Perceived versus Calibrated Income Risks in Heterogeneous-agent Consumption Models

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Abstract

Models of microeconomic consumption (including those used in HA-macro models) typically calibrate the size of income risk to match panel data on household income dynamics. But, for several reasons, what is measured as risk from such data may not correspond to the risk perceived by the agent. This paper instead uses data from the New York Fed's *Survey of Consumer Expectations* to directly calibrate perceived income risks. One of several examples of the implications of heterogeneity in perceived income risks is increased wealth inequality stemming from differential precautionary saving motivations. I also explore the implications of the fact that the perceived risk is lower than the calibrated level.

Keywords: Income risks, Incomplete market, Perceptions, Precautionary saving

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1 Introduction

Income risks matter for both individual behavior and macroeconomic outcomes. Given identical expected income and homogeneous risk preferences, different degrees of risks lead to different saving/consumption and portfolio choices. This is well understood in models in which either the prudence in the utility function (Kimball (1990), Carroll and Kimball (2001)), or occasionally binding constraint induces precautionary savings. It is widely accepted on the basis of empirical research indicating that idiosyncratic income risks are at most partially insured (Blundell et al. (2008)), and such market incompleteness leads to ex-post wealth inequality[1] and different degrees of marginal propensity to consume (MPC) (Krueger et al. (2016); Carroll et al. (2017)). This also changes the mechanisms by which macroeconomic policies can affect economic outcomes.[2] Furthermore, aggregate movements in the degree of idiosyncratic income risks can drive time-varying precautionary saving motives–another source of business cycle fluctuations.[3]

The size and the heterogeneity of the income risks are one of the central inputs in this class of incomplete-market macroeconomic models. One common practice in this literature is that economists typically approximate/estimate risks under a specified income process, relying upon the cross-sectional dispersion in income realizations, and then treat the estimates as the true model parameters known by the agents who make decisions in the model.[4]

But this estimation practice has limitations. The method used by economists to calibrate the size and persistence of income risks as perceived by the agents is subject to problems such as those caused by unobserved heterogeneity or model mis-specification. The intuition behind this is simple: certain information, either the intrinsic heterogeneity of each individual or advance information about future income or risks that enters an agent's information set from time to time, is not

[1] Aiyagari (1994); Huggett (1996); Carroll and Samwick (1997); Krusell and Smith (1998).

[2] Krueger et al. (2016), Kaplan et al. (2018), Auclert (2019).

[3] Challe and Ragot (2016); McKay (2017); Heathcote and Perri (2018); Kaplan and Violante (2018); Den Haan et al. (2018); Bayer et al. (2019); Acharya and Dogra (2020); Ravn and Sterk (2021); Harmenberg and Öberg (2021).

[4] Some recent examples include Krueger et al. (2016), Bayer et al. (2019), Kaplan et al. (2018).

directly observable by economists. If risks calibrated by economists based on flawed estimations differ from those perceived by agents, the model's implications will fail to match behavior even if the model is right (except for the miscalibration).

This paper addresses this issue by utilizing the recently available density forecasts of labor income surveyed by New York Fed's Survey of Consumer Expectation (SCE). Compared to the previous work that has studied partial insurance with expectational surveys [5], this paper's most important innovation is its use of the SCE's density survey which contains directly perceived risks. In the density survey, respondents are asked to provide histogram-type forecasts of their wage growth over the next 12 months, and they also report perceived job-finding and separation probabilities and answers to a set of expectational questions about the macroeconomy. When the individual density forecast is available, a parametric density distribution can be fit to obtain the individual-specific subjective distribution. Then, the second moment, the implied variance of the subjective distribution, allows me to directly characterize the perceived risk profile without relying on external estimates from cross-sectional microdata. This provides a direct measure of the perception of risk that presumably guides individual decisions.

With the individual-specific reported perceived risk (PR) in hand, I first confirm that the differences in mean risk across groups (age; gender; education; etc) measured by the conventional method correspond to differences in the mean self-reported perceptions (e.g., low-income young females are measured as, and perceive themselves as, facing higher risk than middle-aged middle-income males). But within every such group, there is also a great deal of heterogeneity in PR that is not captured by the conventional approach. The R^2 of a regression of PR on the conventional explanatory variables is only about 0.1, indicating that 90 percent of the heterogeneity in perceived risk is not captured by the traditional method.

In addition, the paper also finds that the perceived income risk, on average, is lower than the indirectly calibrated size of risks even within groups. Specifically, the perceived annual real wage risk

[5] For instance, Pistaferri (2001), Kaufmann and Pistaferri (2009).

is around 3%-4% in terms of standard deviations, while the estimation following the conventional approach (consistent with the finding of Low et al. (2010)) is at least 10%. I confirm that this finding is robust to alternative specifications of wage process and different frequency of income shocks.

This evidence motivates me to utilize survey-implied risks as truly perceived by agents to calibrate income risks in a standard incomplete market, overlapping-generation, and general equilibrium model to quantify these effects. The baseline model blends Huggett (1996), the income structure of Carroll and Samwick (1997), and persistent unemployment spells and unemployment benefits, a la Krueger et al. (2016) and Carroll et al. (2017). Contrasting with conventional practice, I show that calibrating risks using surveyed PRs helps explain two well-documented discrepancies between standard model prediction and what is seen in the data: a higher concentration of households with little liquid wealth, and a higher degree of wealth inequality in the data than in the model.

The mechanisms behind these two results are straightforward. First, allowing for heterogeneity in perceived income risks introduces a straightforward force that increases the wealth inequality, as different risks induce different precautionary saving motives, and heterogeneous buffer-stock savings. Second, a lower size of the perceived risk than in the baseline model implies less precautionary saving motive, hence a lower level of wealth accumulation by all agents in the economy.

The benchmark model maintains the full-information-rational-expection (FIRE) assumption in that the perceived risks from the survey are used to calibrate the true model parameters, but in the extended model, I deviate from this assumption. [6] In particular, the extension allows the perceived risks (subjective risks) to be different from the underlying income process (objective risks). This extension achieves two purposes within a single model. On the one hand, it serves as a robustness check with an alternative model assumption deviating from FIRE. On the other hand, it is an experiment model that break down the model implications into two channels: one via ex-ante

[6] There is mounting evidence in macroeconomics that people form expectations in ways deviating from FIRE. See, for example, Mankiw et al. (2003), Reis (2006), Coibion and Gorodnichenko (2012), Wang (2022). But most of such evidence is based on macroeconomic expectations, such as that of inflation.

saving behavior resulting from risk perceptions, or the "choice" channel, and the other via ex-post realized income inequality, the "outcome" channel.

The key finding from this extension is that let consumption/saving decisions be driven by survey-reported risks alone, even if the objective risks remain the same as the conventional calibration, is sufficient to yield a closer match of the model with the empirically measured wealth inequality and fraction of low-liquid-holding consumers in the data.

1.1 Related literature

This paper is related and contributes to several themes in the literature. First, it closely builds on the literature estimating both cross-sectional and time trends of labor income risks and the degree of consumption insurance. Early work by MaCurdy (1982), Abowd and Card (1989), Gottschalk et al. (1994) and Carroll and Samwick (1997) initiated what is now the common practice in the literature of estimating income risks by decomposing it into components of varying persistence on the basis of panel data. Subsequent work has explored time-varying and macro trends of idiosyncratic income risks. For instance, Meghir and Pistaferri (2004) allowed for time-varying risks or conditional heteroscedasticity in the traditional permanent-transitory model. Blundell et al. (2008) used the same specification of income process to estimate partial insurance in conjunction with consumption data. More recently, Bloom et al. (2018) found that idiosyncratic income risks have declined in recent decades.[7] Moreover, recent evidence that relied upon detailed administrative records and larger data samples highlighted the asymmetry and cyclical behaviors of idiosyncratic earning/income risks (Storesletten et al., 2004; Guvenen et al., 2014; Arellano et al., 2017; Guvenen et al., 2019; Bayer et al., 2019; Guvenen et al., 2021). Additionally, a separate literature has focused on job-separation and unemployment risks (Stephens Jr, 2004; Low et al., 2010; Davis and Von Wachter, 2011; Jäger et al., 2022). Table A.6 in the Appendix summarizes the income process and estimated

[7]Synthesizing various data sources, Moffitt (2020) found no such obvious trend for the same period.

risks in selected papers from this literature. Compared to this work, the novelty of this paper lies in its focus on the directly reported perceptions of income risks and how they are correlated with the realized income risks estimated from the panel data.[8]

Second, my paper is most closely related to the well-documented issue of "insurance or information" in the income risk/partial insurance literature (Pistaferri (2001), Kaufmann and Pistaferri (2009), Meghir and Pistaferri (2011), Kaplan and Violante (2010), Stoltenberg and Uhlendorff (2022)). In any empirical tests of consumption insurance or consumption response to income shocks, there is always a concern that what is interpreted as the shock has actually already entered the agents' information set. If so, this may lead to the finding of "excess smoothness" of supposedly unanticipated shocks (Flavin (1988)). My paper shares the spirit with these studies in that we all use surveyed expectations to tackle the identification problem.[9] That is, I directly use the expectation data and explicitly control for the truly conditional expectations of the agents. This helps economists avoid making assumptions about what is exactly in the agents' information set. What differentiates my work from that of others is that I directly use survey-reported income risks, which are available from density forecasts, rather than the estimated risks using the difference between expectations and realizations. An advantage of my approach is that I can directly study individual-specific risks instead of that at the group level.

Third, the paper speaks to an old but recently revived trend in the literature of studying consumption/saving behaviors in models that incorporate imperfect expectations and perceptions. For instance, Pischke (1995) explored the implications of the incomplete information about aggregate/individual income innovations by modeling agent's learning about permanent income component as a signal extraction problem. Wang (2004) studied how such forecasting uncertainty affects consumption via precautionary saving motives. In a similar spirit, to reconcile low MPCs in microdata and the high MPC in the macro level, Carroll et al. (2018) introduce the information

[8] Koşar and Van der Klaauw (2022) is a recent exception, that documents the cross-sectional/life-cycle/business heterogeneity in perceived earning risks using SCE data.

[9] See a recent New York Fed blog for a similar exercise.

rigidity of households with learning about macro news while they are fully updated about micro news. Rozsypal and Schlafmann (2017) found that households' expectation about income exhibits an over-persistent bias. More recently, Broer et al. (2021) have incorporated information choice in a standard consumption/saving model to explore its implication for wealthy inequality. My paper has a similar flavor to all these studies in that it too, emphasizes the role of perceptions. But my work differs from those previous studies in two regards. First, it focuses on the second moment, namely income risks. Second, although most of this existing work explicitly specifies a mechanism of expectation formation that deviates from the full-information-rational-expectation benchmark, this paper advocates for disciplining the model assumptions regarding belief heterogeneity by directly using survey data. [10]

This paper is also directly related to the research that advocates for eliciting probabilistic questions to measure subjective uncertainty in economic surveys (Manski (2004), Delavande et al. (2011), Manski (2018)). Although the initial suspicion about people's ability to understand, use, and answer probabilistic questions was understandable, Bertrand and Mullainathan (2001) and others have shown respondents have the consistent ability and willingness to assign a probability (or "percent chance") to future events. Armantier et al. (2017) thoroughly discuss designing, experimenting, and implementing consumer expectation surveys to ensure the quality of responses. Broadly speaking, the advocates have argued, first, that analysts must go beyond the "revealed preference" approach and, second, the availability of survey data provides economists with direct information about agents' expectations and helps them avoid imposing arbitrary assumptions.(Manski (2004)) This insight holds for not only point forecast but also for risk/uncertainty, because for any economic decision made by a risk-averse agent, both the expectation but also the perceived risks matter a great deal.

Finally, this paper is related empirically to the literature that studies expectation formation using

[10] See Bhandari et al. (2019) for another example of directly using survey data to discipline subjective beliefs in standard macro models.

subjective surveys. In recent decades, a long list of theories of "expectations formation" alternative to FIRE has been developed, each of which examines how agents deviate from full-information rationality benchmarks, such as sticky expectations, noisy signal extraction, least-square learning, etc. Also, empirical work has been devoted to testing these theories comparably (Coibion and Gorodnichenko (2012), Fuhrer (2018)). Yet it is fair to say that thus far, relatively little work has been done on individual variables such as labor income, which might well be more relevant to individual economic decisions. This paper shows that understanding the patterns of beliefs about individual variables, and, in particular, mean and higher moments, is fruitful for macroeconomic modeling.

2 Theoretical framework

2.1 Wage process and perceived risks

To be consistent with the survey-elicited question in SCE, I primarily focus on the wage risk. Conditional on being employed at the same job and same position, and the same hours of work, the log idiosyncratic earning, or the wage rate, of an individual i at time t, $w_{i,t}$ consists of a predictable component, $z_{i,t}$ and a stochastic component, $e_{i,t}$. (Equation 1)

$$w_{i,t} = z_{i,t} + e_{i,t} \tag{1}$$

There is an extensive discussion in the literature about the exact time-series nature of the stochastic component e. For instance, it may consist of a permanent and a transitory component.[11] Or some of the literature replaces the permanent component with a stationary/persistent component

[11] Abowd and Card (1989),Gottschalk et al. (1994), Carroll and Samwick (1997), Blundell et al. (2008), and Kaplan and Violante (2010).

in the form of an AR process.[12] The transitory component could be moderately serially correlated following a moving-average (MA) process.[13] I first proceed with the generic structure, as in Equation 1 without differentiating these various specifications. I defer that discussion to Section 4.2.

Hence, wage growth from t to $t+1$ consists of the predictable change in $z_{i,t+1}$, and the change in the stochastic component $e_{i,t}$.

$$\Delta w_{i,t+1} = \Delta z_{i,t+1} + \Delta e_{i,t} \tag{2}$$

Under the assumption of the full-information rational expectation (FIRE), all shocks realized until t are observed by the agent at time t. Therefore, the expected volatility under FIRE (with a superscript $*$), or what this paper hereafter refers to as perceived risks (PR), is the conditional variance of income growth from t to $t+1$.

$$Var^*_{i,t}(\Delta w_{i,t+1}) = Var^*_{i,t}(\Delta e_{i,t+1}) \tag{3}$$

The predictable changes do not enter PR. Hence, the PR is the *conditional* variance of the change in the stochastic component, $Var^*_{i,t}(\Delta e_{i,t+1})$. Notice that it crucially depends on the time-series nature of $e_{i,t}$.

The size of the true PR is not directly observed by economists. To estimate it, researchers usually start from obtaining an approximation of the stochastic component $e_{i,t}$, denoted as $\hat{e}_{i,t}$, by subtracting observed wage growth in panel data, $\Delta w_{i,t}$, by the approximated predictable change, $\Delta \hat{z}_{i,t}$, that is $\Delta \hat{e}_{i,c,t} = \Delta w_{i,c,t} - \Delta \hat{z}_{i,c,t}$. To mimic $z_{i,t}$ from the agent's point of view, $\hat{z}_{i,t}$ commonly includes factors such as age polynomials, gender, education, occupation, etc. Hence, $\hat{e}_{i,t}$ are, essentially, the regression residuals of the first-step wage regression controlling for a limited number

[12] Storesletten et al. (2004), Guvenen (2007), Guvenen (2009).
[13] Meghir and Pistaferri (2004).

of observable variables measured in the panel data. Then, the cross-sectional variance of $\Delta \hat{e}_{i,t}$ is the input for estimating income risks. It is usually referred to as the "income volatility" in the literature, [14]

Note that the common practice usually estimates income risks at the group level, denoted as c, (such as age, education, and cohort, etc), although in theory, the risk as perceived by a FIRE agent could be totally individual-specific. That is so because at individual level, there are no realizations of risks, but a particular draw of income shock. (Equation 4) The within-group cross-sectional variation of a sufficiently large group size is needed for such an estimation.

$$Var_c(\Delta \hat{e}_{i,c,t}) = Var_c(\Delta w_{i,c,t} - \Delta \hat{z}_{i,c,t}) \tag{4}$$

Unlike the PR of the agent, the term above is an *unconditional* variance at the group level. The distinction between the *conditional* PR by the agent and the *unconditional* volatility approximated by the economists is crucial. Two important issues affect the comparability of the two objects.

First, it is very likely that what is controlled for in the first-step income regression, namely $\hat{z}_{i,c,t}$, does not perfectly coincide with what is *predictable* from the point of view of an FIRE[15] agent at time t. That is so primarily because econometricians with the panel data of earnings cannot control for other "unobserved heterogeneity" not measured in the data. This is equivalent to the "superior information" problem, [16] which refers to the possibility that agents have advance information or foresight regarding their wage growth, and this information is available to econometricians. For instance, a worker might be concerned that a recent dispute with her boss may negatively affect her wage next year, but econometricians have no way of knowing this.

Second, the comparison is sensitive to the time-series nature of $e_{i,c,t}$. Again, this occurs because

[14]For instance, Gottschalk et al. (1994), Moffitt and Gottschalk (2002), Sabelhaus and Song (2010), Dynan et al. (2012), Bloom et al. (2018).

[15]In later sections of the paper, I relax the FIRE assumption, which it makes it possible that PR reported in the survey is also subject to incomplete information and behavioral bias of the agents.

[16]Pistaferri (2001); Kaufmann and Pistaferri (2009).

economists' estimated volatility is unconditional, while the perception is conditional on the information until time t. To illustrate this point, imagine a very persistent component in the income shock. Under the aforementioned process, the estimated income volatility also includes the variance of the realized shock until t, which has already entered the information set of the agent. Therefore, even if the econometricians perfectly recover the $e_{i,t}$ in the first-step regression, the presence of a persistent component in income changes would result in differences between PR and estimated income volatility. Therefore, to approximate the true PR from the point of view of agents, economists need to recover a conditional variance using information from the unconditional variance, typically by assuming a particular time-series structure of the stochastic component e and using cross-sectional moments restrictions to estimate its size. I return to this discussion in Section 4.2.

To summarize, for two reasons, survey-elicited PR has an invaluable use and is preferable to the conventional income risk estimation based on cross-sectional realizations, which is also used to parameterize macro models. First, survey-reported PR is, by construction, conditional on the information set of each agent i, which is likely to include intrinsic heterogeneity specific to the individual or the advance information useful to forecasting that individual's own wage growth.[17] Economists who try to approximate the PR cannot do as well as the agents who answer the questions, because the latter's information is not necessarily available to economists. Second, survey-implied PR provides direct identification of the degree of heterogeneity of income risks across individuals in the economy. This prevents modelers from making possibly imperfect assumptions when they estimate group-specific income risks, by grouping individuals on the basis of very limited dimensions of observable factors, such as education and age.

It is worth pointing out that despite these advantages, survey-implied PRs may reflect the risk perceptions of agents subject to certain behavioral biases, such as overconfidence, in contrast to those assumed by FIRE. I explore below the robustness of the model results of the paper with

[17] For the same reason, the literature on partial insurance uses expectational surveys to resolve the superior information problem. See Pistaferri (2001), Kaufmann and Pistaferri (2009) and others for examples.

respect to these alternative assumptions. The key takeaway is that even if the survey-implied PRs don't align with the true objective size of income risks, they prove to be a better input for predicting individual decisions than the calibrated income risks as done in the conventional approach.

3 Data, variables and density estimation

3.1 Data on perceived risks

The data used for this paper were obtained from the core module of the Survey of Consumer Expectation(SCE) conducted by the New York Fed, a monthly online survey for a rotating panel of around 1,300 household heads during the period June 2013 to July 2021, or 97 months.

I primarily rely upon the density forecast of individual earnings by each respondent in the survey to estimate perceived income risks. The main question used is framed as follows: "Suppose that 12 months from now, you are working in the exact same ["main" if Q11>1] job at the same place you currently work and working the exact same number of hours. In your view, what would you say is the percentage chance that 12 months from now, your earnings on this job, before tax and deductions, will increase by x%?".[18] Then, I fit the bin-based density forecast in each survey response with a parametric distribution.[19] The variance of the estimated distribution naturally represents an individual-specific perceived risk. To obtain the wage risk in real terms, I further add the individual-specific inflation uncertainty estimated by the same procedure and use the same individual's density forecasts of inflation in SCE. This procedure is predicated on the assumption that agents regard individual wage growth and aggregate inflation as independent random variables. This assumption is by no means perfect. For the robustness of the results, I use both adjusted PR in real terms, and nominal PR for the empirical results below.

[18]In the online survey, the respondent can move on to the next question only if the probabilities filled in all bins add up to one. This ensures the basic probabilistic consistency of the answers, which is crucial for any further analysis.

[19]This follows the approach employed by Engelberg et al. (2009) and researchers in the New York Fed (Armantier et al. (2017)). Appendix A.1 documents in detail the estimation methodology and its robustness.

Crucially, because the survey question regards the expected earning growth conditional on the same job position, same hours, and the same location, it can be clearly interpreted as the wage. It becomes immediately clear that wage risk only constitutes a part of income risk, and this has two important implications.

