0.00564)	0.000773 (0.000743)	0.00205*** (0.000516)	0.000566 (0.000744)	0.00183*** (0.000515)	0.000614 (0.000745)	0.00184*** (0.000516)
recently unemployed				0.511* (0.260)	0.228*** (0.0330)	0.0895*** (0.0200)				
unemployed since m-8							0.161*** (0.0207)	0.0783*** (0.0121)		
unemployed since y-1									0.138*** (0.0193)	0.0701*** (0.0113)
Observations	3662	3662	3662	3662	3701	1871	3701	1871	3701	1871
R-squared	0.004	0.013	0.016	0.017	0.015	0.030	0.019	0.041	0.016	0.039

Standard errors are clustered by household. *** $p < 0.001$, ** $p < 0.01$ and * $p < 0.05$.

This table reports regression of perceived risks and perceived unemployment risks on recent experiences of income volatility and the dummy indicating if the individual has recently experienced an unemployment.

Table 4: Perceived Income Risks, Experienced Volatility and Individual Characteristics

	incvar I	incvar II	incvar III	incvar IIII	incvar IIIII	incvar IIIII
IdExpVol	4.58*** (0.33)	2.23*** (0.36)	2.69*** (0.39)	2.75*** (0.39)	2.95*** (0.38)	2.94*** (0.39)
AgExpVol	0.04 (0.04)	0.28*** (0.04)	0.34*** (0.05)	0.32*** (0.05)	0.18*** (0.05)	0.20*** (0.05)
AgExpUE	0.14*** (0.02)	0.08*** (0.02)	0.05** (0.02)	0.05* (0.02)	0.04* (0.02)	0.05** (0.02)
age		-0.02*** (0.00)	-0.02*** (0.00)	-0.02*** (0.00)	-0.02*** (0.00)	-0.02*** (0.00)
gender=male			-0.36*** (0.02)	-0.35*** (0.02)	-0.32*** (0.02)	-0.30*** (0.02)
nlit_gr=low nlit			0.09*** (0.02)	0.09*** (0.02)	0.10*** (0.02)	0.09*** (0.02)
parttime=yes					-0.01 (0.02)	-0.02 (0.02)
selfemp=yes					1.25*** (0.03)	-0.00*** (0.00)
UEprobAgg						0.02*** (0.00)
UEprobInd						0.02*** (0.00)
HHinc_gr=low income					0.16*** (0.02)	0.16*** (0.02)
educ_gr=high school				-0.10*** (0.02)	-0.13*** (0.02)	-0.09*** (0.02)
educ_gr=hs dropout				0.08 (0.11)	0.11 (0.11)	0.29*** (0.11)
N	41422	41422	34833	34833	33480	29687
R2	0.01	0.02	0.04	0.04	0.11	0.06

Standard errors are clustered by household. *** p<0.001, ** p<0.01 and * p<0.05.

This table reports results associated a regression of logged perceived income risks (incvar) on logged idiosyncratic(IdExpVol), aggregate experienced volatility(AgExpVol), experienced unemployment rate (AgExpUE), and a list of household specific variables such as age, income, education, gender, job type and other economic expectations.

Table 5: Perceived Income Risks and Household Spending Plan

	(1)	(2)	(3)	(4)	(5)	(6)
perceived earning risk	8.394*** (1.175)	8.399*** (1.176)	3.642*** (0.533)	3.243*** (0.537)		
perceived earning risk (nominal)					3.656*** (0.990)	
perceived ue risk						0.353*** (0.0553)
R-squared	0.0010	0.00282	0.928	0.928	0.941	0.633
Sample Size	53178	53178	53178	53178	54584	6269
Time FE	No	Yes	No	Yes	Yes	No
Individual FE	Yes	No	Yes	Yes	Yes	Yes

Standard errors are clustered by household. *** $p < 0.001$, ** $p < 0.01$ and * $p < 0.05$.

This table reports regression results of expected spending growth on perceived income risks (incvar for nominal, rincvar for real).

