Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints

Rui Yao
Baruch College, City University of New York

Harold H. Zhang
The University of North Carolina at Chapel Hill

We examine the optimal dynamic portfolio decisions for investors who acquire housing services from either renting or owning a house. Our results show that when indifferent between owning and renting, investors owning a house hold a lower equity proportion in their net worth (bonds, stocks, and home equity), reflecting the substitution effect, yet hold a higher equity proportion in their liquid portfolios (bonds and stocks), reflecting the diversification effect. Furthermore, following the suboptimal policy of always renting leads investors to overweigh in stocks, while following the suboptimal policy of always owning a house causes investors to underweigh in stocks.

For many investors, a house is the largest and most important asset in their portfolios. The 2001 Survey of Consumer Finances (SCF) shows that about two-thirds of U.S. households own their primary residences and home value accounts for 55% of a homeowner's total assets, on average. At the same time, approximately 50% of U.S. households hold stocks and/or stock mutual funds (including holdings in their retirement accounts), and stock investment accounts for less than 12% of household assets. Even for households owning stocks, they account for less than 40% of household assets. Housing differs from other financial assets in that housing serves a dual purpose. It is both a durable consumption good from which the owner derives utility and also an investment vehicle that allows the investor to hold home equity. Further, compared with other financial assets such as bonds and stocks, the housing investment is often highly

We would like to thank Dong-Hyun Ahn, Tony Ciochetti, Henry Cao, Joao Cocco, Jennifer Conrad, Joshua Coval, Bin Gao, Eric Ghysels, Francisco Gomes, Adam Reed, Jim Shilling, Steve Slezak, Richard Stanton, and seminar participants at Cheung Kong Graduate School of Business, CUNY-Baruch College, Fannie Mae, University of Colorado, University of North Carolina at Chapel Hill, University of Texas at Dallas, the 2002 AERUEA annual meetings in Atlanta, and the 2002 Western Finance Association meetings in Park City for helpful comments. We also thank an anonymous referee and Kenneth Singleton (the editor) for very constructive suggestions. We gratefully acknowledge North Carolina Supercomputing Center (NCSC) for providing computing resources and Ekaterini Kyriazidou for providing us the GAUSS code used in our empirical analysis. This article was previously criculated under the title "Optimal Consumption and Portfolio Choices with Risky Housing and Stochastic Labor Income." All errors are our own. Address correspondence to: Harold Zhang, Kenan-Flagler Business School, The University of North Carolina at Chapel Hill, Chapel Hill, NC 27599, or e-mail: zhangha@kenan-flagler.unc.edu.

The Review of Financial Studies Vol. 18, No. 1
doi:10.1093/rfs/hhh007 Advance Access publication March 26, 2004

leveraged and relatively illiquid. Despite the importance of housing assets in investors' portfolios, the interaction of investors' housing choice with their other financial asset holdings is largely avoided by financial advisors, who focus primarily on liquid financial assets. It is also largely unexplored in the academic literature because of the difficulties of dealing with various frictions in housing market, such as collateral requirements and liquidation cost. While little guidance is provided on the issue, the decisions are crucial to the investors' wealth accumulation and welfare over their lifetime.

In this article, we examine the optimal dynamic consumption, housing, and portfolio choices for an investor who receives stochastic labor income and faces substantial housing risk, collateral requirements, and liquidation cost. By explicitly incorporating risky housing, we investigate how the investor chooses his housing services and how his investment decisions interact with his housing choice. Grossman and Laroque (1990) study investment decisions when consumption is derived from a single, riskless, indivisible durable good that is costly to adjust (such as a house). They show that housing choice exhibits a deferred adjustment due to transaction cost. The agent adjusts his housing consumption only when his house value–wealth ratio deviates substantially from the "target" level. They also show that in the presence of the adjustment cost, the agent reduces the proportion of his wealth allocated to risky stocks after he purchases a new house. However, if the house value–wealth ratio is close to the trigger bound of selling the existing house, the agent increases his risk exposure relative to the level just after a new purchasing. In a recent article, Cocco (2004) analyzes the impact of a housing decision on an investor's portfolio choice using simulation. In particular, he focuses on the role of housing consumption in explaining the cross-sectional heterogeneity of investors' portfolio decisions. He finds that the housing asset crowds out stockholding in net worth, and that liquidation cost reduces the frequency of housing adjustment and the investor's exposure to stock.