First, focusing on the wage risks avoids the problem of misconstruing earning changes due to voluntary labor supply decisions as risks. Empirical work estimating income risks is often based on data from total earning or even household income, in which voluntary labor supply decisions inevitably confound the true degree of uninsured idiosyncratic risks. This survey-based measure used here is not subject to this problem. Second, the wage risk also excludes important sources of income fluctuations, such as unemployment and job switching. As demonstrated by research (e.g., Low et al. (2010)), major job transitions often are the dominant source of income risks individual workers face. Therefore, I separately examine unemployment risk expectations, surveyed as perceived job-separation and finding probabilities in SCE, in Section 4.4.

3.2 Wage data

I examine longitudinal data on individual labor earnings from the 2014-2017 and 2018-2020 panels of the Survey of Income and Program Participation (SIPP).[20] Each panel of SIPP, which surveys thousands of workers, is designed to be a nationally representative sample of the U.S. population. The interviews, conducted once a year, collect data on individuals' monthly earnings, hours of work, and other labor market outcomes.[21] On average, each individual is surveyed for 33 months over multiple waves of the survey.

For the purpose of this paper, using SIPP to estimate wage risk has obvious advantages over

[20]Other recent work that estimates income risks using SIPP includes Bayer et al. (2019), who, in contrast to this paper, use quarterly total household income rather than the monthly job-specific earning of individuals.

[21]This causes the "seam" issue documented by Moore (2008), which states that reported changes in answers (e.g. wage growth) within survey waves are systematically smaller than cross-wave changes. For the baseline estimation, I exclude the cross-wave earning growth, which produces a lower-bound estimate of wage risks. See Appendix A.3 for more inspection of this issue.

other commonly used datasets, the most notable of which is the Panel Study of Income Dynamics (PSID). SIPP contains information that allows me to work with wage changes conditional on staying at the same job with the same employer, thanks to its detailed records of job transitions and a unique employer identifier. In contrast, PSID only provides biennial records of labor earnings for years since 1997. For the overlapping periods between SIPP and SCE, it is possible to make a direct comparison between realized wage risks at the annual frequency and the ex-ante perceptions of the wage risks. This is particularly crucial if wage risks are time-varying and dependent on macroeconomic conditions.

For an apples-to-apples comparison, I obtain the hourly wage of workers employed by the same employer by dividing the total monthly earnings from the *primary job* by the average hours of work for the same job for only those who stay with the same employer for at least 2 years. To identify job stayers, I follow the same approach by Low et al. (2010) and I impose five criteria. I only include (1) the working-age population between 25-65; (2) private-sector jobs, excluding workers from government or other public sectors; (3) the same job as the last year; (4) monthly wage rates that are no greater than 10 times or smaller than 0.1 times of the average wage; (5)those who don't have days away from work during the reference month without the pay. This leaves me with a monthly panel of 350-1000 individual earners for the sample period 2013m3-2019m12. Appendix A.3 discusses in greater details the data selection procedure and reports summary statistics.

4 Basic facts about perceived income risks

4.1 Observable and unobservable heterogeneity

In both income risk estimation and parameterization of the incomplete market macro models, it is common practice to assume, first, that idiosyncratic risks differ as a function of certain observable factors such as education, gender, and age, and second, there is no additional within-group heterogeneity in the degree of the risk.[22] This section reports my finding that although the observed heterogeneity in PR across individuals does reflect between-group differences–along dimensions economists have commonly assumed, a dominant fraction of the differences in PR can be attributed to other unobservable heterogeneity. Furthermore, even in those observable dimensions, the group heterogeneity seen in PR does not coincide with that seen in estimated risks.

Figure 1 plots the group average of PRs (both in real and nominal terms), approximated wage volatility, $Var(\Delta \hat{e})$, as defined in Equation 4, and the estimated conditional risk, $Var_t(\Delta \hat{e})$, (see the next section for the exact procedure of generating it), by age, gender and education. Regarding the education-risk profile, both wage volatility and approximated risks are higher for more educated workers. This is consistent with the finding of Meghir and Pistaferri (2004), who examine total labor income instead of wage. In contrast, risk perceptions exhibit the opposite pattern with respect to education level: less-educated workers report higher PRs than more-educated workers. Regarding the life-cycle pattern of risks, neither wage volatility nor estimated risks show a monotone pattern over the life cycle.[23] In contrast, perceived risks almost monotonically decline over the life cycle for both males and females. These findings are confirmed in Table 1, which reports the group average of PR, wage volatility and estimated risks.

Another salient fact is that PR is always *smaller* than estimated risks. In particular, both wage volatility and the estimated risk of different groups fall in the range of 5-15% per year (in standard deviation terms), which aligns with the estimates in a large literature and that used in models, as summarized in Table A.6.[25] But the average perceived risks reported in the survey are only about 3-4%, and at least 50% smaller. For instance, a male high school graduate on average perceives

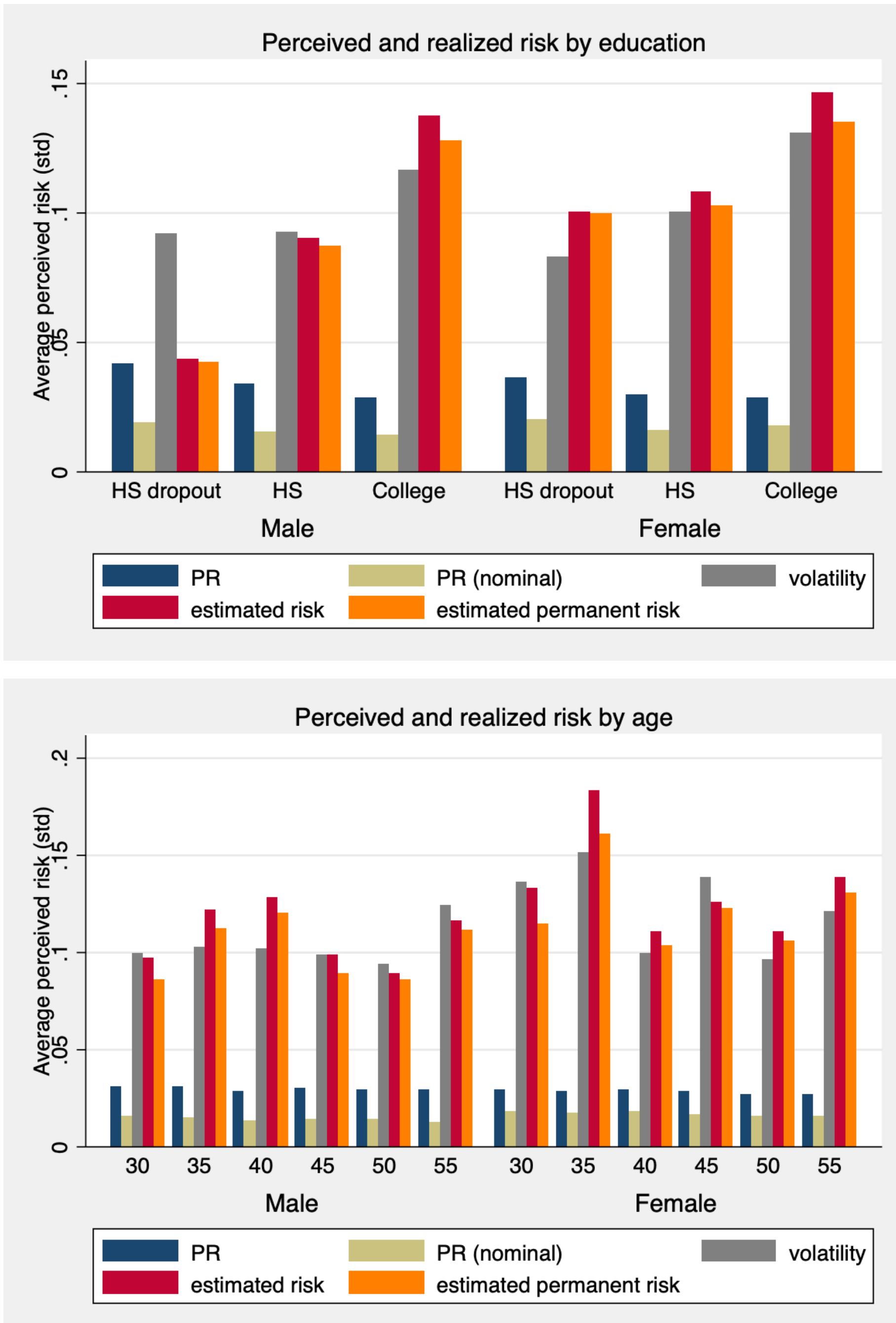
[22]For instance, Meghir and Pistaferri (2004) found that more educated workers face higher income risks than the less educated ones. Sabelhaus and Song (2010) and Bloom et al. (2018) documented that income risks decrease with age, and vary with the current income level in a non-monotonic U-shape. In their models, Cagetti (2003), Blundell et al. (2008), and Carroll et al. (2017) allowed for heterogeneous risks across different demographic variables.

[23]The homogenous age pattern of wage risks is not necessarily contradictory with the well documented declining pattern estimated using data on household income or total earning[24]. It is likely that the decline of income risks over the life cycle has to do with non-wage risks or better insurance via work arrangements over the life cycle.

[25]The most comparable estimates in the literature are by Low et al. (2010), as it explicitly estimates the wage risk of job stayers separately from job switching and unemployment spells. The authors report annual permanent and a transitory risk of 10%, respectively. This implies a total risk of approximately 35%-40%.

his annual wage risk to be 4 percentage points in terms of standard deviation, while the wage risk implied by wage panel data for the same group is above 9-10 percentage points.

Figure 1: Perceived Risks, Wage Volatility and Estimated Wage Risks by Observable Factors



Note: Real and nominal perceived risk (from SCE), average estimated wage volatility (from SIPP), approximated wage risk (from SIPP), and permanent risk (from SIPP) of each education-gender (upper panel) or age-gender (bottom panel) group. The volatility is approximated by the within-group cross-sectional standard deviation of log changes in unexplained wage residuals, as defined in Equation 4. The estimated risk is based on the process specified in Equation 5.

Such a size difference is also evident in the top Figure 2 which plots the distribution of PRs against the distribution of individual-level annual wage volatility in SIPP. The latter is computed as the standard deviation of annual/12-month log growth rates of unexplained wage residuals. The figure shows that PRs are concentrated at a much lower range of values around (2-4%), while in contrast, the average size of wage volatility falls in the range of 10-20% or even higher.

Despite controlling for all the observable factors of individuals, there remains a large degree of heterogeneity that probably can be attributed to other unobserved factors. The bottom panel of Figure 2 shows the sizable dispersion of the unexplained residuals of PRs before and after controlling for observable individual characteristics, including age, age polynomial, gender, education, type of work, and time fixed effects.[26] Controlling for the time fixed effects is important because the focus here is on the idiosyncratic rather than aggregate risks perceived by agents.

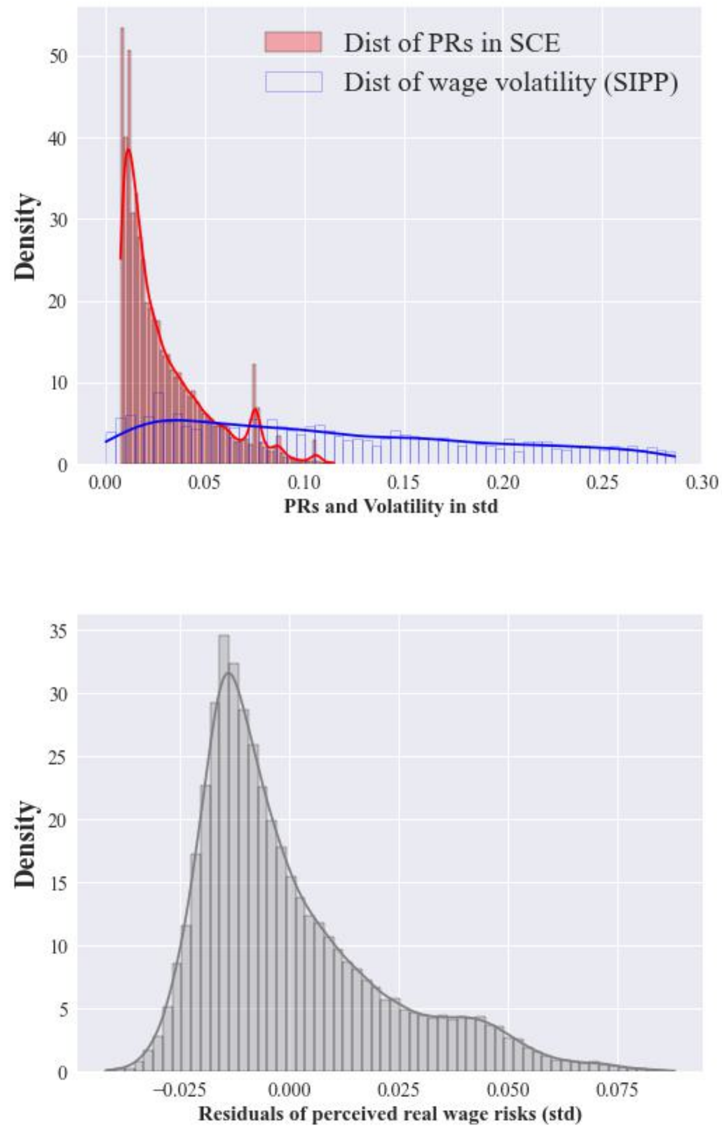
The R^2 of the regression of PRs on all observable factors in SCE, without individual fixed effects, is at most 10%, while including fixed effects increases R^2 to 70%. This finding has two implications. First, the role of within group heterogeneity suggests that the conventional practice of estimating and modeling income risks as only differing by demographic dimensions has limitations. Second, heterogeneity in PR can be directly put into use to model heterogeneous income risks without identifying the source of heterogeneity. Therefore, in Section 5, my model calibration adopts such an approach.

4.2 Decomposed risks of different persistency

As previewed in Section 2, a crucial aspect of income risk estimation is the time-series nature of the shocks. A realized permanent/persistent shock contains information about the future wage growth, while an entirely transitory shock does not. Therefore, in the two scenarios, agents perceive different degree of risks. This is crucial to making an apples-to-apples comparison between survey-reported

[26] Appendix A.2.1 also inspects the unexplained residuals of expected wage growth and higher-order perceived risks such as skewness, which likewise shows sizable heterogeneity.

Figure 2: Dispersion in Perceived Wage Risks



Note: Distributions of PRs regarding real wage growth (in standard deviation terms) (upper) and regression of residuals of PRs unexplained by age, age polynomials, gender, education, type of work arrangement, and time fixed effects (bottom).

PRs and the calibrated risks using the conventional methods.

To proceed, I adopt a commonly used income/wage process in a large body of literature. [27]

I specify that the stochastic component $e_{i,t}$ consists of a permanent component p that follows a

[27] MaCurdy (1982), Abowd and Card (1989), Gottschalk et al. (1994), Carroll and Samwick (1997), Blundell et al. (2008), etc. Crawley et al. (2022) presents a more parsimonious process to resolve the possible model mis-specification caused by "time-aggregation" problem.

random walk and a transitory component θ that is i.i.d. The shocks to both components are log normally distributed, with mean zero and potentially time-varying variances σ^2_ψ and σ^2_θ.[28]

$$\begin{aligned} e_{i,t} &= p_{i,t} + \theta_{i,t} \\ p_{i,t} &= p_{i,t-1} + \psi_{i,t} \end{aligned} \tag{5}$$

Under this specific wage process, the PRs of an FIRE agent is equal to the summation of the variance of the two components $Var^*_{i,t}(\Delta w_{i,t+1}) = \sigma^2_{\psi,t} + \sigma^2_{\theta,t}$. But, in contrast, the income volatility estimated from panel data, as defined as in Equation 4, is a sample analogue of $Var(\Delta e_{i,t}) = \sigma^2_{\psi,t} + \sigma^2_{\theta,t-1} + \sigma^2_{\theta,t}$. It differs from the PR by $\sigma^2_{\theta,t-1}$, exactly due to its unconditional nature.

Therefore, a more comparable counterpart of PR that is from panel data estimation is the sum of *estimates of* permanent and transitory risks, $\hat{\sigma}^2_\psi + \hat{\sigma}^2_\theta$. Denote it as $Var_t(\Delta \hat{e}_{i,t})$. To do so, I follow the same GMM estimation procedure in the literature [29] to identify the time-averaged variances of the permanent and transitory component of the monthly wage growth using SIPP's wage data for the same period. Then I convert these monthly risk parameters into annual frequency so that they are comparable to perceived risks about annual wage growth.[30]

Table 1 reports the group-specific estimates of total, permanent, and transitory wage risks based on wage panel data in comparison with the average and median perceived risks of the same group. The main finding from this comparison is that within each group, the perceived risks are systematically lower than that indirectly estimated income risks, even if the latter is at least one step closer to the perceived risk compared to the unconditional wage volatility. In addition, Figure A.6 in the Appendix compares the two, allowing for time-variation of the risks. The size difference

[28]This also corresponds to the model specification as in Equation 11.

[29]See Appendix A.4.1 for details. The estimation procedure follows Abowd and Card (1989), Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Blundell et al. (2008), which have minor differences depending on the model specification.

[30]For permanent risks, the annual earning risk is the summation of monthly permanent risks over the next 12 months. The transitory risks of annual earnings, in contrast, is the sample average of monthly risks over the next 12 months. Appendix A.4.3 provides alternative estimates for quarterly and yearly frequency.

Table 1: Perceived risk, estimated volatility and risks of each group

	PR(mean)	PR(median)	Volatility	RealizedRisk	PRisk	TRisk
gender						
male (50%)	0.03	0.022	0.105	0.115	0.109	0.0238
female (49%)	0.028	0.022	0.118	0.131	0.122	0.0322
education group						
HS dropout (0%)	0.036	0.022	0.088	0.071	0.07	0.0063
HS graduate (42%)	0.03	0.022	0.096	0.098	0.094	0.0176
College/above (56%)	0.028	0.021	0.124	0.142	0.132	0.0357
5-year age						
20 (2%)	0.037	0.031	0.094	0.069	0.068	0.0061
25 (12%)	0.032	0.027	0.111	0.157	0.156	0.0083
30 (12%)	0.03	0.023	0.116	0.112	0.098	0.0372
35 (13%)	0.029	0.021	0.125	0.149	0.134	0.0524
40 (13%)	0.028	0.02	0.1	0.119	0.111	0.0287
45 (14%)	0.028	0.02	0.119	0.113	0.106	0.0224
50 (15%)	0.027	0.019	0.095	0.1	0.096	0.0203
55 (15%)	0.027	0.018	0.122	0.128	0.121	0.0283
Full sample (100%)	0.029	0.021	0.112	0.123	0.115	0.0279

The mean and median PRs, estimated annual wage volatility, and estimated risks of permanent and transitory components of different groups. All are expressed in standard deviation units.

and disconnect between perceived risks and indirectly estimated risk remains.

The most likely explanation for this disconnect in both size and time-varying patterns between the two series is either unobservable heterogeneity or superior information, a point I will formally elaborate in the next section. For the common panel-data-based estimation to correctly identify idiosyncratic wage risks relevant to heterogeneous individuals, two requirements need to be satisfied. First, economists need to perfectly exclude the predictable changes in wage growth from the point of the agent, by both correctly approximating $z_{i,t}$ in the first-step regression, and by correctly recovering the persistent/permanent components contained in $e_{i,t}$. Second, they also need to correctly assume the dimensions by which risks differ across workers. Given the stringency of these requirements, directly reported PRs may provide a better alternative to calibrating income risks that are truly relevant from the point of view of heterogeneous individuals.

4.3 Accounting for the evidence

This section proposes my preferred explanation for the size differences between individual PRs reported in the survey and those estimated using panel data, which is the role of unobserved heterogeneity or advance information. In the model section 6, I explore alternative hypotheses, such as misperception of risks by agents because of behavioral biases.

For clarity, I follow the same wage process specified in Equation 5 but assuming away time-variation of risk parameters. Furthermore, all agents have individual-specific permanent $\sigma^2_{i,\psi}$ and transitory risks $\sigma^2_{i,\theta}$, hence, perceived income risks, but they have the same relative size of the two κ. Individual PRs follow a log-normal distribution with mean μ_{PR} and standard deviation σ_{PR}.

$$\log(\text{PR}_i) \sim N(\mu_{\text{PR}}, \sigma^2_{\text{PR}}) \tag{6}$$

The two parameters can be straightforwardly estimated by fitting a truncated log-normal distribution to the cross-sectional distribution of the time-average PRs in SCE, as shown in Figure 3.