Table 6: Model parameters

block	parameter name	values	source
risk	σ_ψ	0.10	Median estimates from the literature
risk	σ_θ	0.15	Median estimates from the literature
risk	$U2U$	0.18	Median estimates from the literature
risk	$E2E$	0.96	Median estimates from the literature
initial condition	$\sigma_\psi^{\text{init}}$	0.629	Estimated for age 25 in the 2016 SCF
initial condition	bequest ratio	0	assumption
life cycle	T	40	standard assumption
life cycle	L	60	standard assumption
life cycle	$1 - D$	0.994	standard assumption
preference	ρ	1	standard assumption
preference	β	0.98	standard assumption
policy	\mathbb{S}	0.65	U.S. average
policy	λ	N/A	endogenously determined
policy	λ_{SS}	N/A	endogenously determined
policy	μ	0.15	U.S. average
production	W	1	target values in steady state
production	K2Y ratio	3	target values in steady state
production	α	0.33	standard assumption
production	δ	0.025	standard assumption

This table reports parameters used in the benchmark objective model. All parameters, whenever relevant, are at the annual frequency.

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A Online Appendix

A.1 Income risk decomposition using SIPP data

A.1.1 Sample selection

To estimate the risks to the labor earnings conditional on working for the same hours and staying in the same job, I restrict the universe of the SIPP sample according to this definition for the worker's primary job (JB1). The specific filtering criteria is listed as below, and it is approximately identical to that in [Low et al. \(2010\)](#) for computing the wage rate of the same job using 1993 panel of SIPP.

- Time: January 2013-December 2020
- Age: 20 - 60
- Work-arrangement: employed by someone else (excluding self-employment and other work-arrangement): $EJB1_JBORSE == 1$.
- Employer: staying with the same employer for a tenure longer than 4 months: the same $EJB1_JOBID$ for 4 or more consecutive months.
- Wage: total monthly earning from the primary job divided by the average number of hours worked in the same job, $wage = TJB1_MSUM / TJB1_MWKHS$.
- Outliers: drop observations with wage rate lower than 0.1 or greater than 2.5 times of the individual's average wage.
- No days off from work without pay: $EJB1_AWOP1 = 2$.
- Continued job spell since December of the last year: $RJB1_CFLG = 1$.
- Drop imputed values: $EINTTYPE == 1$ or 2 .

- Drop government/agriculture jobs: drop if TJB1_IND \geq 9400.

The selected sample has summary statistics as reported in Table A.1 below.

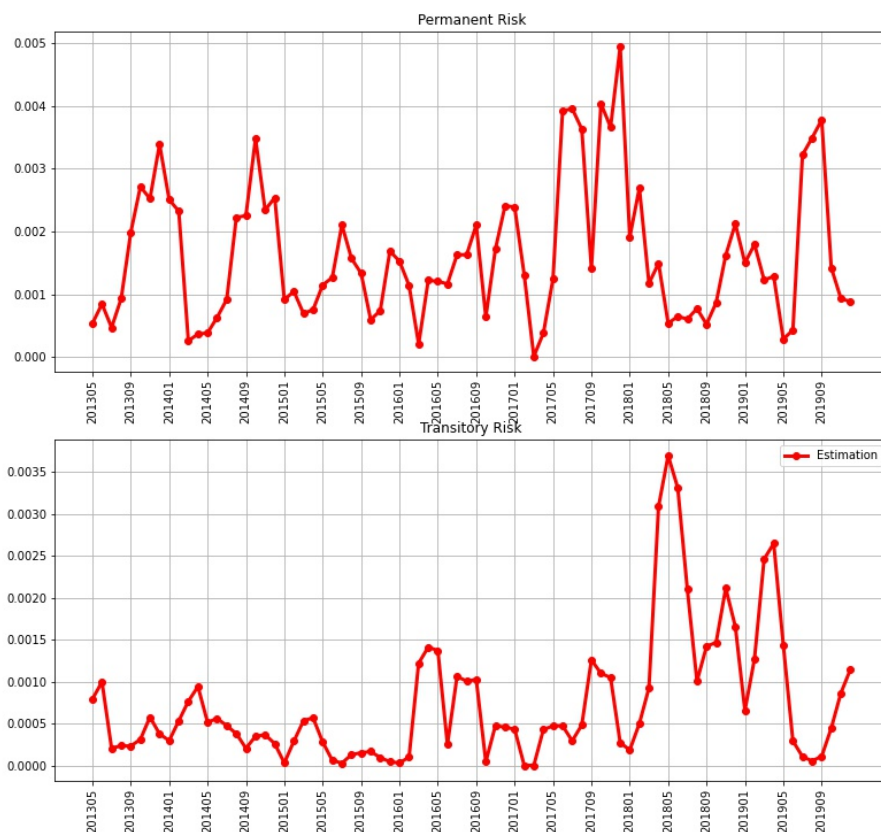
Table A.1: Summary statistics of SIPP sample