Our article differs from the previous studies in several important dimensions. First, we explicitly incorporate the rental market for housing services.[1] Recognizing the existence of the house rental market is crucial to understanding the impact of housing choices on investors' portfolio decisions. It allows investors to separate their housing consumption choice from their housing investment choice and to consume housing

[1] Besides explicitly introducing the house rental market, our model also extends Grossman and Laroque (1990) by incorporating a nondurable numeraire consumption good, housing price risk, collateral requirements, and an uninsurable stochastic labor income. Hu (2002) studies portfolio choices for home-owners in the presence of a house rental market in a five-period model. In her setup, investors make portfolio and housing adjustments every 10 years. They are not allowed to own in the first period so that they can accumulate enough wealth. Further, there is no mortality prior to the final period or bequest motive. However, Hu (2002) explicitly models refinancing charges, which are absent from our model.

services while saving toward the down payment for a house of the desired size. Second, we quantitatively assess the utility cost and biases in portfolio choices when the investor follows an alternative suboptimal policy of either (1) acquiring housing services only from renting, as implicitly assumed in most existing studies on portfolio choices such as Heaton and Lucas (2000b) and Cocco, Gomes, and Maenhout (2004), or (2) acquiring housing services only from owning a house, as in Cocco (2004). Finally, using Panel Study of Income Dynamics (PSID) data, we perform an in-depth empirical analysis that simultaneously accounts for both the sample selection in stock market participation and also the fixed effects among investors that affect both participation and equity-proportion decisions.

In our model, numeraire consumption goods and housing services are substitutable both intratemporally and intertemporally. Investors receive stochastic labor income calibrated to the lifetime earnings profile of a college or high school graduate. Because of the tax advantage of mortgage debt and/or the consumption preference associated with home ownership, as well as the moral hazard concern associated with renting, holding everything else equal, investors prefer owning a house to renting in our model. However, a down payment is required to buy a house. Further, homeowners are required to maintain a positive home equity position and incur a significant transaction cost when selling their house.

Our results indicate that investors rent housing services when their level of liquid assets is low. However, investors buy a house to benefit from home ownership when they are no longer liquidity-constrained. When indifferent between owning and renting, investors choose substantially different portfolio compositions when owning a house versus when renting housing services. When owning a house, investors reduce the equity proportion in their net worth (bonds, stocks, and home equity), reflecting the substitution effect of home equity for risky stocks. However, when owning, investors hold a higher equity proportion in their liquid financial portfolio (bonds and stocks). This reflects the diversification benefit afforded the homeowner who can use home equity to buffer financial and labor-income risks. While existing studies have emphasized the substitution effect, our study identifies and quantitatively assesses the effect of this diversification on investors' holdings of other risky assets.

We also find that the presence of liquidation cost creates a no-adjustment region for housing services. Within the no-adjustment region, investors adjust their numeraire good consumption intratemporally to maximize utility. Further, when close to the trigger bounds of the no-adjustment region, investors hold a *higher* equity proportion in their liquid financial portfolios to achieve the optimal risk-return tradeoff. This can be attributed to investors' lower relative risk aversion on the trigger bounds of the no-adjustment region.

Our analysis of alternative housing-choice policies indicates that housing choice has a significant impact on the investors' portfolio decisions. Compared with the optimal portfolio choice, which allows investors to endogenously choose renting versus owning a house, investors overweigh in equity when following the suboptimal policy of always renting housing services and underweigh in equity when following the suboptimal policy of acquiring housing services only by owning. The former reflects the investors' incentive to hold a *safer* liquid portfolio when saving for housing down payment. The latter reflects the motive to save, which when investors forgo the opportunity to rent housing services in poor economic climates, is excessively precautionary.

Further, we find that while investors with substantial net worth suffer the largest welfare losses when always renting housing services, investors with very little net worth or old investors approaching the terminal date lose the most when they forgo the opportunity to rent. The former prevents investors with high net worth from taking advantage of home ownership. The latter, on the other hand, forces investors facing binding liquidity constraints or imminent liquidation to consume a disproportionate level of housing services.

When stock and housing returns are correlated, there is a hedging demand for holding stocks. We find that the hedging demand induced by a positive correlation between stock and housing returns *reduces* homeowners' stockholding yet *raises* renters' equity proportion. This is attributed to the effective long position in housing assets held by homeowners and the short position held by renters. Introducing exogenous moving shocks shifts the trigger bound of owning versus renting upward, particularly for young investors who face the highest mobility rate. However, the exogenous moving shock has little direct impact on renters' or homeowners' portfolio choices.