To capture unobserved heterogeneity explicitly, I allow for the unexplained income residual change $\Delta\hat{e}_{i,t}$ to be different from that which is truly unpredictable from the individual i's point of view, $\Delta e_{i,t}$, by exactly $\xi_{i,t}$. To be entirely consistent with the wage process, I also assume that $\xi_{i,t}$ consists of a corresponding permanent component $\xi^{\psi}_{i,t}$ and a change in transitory component $\Delta\xi^{\theta}_{i,t}$.[31] For instance, one reasonable interpretations of the unobserved permanent component of $\xi^{\psi}_{i,t}$ is the individual-specific growth rate in permanent income, which cannot be observed by researchers. An example of the transitory changes in the unobserved component is temporary wage cuts.

$$\Delta\hat{e}_{i,t} = \Delta e_{i,t} + \xi_{i,t} = \psi_{i,t} + \Delta\theta_{i,t} + \xi_{i,t} = \psi_{i,t} + \Delta\theta_{i,t} + \xi^{\psi}_{i,t} + \Delta\xi^{\theta}_{i,t} \tag{7}$$

[31] This is similar to the specification of unobserved heterogeneity in *income* as in Primiceri and Van Rens (2009), which only allows for a permanent component of the unobserved heterogeneity.

When economists estimate wage risks using panel data, they typically identify the *average* permanent and transitory risks at the population, or group level. It is easy to show that, except for a special case absent of such unobserved heterogeneity captured by $\sigma^2_{\xi,\psi} = \sigma^2_{\xi,\theta} = 0$, the common methods-of-moment estimation procedure used in the literature can only recover an upward-biased PR from these estimates, with the difference being exactly the variance due to the unobserved heterogeneity.[32]

$$\widehat{\mathrm{PR}} = \hat{\sigma}^2_{\psi} + \hat{\sigma}^2_{\theta} = \int \mathrm{PR}_i di + \sigma^2_{\xi} = \int \sigma^2_{i,\psi} di + \int \sigma^2_{i,\theta} di + \underbrace{\sigma^2_{\xi,\psi} + \sigma^2_{\xi,\theta}}_{\text{unobserved heterogeneity}} \tag{8}$$

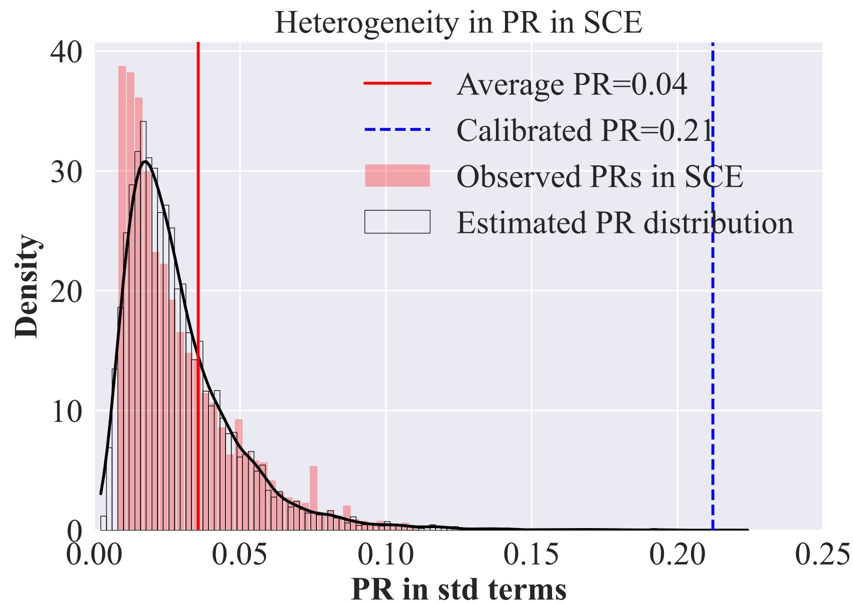
Therefore, the size of the unobserved heterogeneity $\sigma^2_{\xi} \equiv \sigma^2_{\xi,\psi} + \sigma^2_{\xi,\theta}$ can be directly identified by taking the difference between average PR in SCE and what is the average estimated risk ($\hat{\sigma}^2_{\psi}$ and $\hat{\sigma}^2 + \theta$) using panel data (the difference between the two vertical lines in Figure 3). Furthermore, with an auxiliary assumption that the two unobserved terms have the same ratio as the ratio of permanent and transitory risk, we can further decompose the estimated heterogeneity into $\sigma^2_{\xi,\psi}$, and $\sigma^2_{\xi,\theta}$, which represents the size of unobserved heterogeneity in permanent and transitory wage changes, respectively.

With the benchmark wage risk estimates of $\sigma_{\psi} = 0.15$ and $\sigma_{\psi} = 0.15$ (used to calibrated the baseline model in Section 5), hence a conventionally calibrated $\widehat{PR} = 0.41$, and $\kappa = 1$, the procedure produces the estimated unobserved heterogeneity: $\sigma_{\xi,\psi} = 0.13$ and $\sigma_{\xi,\theta} = 0.13$, and a fitted truncated-log-normal distribution of PRs, as plotted in Figure 3. In Section 5, I use these estimates to calibrate the heterogeneous perceived wage risks in the model. Using the wage risk estimates by Low et al. (2010), $\sigma_{\psi} = 0.10$ and $\sigma_{\psi} = 0.09$ yield a smaller size estimate of the unobserved heterogeneity $\sigma_{\xi,\psi} = 0.08$, $\sigma_{\xi,\theta} = 0.07$. In both cases, the estimates imply that a dominant fraction of observed wage inequality and volatility is attributed to to observed heterogeneity, instead of risks,

[32] The common GMM procedure produces an estimated transitory risk with a size of $\hat{\sigma}^2_{\epsilon} = -cov(\Delta\hat{e}_{i,t}, \Delta\hat{e}_{i,t+1}) = -cov(\Delta e_{i,t} + \xi_{i,t}, \Delta e_{i,t+1} + \xi_{i,t+1}) = -\int cov(\Delta e_{i,t} + \xi_{i,t}, \Delta e_{i,t+1} + \xi_{i,t+1}) = \int \sigma^2_{i,\theta} + \sigma^2_{\xi,\theta}$, and an estimated permanent risk of $\hat{\sigma}^2_{\psi} = var(\Delta\hat{e}_{i,t}) - 2\hat{\sigma}^2_{\theta} = var(\Delta e_{i,t}) + (\sigma^2_{\xi,\psi} + 2\sigma^2_{\xi,\theta}) - 2\hat{\sigma}^2_{\theta} = \int var(\Delta e_{i,t}) + (\sigma^2_{\xi,\psi} + 2\sigma^2_{\xi,\theta}) - 2\hat{\sigma}^2_{\theta} = \int(\sigma^2_{i,\psi} + 2\sigma^2_{i,\theta}) + (\sigma^2_{\xi,\psi} + 2\sigma^2_{\xi,\theta}) - 2\hat{\sigma}^2_{\theta} = \int(\sigma^2_{i,\psi} + 2\sigma^2_{i,\theta}) + \sigma^2_{\xi,\psi} + 2\sigma^2_{\xi,\theta} - 2(\int \sigma^2_{i,\theta} + \sigma^2_{\xi,\theta}) = \int \sigma^2_{i,\psi} + \sigma^2_{\xi,\psi}$.

as the conventional calibration of the model.

Figure 3: Estimated Heterogeneity in PRs



Note: The observed distribution of perceived income risks from SCE and the fitted truncated log-normal distribution estimation.

4.4 Unemployment risk perceptions

My analysis has so far focused only on wage risks conditional on staying in the same job. But it only constitutes a part of the income risks, given that major labor market transitions, such as job loss and switching, usually result in more significant changes in labor income.[33] In addition, unemployment risks are usually another central input of the incomplete-market macroeconomic models.[34] In these models, as in the approach to wage risks, the common practice is to model the process of labor market transitions on the basis of on externally estimated stochastic processes32.[35]. This section shows that, although, on average, the survey-reported expectations of job separation/finding probabilities

[33] Low et al. (2010), Davis and Von Wachter (2011).

[34] For examples, see Krueger et al. (2016) and Bayer et al. (2019), etc.

[35] The exceptions are models that endogenize job search & match mechanisms, such as Ravn and Sterk (2017), Ravn and Sterk (2021), McKay (2017) in which job-separation rates typically remains exogenous and externally calibrated.

track realized aggregate dynamics computed through a panel data in a standard approach in search & match labor literature, as in Fujita and Ramey (2009), it masks a huge amount of heterogeneity, which is not assumed in standard models.

To achieve a fair comparison between perceptions and realizations measured for different horizons, I cast both probabilities into a continuous-time rate for a Poisson point process. [36] Figure 4 plots the converted realizations of job-separation/finding rates, respectively, against the corresponding average, and the 25/75 percentile of the expectations across all survey respondents at each point in time. A number of straightforward findings emerge. First, although the two series are constructed independently of one another, on average, perceptions track the aggregate realizations relatively well. The most notable deviation between the belief and realization occured during March 2020, which saw an unprecedented increase in one-month job separations[37] and a dramatic decrease in job findings. Second, however, as shown by the wide 25/75 inter-range-percentile around mean expectations, individual respondents vastly disagree on their individual separation and finding probabilities. Because the question in the survey concerns the individual-specific transitions, it is reasonable to assume that this reflects either the unobserved heterogeneity or information available to their individual status, which economists cannot directly observe.

4.5 Perceived income risk and consumption spending

Due to precautionary saving motives, higher perceived risks induce households to lower current consumption, thus increasing expected consumption growth. Despite such a clear directional prediction in theory, identifying the exact size of such an effect(i.e., perceived risks on ex-ante consumption/saving decisions that are separate from the ex-post income impacts) has been challenging

[36] Assuming the reported probability of separation from the current job in the next 12 months be $P_{i,t}(ue_{t+12}|e_t)$, then the corresponding monthly Poisson rate of job-separation is $-log(1-P_{i,t}(u_{t+12}|e_t))/12$. This follows from the fact that for a continuous-time Poisson point process with an event rate of θ, the arrival probability over a period of Δt units of time is equal to $1-exp^{-\theta\Delta t}$. With the realized month-to-month flow rate estimated from CPS $P(ue_{t+1}|e_t)$, the corresponding realized Poisson rate is $-log(1-P(ue_{t+1}|e_t))$.

[37] The observation of March 2020 was dropped in the graph, otherwise, it overshadows all other observations in the sample.

Figure 4: Expected and Realized Job-separation/finding Rate

Note: Realized job separation/finding rates are computed from CPS following the method of Fujita and Ramey (2009). Both realizations and perceived probabilities are expressed as Poisson point rates in continuous time, with one month as the unit of time. 3-month moving average of each series is plotted.

when conventional data source are used as it does not directly elicit ex-ante plans and perceptions at the individual level. This section shows that the coexistence of individual-specific perceived risks and the consumption plan of the same individual in SCE provides a rare opportunity to resolve this problem.[38] This contrasts with the best practice to date, which is to impute ex-ante unemployment risks to a particular individual on the basis of only a number of observable factors from realizations(Harmenberg and Öberg, 2021).

I run on the same individual's expected wage growth and perceived income risks a regression of expected consumption growth, specified below, under a range of specifications.

$$E_{i,t}(\Delta c_{i,t+1}) = u_0 + u_1 E_{i,t}(\Delta w_{i,t}) + u_2 \text{Var}_{i,t}(\Delta w_{i,t+1}) + \xi_{i,t}$$

In the past, the literature took it for granted that such a reduced-form regression has a clear correspondence to the commonly used approximated Euler Equation to the second-order (for instance, Parker and Preston (2005)), where the expected consumption growth is equal to the sum of intertemporal substitution and the precautionary saving motive. But a linearly approximated Euler equation is reasonable only under a set of unrealistic and stringent assumptions, such as the absence of external borrowing constraint, the absence of the buffer-stock-saving behavior as elaborated in Carroll and Samwick (1997), and mild-sized income fluctuations, a point forcefully made by Carroll (2001). Therefore, in the regression results below, I primarily focus on testing the significance and the qualitative effects of precautionary saving motives, without providing a structural interpretation of the size of the estimated coefficient.

Across all specifications, as reported in Table 2, in addition to the significantly positive coefficient of expected wage growth, which is consistent with the buffer-stock-saving behavior, perceived risk is positively correlated with the expected spending growth, as the precautionary saving motive predicts. Specifically, after controlling for individual fixed effect (e.g., the discount rate), and time

[38] Other work which examines the impacts of expectations on readiness to spend include Bachmann et al. (2015) and Coibion et al. (2020). Recently, in closely related studies, Fuster et al. (2020) and Bunn et al. (2018) have relied on survey answers to measure stated marginal propensity to consume.

fixed effect (e.g., interest rate), each unit increase in perceived variance leads to around a 1.7 percentage point increase in expected spending growth. Additionally, for the same individual, the perceived unemployment probability, measured by perceived job separation probability, in the next 4 months also has a significantly positive correlation with expected consumption growth. [39]

Table 2: Perceived Income Risks and the Household Spending Plan

	(1)	(2)	(3)	(4)	(5)
expected wage growth	0.324***	0.306***	0.254***	0.243***	
	(0.0825)	(0.0828)	(0.0334)	(0.0334)	
perceived wage risk	6.127***	6.185***	2.096***	1.711***	
	(1.163)	(1.165)	(0.439)	(0.442)	
perceived UE risk next 4m					0.353***
					(0.0553)
R-squared	0.000939	0.00318	0.953	0.953	0.633
Sample Size	56046	56046	56046	56046	6269
Time FE	No	Yes	No	Yes	Yes
Individual FE	No	No	Yes	Yes	Yes

Regression results of expected spending growth on perceived income risks. Standard errors are clustered by household. *** p<0.001, ** p<0.01 and * p<0.05.

5 Perceived risks and wealth inequality

5.1 An overlapping-generation model

I set up a standard incomplete market/life-cycle/general-equilibrium model without aggregate risks. The model structure resembles that of Huggett (1996), and it embeds a more realistic income risk profile and economic environment a la Carroll and Samwick (1997), Krueger et al. (2016) and Carroll et al. (2017).

[39] One common econometric concern with running regressions of this kind is the measurement error in the regressor, i.e. the perceived risks. In a typical OLS regression in which the regressor has i.i.d. measurement errors, the coefficient estimate for the imperfectly measured regressor has a bias toward zero. For this reason, if I find that expected spending growth is indeed positively correlated with perceived risks, taking into account the bias, it implies that the correlation between the two is greater.

In each period, a continuum of agents is born. Each agent i lives for L and works for T ($T \leq L$) periods since entering the labor market, during which he/she earns stochastic labor income y_τ at the work-age of τ. After retiring at age of T, the agent lives for another $L - T$ periods of life and receive social security benefits. Without aggregate risks, there is no need to treat calendar time t and the working age τ as two separate state variables, hence I suppress time script t from now on. All shocks are idiosyncratic.

5.1.1 Consumer's problem

The consumer chooses the whole future consumption path to maximize expected life-long utility under a discount factor β and constant survival probability $(1 - D)$.

$$\max \quad \mathbb{E}\left[\sum_{\tau=0}^{\tau=L-1}(1-D)^\tau \beta^\tau u(c_{i,\tau})\right] \tag{9}$$

where $c_{i,\tau}$ represents consumption at the work-age of τ. The felicity function $u(c)$ takes a standard CRRA form with a relative risk aversion coefficient of ρ: $u(c) = \frac{c^{1-\rho}}{1-\rho}$. [40]

Denote total cash in hand at the beginning of the period τ as $m_{i,\tau}$, the end-of-period saving in period τ after consumption as $a_{i,\tau}$, and the bank balance in period τ as $b_{i,\tau}$. Labor income y_τ is taxed at an income rate of λ and social tax rate λ_{SS}. Also, assume R is the gross real interest factor. The consumer starts with some positive bank balance in the first period of life, b_1, which may partly come from a lump-sum accidental bequest from the deceased population each period. The household makes consumption and saving decisions subject to the following intertemporal budget constraint.

[40] There is no bequest motive or preference-shifter along life cycle, but these features can be easily incorporated.

$$
\begin{aligned}
& a_{i,\tau} = m_{i,\tau} - c_{i,\tau} \\
& b_{i,\tau+1} = a_{i,\tau} R \\
& m_{i,\tau+1} = b_{i,\tau+1} + (1-\lambda)(1-\lambda_{SS}) y_{i,\tau+1} \\
& a_{i,\tau} \geq 0
\end{aligned}
\tag{10}
$$

The last line of the equation above is the no-borrowing constraint.

5.1.2 Income process

Each agent receives stochastic labor income during her working age from $\tau = 0$ to $\tau = T$ and receives a social security benefit after retirement. The income processes in both sub-periods can be defined in a generic manner as described below. By allowing the possibility of persistent unemployment spells, the process is assumed to follow a slight variant of the standard permanent/transitory income process used in the literature[41]. Specifically, $y_{i,\tau}$ is a multiplication of the idiosyncratic wage rate[42] $n_{i,\tau}$ and the economy-wide wage rate W. The former consists of one permanent component $p_{i,\tau}$ and one potentially persistent or transitory $\xi_{i,\tau}$. The aggregate wage is to be determined by the forces of general equilibrium.

$$
\begin{aligned}
y_{i,\tau} &= n_{i,\tau} W \\
n_{i,\tau} &= p_{i,\tau} \xi_{i,\tau}
\end{aligned}
\tag{11}
$$

During the work, the permanent wage component is subject to a mean-one white-noise shock ψ in each period and grows according to a deterministic life-cycle profile governed by $\{G_\tau\}_{\tau=1\ldots L}$,

[41] Carroll et al. (2017), Kaplan and Violante (2018), etc.

[42] This is equivalent to the usual interpretation of it in the literature as idiosyncratic productivity under the implicit assumption of a perfectly inelastic labor supply.

which, according to existing estimates, usually follows a hump-shape (e.g. Gourinchas and Parker (2002)).

$$\begin{aligned} p_{i,\tau} &= G_\tau p_{i,\tau-1}\psi_{i,\tau} \\ log(\psi_{i,\tau}) &\sim N(-\frac{\sigma_\psi^2}{2}, \sigma_\psi^2) \quad \forall \tau \leq T \end{aligned} \tag{12}$$

The persistent/transitory shock $\xi_{i,\tau}$ takes different values depending on the transitory or persistent state of unemployment, which follows a Markov process.[43]

$$\begin{aligned} \xi_{i,\tau} &= \begin{cases} \theta_{i,\tau} \quad \text{if} \quad \nu_{i,\tau} = e \quad \& \quad \tau \leq T \\ \zeta \quad \text{if} \quad \nu_{i,\tau} = u \quad \& \quad \tau \leq T \\ \mathbb{S} \quad \text{if} \quad \tau > T \end{cases} \\ log(\theta_{i,\tau}) &\sim N(-\frac{\sigma_\theta^2}{2}, \sigma_\theta^2) \end{aligned} \tag{13}$$

where ζ is the replacement ratio of the unemployment insurance and $\theta_{i,\tau}$ is the i.i.d. mean-one white noise shock to the transitory component of the income conditional on staying employed. Notice that this process also embodies the income process after retirement after $\tau = T$. The agent receives social security with a replacement ratio $\mathbb{S}$, and proportional to her permanent income and aggregate wage rate. That is, the effective pension benefit received is $\mathbb{S}p_{i,\tau}W$. I assume that the permanent income component after retirement follows a deterministic path without additional stochastic shocks.

During the work age of any individual i, the transition matrix between unemployment ($\nu_{i,\tau} = u$) and employment ($\nu_{i,\tau} = e$) is the following.

[43] This formulation follows Krueger et al. (2016).

$$\pi(\nu_{\tau+1}|\nu_\tau) = \begin{bmatrix} \mho & 1-\mho \\ 1-E & E \end{bmatrix} \tag{14}$$

In general, this assumption implies to some degree unemployment risks persist, but it conveniently nests the special case in which the unemployment risk is purely transitory when $\mho = 1 - E$, meaning the probability of unemployment is not dependent on the current status.

Unemployment risks are idiosyncratic. Hence, by the law of large numbers, the fraction of the population that is unemployed and employed at each age, denoted by $\Pi^{\mho}_\tau$ and Π^{E}_τ, respectively, are essentially deterministic, and is not dependent on age.

Notice that in the benchmark model laid out here, I assume the all parameters of income risks σ_ψ, σ_θ, $\mho$, and E are age-invariant (equivalent to time-independent in this setting). Doing this allows me to avoid restricting the heterogeneity in income risks only by only the dimension of age. I allow for income risks to be stochastic/state-dependent in the extensions of the model discussed in Appendix A.11.