	Obs	Volatility
Year		
2013 (17%)	9,815	0.06
2014 (20%)	12,672	0.11
2015 (15%)	9,543	0.1
2016 (9%)	6,128	0.11
2017 (13%)	7,533	0.07
2018 (15%)	9,378	0.13
2019 (8%)	5,507	0.12
Education		
HS dropout (22%)	13,846	0.09
HS graduate (46%)	28,385	0.1
College/above (30%)	18,345	0.12
Gender		
male (55%)	33,842	0.1
female (44%)	26,734	0.11
Full sample (100%)	60,576	0.1

A.2 Income risk decomposition under alternative assumptions

A.2.1 Baseline estimation

Figure A.1: Monthly permanent and transitory income risks



Note: this figure plots the 3-month moving average of the estimated monthly permanent and transitory risks (in variance) using the SIPP panel data on wage between 2013m3-2019m12.

A.2.2 Infrequent arrival of the transitory shocks

A.3 Results with the first moment (the expected and realized wage growth)

Although the main focus of this paper is on income/wage risks, specifically the second moment of wage growth, it is natural to ask if the expected wage growth revealed in SCE aligns with what is realized as seen in SIPP. It is not surprising that both expected and the realized average wage growth rate conditional on education and gender decline over the life-cycle, as shown in the downward fitted

Table A.2: Perceived risk, realized volatility and approximated risks of each group

	PR(mean)	PR(median)	Volatility	RealizedRisk	PRisk	TRisk
gender						
1 (50%)	0.03	0.022	0.105	0.115	0.109	0.0238
2 (49%)	0.028	0.022	0.118	0.131	0.122	0.0322
education group						
HS dropout (0%)	0.036	0.022	0.088	0.071	0.07	0.0063
HS graduate (42%)	0.03	0.022	0.096	0.098	0.094	0.0176
College/above (56%)	0.028	0.021	0.124	0.142	0.132	0.0357
5-year age						
20 (2%)	0.037	0.031	0.094	0.069	0.068	0.0061
25 (12%)	0.032	0.027	0.111	0.157	0.156	0.0083
30 (12%)	0.03	0.023	0.116	0.112	0.098	0.0372
35 (13%)	0.029	0.021	0.125	0.149	0.134	0.0524
40 (13%)	0.028	0.02	0.1	0.119	0.111	0.0287
45 (14%)	0.028	0.02	0.119	0.113	0.106	0.0224
50 (15%)	0.027	0.019	0.095	0.1	0.096	0.0203
55 (15%)	0.027	0.018	0.122	0.128	0.121	0.0283
Full sample (100%)	0.029	0.021	0.112	0.123	0.115	0.0279

This table reports estimated realized annual volatility, risks of different components, and the expected income volatility of different groups. All are expressed in standard deviation units.

lines in Figure A.2. But In the sample of 2013-2019, expected wage growth seems to be persistently downward biased compared to its realization. This was not driven by the widely-documented fact of upward biased inflation expectation (See for instance, Wang (2021)), as even the same pattern shows up in the nominal wage growth.

A.4 Life-cycle wage profile

[INSERT FIGURE A.3 HERE]

A.5 Income risks in the existing literature

[INSERT TABLE A.3 HERE]

A.6 Persistent/permanent effect of job loss in the existing literature

[INSERT TABLE A.4 HERE]

A.7 Estimation of the 2-regime switching model of risk perceptions

For each individual i , we observe at most 12 observations of their perceived income volatility over the earning growth next year $\tilde{v}ar_{i,t}$ from $t = 1$ to $t = 12$. We assume the following relation between observed survey reported volatility and underlying perceived monthly permanent/transitory risks by the individual i at time t .