Comparative static analysis confirms the robustness of our qualitative findings to various perturbations of the baseline model. It demonstrates that when the benefit of owning a house is reduced, investors remain as renters longer and hold a *riskier* liquid portfolio once they become homeowners. Eliminating housing risk or the positive correlation between housing return and the labor-income growth rate lowers the trigger bound of becoming a homeowner. It also leads to a *riskier* liquid portfolio for homeowners.

Our empirical analysis demonstrates that renters' and homeowners' portfolio choices have different determinants. They also react very differently to key common variables identified in our theoretical analysis, such as net-worth–income ratio and age. Together these lead to very different portfolio choices between renters and homeowners. Overall the empirical findings provide some support to our model predictions based on policy function and simulation analysis.

The rest of the article is organized as follows. Section 1 describes our economic model. Section 2 first discusses investors' optimal consumption and portfolio choices with and without a housing endowment. We also conduct a welfare analysis, an analysis of the effects of hedging demand and moving shocks, a comparative static analysis, and a simulation analysis in this section. Section 3 presents empirical evidence on investors' portfolio decisions. Finally, Section 4 concludes the discussion.

1. The Economic Model

The economy consists of investors living for at most T periods, where T is a positive integer. Let λ_j be the probability that the investor is alive at time j for $j = 0, \ldots, T$, conditional on being alive at time $j - 1$. We assume that $\lambda_j > 0$ for all j and that $\lambda_T = 0$. The probability that an individual investor lives up to period t ($t \leq T$) is given by the following survival function:

$$F(t) = \prod_{j=0}^{t} \lambda_j, \tag{1}$$

where $0 < F(t) < 1$ for all $0 \leq t < T$, and $F(T) = 0$.

The investor in the economy derives utility from consuming a numeraire good C_t and housing services H_t. In each time period, the investor also receives nonfinancial income Y_t. Before retirement at age J, Y_t represents labor income with real growth rate given by:

$$\Delta \log Y_t = f(t) + \epsilon_t, \quad \text{for } t = 0, \ldots, J - 1 \tag{2}$$

where $f(t)$ is an age-dependent deterministic function and ϵ_t is a shock to the labor-income growth rate. After retirement at age J, Y_t represents payments from pension and social security, at a constant fraction (θ) of his preretirement labor income Y_{J-1}. For simplicity, we assume that labor is inelastically supplied until retirement. The detailed specification on $f(t)$ and ϵ_t is discussed in Section 2.

Similar to Carroll and Dunn (1997), we allow the investor to acquire housing services by either renting or owning a house. Owning a house in our model serves a dual purpose. It not only provides the investor housing services, but also allows the investor to hold home equity. The investor, however, can separate his housing consumption choice from his housing investment choice and avoid most housing price risks by renting. If the investor rents housing services in the previous period, he can either keep renting, or buy a house and become a homeowner at period t. To rent, the investor pays a fraction (α) of the market value of the rental house ($P_t^H H_t$, where P_t^H is the time t price per unit of housing services and H_t is the unit of housing services) to the landlord. To become a homeowner, the investor needs to pay at least a fraction (δ) of the house value as a down payment and finance the rest through a mortgage.

If the investor owns his housing services in the previous period, he first needs to decide whether to sell his house or stay in the same house for the coming period. Selling a house entails a substantial liquidation cost—assumed to be a fraction (ϕ) of the market value of the house—which is borne by the seller. A homeowner upon selling his house faces the same decision as a renter: whether to rent or to buy a house for the coming time period. A homeowner also needs to spend a fraction (ψ) of the house value on repair and maintenance to keep housing quality constant. A renter, however, does not pay for maintenance.

At the beginning of each period the investor incurs an exogenous moving shock, represented by D_t^m, which takes the value of 1 if the investor has to move for exogenous reasons and zero otherwise. A homeowner who experiences the moving shock ($D_t^m = 1$) is forced to sell his house and incurs housing liquidation costs. A renter can move without incurring any transaction costs. We assume that the real return on housing assets ($\tilde{R}_t^H$) follows a stochastic (binomial) process, which can be correlated with stock returns or the preretirement labor-income growth rate.