5.1.3 Value function and consumption policy

The following value function characterizes the consumer's problem.

$$V_\tau(\nu_{i,\tau}, m_{i,\tau}, p_{i,\tau}) = \max_{\{c_{i,\tau}, a_{i,\tau}\}} \quad u(c_{i,\tau}) + (1-D)\beta\mathbb{E}_\tau\left[V_{\tau+1}(\nu_{i,\tau}, m_{i,\tau+1}, p_{i,\tau+1})\right] \tag{15}$$

where the three state variables for the agents are current employment status $\nu_{i,\tau}$, total cash in hand $m_{i,\tau}$ and permanent income $p_{i,\tau}$. $\nu_{i,\tau}$ drops from the state variables in the special case of a purely transitory unemployment shock ($\mho = 1 - E$).[44]

[44]Another trick used in the literature to reduce the number of state variables is to normalize the value function by permanent income level p_τ, so that it drops from the state variable. I also use the endogenous grid method (EGM) developed by Carroll (2006).

The solution to the stated problem above is a set of age-specific optimal consumption policies, $c^*_\tau(u_{i,\tau}, m_{i,\tau}, p_{i,\tau})$, and the saving policies, $a^*_\tau(u_{i,\tau}, m_{i,\tau}, p_{i,\tau})$. Both are functions of all state variables.

5.1.4 Technology

The economy has a standard CRS technology that turns the capital and supplied efficient units of labor into aggregate output.

$$Y = ZK^\alpha N^{1-\alpha} \tag{16}$$

The capital depreciates at a rate of δ each period. The factors of input markets are fully competitive.

5.1.5 Demographics

For the sake of simplicity, I assume there is no population growth. With a deterministic life-cycle profile of survival probabilities, there exists a stable age distribution $\{\mu_\tau\}_{\mu=1,2,..L}$ such that $\mu_{\tau+1} = (1-D)\mu_\tau$ and $\sum_{\tau=1}^{L} \mu_\tau = 1$. The former condition reflects the probability of survivals at each age and the latter is a normalization that guarantees the fraction of all age groups sum up to 1.[45]

5.1.6 Government

Government runs a balanced budget in each period. Therefore, outlays from unemployment insurances are financed by the income tax that is levied on both labor income and unemployment benefits. Given a replacement ratio ζ and the proportion of employed population $1 - \Pi^{\mho}$, the

[45] In a more general setting with a constant population growth rate n and age-specific survival probability $1 - D_\tau$, the condition becomes $\mu_{\tau+1} = \frac{(1-D_{\tau+1})}{1+n}\mu_\tau \quad \forall \tau = 1, 2...L$, as discussed in Ríos-Rull (1996) and Huggett (1996).

corresponding tax rate λ can be easily pinned down on the basis of the equation below. [46]

$$\lambda \left[1 - \Pi^{\mho} + \zeta\Pi^{\mho}\right] = \zeta\Pi^{\mho} \tag{17}$$

The Social Security tax rate λ_{SS} is also determined in the model by the pension replacement ratio $\mathbb{S}$, the permanent income ratio, the relative population size of the retired and the working age, and the aggregate employment rate.

$$\lambda_{SS} \sum_{\tau=1}^{T} \mu_\tau G_\tau (1 - \Pi^{\mho}) = \mathbb{S} \sum_{\tau=T+1}^{L} \mu_\tau G_\tau \tag{18}$$

5.1.7 Stationary equilibrium

Denote $x = \{m, p, \nu\} \in X$ as the idiosyncratic state of individuals. At any point in time, agents in the economy differ in age τ and their idiosyncratic state x. The former is given by $\{\mu_\tau\}_{\mu=1,2,..L}$. For the latter, $\psi_\tau(B)$ is used to represent the fraction of agents at age τ whose individual states lie in B as a proportion of all age τ agents. The distribution of age $\tau = 1$ agents depends on the initial condition of labor income outcomes and the size of accidental bequests, if any. For any other age $\tau = 2...L$, the distribution $\phi_\tau(B)$ evolves as the following.

$$\psi_\tau(B) = \int_{x \in X} P(x, \tau - 1, B) \mathrm{d}\psi_{\tau-1} \quad \text{for all} \quad B \in B(X) \tag{19}$$

where $P(x, \tau - 1, B)$ is the probability that an agent will transit to B in the next period, conditional on the individual state x at age $\tau - 1$. The transition function depends on the optimal consumption policy $c^*(x, \tau)$ at age τ and the exogenous transition probabilities of income shocks.[47]

In the absence of the aggregate risk, I focus on the stationary equilibrium of the economy (StE),

[46] This convenient result crucially depends on the assumption that the unemployment insurance benefit is paid proportionally to permanent income.

[47] In the model computation, the P functions correspond to age-specific transition matrices over a finite number of discretized grid points of multiple state variables. The age-specific distributions $\psi_\tau(B)$ are generated by forward iteration of multiplying the distribution of age $\tau - 1$ by the transition matrix of age τ.

which consists of consumption and saving policies $c(x,\tau), a(x,\tau)$, as well as constant production factor prices, including the real interest rate R and the wage W, the initial wealth of newborn b_1, unemployment benefit ζ, tax rate λ, and the time-invariant distribution $(\psi_1, \psi_2, \ldots, \psi_L)$ such that

1. Consumption and saving policies are optimal given the real interest rate R, wage W, the tax rate λ.

$$c(x,\tau) = c^*(x,\tau)$$

$$a(x,\tau) = a^*(x,\tau)$$

2. Distributions $(\psi_1, \psi_2, \ldots, \psi_L)$ are consistent with optimizing behaviors of household, as described in Equation 19.

3. The factor markets are clearing.

$$\begin{aligned} &\sum_\tau \mu_\tau \int_X a(x,\tau)\mathrm{d}\psi_\tau = K \\ &\sum_{\tau=0}^{T-1} \mu_\tau \Pi_\tau^E = N \end{aligned} \tag{20}$$

4. Firm optimization under competitive factor markets.

$$W = Z(1-\alpha)(K/N)^\alpha$$

$$R = 1 + Z\alpha(K/N)^{\alpha-1} - \delta$$

5. Initial bank balance of new borns are equal to accidental bequests.

$$b_1 = \sum_\tau \mu_\tau D \int_{x\in X} a(x,\tau) R \mathrm{d}\psi_\tau$$

6. The government budget is balanced as described in Equation 17 and 18.

The economy may potentially arrive at different stationary equilibria, depending on the specific assumptions about the size and heterogeneous income risks, which in this model, include σ_ψ, σ_θ, E, and $\mho$.

5.2 Calibration

The central inputs of the model in this paper–the size and the heterogeneity in perceived income risks–are estimated from the survey, using the auxiliary model laid out in Section 4.3. Here I discuss other model parameters in great detail.

Life-cycle. The model is set at a yearly frequency. The working age spans 25 years old to 65 years old ($T = 40$) and the agent dies with certainty at age of 85 ($L = 60$). The constant death probability before the terminal age is set as $D = 0.625\%$.

Regarding the deterministic permanent income profile over the life-cycle, G_τ, I draw on an age polynomial regression of the wage growth from SIPP for workers aged 25-65 while controlling for other observable demographic variables such as education, gender, occupation, and time fixed effects, etc. This yields estimation results very similar to those obtained by Gourinchas and Parker (2002), Cagetti (2003) and Kaplan and Violante (2014). The estimated wage profile is plotted in Appendix A.13. For the retirement phase, I assume a one-time drop of 20% in permanent wage at age 66, i.e. $G_{41} = 0.8$, and then the permanent wage stays flat till death. This produces an average expected growth rate of permanent income over the entire life-cycle exactly equal to one. This serves as a normalization. Note that although alternative assumptions, such as a more smooth decline of income after retirement, do change the wealth distribution across generations among the retired, they do not change consumption/saving decisions because such a profile is entirely deterministic.

Initial conditions. Assumptions about the cross-sectional distribution of the initial permanent productivity and liquid asset holdings matter for the subsequent wealthy inequality. I set the standard deviation of the log-normally-distributed initial permanent wage $p_{i,\tau}$ to be 0.6 in order to match the the heterogeneity in "usual income" (an approximation of the permanent income) at age 25 from the SCF. Initial liquid assets holdings at $\tau = 0$ are assumed to have a cross-sectional standard-deviation of 0.50.

Income risks. Given the critical importance of the income risks assumption in my model, in addition to my estimates from SIPP (as reported in Table 1), I thoroughly survey the risk estimates used in the existing incomplete market macro literature, as summarized in Table A.6 in the Appendix. For comparison, I convert all risks into the annual frequency (because some of the estimates are for a different frequency). Whenever group-specific risks are assumed (depending on education and age), I summarize them as a range. Also, for models that assume a persistent instead of a permanent component, I treat the assumed size of the persistent risks as a lower bound for the permanent risk.[48] For models with income risks dependent on aggregate business cycles, a la Krusell and Smith (1998), I compute the steady-state size of idiosyncratic risks using the transition probabilities of the aggregate economy employed in the paper.

Regardless of the disagreement in these estimates, the income risks used in these models are constantly larger than those reported in the survey. This is true for presumably the most comparable one to the surveyed PR among them, the wage risk estimate by Low et al. (2010). I use the median values of each parameter in the literature as the benchmark income risks profile, which is a combination of $\sigma_\psi = 0.15$, $\sigma_\theta = 0.15$. And following the calibration of Krueger et al. (2016), the yearly probability of staying on unemployment is $\mho = 0.18$ and that of staying employed $E = 0.96$.

Technology. The annual depreciation rate is set to be $\delta = 2.5\%$. The capital share takes a standard value of $\alpha = 0.36$, for the U.S. economy. Without aggregate shocks, Z is simply a normalizer. Therefore, I set its value such that the aggregate wage rate W is equal to one under a capital/output ratio $K/Y = 3$ at the steady-state level of employment in the model.

Government policies. As in Krueger et al. (2016), unemployment insurance replacement

[48]One can think of the permanent income shock as a limiting case of AR(1) shock, with the persistence parameter infinitely close to 1. The effective income risks increase with the persistence of the shock.

ratio is set to be $\mu = 0.15$. The pension income relative to the permanent income is assumed to be $\mathbb{S} = 60\%$. This, plus the 20% drop in permanent income, gives an effective deterministic wage drop of 48% from the working-age to retirement, which corresponds to an empirical replacement ratio estimated for the U.S. economy. The corresponding tax rates that finance the unemployment insurance and social security is determined by the equilibrium within the model.

Preference. The coefficient of relative risk aversion $\rho = 2.0$, which is common in this literature. Following the common practice in the literature, the discount factor β is internally calibrated to generate a mean wealth/income ratio approximately comparable to the average net liquid wealth to permanent income ratio of 0.69 in SCF. [49]

Table 3 summarizes the parameters used in the calibration of the baseline model. This is nearly identical to what would be considered as a standard calibration of an incomplete market liquid-assets calibration. (Kaplan and Violante (2022))

6 Model results

6.1 Baseline model

I first examine the wealth accumulation and wealth inequality generated from a benchmark calibration, as reported above. In particular, under a set of standard parameterization on permanent and transitory risks at the annual frequency to be $\sigma_\psi = 0.15$ and $\sigma_\theta = 0.15$, and the unemployment risks to be $U2U = 0.18$ and $E2E = 0.96$, the baseline of Figure 5 reproduces the well-known result[50] that a carefully calibrated standard one-asset incomplete market model without additional heterogeneity, such as that in time discount rates predicts less wealth inequality (a Gini coefficient

[49] Kaplan and Violante (2022) discusses in details how the internally calibrated discount factors in one-asset models differ depending on targeting liquid wealth or total net worth. Their calibration of β is 0.945 for a targeted liquid-asset-to-income ratio of 0.6, and 0.98 for a targeted net-worth-to-income ratio of 4.6. This is the same as the average value estimated in the models with heterogeneous time preferences, as in Carroll et al. (2017) and Krueger et al. (2016).

[50] See Guvenen (2011), De Nardi (2015), and Kaplan and Violante (2018) for a thorough survey on this topic.

Table 3: Model parameters

block	parameter name	values	source
risk	σ_ψ	0.15	Median estimate from the literature
risk	σ_θ	0.15	Median estimate from the literature
risk	$U2U$	0.18	Median estimate from the literature
risk	$E2E$	0.96	Median estimate from the literature
initial condition	$\sigma_\psi^{\text{init}}$	0.629	Estimated for age 25 in the 2016 SCF
initial condition	bequest ratio	0	assumption
life cycle	T	40	standard assumption
life cycle	L	60	standard assumption
life cycle	$1-D$	0.994	standard assumption
preference	ρ	2	standard assumption
preference	β	0.96	calibrated to match liquid-wealth-to-income ratio
policy	$\mathbb{S}$	0.65	U.S. average
policy	λ	N/A	endogenously determined
policy	λ_{SS}	N/A	endogenously determined
policy	μ	0.15	U.S. average
production	W	1	target values in steady state
production	K2Y ratio	3	target values in steady state
production	α	0.33	standard assumption
production	δ	0.025	standard assumption

Parameters used in the baseline model. All parameters, whenever relevant, are at the annual frequency.

of 0.63 in partial and 0.64 in general equilibrium) than the liquid wealth inequality in the data. For instance, the distribution of net liquid wealth based on the definition of Kaplan et al. (2014) and Carroll et al. (2017) [51] has a Gini coefficient of 0.88 in the 2016 SCF.[52]

The second major discrepancy between the model and data is that the former significantly underpredicts the share of agents who are close to borrowing constraints. In particular, the baseline model predicts a share of hands-to-month households (H2M) (defined as agents whose ratios of wealth to annual permanent income is below 1/24) less than 1%, which is significantly lower than 0.31, the share computed based on net liquid wealth in SCF. It is known that the strong precautionary saving motives in this model incentivize agents to build buffer-stocks and stay away from their borrowing constraints.

[51] According to this definition, liquid asset includes checking, saving, money market funds, government bonds, directly held mutual funds, stocks and corporate bonds, and liquid debt is the sum of all credit card balances that accrue interest, after the most recent payment.

[52] I exclude the households in SCF with negative net liquid wealth and the top 5% in terms of total net worth. The former is meant to be consistent with the no-borrowing constraint assumption. The latter is also a common practice in the literature (for instance, Kaplan and Violante (2022)) because the one-asset model has been found to poorly explain the consumption/saving behaviors of the super rich.

The baseline model proves to be a more successful starting point in terms of matching the life-cycle patterns of wealth accumulation. The baseline in the bottom panel of Figure 5 plots the hump-shaped average wealth over the life cycle implied by the model against the average life-cycle profile of net liquid wealth (for PE) and net worth (for GE) computed from SCF. I use total net worth to benchmark the results from GE primarily because this is more consistent with the model assumption that savings are used as productive capital of the economy. The model-implied wealth accumulation over the life cycle roughly resembles the observed pattern from the data. In particular, allowing the voluntary bequest in the last period of life helps me match the saving behaviors better after retirement.

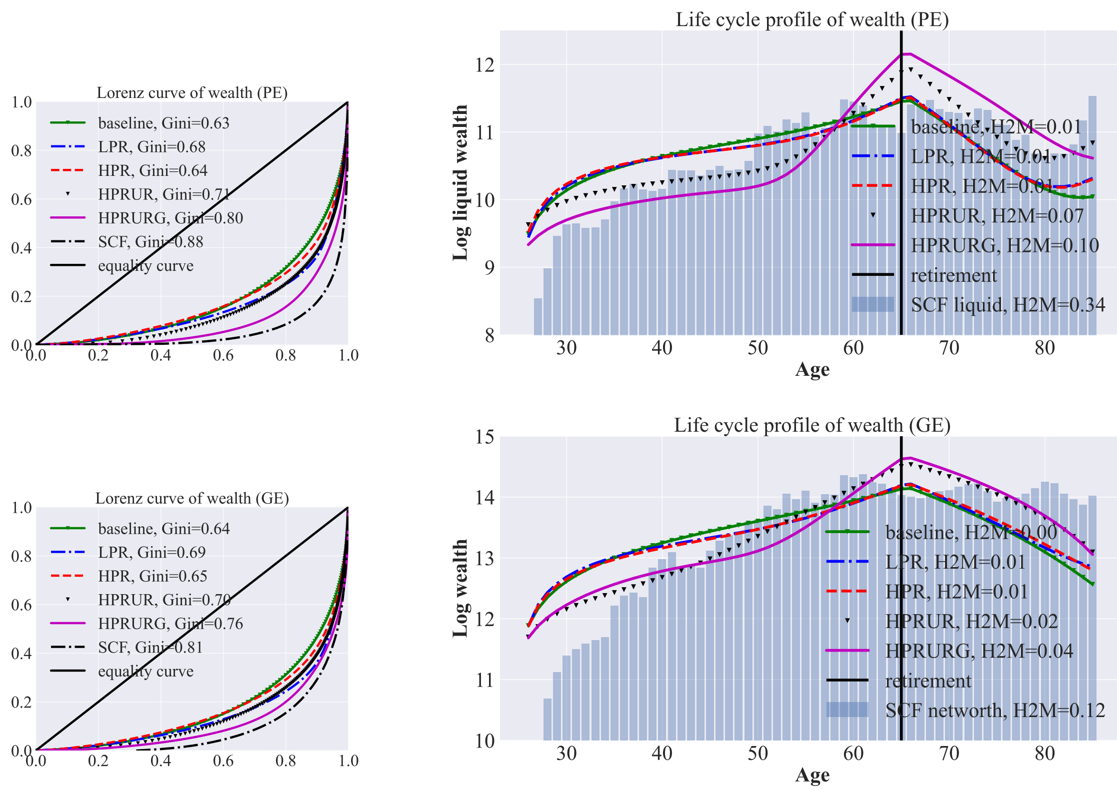
6.2 Model results with survey-implied risks

In this section, I sequentially add the following five features of income risks, estimated from the survey-reported PRs, and show that it generates a higher wealth Gini and a fraction of H2M, compared to in the baseline model, which are closer to that observed in the data. First is an average lower wage risk (LPR). Second is heterogeneous perceived wage risks in addition to the average lower size (HPR). Third is heterogeneous unemployment risks ($HPRUR$) as revealed in perceived U2U and E2E probabilities. Fourth, in addition to heterogeneous risks, I allow for heterogeneous growth rates of wage ($HPRURG$). Finally, I allow for time preference heterogeneity in addition to perceived income risks ($HPRURGTP$).

6.2.1 Lower wage risks (LPR)

For LPR calibration, I keep everything the same as in the baseline calibration above, except that I make the permanent and transitory risks smaller on the basis of an average perceived risk of 0.04, i.e. $\sigma_\psi = 0.03$ and $\sigma_\theta = 0.03$. In the meantime, I calibrate the ex-ante unobserved heterogeneity anticipated by the agents using the estimates of $\sigma^2_{\xi,\psi}$ and $\sigma^2_{\xi,\theta}$ produced in Section 4.3.

Figure 5: Wealth Inequality in Partial and General Equilibrium: A Model Comparison



Note: The panel shows, under various model assumptions, the Lorenz curve of households' wealth (left) and the model-generated life-cycle profile of log average wealth compared to the average net liquid wealth by age in the 2016 vintage of Survey of Consumer Finance (SCF) (right), both in the partial equilibrium (upper panel) and in general equilibrium (bottom panel).

This additional step of reconfiguration of the relative importance of risks and heterogeneity is crucial to ensure comparability with the baseline model. Other things equal, a smaller size of income risks would have mechanically lowered realized income inequality in the model. In order for the model to still generate realistic income inequality as seen in the labor income data, the differences between the previous and new calibrated risks have to be "attributed" to unobserved heterogeneity.

The LPR in Figure 5 shows two implications of a smaller size of risks and a larger role of anticipated heterogeneity. First, a lower PR induce a milder precautionary saving motive and reduces buffer-stock savings of all working agents, as indicated by a lower level of wealth to income ratio than in the baseline model. This also results in a slightly larger fraction of H2M agents (1% in both PE and GE) compared nearly zero in the baseline model.

Second, allowing for a larger role in heterogeneity instead of risks unambiguously leads to *more* wealth inequality than in the baseline model (A Gini coefficient of 0.68 in PE and 0.65 in GE), as shown in Figure 5.

6.2.2 Heterogeneous wage risks (HPR)

As shown in Section 4.1, a large degree of heterogeneity in PRs is attributable to individual fixed effects, which might reflect the true ex-ante heterogeneity in wage risks that different individuals face beyond common observable factors. Hence, I directly calibrate the heterogeneity in wage risks using the estimated distribution of PRs, which is detailed in Section 4.3.

I use three equally probable values $[0.01, 0.02, 0.04]$ for σ_ψ and σ_θ, which are discretized from the estimated log-normal distribution of PRs to calibrate such heterogeneity. On the top of LPR, allowing heterogeneity in PRs unambiguously contributes to more wealth inequality because it induces different precautionary saving motives and buffer stock savings. But this is counteracted by a lower average risk, which objectively induces less income and wealth inequality, as discussed in LPR. As a result of the two competing forces, the wealth Gini coefficient increases by only one percentage point to 0.64 in HPR, and from 0.64 to 0.65 in GE.