Table A.3: The size and nature of idiosyncratic income risks in the literature

	σ_ψ	σ_θ	\bar{U}	E	Earning Process	Unemployment	Source
Huggett (1996)	[0.21, +]	N/A	N/A	N/A	AR(1)	No	Page 480
Krusell and Smith (1998)	N/A	N/A	[0.04, 0.1]	[0.9, 0.96]	N/A	Persistent	Page 876
Cagetti (2003)	[0.264, 0.348]	N/A	N/A	N/A	Random +MA innovations	No	Page 344
Gourinchas and Parker (2002)	[0.108, 0.166]	[0.18, 0.256]	0.003	0.997	Permanent +transitory	Transitory	Table 1
Meghir and Pistaferri (2004)	0.173	[0.09, 0.21]	N/A	N/A	Permanent +MA	No	Table 3
Storesletten et al. (2004)	[0.094; +]	0.255	N/A	N/A	Persistent + transitory	No	Table 2
Blundell et al. (2008)	[0.1, +]	[0.169, +]	N/A	N/A	Permanent + MA	No	Table 6
Low et al. (2010)	[0.095, 0.106]	0.08	0.028	N/A	Permanent+transitory with job mobility	Persistent	Table 1
Kaplan and Violante (2014)	0.11	N/A	N/A	N/A	Persistent	No	Page 1220
Krueger et al. (2016)	[0.196, +]	0.23	[0.046, 0.095]	[0.894, 0.95]	Persistent +transitory	Persistent	Page 26
Carroll et al. (2017)	0.10	0.10	0.07	0.93	Permanent+transitory	Transitory	Table 2
Bayer et al. (2019)	0.148	0.693	N/A	N/A	Persistent time+MA	No	Table 1
My Estimates based on SIPP	0.10	0.016	N/A	N/A	Permanent +transitory	No	Table A.1

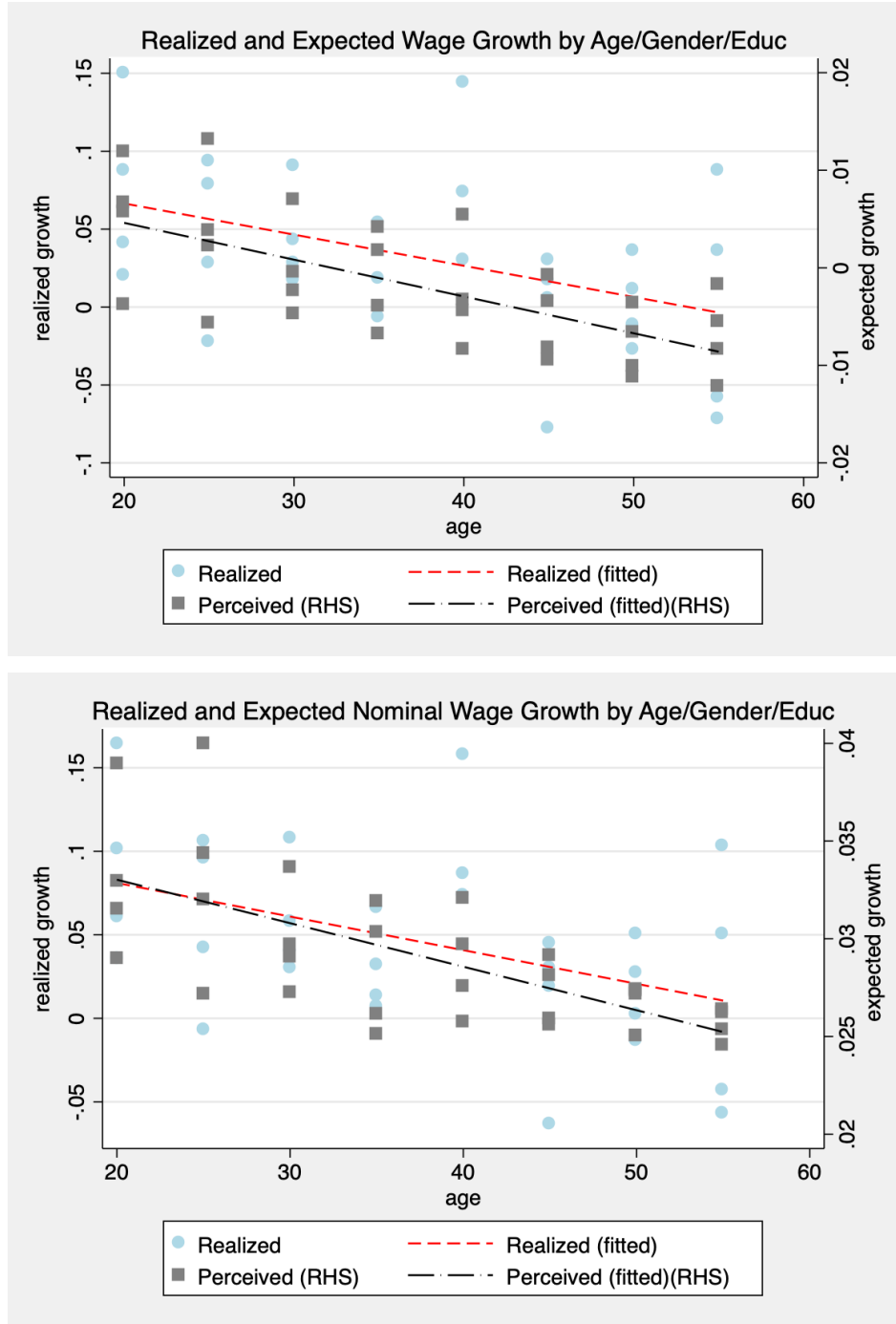
This table summarizes the conservative (lower bound) estimates/parameterization on idiosyncratic income risks at the annual frequency seen in the literature.