We assume that the investor can invest in two financial assets: a riskless bond (B_t) and a risky stock (S_t). No transaction costs are incurred for trading these assets. The real gross return on the riskless bond is denoted R_f and is assumed to be constant over time. The real gross return on the risky stock is denoted $\tilde{R}_t^s$ and is assumed to follow a stochastic (binomial) process that can be contemporaneously correlated with the labor-income growth rate and the housing return. Short sale of stock is not allowed and borrowing is allowed at the riskfree rate, but only through collateralizing the investor's house.[2] We also assume that in each period, the investor can costlessly adjust the amount of mortgage through refinancing, a second mortgage, or home equity loans. Denote M_t as the investor's mortgage balance at time t. The investor's bond holdings and mortgage balance then have to satisfy the following constraints:[3]

$$B_t \geq 0 \quad \text{and} \quad 0 \leq M_t \leq D_t^o(1-\delta)P_t^H H_t, \quad \text{for } t = 0, \ldots, T-1 \tag{3}$$

where $1-\delta$ is the maximum proportion of the house's value that can be borrowed in the form of a mortgage against the investor's house, and D_t^o is

[2] We assume that the investor cannot directly borrow against his future labor income because of the moral hazard concern. We also rule out margin account borrowing against one's stockholdings.

[3] In practice, the loan-to-value (LTV) ratio requirement, $1-\delta$, only applies at loan origination. In the event of a housing market downturn, the investor can carry a mortgage larger than the market value of his house. In this case, the investor theoretically would be better off defaulting on his mortgage obligation. However, in reality, residential mortgage defaults are rare and less than 2% [Deng, Quigley, and Van Order (2000)], implying a high credit cost. Furthermore, in practice, refinancing is not costless either. So the investor will refinance to cash out equity only when the benefit of borrowing additional debt outweighs the closing charges. Modeling costly default and refinancing would introduce a separate continuous state variable to keep track of the investor's mortgage balance, which would greatly increase the computational burden. We investigate these interesting issues in Yao and Zhang (2004).

the home-ownership status dummy, which takes the value of 1 if the investor owns his residence and zero otherwise.

We further assume that the after-tax mortgage rate is the same as the after-tax rate of return on the riskless bond. From the investor's perspective, paying down the mortgage by $1 is equivalent to increasing his bond holding by the same amount, as long as the borrowing constraint is satisfied [Equation (3)].[4]

In our model, the investor has a bequest motive represented by a function of bequeathed wealth net of house liquidation cost. Following Dammon, Spatt, and Zhang (2001), we assume that upon an investor's death, the liquidated wealth is used to purchase an L-period annuity to pay for his beneficiary's numeraire good consumption and housing services, with the annuity factor A_L defined as

$$A_L \equiv \frac{r_f(1+r_f)^L}{(1+r_f)^L - 1},$$

where $r_f = R_f - 1$ is the riskfree rate.

The investor's problem is to maximize his discounted expected utility of lifetime numeraire good and housing-service consumption and bequest, subject to the intertemporal budget constraint, given his initial endowment and asset holdings. The investor's problem at time $t = 0$ can now be represented as follows

$$\max_{A(t)} E\left\{\sum_{t=0}^{T} \beta^t [F(t)u(C_t, H_t) + [F(t-1) - F(t)]B(Q_t)]\right\}$$
$$A(t) = \{C_t, H_t, B_t, S_t, D_t^o, D_t^s\}, \quad t = 0, \ldots, T-1 \tag{4}$$

s.t.

$$W_t = B_{t-1}R_f + S_{t-1}\tilde{R}_t^S + D_{t-1}^o P_{t-1}^H H_{t-1}[\tilde{R}_t^H(1-\phi) - (1-\delta)R_f], \tag{5}$$
$$Q_t = W_t + Y_t, \tag{6}$$
$$\begin{aligned} Q_t = {} & C_t + B_t + S_t + (1 - D_{t-1}^o)[(1-D_t^o)\alpha(P_t^H H_t) + D_t^o(\psi+\delta)(P_t^H H_t)] \\ & + D_{t-1}^o[D_t^m + (1-D_t^m)D_t^s][(1-D_t^o)\alpha(P_t^H H_t) + D_t^o(\psi+\delta)(P_t^H H_t)] \\ & + D_{t-1}^o(1-D_t^m)(1-D_t^s)[(\psi+\delta-\phi)(P_t^H H_{t-1})], \end{aligned} \tag{7}$$
$$Y_{t+1} = Y_t \exp\{f(t+1) + \epsilon_{t+1}\}, \tag{8}$$
$$C_t > 0, \quad H_t > 0, \quad B_t \geq 0, \quad S_t \geq 0, \tag{9}$$