Recalibrations in both LPR and HPR scenarios do take the baseline model closer to matching the data, but it is worth noting that the improvements in model performance are not sufficiently large. This is particularly so especially when it comes to matching the size of H2M agents. It suggests that only incorporating heterogeneity in wage risks can be complemented by recalibrating another important source of heterogeneity, namely the unemployment risks.

6.2.3 Heterogeneous unemployment risks (HPRUR)

Just like the calibration of wage risks, a common calibration strategy of incomplete market models with unemployment spells typically parameterizes the model with one homogenous pair of $U2U$

($\mho$ in the model) and $E2E$ (E in the model) probabilities (e.g., Krueger et al. (2016)). But this assumption may mask the unobserved heterogeneity among agents and their true perceived unemployment risks given the information they have about their own idiosyncratic circumstances (Mueller and Spinnewijn (2021)).

To capture the heterogeneity in unemployment risks, I adopt the same approach that in Section 4.3 I apply to perceived wage risks to fit a truncated log normal distribution to the survey-reported perceived U2U and E2E probabilities (See Figure A.15). The estimated distribution is further discretized into three equally probable grid points $[0, 0.02, 0.24]$ of $U2U$ and $[0.96, 0.99, 1.0]$ of $E2E$.

The resulting model, which has both heterogeneous wage risks and unemployment risks ($HPRUR$) generates a significantly higher degree of wealth inequality (an increase of 8 percentage points in Gini coefficient to 0.71 in PE and 6 percentage points increase in GE). In addition, the fraction of H2M households also substantially increase to 7% in PE and 2% in GE.

Particularly noteworthy is the fact that calibrating unemployment risks produces a better fit of the life-cycle profile of wealth to data than the baseline model and LPR.

6.2.4 Heterogeneous growth rates (HPRURG)

Although the main theme of this paper is the heterogeneity in income risks, the density forecasts also provide individual-specific expected wage growths that exhibit substantial heterogeneity: a standard deviation across respondents of 0.03, or 3 percentage points in annual growth rates (See Figure A.15).

Here, I further extend the $HPRUR$ model to incorporate this heterogeneity. In particular, I allow the heterogeneity in the 1-year-ahead wage growth expectations to be translated into three equally probable distinctive deterministic wage profiles, which are plotted in Figure A.14. The mean profile corresponds to the baseline model calibration of $\{G_\tau\}_{\tau=1...L}$. Because the direct survey inputs needed to capture heterogeneity in the deterministic wage growth path–expected wage growths over the life-cycle– are not available, the calibration above essentially makes the assumption that the heterogeneity in wage growth from one year to another completely reflects perceived differences in the permanent component of the wage growth.

As shown in Table 4, the implied wealth Gini from allowing for heterogeneous growth rates ($HPRURG$) in addition to heterogeneous income risks increases from 0.71 to 0.80 in PE and from 0.70 to 0.76 in GE. Moreover, the H2M ratios further see a sizable increase to 10% in PE and 4% in GE.

Table 4: Summary of models results and data

Model/Data	Gini	Top 0.05	Top 0.1	Top 0.5	Mean wealth/income ratio	H2M share
SCF (liquid)	0.88	0.72	0.82	0.99	0.67	0.34
baseline (PE)	0.63	0.40	0.53	0.89	1.17	0.01
HPR (PE)	0.64	0.43	0.57	0.89	0.84	0.01
HPRUR (PE)	0.71	0.48	0.62	0.93	0.51	0.07
HPRURG (PE)	0.80	0.56	0.70	0.97	0.63	0.10
SCF (net worth)	0.81	0.57	0.71	0.98	6.72	0.12
baseline (GE)	0.64	0.40	0.53	0.90	1.65	0.00
HPR (GE)	0.65	0.43	0.57	0.89	1.23	0.01
HPRUR (GE)	0.70	0.47	0.61	0.92	1.12	0.02
HPRURG (GE)	0.76	0.52	0.65	0.95	0.99	0.04

Model-implied Gini coefficients, the wealth shares owned by the top 5, 10 and 50 percent of the agents, mean wealth-to-income ratio, and shares of hand-to-mouth agents (H2M), in the stationary distribution of partial and general equilibrium. H2M is defined as those whose liquid wealth is no more than two weeks of(1/24 of annual) income. The same statistics in the data are computed for both net liquid wealth, and total net worth using 2016 SCF.

6.2.5 The role of preference heterogeneity

One of the widely used additional features added to the baseline model in the existing literature to account for the model-data gap in wealth inequality is heterogeneity in preferences (more commonly, that in time discount rates, β in the model) (Krusell and Smith (1998), Krueger et al. (2016), Carroll et al. (2017).) Such an assumption has been gradually supported by some empirical evidence and laboratory experiments.[53] However, despite such indirect evidence, the exact degree

[53] For instance, Epper et al. (2020) directly elicited time preferences of individuals via experiments and show that they have real effects on wealth accumulation.

of time preference heterogeneity in the model cannot be directly observed and estimated. Thus, the authors commonly adopt the "revealed preference" approach to indirectly infer the model-implied heterogeneity in preferences.

In contrast, this paper argues that survey-implied heterogeneity has the advantage of being directly observable and useful in the model. Hence, the heterogeneity of income risks and growth rates is another observable factor that should be first accounted for before attributing the unexplained wealth inequality to solely preference heterogeneity. Another advantage (not explored in this paper), is that disciplining the model with observed heterogeneity, such as in income risks perceived by agents, makes the model more transparent and allows welfare analysis to be carried out with greater clarity than in the unobserved preference heterogeneity approach.

It would have been a straightforward exercise for this paper to quantitatively compare the estimated preference heterogeneity from the baseline model and the preferred model that additionally accounts for the observable heterogeneity in income risks. As shown in Table 4, an incremental recalibration of the baseline model gradually reduces the model residuals in comparison with the data. So it shall be no surprises that the indirectly estimated preference heterogeneity will be less.

6.3 Subjective perceived risks

So far, all of the model experiments have maintained the assumption of full-information-rational-expectation. I allow for heterogeneity in these risk parameters across agents, but I treat the survey-implied risks of as the true model risk parameters that determine the dispersion of income shocks–a calibration alternative to the conventional assumptions.

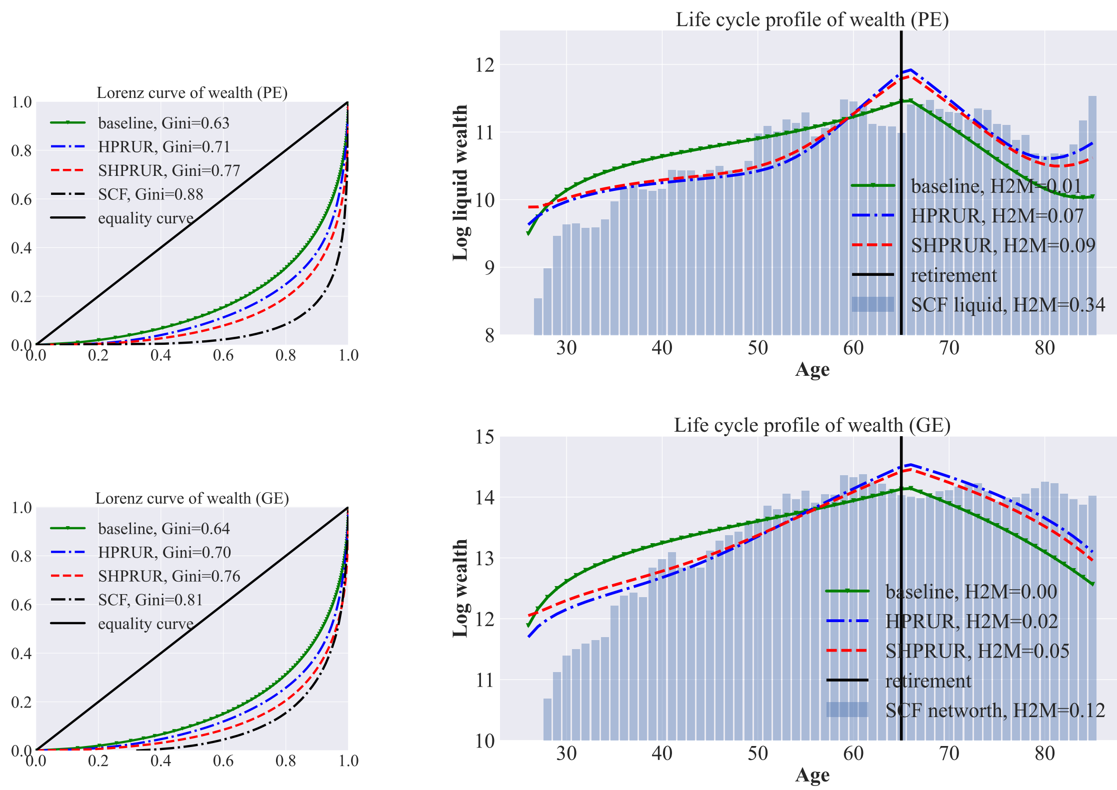
But it is critical to consider how robust the results are if we adopt a different assumption that agents' perceive risks as reported in the survey only shape their consumption/saving decisions (as calibrated $HPRUR$), but are somehow different from the true underlying risk parameters, which objectively govern the distribution of income shocks (as calibrated in the baseline model).

Such a model exercise is actually not just a robustness check, but also an experiment model that breaks down the model effects of heterogeneous and lower income risks on wealth inequality into two channels. The first channel can be called the "choice" channel because it is via ex-ante consumption/saving decisions of the agents based on certain perceived income risks. The second channel can be called the "outcome" channel because it is a function of the ex-post realized dispersion of income shocks.

The key finding of this exercise is that the additional wealth inequality from heterogeneous income risks is driven by the ex-ante choice channel, which is counteracted by the outcome channel, while the higher H2M share is a consequence of both the ex-ante and the ex-post mechanisms. Figure 6 compare the subjective model $SHPRUR$ with both the baseline and the $HPRUR$ model as calibrated above. The subjective model shifts the Lorenz curve further outward (a Gini of 0.77 in PE and 0.76 in GE) than the baseline model, and the shift is greater than in the objective model. Such a shift only comes from changes in ex-ante saving behaviors when a heterogeneous and lower income risk profile is added to the baseline model. This suggests that even if we don't recalibrate the objective income risks in the baseline model, but, instead, allow the survey-implied risks to serve as a better input when predicting consumption/saving choices, it reduced the difference in wealth inequality unexplained between the model and data. At the same time, the fraction of H2M consumers implied by the subjective model (9% in PE) is also bigger than the objective model, in both PE and GE models.

To summarize, the subjective model results reinforce the key argument of this paper: it shows that even if the perceived income risks reported in the survey are not perfectly "correct" compared to what objectively governs the size of stochastic income shocks, to the extent that household saving decisions are made based on such perceptions, they help general model predictions about wealth accumulation behaviors which are better aligned with the data.

Figure 6: Wealth Inequality in Partial and General Equilibrium: Objective v.s. Subjective



Note: The panel shows, under objective ($HPRUR$) and subjective assumptions ($SHPRUR$), the Lorenz curve of households wealth (left) and the model-generated life-cycle profile of log average wealth compared to the average net liquid wealth by age in the 2016 vintage of Survey of Consumer Finance (SCF) (right) in the partial equilibrium (upper panel) and in the general equilibrium (bottom panel).

7 Conclusion

A large class of incomplete-market macroeconomic models that features uninsured idiosyncratic income risks and resulting wealth inequality does not incorporate one observable dimension of heterogeneity in income risks. Utilizing the New York Fed's *Survey of Consumer Expectations* which elicits density forecasts of wage growth, I explore the model implications of two major empirical findings. The survey-reported perceived risks are more heterogeneous than that is assumed by common calibration of these models, and prove to be another observable factor useful for matching the model-predicted wealth inequality with empirical patterns. Furthermore, perceived risks are lower than the conventional estimates/ calibration, which helps explain why these models usually predict higher buffer stock savings than in the actual data.

This paper demonstrates the rich research potential of incorporating into heterogeneous-agent models survey data that reflects realistic heterogeneity in expectations/perceptions. In a world that offers increasingly rich survey data that directly measures expectations, economists no longer are obligated to calibrate important model parameters such as income risks indirectly from the panel data and or adopting the stringent assumption of rational expectations. The use of survey-implied heterogeneity establishes a direct link between expectations and behaviors, and helps economists match empirical patterns within the macroeconomy better.

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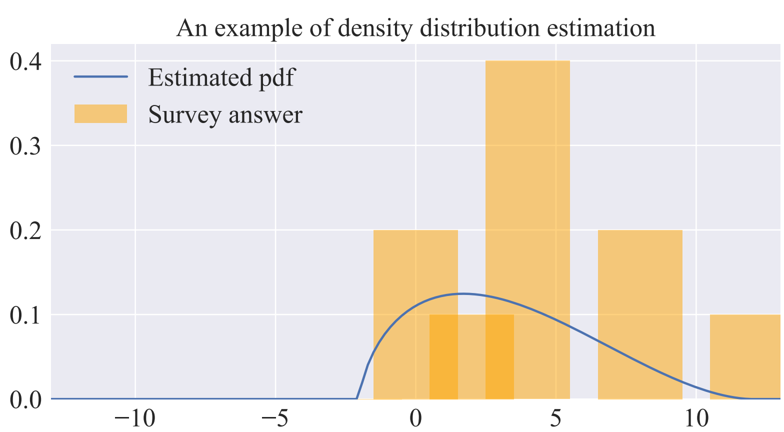
A Online Appendix

A.1 Density estimation of survey answers

With the histogram answers for each individual in hand, I follow Engelberg et al. (2009) to fit each of them with a parametric distribution accordingly for three following cases. (See Figure A.1 for an example.) In the first case, when there are three or more intervals filled with positive probabilities, it was fitted with a generalized beta distribution. In particular, if there is no open-ended bin on the left or right, then a two-parameter beta distribution is sufficient. If there is an open-ended bin with positive probability on either left or right, since the lower bound or upper bound of the support needs to be determined, a four-parameter beta distribution is estimated. In the second case, in which there are exactly two adjacent intervals with positive probabilities, it is fitted with an isosceles triangular distribution. In the third case, if there is only one positive-probability of interval only, i.e. equal to one, it is fitted with a uniform distribution.

Figure A.1: An illustration of the density estimation of the survey answer



Note: This is one example of the bin-based forecast of wage growth at SCE and how it is fit by a parametric distribution. The horizontal axis is the values of expected wage growth and the vertical axis is the probability assigned by the respondent.

For all the moment's estimates, there are inevitably extreme values. This could be due to the idiosyncratic answers provided by the original respondent, or some non-convergence of the numerical estimation program. Therefore, for each moment of the analysis, I exclude top and bottom 1% observations, leading to a sample size of around 53,180.

I also recognize what is really relevant to many economic decisions such as consumption is real income instead of nominal income. I use the inflation expectation to convert expected nominal earning growth to real growth expectations.

The real earning risk, namely the variance associated with real earning growth, if we treat inflation and nominal earning growth as two independent stochastic variables, is equal to the summed variance of the two. The independence assumption is admittedly an imperfect assumption because of the correlation of wage growth and inflation at the macro level. Therefore, throughout the paper, I also report results with nominal wage growth forecast directly.

A.2 Other facts about PR

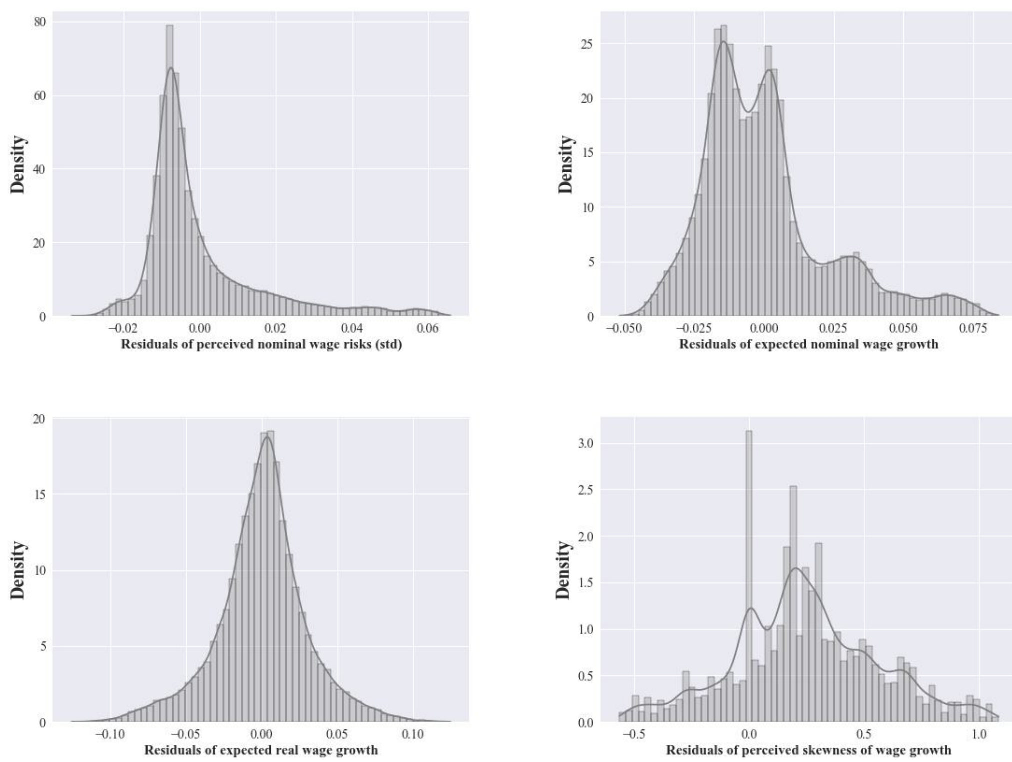
A.2.1 Heterogeneity of expectations in other moments

In addition to Figure 2, Figure A.2 shows the within-group heterogeneity of nominal PRs, expected real and nominal wage growth rates, and perceived skewness controlling for observable demographic variables in SCE.

A.2.2 PR by realized earnings

Standard models with idiosyncratic income risks do not assume heterogeneity by permanent income in addition to the observed group factors that may affect permanent income, such as education. Is it so in risk perceptions? It turns out that PR does correlate with the realized outcomes of the individuals. For a subsample of around 4000 observations, SCE surveys the annual earning of the respondent along with their risk perceptions. I group individuals into 10 groups based on their

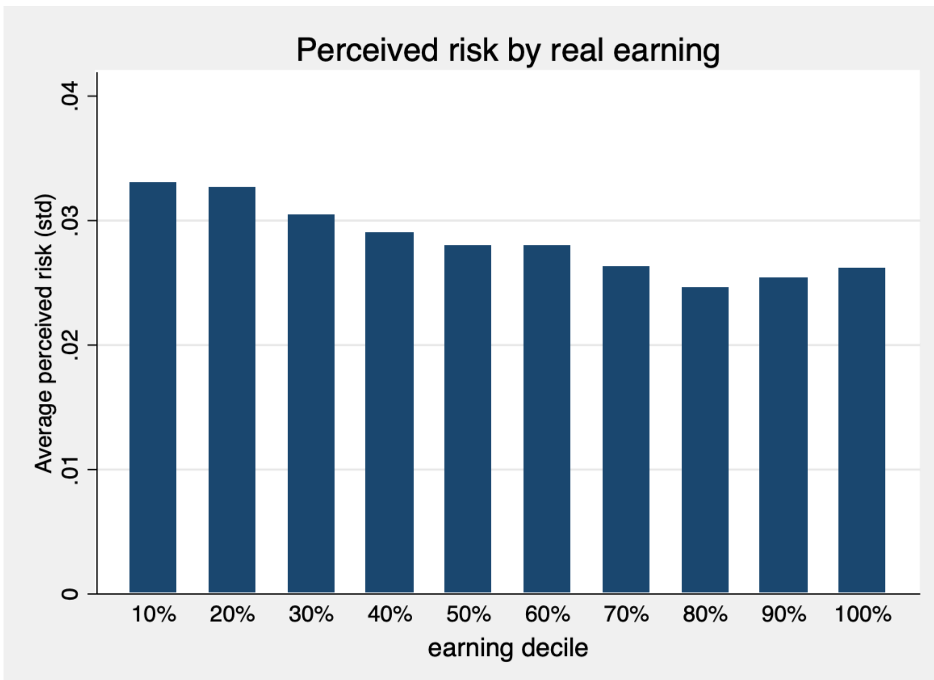
Figure A.2: Dispersion in Expected Wage Growth and Perceived Skewness



Note: The distributions of residuals of nominal PR (in standard deviation terms), expected nominal and real wage growth rates, and perceived skewness of 1-year-ahead wage growth in SCE unexpected by observable demographic variables.

reported earning (within the same time) and plot the average risk perceptions against the decile rank in Figure A.3. Perceived risks decline as one's earnings increase. This is not exactly consistent with the uptick in income risks for the highest income group, as documented by Bloom et al. (2018) using tax records of income. The most likely explanation is that the small sample I used from SCE does not cover actual top earners. The average annual earning of the top income group is between $45,000 and $120,000 in our sample.