Table A.4: Summary of the literature on persistent/permanent effect from job displacement

	Loss (years after displacement)	Income risks	Period	Variables	Data/Sample
Ruhm (1991)	10%-13%(4)	NA	1969-1982	Earning	PSID
Jacobson et al. (1993)	25%(6)	NA	1974-1986	Earning	Administrative records of Pennsylvanian.
von Wachter et al. (2009)	21%-27%(20)	NA	1978-2004	Earning	Social security records, and firm-level employment data.
Couch and Placzek (2010)	13%-15% (6)	NA	1993-2004	Earning	Administrative data of Connecticut
Low et al. (2010)	6%-9%(1)	20%	Model	Wage rate	Model
Davis and Von Wachter (2011)	10%-20%(20)	NA	1980-2005	Earning	Social security records
Farber (2017)	6.2% (0)	Lower E2E rate	1984-2016	Wage rate	Displaced Workers Surveys (DWS)
Lachowska et al. (2020)	16%(5)	NA	2002-2014	Wage rate	Employment Security Department of Washington state.
Pytka and Gulyas (2021)	6% (11) (median)	NA	1984-2017	Earning	Austrian social security records

This table summarizes the empirical estimates on earning/wage loss from job-displacement. For Farber (2017), the loss is computed as the combined effect for those re-employed at a full-time and a part-time job. For Pytka and Gulyas (2021), I converted the accumulated loss into an annual percentage loss.

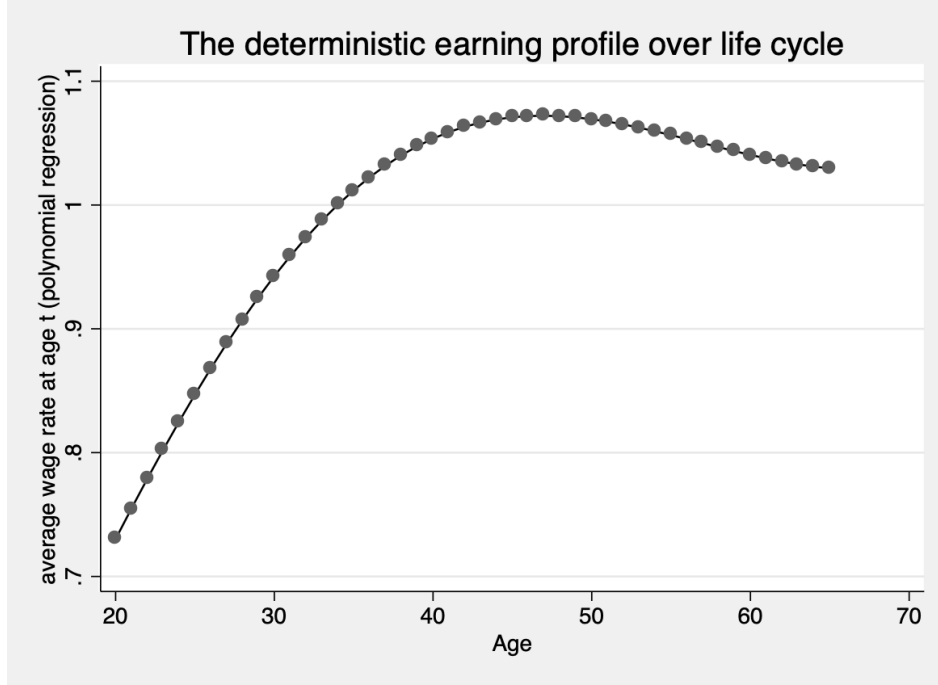
Figure A.2: Realized and Perceived Income Growth over the Life Cycle