[4] Under the assumption of costless refinancing, the investor will *never* simultaneously hold both bonds and a mortgage if different lending and borrowing rates are allowed. When the lending and borrowing rates are the same, there is an indeterminacy with respect to bond and mortgage holdings. To pin down the investor's bond holding, in our subsequent analysis, we assume that the investor always carries the maximum mortgage balance allowed, i.e., $M_t = D_t^o(1-\delta)P_t^H H_t$.

given the initial home ownership status $D^{o}_{(-1)}$, realization of exogenous moving shock D^{m}_{0}, net worth before labor income (net of house liquidation cost if applicable) W_0, labor income Y_0, housing price P^{H}_{0}, and housing stock $H_{(-1)}$. The expression inside the square brackets in Equation (4) is the investor's probability-weighted utility at time t. The first term measures the investor's utility of numeraire good and housing-service consumption in period t weighted by the probability of living through period t, while the second term is the investor's utility of bequest weighted by the probability of dying in period t. $u(\cdot)$ and $B(\cdot)$ denote the investor's utility function and bequest function, respectively. β is the subjective time discount factor. $F(-1)$ is set to 1 to indicate that the investor has survived up to period 0. Due to house liquidation costs, home ownership choice at time period $t-1$, D^{o}_{t-1}, is also a state variable at time t. D^{s}_{t} is a binary choice variable that takes the value of 1 if the investor sells his house at time t and zero otherwise.

Equation (5) defines the evolution of W_t, the investor's net worth at the beginning of the period, net of house liquidation cost. Equation (6) defines Q_t, the investor's total spendable resources available for consumption and investment at time t.[5] Equation (7) defines the investor's budget constraints at period t, and Equation (8) defines the evolution of his labor income.

We assume that the investor's preferences over numeraire good consumption and housing services are represented by the Cobb–Douglas utility function:

$$u(C_t, H_t) = \frac{(C_t^{1-\omega} H_t^{\omega})^{1-\gamma}}{1-\gamma}, \tag{10}$$

where ω measures the relative importance of housing services versus numeraire good consumption and γ is the curvature parameter.[6] An investor with the Cobb–Douglas utility will spend on C_t and H_t in a fixed proportion in a one-period model. This property still holds in a multiperiod setup for the periods that the investor rents housing services. This is because renting does not trigger housing-related transaction costs in any subsequent periods, and a renter will adjust his current consumption to the point where the marginal utilities of an additional dollar spent on the numeraire good and rental housing are equated. Therefore, the existence of the rental market eliminates housing as a separate choice variable when the investor rents housing services. In fact, if renting always dominates owning and housing price is nonstochastic, our setup can be

[5] For ease of exposition, for the rest of the article, we refer to Q_t as investor's total wealth or simply wealth.

[6] In the presence of house liquidation cost, the investor's relative risk aversion is not identical to the curvature parameter. In fact, the investor's relative risk aversion varies within the no-adjustment region [see Damgaard, Fuglsbjerg, and Munk (2003)].

simplified to the portfolio choice problem with nontradable labor income and a single numeraire consumption good, such as those modeled in Heaton and Lucas (2000b) or Cocco, Gomes, and Maenhout (2004), among others. However, when renting is suboptimal to owning at least in some stage of an investor's life, house liquidation cost and endogenous borrowing constraints break down the fixed proportion of expenditure on C_t and H_t. The value of the endowed housing asset in this case becomes a state variable that affects an investor's consumption and portfolio choices.

We assume that the annuity income from a bequest is used to pay for the beneficiary's numeraire good consumption and housing rental costs. Further, the beneficiary's numeraire good and housing-service consumption is set at the fixed proportion of $(1-\omega)/\omega$, the optimal level for the Cobb–Douglas utility function when renting. Hence, the bequest function can be defined as

$$B(Q_t) \equiv \sum_{k=t+1}^{t+L} \beta^{k-t} \frac{\left[(A_L Q_t)\omega^{\omega}(1-\omega)^{1-\omega}\right]^{1-\gamma}}{(1-\gamma)(\alpha P_t^H)^{\omega(1-\gamma)}}$$
$$\equiv \frac{\beta(1-\beta^L)\left[(A_L Q_t)\omega^{\omega}(1-\omega)^{1-\omega}\right]^{1-\gamma}}{(1-\beta)(1-\gamma)(\alpha P_t^H)^{\omega(1-\gamma)}}. \tag{11}$$