Figure A.3: Perceived Wage Risks by Earning Decile



Note: this figure plots average perceived income risks by the decile of annual earning of the same individual.

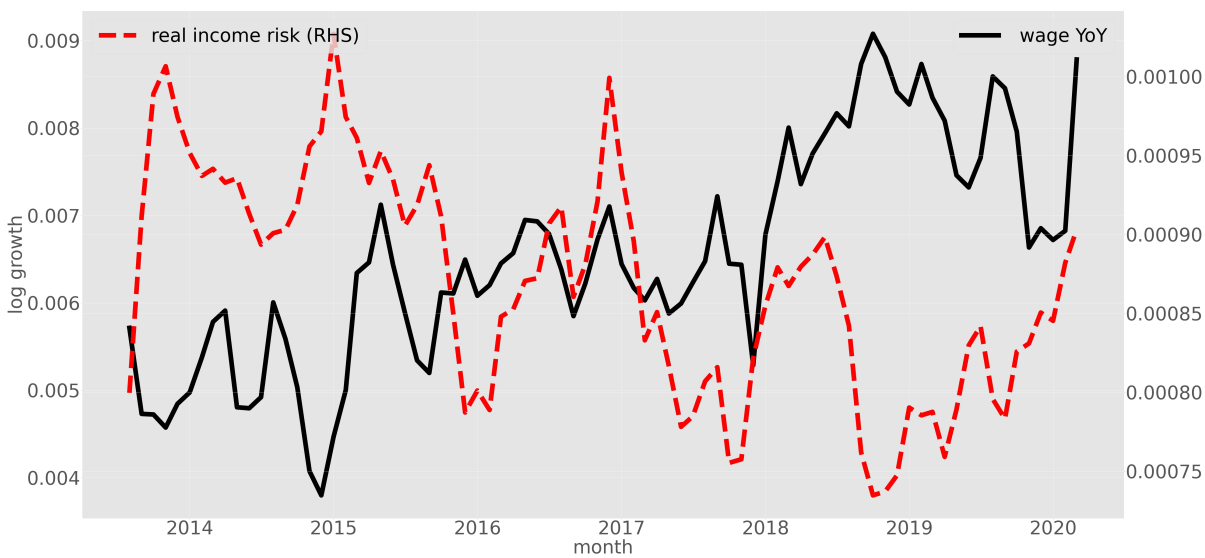
A.2.3 Counter-cyclicality of perceived risk

Some studies have documented that income risks are counter-cyclical based on cross-sectional income data. [54] It is worth inspecting if the subjective income risk profile has a similar pattern. Figure A.4 plots the average perceived income risks from SCE against the YoY growth of the average hourly

[54]But they differ in exactly which moments of the income are counter-cyclical. For instance, Storesletten et al. (2004) found that variances of income shocks are counter-cyclical, while Guvenen et al. (2014) and Catherine (2019), in contrast, found it to be the left skewness.

wage across the United States, which shows a clear negative correlation. Table A.1 further confirms such a counter-cyclicality by reporting the regression coefficients of different measures of average risks on the wage rate of different lags. All coefficients are significantly negative.

Figure A.4: Recent Labor Market Conditions and Perceived Risks



Note: Recent labor market outcome is measured by hourly wage growth (YoY). The 3-month moving average is plotted for both series.

The pattern can also be seen at the state level. Table A.2 reports the regression coefficients of the monthly average perceived risk within each state on the state labor market conditions, measured by either wage growth or the state-level unemployment rate, respectively. It shows that a tighter labor market (higher wage growth or a lower unemployment rate) is associated with lower perceived income risks. Note that our sample stops in June 2019 thus not covering the outbreak of the pandemic in early 2020. The counter-cyclicality will be very likely more salient if it includes the current period, which was marked by catastrophic labor market deterioration and increase market risks.

The counter-cyclicality in subjective risk perceptions seen in the survey may suggest the standard assumption of state-independent symmetry in income shocks is questionable. But it may

Table A.1: Current Labor Market Conditions and Perceived Income Risks

	mean:var	mean:iqr	mean:rvar	median:var	median:iqr	median:rvar
0	-0.28**	-0.42***	-0.48***	-0.16	-0.16	-0.53***
1	-0.44***	-0.54***	-0.51***	-0.02	-0.02	-0.53***
2	-0.39***	-0.44***	-0.43***	-0.05	0.0	-0.45***
3	-0.44***	-0.47***	-0.41***	-0.09	-0.06	-0.5***
4	-0.29**	-0.38***	-0.32***	-0.19	-0.14	-0.5***

*** p<0.001, ** p<0.01 and * p<0.05.

This table reports correlation coefficients between different perceived income moments(inc for nominal and rinc for real) at time t and the quarterly growth rate in hourly earning at $t, t-1, ..., t-k$.

well be, alternatively, because people's subjective reaction to the positive and negative shocks are asymmetric even if the underlying process being symmetric.

A.2.4 Experiences and perceived risk

Different generations also have different perceived income risks. Let us explore to what extent the cohort-specific risk perceptions are influenced by the income volatility experienced by that particular cohort. Different cohorts usually have experienced distinct macroeconomic and individual histories. On one hand, these non-identical experiences could lead to long-lasting differences in realized life-long outcomes. An example is that college graduates graduating during recessions have lower life-long income than others. (Kahn (2010), Oreopoulos et al. (2012), Schwandt and Von Wachter (2019)). On the other hand, experiences may have also shaped people's expectations directly, leading to behavioral heterogeneity across cohorts (Malmendier and Nagel (2015)). Benefiting from having direct access to the subjective income risk perceptions, I could directly examine the relationship between experiences and perceptions.

Individuals from each cohort are borned in the same year and obtained the same level of their

Table A.2: Average Perceived Risks and Local Labor Market Conditions

	(1) log perceived risk	(2) log perceived risk	(3) log perceived iqr	(4) log perceived iqr
Wage Growth (Median)	-0.05***		-0.03***	
	(0.01)		(0.01)	
UE (Median)		0.04*		0.04***
		(0.02)		(0.01)
Observations	3589	3589	3596	3596
R-squared	0.021	0.019	0.025	0.027

*** p<0.001, ** p<0.01 and * p<0.05.

Regression coefficient of the average perceived income risk of each state in different times on current labor market indicators–i.e., wage growth and the unemployment rate. The monthly state wage series is from Local Area Unemployment Statistics (LAUS) of BLS. The quarterly state unemployment rate is from the Quarterly Census of Employment and Wage (QCEW) of BLS.

Table A.3: Extrapolation from Recent Experience

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
income shock squared	0.0225***	0.0222***	0.0217***	0.0207***	0.000773	0.00205***	0.000566	0.00183***	0.000614	0.00184***
	(0.00562)	(0.00570)	(0.00562)	(0.00564)	(0.000743)	(0.000516)	(0.000744)	(0.000515)	(0.000745)	(0.000516)
recently unemployed				0.511*	0.228***	0.0895***				
				(0.260)	(0.0330)	(0.0200)				
unemployed since m-8							0.161***	0.0783***		
							(0.0207)	(0.0121)		
unemployed since y-1									0.138***	0.0701***
									(0.0193)	(0.0113)
Observations	3662	3662	3662	3662	3701	1871	3701	1871	3701	1871
R-squared	0.004	0.013	0.016	0.017	0.015	0.030	0.019	0.041	0.016	0.039

Standard errors are clustered by household. *** p<0.001, ** p<0.01 and * p<0.05.

This table reports regression of perceived risks and perceived unemployment risks on recent experiences of income volatility, and the dummy indicates whether the individual has recently experienced unemployment.

respective highest education. The experienced volatility specific to a certain cohort c at a given time t can be approximated as the average squared residuals from an income regression based on the historical sample only available to the cohort's life time. This is approximately the unexpected income changes of each person in the sample. I use the labor income panel data from PSID to estimate the income shocks. [55] In particular, I first undertake a Mincer-style regression using major demographic variables as regressors, including age, age polynomials, education, gender and

[55] I obtain the labor income records of all household heads between 1970-2017. Farm workers, youth and olds and observations with empty entries of major demographic variables are dropped.

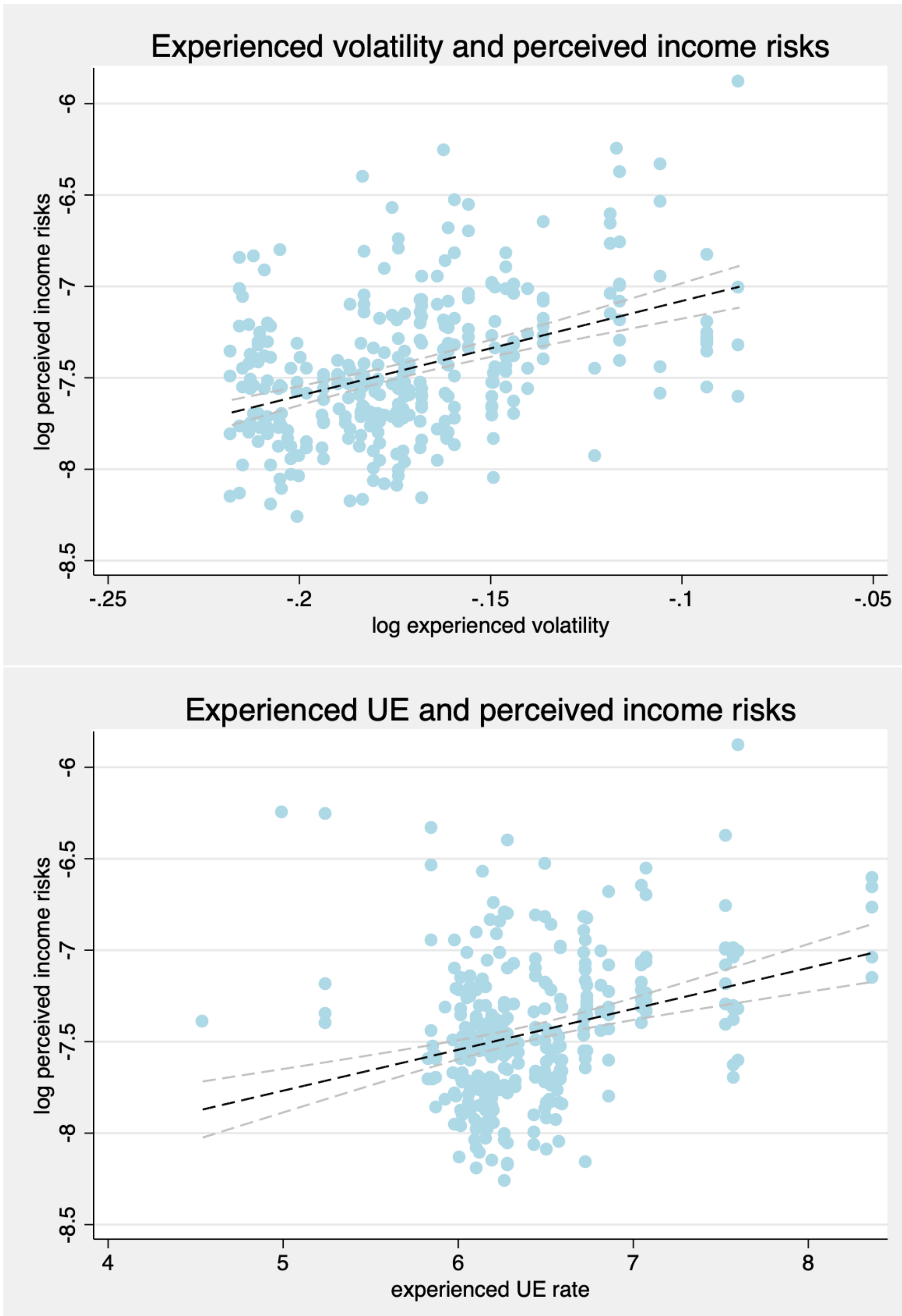
time-fixed effect. Then, for each cohort-time sample, the regression mean-squared error (RMSE) is used as the approximate to the cohort/time-specific income volatility.

There are two issues associated with such an approximation of experienced volatility. First, I, as an economist with $PSID$ data in my hand, am obviously equipped with a much larger sample than the sample size facing an individual that may have entered her experience. Since larger sample also results in a smaller RMSE, my approximation might be smaller than the real experienced volatility. Second, however, the counteracting effect comes from the superior information problem, i.e. the information set held by earners in the sample contains what is not available to econometricians. Therefore, not all known factors predictable by the individual are used as a regressor. This will bias upward the estimated experienced volatility. Despite these concerns, my method serves as a feasible approximation sufficient for my purpose here.

The right figure in Figure A.5 plots the (logged) average perceived risk from each cohort c at year t against the (logged) experienced volatility estimated from above. It shows a clear positive correlation between the two, which suggests that cohorts who have experienced higher income volatility also perceived future income to be riskier. The results are reconfirmed in Table A.4, for which I run a regression of logged perceived risks of each individual in SCE on the logged experienced volatility specific to her cohort while controlling individuals age, income, educations, etc. What is interesting is that the coefficient of *expvol* declines from 0.73 to 0.41 when controlling the age effect because that variations in experienced volatility are indeed partly from age differences. While controlling more individual factors, the effect of the experienced volatility becomes even stronger. This implies potential heterogeneity as to how experience was translated into perceived risks.

How does experienced income shock per se affect risk perceptions? We can also explore the question by approximating experienced income growth as the growth in unexplained residuals. As shown in the left figure of Figure A.5, it turns out that a better past labor market outcome experienced by the cohort is associated with lower risk perceptions. This indicates that it is not just the volatility, but also the change in level of the income, that is asymmetrically extrapolated into their perceptions of risks.

Figure A.5: Experience and Perceived Income Risk



Note: The experienced income volatility is the cross-sectional variance of log change in income residuals estimated using a sub sample restricted to the lifetime of a particular group. For instance, the life experience of a 25-year old until 2015 spans 1990 to 2015. The perceived income risk is the average across all individuals from the cohort in that year. Cohorts are time/year-of-birth specific and all cohorts sized 30 or smaller are excluded.

A.2.5 Perceived risks and individual characteristics

Table A.4 reports the regression results of individual PRs on individual variables with various specifications.

A.2.6 Time-varying patterns of PRs

Figure A.6 plots the time-varying 1-year-ahead perceived risks and corresponding realizations of the risks of the total, permanent and transitory wage components, based on the estimates of SIPP data. Under correct model specification and FIRE of the agents, one may expect the PRs and estimated risk to be, if not equal, at least comove with each other. But the results suggest a negligible correlation between the two series. It is also obvious that the magnitudes of the PRs are significantly lower than the estimated risk using SIPP, reinforcing the finding in Section 4.1. For instance, the latter, which is based on the full sample, should be 10% in standard deviation a year, while the average earning risk perception in SCE is only 2%.

A.3 Wage risk estimation using SIPP data

A.3.1 Sample selection

To estimate the wage risks, or risks to the earning conditional on working for the same hours and staying in the same job, I restrict the universe of the SIPP sample according to this definition for the worker's primary job (JB1). The specific filtering criteria is listed as below, and it is approximately identical to that in Low et al. (2010) for computing the wage rate of the same job using 1993 panel of SIPP.

- Time: January 2013-December 2020

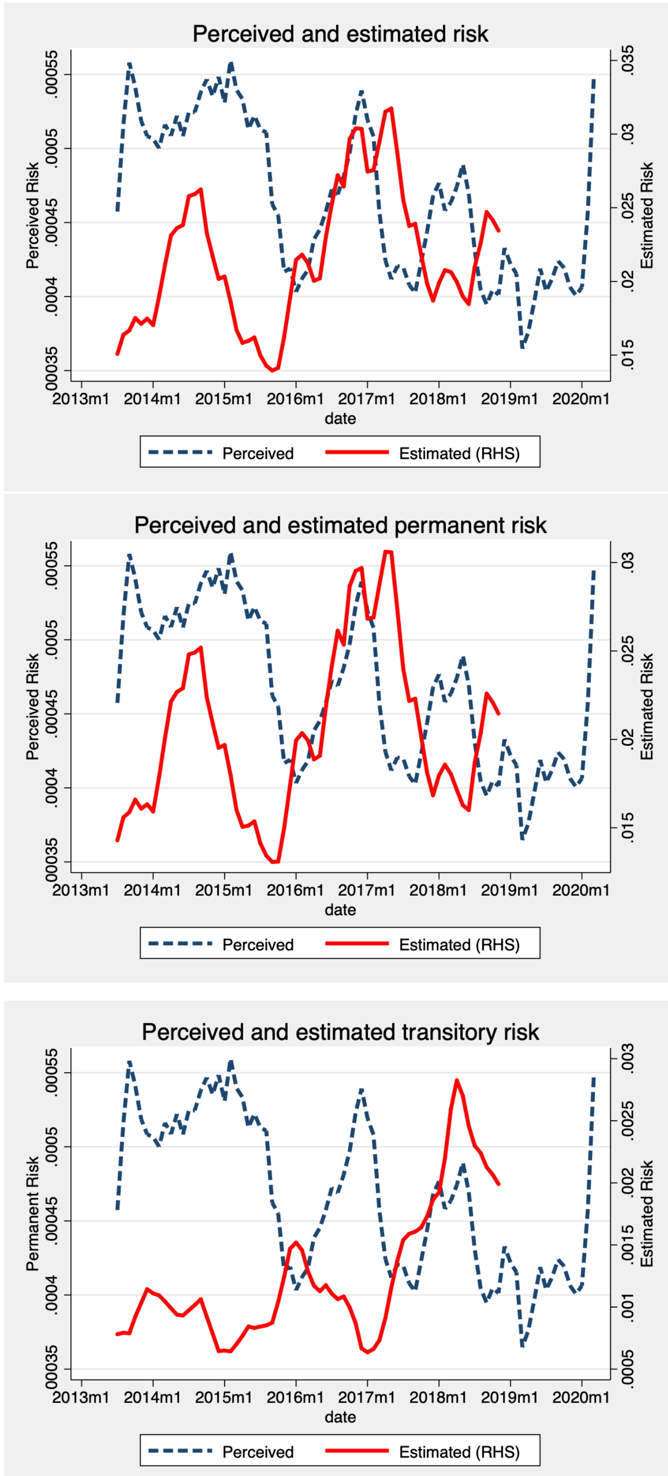
Table A.4: Covariants of Perceived Wage Risks

	incvar I	incvar II	incvar III	incvar IIII	incvar IIIII	incvar IIIIII
IdExpVol	4.58***	2.23***	2.69***	2.75***	2.95***	2.94***
	(0.33)	(0.36)	(0.39)	(0.39)	(0.38)	(0.39)
AgExpVol	0.04	0.28***	0.34***	0.32***	0.18***	0.20***
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
AgExpUE	0.14***	0.08***	0.05**	0.05*	0.04*	0.05**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
age		-0.02***	-0.02***	-0.02***	-0.02***	-0.02***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
gender=male			-0.36***	-0.35***	-0.32***	-0.30***
			(0.02)	(0.02)	(0.02)	(0.02)
nlit_gr=low nlit			0.09***	0.09***	0.10***	0.09***
			(0.02)	(0.02)	(0.02)	(0.02)
parttime=yes					-0.01	-0.02
					(0.02)	(0.02)
selfemp=yes					1.25***	-0.00***
					(0.03)	(0.00)
UEprobAgg						0.02***
						(0.00)
UEprobInd						0.02***
						(0.00)
HHinc_gr=low income					0.16***	0.16***
					(0.02)	(0.02)
educ_gr=high school				-0.10***	-0.13***	-0.09***
				(0.02)	(0.02)	(0.02)
educ_gr=hs dropout				0.08	0.11	0.29***
				(0.11)	(0.11)	(0.11)
N	41422	41422	34833	34833	33480	29687
R2	0.01	0.02	0.04	0.04	0.11	0.06

Standard errors are clustered by household. *** p<0.001, ** p<0.01 and * p<0.05.

Results associated with a regression of logged perceived income risks (incvar) on logged idiosyncratic(IdExpVol), aggregate experienced volatility(AgExpVol), experienced unemployment rate (AgExpUE), and a list of household specific variables, such as age, income, education, gender, job type, and other economic expectations.

Figure A.6: Perceived versus Estimated Risks over Time



Note: Median 1-year-ahead perceived wage risks (in variance terms) in the whole SCE sample against the estimated total, permanent, and transitory risks over the *same* period. Both series concern real wage growth. The realized risks are first estimated monthly from SIPP and then aggregated into annual frequency.