Note: this figure plots the average real (upper panel) and nominal (bottom panel) realized and perceived wage growth of different age groups, conditional on the gender and education of the individual. The realized wage growth is approximated by the average log changes in real wage of each age/education/gender group based on SIPP.

$$\log \tilde{\text{var}}_{i,t} = \log(12\tilde{\sigma}_{i,t,\psi}^2 + 1/12\tilde{\sigma}_{i,t,\theta}^2) + \xi_t + \eta_i + \epsilon_{i,t}$$

Figure A.3: Estimated Deterministic Earning Profile over the Life-Cycle



Note: this figure plots the estimated average age profile of real earnings using SIPP between 2013m3-2019m12. It is based on fourth-order age polynomials regressions controlling time, education, occupations, gender, etc.

η_i and ξ_t are individual and time fixed effect, respectively. The i.i.d shock $\epsilon_{i,t}$ represents any factor that is not available to economists working with the survey, but affects i 's survey answers at the time t . We assume it is normally distributed.

Notice that $\tilde{\text{var}}_{i,t}$ alone is not enough to separately identify the perceived permanent and transitory risks. To proceed, I make the following auxiliary assumption: the agent adopts a constant ratio of decomposition between permanent and transitory risks, $\kappa = \frac{\tilde{\sigma}_{i,t,\psi}}{\tilde{\sigma}_{i,t,\theta}}$, the value of κ is externally estimated from the realized income data.

With the additional assumption, we can rewrite the equation above, utilizing the fact that risks for one year are the cumulative sum of monthly ones for permanent shocks and the average of monthly ones for transitory shocks.

$$\log(\tilde{\text{var}}_{i,t}) = \log\left[\left(12 + \frac{1}{12\kappa^2}\right)\tilde{\sigma}_{i,t,\psi}^2\right] + \xi_t + \eta_i + \epsilon_{i,t}$$

We *jointly* estimate a Markov-switching model on perceived volatility $\log(\hat{\text{v\ddot{a}r}}_{i,t})$, perceived probability on unemployment status $\tilde{\mathcal{U}}_{i,t}$, and perceived probability on employment status $\tilde{E}_{i,t}$. The vector model to be estimated can be represented as below.

$$\hat{\Gamma}_{i,t}^s = \tilde{\Gamma}^l + \mathbb{1}(J_{i,t} = 1)(\tilde{\Gamma}^h - \tilde{\Gamma}^l) + \tau_{i,t}$$

where $\hat{\Gamma}_{i,t}^s = [\log(\hat{\text{v\ddot{a}r}}_{i,t}), \hat{\mathcal{U}}_{i,t}, \hat{E}_{i,t}]'$ is a vector of sized three, consisting of properly transformed reported risk perceptions from the survey, excluding the time and individual fixed effects in a first step regression. $J_{i,t} = 1$ for high risk state and $= 0$ if at the low risk state. $\tau_{i,t}$ is a vector of three i.i.d. normally distributed shocks.

The estimation of 2-regime Markov switching models produces estimates of $\tilde{\Gamma}^l$, $\tilde{\Gamma}^h$, the staying probability q , and p , and the variance of $\tau_{i,t}$. Then the following relationship can be used to recover perceived permanent and transitory risks respectively.

$$\tilde{\Gamma}^l = [\log[(12 + \frac{1}{12\kappa^2})\tilde{\sigma}_\psi^{l2}], \tilde{\mathcal{U}}_l, \tilde{E}_l]'$$

$$\tilde{\Gamma}^h = [\log[(12 + \frac{1}{12\kappa^2})\tilde{\sigma}_\psi^{h2}], \tilde{\mathcal{U}}_h, \tilde{E}_h]'$$

Estimation sample I restrict the sample to SCE respondents who were surveyed for at least 6 consecutive months with non-empty reported perceived earning volatility, separation and job-finding expectations. This left with me 6457 individuals.

A.8 Model extension: subjective risk perceptions

In the benchmark model, I maintain the FIRE assumption that the agents perfectly know the underlying parameters of income risks $\Gamma = \{\sigma_\psi^2, \sigma_\theta^2, \mathcal{U}, E\}$ as assumed by the modelers and behave optimally accordingly.