The value function of the investor's intertemporal consumption and investment problem can be written as

$$V_t(X_t) = \max_{A(t)} \left\{ \lambda_t \left[\frac{(C_t^{1-\omega} H_t^{\omega})^{1-\gamma}}{1-\gamma} + \beta E_t[V_{t+1}(X_{t+1})] \right] \right.$$
$$\left. + (1-\lambda_t) \frac{\beta(1-\beta^L)\left[A_L Q_t \omega^{\omega}(1-\omega)^{1-\omega}\right]^{1-\gamma}}{(1-\beta)(1-\gamma)(\alpha P_t^H)^{\omega(1-\gamma)}} \right\}$$
$$A_t = \{C_t, H_t, B_t, S_t, D_t^o, D_t^s\}, \quad t = 0, \ldots, T-1 \tag{12}$$

and the sufficient vector of state variables consists of the beginning-of-period home ownership status dummy, moving shock dummy, price per unit of housing services, size of the existing house, and the investor's levels of labor income and net worth, that is, $X_t \equiv \{D_{t-1}^o, D_t^m, P_t^H, H_{t-1}, Y_t, W_t\}$.

The above problem can be simplified by using the investor's wealth, Q_t, as a normalizer to reduce the dimension of the state space. The details of this normalization scheme and the procedure for finding a numerical solution are given in the appendix. As a result of this normalization, the investor's optimization problem involves the following choice variables: The numeraire good-consumption–wealth ratio, $c_t = C_t/Q_t$; the house-value–wealth ratio, $h_t = P_t^H H_t/Q_t$; the fraction of wealth allocated to bonds, $b_t = B_t/Q_t$; the fraction of wealth allocated to stocks, $s_t = S_t/Q_t$; the housing tenure choice, D_t^o; and the house liquidation decision, D_t^s. The relevant state variables for the normalized optimization problem are home ownership status, D^o_{t-1}; moving shock, D^m_t; beginning-of-period net-worth–labor-income ratio, $w_t = W_t/Y_t$; and beginning-of-period house-value–net worth ratio, $\bar{h}_{t-1} = P^H_t H_{t-1}/W_t$.

2. Numerical Results

In our numerical analysis, we establish a baseline case with the following parameter values. We assume that the investor makes decisions annually starting at age 20 ($t = 0$) and lives for at most for another 80 years until age 100 ($T = 100$). The annual mortality rate is calibrated to the 1998 life table for the total U.S. population from the National Center for Health Statistics [Anderson (2001)]. The age-dependent deterministic labor-income growth rate before retirement $f(t)$ is based on the empirical estimation of Cocco, Gomes, and Maenhout (2004) by fitting a third-order polynomial to the labor income of college graduates using the PSID data. The profile demonstrates a hump shape over the life cycle before retirement. Following Cocco, Gomes, and Maenhout (2004), the investor is assumed to retire at age $J = 65$ and receives constant annual nonfinancial income, including a pension, social security payments, and distributions from retirement accounts, that is equal to $\theta = 60\%$ of his labor income at age 64.[7] Similar to Viceira (2001), we only consider transitory shocks to the labor-income growth rate and set the standard deviation of the shocks at 13% per year.

We set the annual discount factor at $\beta = 0.96$ and the curvature parameter at $\gamma = 5$. We also set $L = 20$ in the bequest function, assuming that the investor wishes to provide his beneficiary with 20 years of numeraire goods and housing services from the bequest. Housing preference is set at $\omega = 0.2$, consistent with the average proportion of household housing expenditure in the 2001 Consumer Expenditure Survey [U.S. Department of Labor (2003)].

Consistent with the historical real costs of renting and owning, we set the annual rental cost at $\alpha = 6.0\%$ of the market value of the rental property and the annual maintenance and depreciation cost at $\psi = 1.5\%$ of the market value of the owned property.[8] The cost of selling an existing

[7] In the following analysis, we continue to use the term "labor income" even after retirement to avoid multiple definitions for the state variable.

[8] The latest Residential Finance Survey [U.S. Census Bureau (1992)] shows that the average rental cost is 7% of the market value of a rental property with one to four housing units. The rental cost is higher for rental properties with more than four housing units. The housing statistics of the United States also show that the median rent as a fraction of median home value is about 7% for the period between 1993 to 1997 [see Hu (2002)]. We adopt a slightly lower value since we abstract from inflation and adopt a low real rate of interest of 2% as the opportunity cost of capital for the landlord. The implied cost differential of renting versus owning in our baseline case is consistent with (but slightly lower than) what is used in Campbell and Cocco (2003), which assumes that the rental premium is 3% above the per-period owning cost to account for moral hazard in the housing rental market.

house is set at $\phi = 6\%$ of the market value of the house, the conventional fee charged by the vast majority of real estate agents. The home equity requirement is set at $\delta = 20\%$ of house value.