- Age: 20 - 60
- Work-arrangement: employed by someone else (excluding self-employment and other work-arrangement): EJB1_JBORSE ==1.
- Employer: staying with the same employer for a tenure longer than 4 months: the same EJB1_JOBID for 4 or more consecutive months.
- Wage: total monthly earning from the primary job divided by the average number of hours worked in the same job, wage = TJB1_MSUM/TJB1_MWKHRS.
- Outliers: drop observations with wage rate lower than 0.1 or greater than 2.5 times of the individual's average wage.
- No days off from work without pay: EJB1_AWOP1 = 2.
- Continued job spell since December of the last year: RJB1_CFLG=1.
- Drop imputed values: EINTTYPE==1 or 2.
- Drop government/agriculture jobs: drop if TJB1_IND>=9400.

Based on the selected sample, Table A.5 reports the size and approximated group-specific wage volatility as defined in Equation 4.

A.3.2 SEAM Effect

One special feature of SIPP is that it collects monthly information by surveying each correspondent every four months before the 2013 wave and once a year afterward (since 2014 wave). This leads to the well-documented issue of SEAM effect (Ryscavage, 1993; Rips et al., 2003; Nekarda, 2008; Callegaro, 2008), which states that reported changes in survey answers are relatively small for adjacent months within a survey wave but much more abrupt between months across surveys. Such

Table A.5: Summary statistics of SIPP sample

	Obs	Volatility
Year		
2013 (17%)	9,815	0.06
2014 (20%)	12,672	0.11
2015 (15%)	9,543	0.1
2016 (9%)	6,128	0.11
2017 (13%)	7,533	0.07
2018 (15%)	9,378	0.13
2019 (8%)	5,507	0.12
Education		
HS dropout (22%)	13,846	0.09
HS graduate (46%)	28,385	0.1
College/above (30%)	18,345	0.12
Gender		
male (55%)	33,842	0.1
female (44%)	26,734	0.11
Full sample (100%)	60,576	0.1

a difference could be either due to underreporting of changes within a reference period (due to reasons such as the recall bias) or overreporting of changes across reference periods.

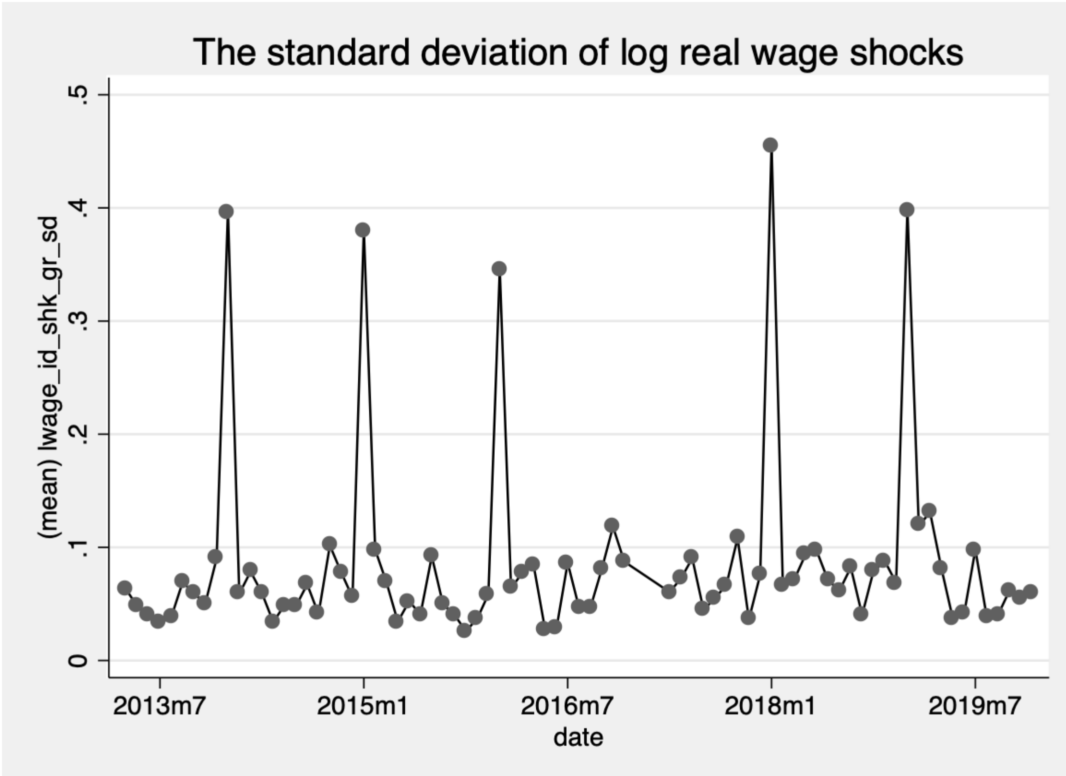
This effect is clearly seen from the time series plot of monthly wage volatility in Figure A.7, where there is always a spike in the size of volatility between December to January in the sample period.[56]

Because of this issue, for monthly risk estimation, I exclude the December-January observations, leading to non-identification of the risks of each January. By doing so, I basically assume that within-wave respondents do not underreport true changes to the wages, while the cross-wave answers overreport these changes. But the opposite assumption might be true, in that respondents underreport changes within the reference year when they retroactively answer survey questions, and the changes across reference periods are correctly reported.

One way to incorporate the cross-wage changes instead of dropping them by brutal force is to estimate risks at a lower frequency, i.e. quarterly and yearly, and construct the quarterly/yearly period such that it covers the cross-wave cutoff month December. Figure A.10 and A.11 in section

[56]Note that the only exception is January 2017, for which no monthly growth rate is not available due to reshuffling of the SIPP sample.

Figure A.7: Estimated monthly wage volatility



Note: The monthly wage volatility as defined in Equation 4 for the entire selected sample, estimated from SIPP.

A.4.3 plot the time-varying risks estimated for quarterly and annual frequency, respectively.

A.4 Wage risk estimation under alternative assumptions

A.4.1 Baseline estimation

Permanent and transitory risks are identified via the following moment restrictions.

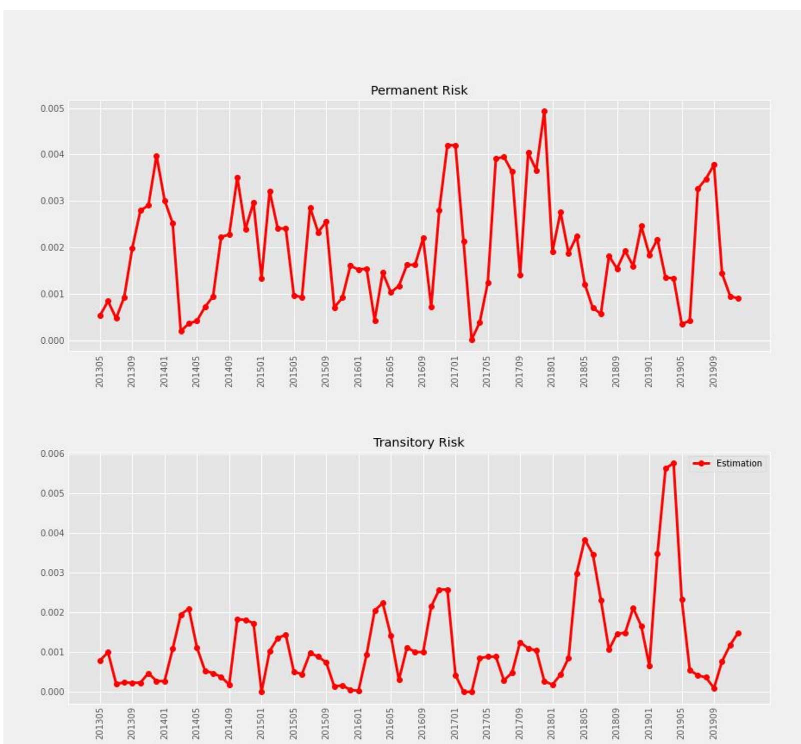
$$
\begin{aligned}
& var(\Delta e_{i,t}) = var(\psi_t + \theta_t - \theta_{t-1}) = \sigma^2_{\psi,t} + \sigma^2_{\theta,t} + \sigma^2_{\theta,t-1} \\
& cov(\Delta e_{i,t}, \Delta e_{i,t+1}) = cov(\psi_t + \theta_t - \theta_{t-1}, \psi_{t+1} + \theta_{i,t+1} - \theta_{i,t}) = -\sigma^2_{\theta,t} \\
& cov(\Delta e_{i,t-1}, \Delta e_{i,t}) = cov(\psi_{t-1} + \theta_t - \theta_{t-1}, \psi_t + \theta_{i,t} - \theta_{i,t-1}) = -\sigma^2_{\theta,t-1}
\end{aligned}
\tag{21}
$$

With four years of wage of individual i from $t-2$ to t, hence three years of first difference Δw, the above three equations can exactly identify the permanent risk specific to time t, $\sigma_{\psi,t}$ and the time-specific transitory risk $\sigma_{\theta,t}$ and $\sigma_{\theta,t-1}$.

Three years of wage data is sufficient under a slightly looser restriction that the transitory risks stay constant over each 3-year horizon, between $t-1$ and $t+1$, call it $\bar{\sigma}_{\theta,t}$. In particular, we have the following identification. With wage growth in year 2014, 2015, 2016, and 2018, 2019, I can identify the year-specific permanent risks for 2014, 2015, 2016, 2018, and 2019, and the average transitory risks for 2014-2016 and 2017-2019, as shown in Figure A.10.

$$
\begin{aligned}
var(\Delta w_{i,t}) &= var(\psi_t + \theta_t - \theta_{t-1}) = \sigma^2_{\psi,t} + 2\bar{\sigma}^2_{\theta,t} \\
cov(\Delta w_{i,t}, \Delta w_{i,t+1}) &= cov(\Delta w_{i,t-1}, \Delta w_{i,t}) = -\bar{\sigma}^2_{\theta,t}
\end{aligned}
\tag{22}
$$

Figure A.8: Monthly permanent and transitory income risks

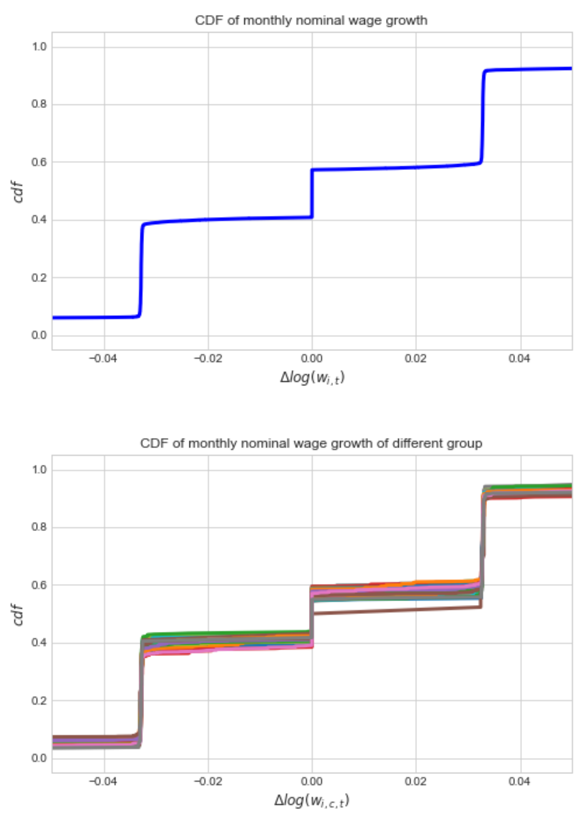


Note: The 3-month moving average of the estimated monthly permanent and transitory risks (variance) using the SIPP panel data on wages between 2013m1-2019m12.

A.4.2 Evidence for the infrequent arrival of the wage shocks

The baseline income process specified as in Equation 2 has been commonly adopted for annual or at most quarterly income/wage data in the literature. But some recent work such as Druedahl et al. (2021) shows that income dynamics at a higher frequency, i.e. monthly, require a modification to such a process to be more consistent with the data. In particular, the authors allow for infrequent arrivals of both transitory and permanent shocks. The assumption of infrequent shocks is primarily motivated by the observed pattern (as confirmed in Figure A.9 using nominal wage growth in SIPP) that a sizable mass of individual monthly wage growth is concentrated around zero.

Figure A.9: CDF of monthly wage growth



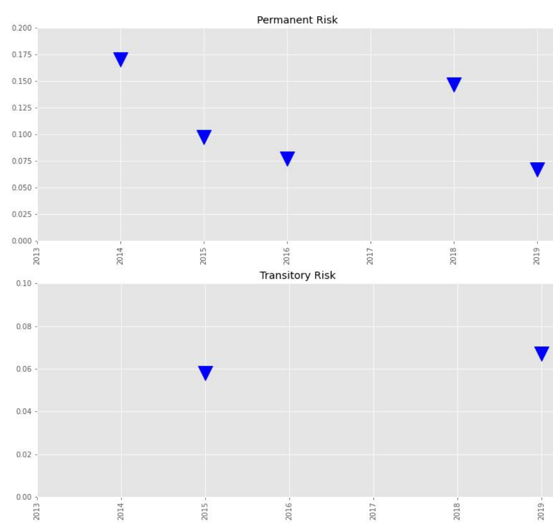
Note: The cumulative distribution function of the monthly wage growth from SIPP for the whole sample (left) and by the gender-education-age-specific group (right).

A.4.3 Estimated risks at a lower frequency

Most of the income risk estimation in the literature is done at a lower frequency, such as yearly and quarterly. Figure A.10 plots the estimated time-varying permanent and transitory risks using the annual growth of the average wage of each year in the sample.[57] Due to the reshuffling of the entire SIPP sample in 2017, no annual wage growth rate can be calculated in 2017, hence, the permanent risks of 2017 and the transitory risks of its adjacent years are unable to be identified.

For the years with identified risks, the estimated risks at annual frequency seem to be much larger than that commonly seen in the literature, as summarized in Table A.6. In particular, the size of the permanent shock ranges from 27% to 41%, in contrast to the standard estimation of 10-15%. And the transitory risks are estimated to be around 25%, which also exceeds the standard estimates of 10% to 20%.

Figure A.10: Yearly permanent and transitory wage risks



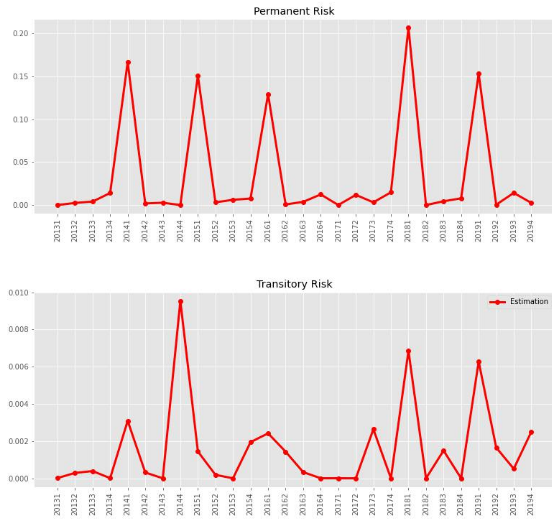
Note: The estimated yearly permanent and transitory risks (variance) using the SIPP panel data on wages between 2013m1-2019m12.

A similar issue can be seen from quarterly estimates using quarterly growth of average wage

[57] A similar size of estimates is obtained when YOY growth of monthly wage is used.

rates.(See Figure A.11) Reminiscent of the seasonal spike in the monthly volatility in January, there is a similar spike in the first quarter of every year in the sample.

Figure A.11: Quarterly permanent and transitory wage risks



Note: The estimated quarterly permanent and transitory risks (variance) using the SIPP panel data on wages between 2013m1-2019m12.

A.5 Results with the first moment (the expected and realized wage growth)

Although the main focus of this paper is on income/wage risks, specifically the second moment of wage growth, it is natural to ask if the expected wage growth revealed in SCE aligns with what is realized as seen in SIPP. It is not surprising that both expected and the realized average wage growth rate conditional on education and gender decline over the life-cycle, as shown in the downward fitted lines in Figure A.12. But In the sample of 2013-2019, expected wage growth seems to be persistently downward biased compared to its realization. This was not driven by the widely-documented fact of upward biased inflation expectation (See for instance, **?**), as even the same pattern shows up in the nominal wage growth.

A.6 Homogenous and heterogeneous life-cycle wage profiles

Figure A.13 plots the deterministic wage profile used to calibrate the baseline model, which is estimated from SIPP for job-stayers. Figure A.14 plots the heterogeneous wage profiles used in the model experiment of $HPRURG$, which is calibrated based on the heterogeneous wage growth rates reported in SCE.

A.7 Calibration heterogeneous income risks/growth rates using the survey

A.8 Income risks in the existing literature

Table A.6 summarizes the most common estimates of income risks seen in the literature.

A.9 Persistent/permanent effect of job loss in the existing literature

Table A.7 summarizes the estimated size of wage loss and change in income risks following job displacement and mass layoff, as reported in the literature.

A.10 Estimating state-dependent PR using the survey

A.10.1 The model

In this appendix section, I provide an alternative approach of estimating perceived risks using survey to that in 4.3. The central idea of this approach is to treat an individual's reported perceived risks as noisy signals of their true underlying yet unobservable risk belief that affects their decisions and assume that the survey answers are masked by measurement errors.

In particular, I assume the belief state is binary, i.e. low and high risks. Then the parameters to be estimated from the panel data of risk perceptions from SCE are the state-dependent risk profile $\tilde{\Gamma}_l = \{\tilde{\sigma}^l_\psi, \tilde{\sigma}^l_\theta, \tilde{\mho}^l, \tilde{E}^l\}$, $\tilde{\Gamma}_h = \{\tilde{\sigma}^h_\psi, \tilde{\sigma}^h_\theta, \tilde{\mho}^h, \tilde{E}^h\}$ and Ω, the transition matrix between the two states.

Denote the reported risk perception of the individual i at time t in the survey by $\tilde{\Gamma}^s_{i,t}$. It consists of the underlying risk perceptions relevant to individual decisions, or the model counterpart $\tilde{\Gamma}_{i,t}$, and an individual-specific, time-specific and an $i.i.d$ shock to the survey responses, respectively. The realization of $\tilde{\Gamma}_{i,t}$ depends on a hidden state $J_{i,t}$ which is non-observable to economists working with the survey data. It takes value of 1 if the individual i is at a high-risk-perception state $\tilde{\Gamma}_{i,t} = \tilde{\Gamma}_h$ and zero if at low-risk-perceptions $\tilde{\Gamma}_{i,t} = \tilde{\Gamma}_l$. The $i.i.d$ shock $\epsilon_{i,t}$ is assumed to follow a mean-zero normal distribution with variance σ^2_ϵ.

$$
\begin{aligned}
&\underbrace{\tilde{\Gamma}^s_{i,t}}_{\text{reported PR}} = \underbrace{\tilde{\Gamma}_l + \mathbb{1}(\overbrace{J_{i,t}}^{\text{Hidden state}} = 1)(\tilde{\Gamma}_h - \tilde{\Gamma}_l)}_{\tilde{\Gamma}_{i,t}} + \xi_t + \eta_i + \epsilon_{i,t} \\
&\text{Prob}(J_{i,t+1} | J_{i,t}) = \Omega
\end{aligned}
$$

Notice that the individuals do not separately report their perceived risks for the permanent and transitory shocks, but instead the overall expected income volatility. Therefore, I make an auxiliary assumption that the agent adopts a constant ratio of decomposition between permanent and transitory risks, $\kappa = \frac{\tilde{\sigma}_{i,t,\psi}}{\tilde{\sigma}_{i,t,\theta}}$, the value of κ is externally estimated from the realized income data.

In addition, since the surveyed risk perceptions is at the monthly frequency, I estimate the underlying risk parameters for monthly shocks. [58]

For each individual i, we observe at most 12 observations of their perceived income volatility of the earning growth next year $\tilde{var}_{i,t}$ from t to $t+12$ and their job-separation and job-finding expectations, respectively. The panel structure allows the individual fixed effect η_i and time-fixed effect ξ_t to be easily identified.

Then the parameters can be estimated with a modified 2-regime Markov switching model a la Hamilton (1989) using the maximum-log-likelihood (MLE). (See the detailed implementation in Appendix A.10.2). Table A.8 reports the baseline estimates of the parameters associated with the 2-state Markov model of subjective perceptions. All parameters are converted from monthly into yearly counterparts to be consistent with the model frequency.

The estimates of subjective profile confirms the key finding we have detailed in the previous section. The estimated staying probabilities at low and high risk perceptions, q and p, are around 0.9, indicating a high degree of persistence in individual risk perceptions. Given these estimated transition probabilities, earning risk perceptions are on average lower than the objective level assumed in the literature.