But here, I relax the FIRE assumption by separately treating the “true” underlying risk parameters Γ and the risk perceptions held by the agents. The latter is denoted as $\tilde{\Gamma}_i$. This extension is meant to capture the four empirical patterns documented in the previous sections.

1. Underestimation of the earning risks (compared to what is assumed to be the truth in the model)
2. Heterogeneity in risk perceptions
3. Extrapolation of recent experiences
4. State-dependence of risk perceptions

The possible approaches of capturing these perceptual patterns are by no means unique. I adopt one simple framework that does not require explicitly specified mechanisms of perception formation but sufficient to reflect these the patterns revealed from the survey data.

Assume that each agent i in the economy cannot directly observe the underlying risk parameters Γ , but instead make his/her best choices based on a subjective risk perceptions $\tilde{\Gamma}_{i,\tau}$, which swing between two states: $\tilde{\Gamma}_l$ (low risk) and $\tilde{\Gamma}_h$ (high risk). The transition between the two states is governed by a Markov process with a transition matrix Ω . In the calibration of the model in latter sections, these subjective parameters can be estimated from survey data relied upon auxiliary assumptions.

Such an assumption automatically allows for heterogeneity in risk perceptions across different agents at any point of the time. All individuals are distributed between low and high risk-perception states. In one of the extensions, I also admit ex-ante heterogeneity, namely permanent differences in risk perceptions due to individual fixed effects.

The transition probability between low-risk and high-risk perception states can be also configured so that the average risk perception is lower than the true level of the risk. If we let the transition matrix Ω to be dependent on individual unemployment status $\nu_{i,\tau}$, or macroeconomic conditions,

we can also easily accommodate the possibility of experience extrapolation and state-dependence feature of risk perceptions.

Under the assumption of subjective perception, the subjective state of the risk perceptions $\tilde{\Gamma}$ becomes an additional state variable entering the Bellman equation of the consumer's problem, restated in below.

$$\tilde{V}_\tau(\tilde{\Gamma}_\tau, \nu_\tau, m_\tau, p_\tau) = \max_{\{c_\tau\}} u(c_\tau) + (1 - D)\beta \mathbb{E}_\tau [\tilde{V}_{\tau+1}(\tilde{\Gamma}_{\tau+1}, \nu_\tau, m_{\tau+1}, p_{\tau+1})] \quad (21)$$

Notice here that I assume that the agents recognize the transition between two subjective perception states and take it into account when making the best choices. This assumption guarantees time-consistency and provides additional discipline to the model assumption.

The consumer's solution to the problem above is the age-specific consumption policy $\tilde{c}_\tau^*(\tilde{\Gamma}_\tau, u_\tau, m_\tau, p_\tau)$ that is also a function of subject risk perception state $\tilde{\Gamma}$.

The distinction between objective and subject risk perception marks the single most important deviation of this paper from existing incomplete-market macro papers.⁴⁵ There is a long tradition of explicitly incorporating various kinds of heterogeneity in addition to uninsured idiosyncratic income shocks in these kinds of models to achieve better match with observed cross-sectional wealth inequality. One of the most notable assumptions used in the literature is the heterogeneity in time preferences (Krusell and Smith (1998), Carroll et al. (2017), Krueger et al. (2016)). My modeling approach shares the spirit with and are not mutually exclusive to these existing assumptions on preferential heterogeneity. But, to some extent, perceptual heterogeneity is more preferable as such patterns are directly observed from the survey data, as I show in the previous part of the paper.

A more fundamental justification for such a deviation from the full information rational ex-

⁴⁵For instance, Bewley (1976), Huggett (1993), Aiyagari (1994), Krusell and Smith (1998), Krueger et al. (2016), Carroll et al. (2017).

pectation assumption is that risk parameters Γ are barely observable objects to agents. This is so no matter if they are exogenously assumed by economists or endogenously determined in the equilibrium of the model.⁴⁶ Therefore, the conventional argument in favor of rational expectation assumption, namely equilibrium outcome drives the agents' perceptions to converge to the "truth", does not apply here.

Incorporating subjective risk perceptions also alters aggregate dynamics of the distributions as described in Equation 19, as restated below.

$$\tilde{\psi}_{\tau-1}(\tilde{B}) = \int_{\tilde{x} \in \tilde{X}} \tilde{P}(\tilde{x}, \tau - 1, \tilde{B}) d\tilde{\psi}_{\tau-1} \quad \text{for all } \tilde{B} \in \tilde{B}(X) \quad (22)$$

The state variable \tilde{x} includes subjective state $\tilde{\Gamma}$ in addition to those contained in x . The transition probabilities \tilde{P} now depend on the optimal consumption policies $c^*(\tilde{x})$ as a function of belief state $\tilde{\Gamma}$, as well as the exogenous transition probabilities of the true stochastic income process Γ .