We set the riskfree rate at $r_f = 2.0\%$ and the risk premium at $\mu = 4.0\%$. Claus and Thomas (2001), Fama and French (2002), and others have argued that the expected future equity risk premium should be substantially lower than the historical average of 7–8%. The standard deviation of the risky asset return is set at $\sigma_S = 15.7\%$, the historical estimate of the Standard & Poors 500 index portfolio. The mean real house price appreciation rate is set at $\mu_H = 0\%$.[9] The volatility of the housing return is set at $\sigma_H = 10.0\%$, a value between the aggregate index level [Goetzmann and Spiegel (2000)] and the upper bound of the empirical estimates at the household level [Flavin and Yamashita (2002)].

The correlation between the housing return and the labor-income growth rate is set at $\rho_{Y,H} = 0.2$. We set the correlation between the labor-income growth rate and stock return at $\rho_{Y,S} = 0.0$, consistent with the empirical (lack of) correlation between labor income and stock market returns at the occupational level, as documented in Cocco, Gomes, and Maenhout (2004) and Davis and Willen (2000). The correlation between the housing and stock returns is also set at $\rho_{H,S} = 0.0$, consistent with the estimate in Flavin and Yamashita (2002) and Goetzmann and Spiegel (2000). Further, we set the probability of incurring an exogenous moving shock to zero. We call the above parameter values the baseline case.

We explore the hedging demand for stocks induced by housing price risk by allowing a positive correlation between stock and housing returns set at $\rho_{H,S} = 0.2$. The effect of exogenous moving shocks on the investor's housing and portfolio decisions is examined by calibrating probabilities of moving to the average annual intercounty migration rate of college graduates between March 2000 and March 2001 as reported in the Current Population Survey (CPS) conducted by the U.S. Census Bureau (2003).[10]

We also consider alternative parameterizations to check the robustness of our findings and examine the effects of varying maintenance cost, housing risks, and labor-income profiles on the investor's housing and portfolio choices. Specifically, we consider the cases of higher house maintenance cost ($\psi = 2.5\%$), zero housing-return risk ($\sigma_H = 0$), zero

[9] Based on 80 quarters of housing index data between March 1980 and March 1999, Goetzmann and Spiegel (2000) estimate real housing returns for the 12 largest Metropolitan Statistical Areas (MSA); the annualized arithmetic/geometric mean housing returns vary from $-1.0\%/-1.1\%$ to 3.46%/3.26%.

[10] In reality, moving can be caused by job- or family-demographics-related reasons such as divorce, marriage, or family expansion, as well as by changes in a family's desired housing consumption level due to changes in net worth and labor income. The latter is endogenously determined and has already been taken into account in our model. Unfortunately, the data did not specify the reasons for moving. We assume that moving to a location in a different county is caused by exogenous reasons. We thank the referee for this valuable point. See www.census.gov/population/www/socdemo/migrate/cps2001.html, Table 5, for details on general mobility by age and education attainment.

correlation between the labor-income growth rate and housing return ($\rho_{Y,H} = 0$), and the labor-income profile of a high school graduate.

2.1 Optimal consumption and investment policies *without* a housing endowment

Our discussion in this section focuses on the optimal consumption and investment decisions of the investor who enters time period t without a housing endowment, so that no liquidation cost is incurred for the current period when the investor makes adjustments to his housing services. These decisions are important because one-third of U.S. households are renters. Also, for the investor with an initial housing endowment, his optimal decisions after house liquidation are identical to those of the investor without a housing endowment but having the same level of spendable resources and labor income.

Figure 1a shows the investor's optimal housing tenure choice as a function of the investor's beginning-of-period net-worth–labor-income ratio and the investor's age. The solid curve represents the net-worth–labor-income ratio trigger bound of owning versus renting. At a given age, the investor with a high net-worth–labor-income ratio purchases a house, while the investor with a low net-worth–labor-income ratio chooses to rent. To own a house, the investor needs to meet the initial down payment requirement and also satisfy subsequent home equity requirements using his wealth on hand. An investor with a high net-worth–labor-income ratio is less liquidity-constrained and can afford a house closer to his desired size. Therefore, he expects to stay in the house longer to reduce liquidation cost and is in a better position to benefit from home ownership.[11]