A.10.2 Details of the estimation

For each individual i, we observe at most 12 observations of their perceived income volatility over the earning growth next year $\tilde{var}_{i,t}$ from $t=1$ to $=12$. We assume the following relation between observed survey reported volatility and underlying perceived monthly permanent/transitory risks by the individual i at time t.

$$\log \tilde{\mathrm{var}}_{i,t} = \log(12\tilde{\sigma}^2_{i,t,\psi} + 1/12\tilde{\sigma}^2_{i,t,\theta}) + \xi_t + \eta_i + \epsilon_{i,t}$$

[58] $\tilde{\mathrm{var}}_{i,t} = (12\tilde{\sigma}^2_{i,t,\psi} + 1/12\tilde{\sigma}^2_{i,t,\theta}) exp^{\xi_t} exp^{\eta_i} exp^{\epsilon_{i,t}} \rightarrow \log \tilde{\mathrm{var}}_{i,t} = \log(12\tilde{\sigma}^2_{i,t,\psi} + 1/12\tilde{\sigma}^2_{i,t,\theta}) + \xi_t + \eta_i + \epsilon_{i,t} \rightarrow \log(\tilde{\mathrm{var}}_{i,t}) = \log[(12 + \frac{1}{12\kappa^2})\tilde{\sigma}^2_{i,t,\psi}] + \xi_t + \eta_i + \epsilon_{i,t}$.

η_i and ξ_t are individual and time fixed effect, respectively. The i.i.d shock $\epsilon_{i,t}$ represents any factor that is not available to economists working with the survey, but affects i's survey answers at the time t. We assume it is normally distributed.

Notice that $\tilde{\text{var}}_{i,t}$ alone is not enough to separately identify the perceived permanent and transitory risks. To proceed, I make the following auxiliary assumption: the agent adopts a constant ratio of decomposition between permanent and transitory risks, $\kappa = \frac{\tilde{\sigma}_{i,t,\psi}}{\tilde{\sigma}_{i,t,\theta}}$, the value of κ is externally estimated from the realized income data.

With the additional assumption, we can rewrite the equation above, utilizing the fact that risks for one year are the cumulative sum of monthly ones for permanent shocks and the average of monthly ones for transitory shocks.

$$\log(\tilde{\text{var}}_{i,t}) = \log[(12 + \frac{1}{12\kappa^2})\tilde{\sigma}^2_{i,t,\psi}] + \xi_t + \eta_i + \epsilon_{i,t}$$

We *jointly* estimate a Markov-switching model on perceived volatility $\log(\tilde{\text{var}}_{i,t})$, perceived probability on unemployment status $\tilde{\mho}_{i,t}$, and perceived probability on employment status $\tilde{E}_{i,t}$. The vector model to be estimated can be represented as below.

$$\hat{\tilde{\Gamma}}^s_{i,t} = \tilde{\Gamma}^l + \mathbb{1}(J_{i,t} = 1)(\tilde{\Gamma}^h - \tilde{\Gamma}^l) + \tau_{i,t}$$

where $\hat{\tilde{\Gamma}}^s_{i,t} = [\widehat{\log(\tilde{\text{var}}_{i,t})}, \hat{\tilde{\mho}}_{i,t}, \hat{\tilde{E}}_{i,t}]'$ is a vector of sized three, consisting of properly transformed reported risk perceptions from the survey, excluding the time and individual fixed effects in a first step regression. $J_{i,t} = 1$ for high risk state and $= 0$ if at the low risk state. $\tau_{i,t}$ is a vector of three i.i.d. normally distributed shocks.

The estimation of 2-regime Markov switching models produces estimates of $\tilde{\Gamma}_l$, $\tilde{\Gamma}_h$, the staying probability q, and p, and the variance of $\tau_{i,t}$. Then the following relationship can be used to recover perceived permanent and transitory risks respectively.

$$\tilde{\Gamma}^l = [\log[(12 + \frac{1}{12\kappa^2})\tilde{\sigma}_{\psi}^{l2}], \tilde{\mho}_l, \tilde{E}_l]'$$

$$\tilde{\Gamma}^h = [\log[(12 + \frac{1}{12\kappa^2})\tilde{\sigma}_{\psi}^{h2}], \tilde{\mho}_h, \tilde{E}_h]'$$

Estimation sample I restrict the sample to SCE respondents who were surveyed for at least 6 consecutive months with non-empty reported perceived earning volatility, separation and job-finding expectations. This left with me 6457 individuals.

Table A.8 reports the estimated parameters.

A.11 Model extension: state-dependent risk perceptions

In the benchmark model, I maintain the FIRE assumption that the agents perfectly know the underlying parameters of income risks $\Gamma = \{\sigma^2_{\psi}, \sigma^2_{\theta}, \mho, E\}$ as assumed by the modelers and behave optimally accordingly.

But here, I relax the FIRE assumption by separately treating the "true" underlying risk parameters Γ and the risk perceptions held by the agents. The latter is denoted as $\tilde{\Gamma}_i$. This extension is meant to capture the four empirical patterns documented in the previous sections.

1. Underestimation of the earning risks (compared to what is assumed to be the truth in the model)

2. Heterogeneity in risk perceptions

3. Extrapolation of recent experiences

4. State-dependence of risk perceptions

The possible approaches of capturing these perceptual patterns are by no means unique. I adopt one simple framework that does not require explicitly specified mechanisms of perception formation but sufficient to reflect these the patterns revealed from the survey data.

Assume that each agent i in the economy cannot directly observe the underlying risk parameters Γ, but instead make his/her best choices based on a subjective risk perceptions $\tilde{\Gamma}_{i,\tau}$, which swing between two states: $\tilde{\Gamma}_l$ (low risk) and $\tilde{\Gamma}_h$ (high risk). The transition between the two states is governed by a Markov process with a transition matrix Ω. In the calibration of the model in latter sections, these subjective parameters can be estimated from survey data relied upon auxiliary assumptions.

Such an assumption automatically allows for heterogeneity in risk perceptions across different agents at any point of the time. All individuals are distributed between low and high risk-perception states.

The transition probability between low-risk and high-risk perception states can be also configured so that the average risk perception is lower than the true level of the risk. If we let the transition matrix Ω to be dependent on individual unemployment status $\nu_{i,\tau}$, or macroeconomic conditions, we can also easily accommodate the possibility of experience extrapolation and state-dependence feature of risk perceptions.

Under the assumption of subjective perception, the subjective state of the risk perceptions $\tilde{\Gamma}$ becomes an additional state variable entering the Bellman equation of the consumer's problem, restated in below.

$$\tilde{V}_\tau(\tilde{\Gamma}_\tau, \nu_\tau, m_\tau, p_\tau) = \max_{\{c_\tau\}} \quad u(c_\tau) + (1-D)\beta\mathbb{E}_\tau \left[\tilde{V}_{\tau+1}(\tilde{\Gamma}_{\tau+1}, \nu_\tau, m_{\tau+1}, p_{\tau+1})\right] \tag{23}$$

Notice here that I assume that the agents recognize the transition between two subjective perception states and take it into account when making the best choices. This assumption guarantees time-consistency and provides additional discipline to the model assumption.

The consumer's solution to the problem above is the age-specific consumption policy $\tilde{c}^*_\tau(\tilde{\Gamma}_\tau, u_\tau, m_\tau, p_\tau)$ that is also a function of subject risk perception state $\tilde{\Gamma}$.

The distinction between objective and subject risk perception marks the single most important deviation of this paper from existing incomplete-market macro papers. [59] There is a long tradition of explicitly incorporating various kinds of heterogeneity in addition to uninsured idiosyncratic income shocks in these kinds of models to achieve better match with observed cross-sectional wealth inequality. One of the most notable assumptions used in the literature is the heterogeneity in time preferences (Krusell and Smith (1998), Carroll et al. (2017), Krueger et al. (2016)). My modeling approach shares the spirit with and are not mutually exclusive to these existing assumptions on preferential heterogeneity. But, to some extent, perceptual heterogeneity is more preferable as such patterns are directly observed from the survey data, as I show in the previous part of the paper.

A more fundamental justification for such a deviation from the full information rational expectation assumption is that risk parameters Γ are barely observable objects to agents. This is so no matter if they are exogenously assumed by economists or endogenously determined in the equilibrium of the model. [60] Therefore, the conventional argument in favor of rational expectation assumption, namely equilibrium outcome drives the agents' perceptions to converge to the "truth", does not apply here.

Incorporating subjective risk perceptions also alters aggregate dynamics of the distributions as described in Equation 19, as restated below.

$$\tilde{\psi}_{\tau-1}(\tilde{B}) = \int_{\tilde{x}\in\tilde{X}} \tilde{P}(\tilde{x}, \tau-1, \tilde{B})\mathrm{d}\tilde{\psi}_{\tau-1} \quad \text{for all} \quad \tilde{B} \in \tilde{B}(X) \tag{24}$$

[59] For instance, Bewley (1976), Huggett (1993), Aiyagari (1994), Krusell and Smith (1998), Krueger et al. (2016), Carroll et al. (2017).

[60] So far, the majority workhorse incomplete market macro literature has not incorporated any endogenous mechanisms that determine the level of income risks. The emerging literature that incorporates labor market search/match frictions in these models have relied upon simplifying assumptions to get tractability. See, for instance, McKay (2017); Acharya and Dogra (2020); Ravn and Sterk (2021), with the only exception being Ravn and Sterk (2017).

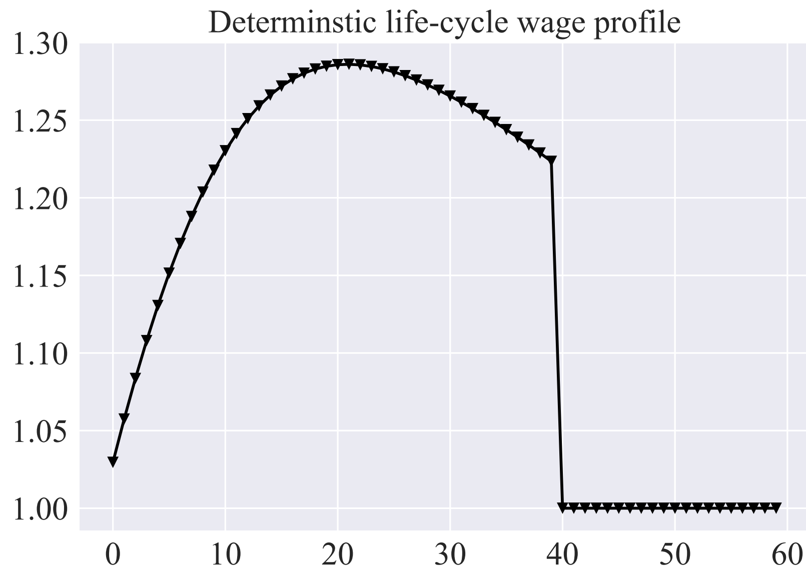
The state variable $\tilde{x}$ includes subjective state $\tilde{\Gamma}$ in addition to those contained in x. The transition probabilities $\tilde{P}$ now depend on the optimal consumption policies $c^*(\tilde{x})$ as a function of belief state $\tilde{\Gamma}$, as well as the exogenous transition probabilities of the true stochastic income process Γ.

Then the new StE under subjective risk perceptions can be defined accordingly.

Figure A.12: Realized and Perceived Income Growth over the Life Cycle

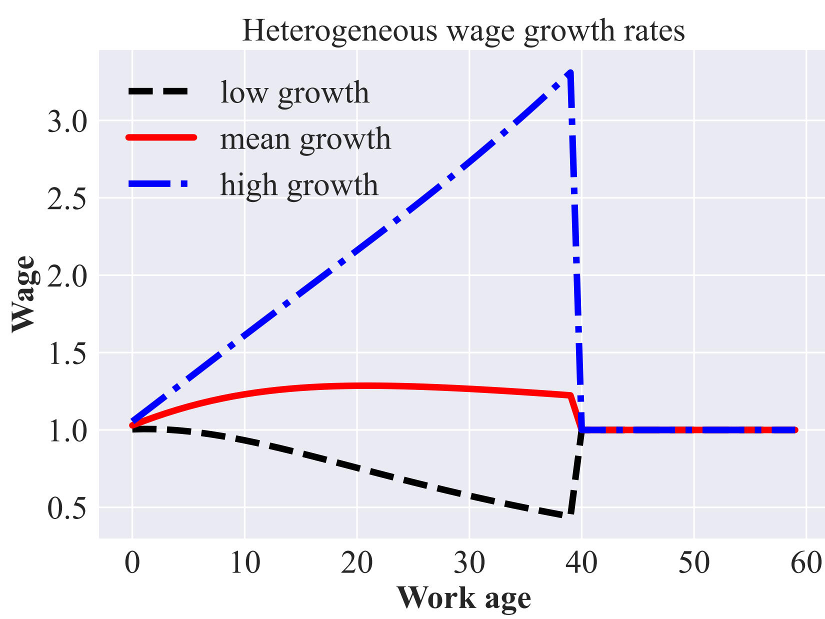
Note: The average real (upper panel) and nominal (bottom panel) realized and perceived wage growth of different age groups, conditional on the gender and education of the individual. The realized wage growth is approximated by the average log changes in real wage of each age/education/gender group based on SIPP.

Figure A.13: Estimated deterministic wage profile over the life-cycle



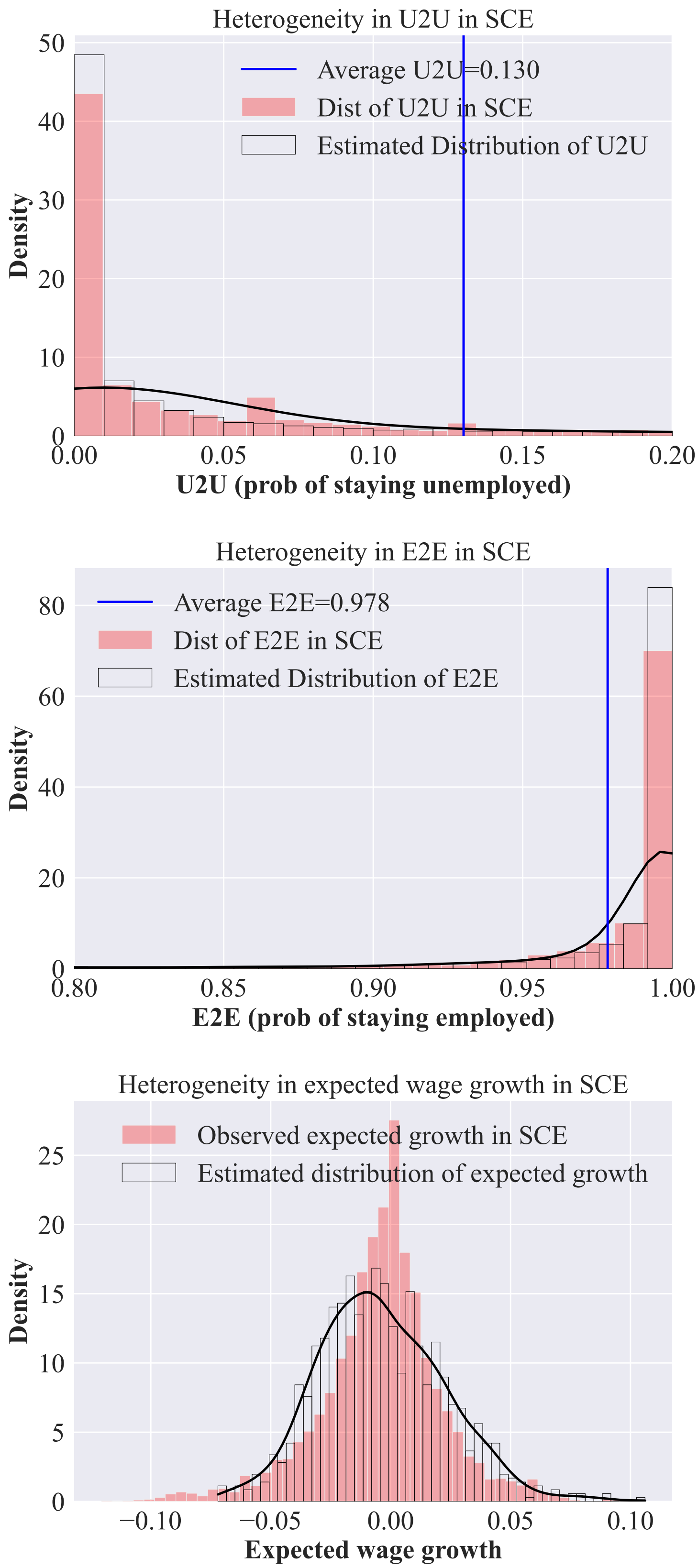
Note: The estimated average age profile of real earnings using SIPP between 2013m3-2019m12. It is based on a regression of fourth-order age polynomials controlling for time, education, occupations, gender, etc.

Figure A.14: Heterogeneous wage profiles over the life-cycle



Note: The heterogeneous deterministic wage profiles implied by the heterogeneous wage growth rates. This is used to calibrate the $HPRURG$ model.

Figure A.15: Calibration of heterogeneous UE risks and wage growth rates from the SCE



This figure illustrates the calibration of unemployment risks, and wage growths using SCE.

Table A.6: The size and nature of idiosyncratic income risks in the literature

	σ_ψ	σ_θ	$\mho$	E	Earning Process	Unemployment	Source
Huggett (1996)	[0.21,+]	N/A	N/A	N/A	AR(1)	No	Page 480
Krusell and Smith (1998)	N/A	N/A	[0.04,0.1]	[0.9,0.96]	N/A	Persistent	Page 876
Cagetti (2003)	[0.264, 0.348]	N/A	N/A	N/A	Random +MA innovations	No	Page 344
Gourinchas and Parker (2002)	[0.108,0.166]	[0.18, 0.256]	0.003	0.997	Permanent +transitory	Transitory	Table 1
Meghir and Pistaferri (2004)	0.173	[0.09, 0.21]	N/A	N/A	Permanent +MA	No	Table 3
Storesletten et al. (2004)	[0.094; +]	0.255	N/A	N/A	Persistent + transitory	No	Table 2
Blundell et al. (2008)	[0.1,+]	[0.169,+]	N/A	N/A	Permanent + MA	No	Table 6
Low et al. (2010)	[0.095,0.106]	0.08	0.028	N/A	Permanent+transitory with job mobility	Persistent	Table 1
Kaplan and Violante (2014)	0.11	N/A	N/A	N/A	Persistent	No	Page 1220
Krueger et al. (2016)	[0.196,+]	0.23	[0.046,0.095]	[0.894,0.95]	Persistent +transitory	Persistent	Page 26
Carroll et al. (2017)	0.10	0.10	0.07	0.93	Permanent+transitory	Transitory	Table 2
Bayer et al. (2019)	0.148	0.693	N/A	N/A	Persistent time+MA	No	Table 1
My Estimates based on SIPP	0.10	0.016	N/A	N/A	Permanent +transitory	No	Table A.1

The conservative (lower bound) estimates/parameterization on idiosyncratic income risks at the annual frequency seen in the literature.

Table A.7: Summary of the literature on persistent/permanent effect from job displacement

	Loss (nb of years after displacement)	Income risks	Period	Variables	Data/Sample
Ruhm (1991)	10%-13%(4)	NA	1969-1982	Earning	PSID
Jacobson et al. (1993)	25%(6)	NA	1974-1986	Earning	Administrative records of Pennsylvanian.
von Wachter et al. (2009)	21%-27%(20)	NA	1978-2004	Earning	Social security records, and firm-level employment data.
Couch and Placzek (2010)	13%-15% (6)	NA	1993-2004	Earning	Administrative data of Connecticut
Low et al. (2010)	6%-9%(1)	20%	Model	Wage rate	Model
Davis and Von Wachter (2011)	10%-20%(20)	NA	1980-2005	Earning	Social security records
Farber (2017)	6.2% (0)	Lower E2E rate	1984-2016	Wage rate	Displaced Workers Surveys (DWS)
Lachowska et al. (2020)	16%(5)	NA	2002-2014	Wage rate	Employment Security Department of Washington state.
Pytka and Gulyas (2021)	6% (11) (median)	NA	1984-2017	Earning	Austrian social security records

The empirical estimates on earning/wage loss from job-displacement. For Farber (2017), the loss is computed as the combined effect for those re-employed at a full-time and a part-time job. For Pytka and Gulyas (2021), I converted the accumulated loss into an annual percentage loss.

Table A.8: Estimated subjective risk perceptions

	baseline
$std(\tilde{\sigma})$	1.203
q	0.565
p	0.565
$\tilde{\sigma}_{\psi}^{l}$	0.897
$\tilde{\sigma}_{\theta}^{l}$	0.021
$\tilde{\sigma}_{\psi}^{h}$	1.140
$\tilde{\sigma}_{\theta}^{h}$	0.027

This table reports estimates of the parameters for the 2-state Markov switching model of subjective risk perceptions. Risks are at the annual frequency.