Then the new StE under subjective risk perceptions can be defined accordingly.

A.9 Model Extension: costly adjustment in consumption

In this section, I extend the benchmark consumption model to incorporate an additional discrete choice of costly extensive adjustment. This is meant to introduce one additional mechanism which helps calibrate the model to match a high level of marginal propensity to consume (MPC) seen in the empirical estimates using natural experiments. One recent example of such a model formulation is Fuster et al. (2021).

Two issues are worth clarifying here. First, this costly adjustment can be explicitly micro-founded by various monetary or mental obstacles that prevent agents from making optimal ad-

⁴⁶So far, the majority workhorse incomplete market macro literature has not incorporated any endogenous mechanisms that determine the level of income risks. The emerging literature that incorporates labor market search/match frictions in these models have relied upon simplifying assumptions to get tractability. See, for instance, McKay (2017); Acharya and Dogra (2020); Ravn and Sterk (2021), with the only exception being Ravn and Sterk (2017).

justments in consumption from period to period. Regardless of its specific micro foundations, it effectively leads to extensive adjustment in consumption. Second, the assumption also conveniently captures, in the one-asset setting, the essence of implications from costly adjustment of illiquid assets in the two-asset setting, which generates wealthy hands-to-mouth behaviors, as formulated in the (Kaplan and Violante, 2014).

Specifically, I assume that there is a utility cost the agents need to incur χ , when changing the consumption in each period τ . Recognizing this, in each period, the agents need to first make a discrete choice of whether making adjustments to the consumption. In the case of adjustment, the agents solve the optimal consumption optimally. In the case of non-adjustment, the consumption stays at the level as the previous period, since it is the default consumption choice. Note that since the consumer always has the choice of adjustment, this naturally guarantees that in the presence of negative income shock when staying at the same level of consumption is no longer feasible, the agents will adjust the consumption to obey the budget constraints.

The change in the nature of the problem can be summarized by the restated value functions below. I restate the problem only for a consumer with objective risk profiles, as the subjective agent only has idiosyncratic risk perceptions $\tilde{\Gamma}_{i,\tau}$ as one additional state variable.

$$\begin{aligned}
V_\tau(c_{i,\tau-1}, u_{i,\tau}, m_{i,\tau}, p_{i,\tau}) &= \max \{V_\tau^A(u_{i,\tau}, m_{i,\tau}, p_{i,\tau}) - \chi, V_\tau^N(c_{i,\tau-1}, u_{i,\tau}, m_{i,\tau}, p_{i,\tau})\} \\
V_\tau^A(u_{i,\tau}, m_{i,\tau}, p_{i,\tau}) &= \max_{\{c_{i,\tau}\}} u(c_{i,\tau}) + (1 - D)\beta\mathbb{E}_\tau[V_{\tau+1}(u_{i,\tau}, R(m_{i,\tau} - c_{i,\tau}) + y_{i,\tau+1}, p_{i,\tau+1})] \\
V_\tau^N(c_{i,\tau-1}, u_{i,\tau}, m_{i,\tau}, p_{i,\tau}) &= u(c_{i,\tau-1}) + (1 - D)\beta\mathbb{E}_\tau[V_{\tau+1}(c_{i,\tau-1}, u_{i,\tau}, m_{i,\tau+1}, p_{i,\tau+1})]
\end{aligned} \tag{23}$$

where V^A and V^N represent value functions associated with adjustment and non-adjustment. Notice that in the case of non-adjustment, the consumption in previous period becomes an additional state variable. But essentially, there is no choice to be made as to consumption in the case of non-adjustment.

Solving consumption policies with the both intensive and extensive margin choices introduces additional computational challenges. In particular, it results in discrete jumps hence discontinuity in the value function over different values of state variables and the first order condition, namely the Euler equation, is no longer sufficient for the optimality of consumption. Although brutal force value function maximization is able to produce solutions to the model, I adopt the “Discrete Choice Endogenous Grid Algorithm (DCEGM)” introduced by [Iskhakov et al. \(2017\)](#) to speed up the computation.