Furthermore, the net-worth–labor-income ratio trigger bound decreases in the age of the investor before the investor reaches his late forties and increases thereafter. It then sharply declines as the investor approaches retirement and his labor income is drastically reduced. After retirement, the trigger bound monotonically increases in investor age. The level of the trigger bound is determined primarily by the investor's earnings profile before retirement and bequest motive afterward. A young investor has a high present value of labor income and wishes to own a large house relative to his current labor income in order to smooth intertemporal housing services and minimize house liquidation cost. Therefore,

[11] For the baseline case, the per-period user cost of home ownership — the sum of per-period maintenance cost, mortgage cost, and the opportunity cost of home equity, minus housing appreciation — is lower than the per-period cost of renting a similar property. Another possible modeling approach to induce home ownership is through consumption motives — the investor derives higher utility from owning than renting the same property — by assigning a higher housing preference parameter for owning a house versus renting. We solved a model in which $\omega = 0.20$ if housing services are acquired through renting and $\omega = 0.25$ if the investor owns his home, while per-period rental and owning costs are the same. The results are qualitatively very similar.

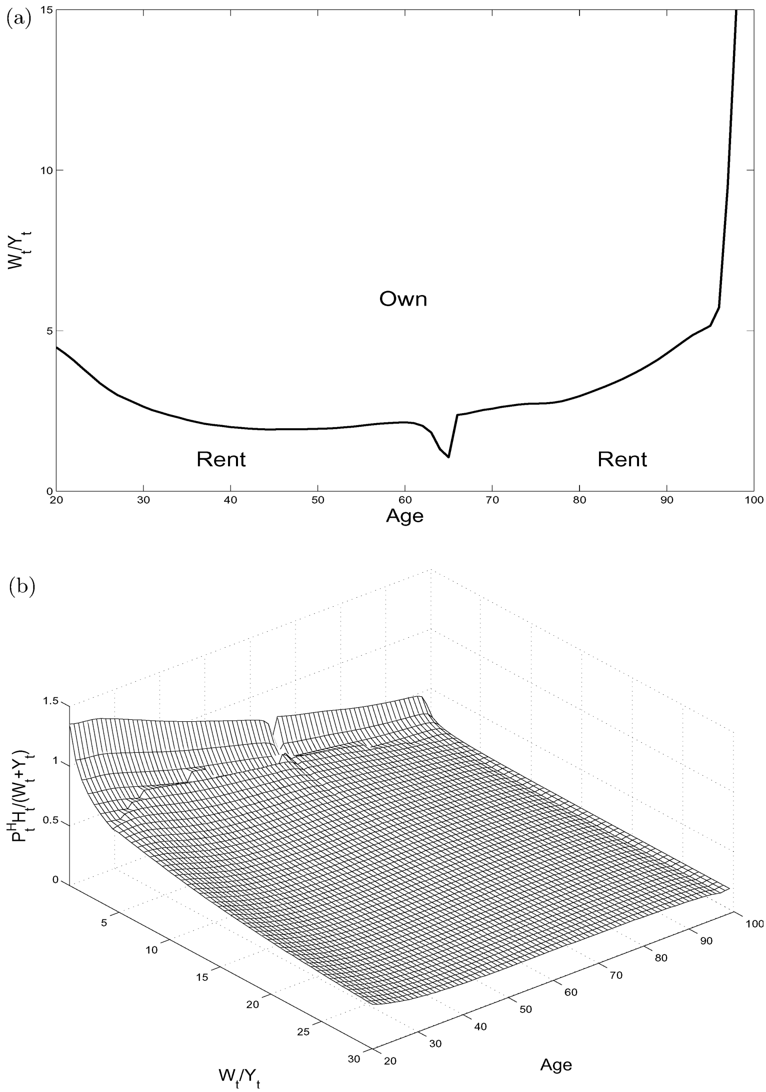

Figure 1
Owning versus renting trigger boundary of the net-worth–labor-income ratio (W_t/Y_t) as a function of investor age (a) and the housing-value–total-wealth ratio as a function of the beginning-of-period net-worth–labor-income ratio (W_t/Y_t) and investor age (b)

a young investor will require a higher net-worth–labor-income ratio to trigger house ownership. As the investor ages and the present value of his earnings declines, his desired house size also decreases relative to his labor income. A lower level of the net-worth–labor-income ratio is then enough to trigger home ownership. After retirement, the mortality rate increases rapidly and the bequest motive gradually dominates the investor's housing decision. Because the investor derives utility from bequeathing his wealth net of house liquidation cost, he is reluctant to purchase a house unless his net-worth–labor-income ratio is so high that the additional benefit of owning a house outweighs the cost of liquidation at the time of death. As the investor ages and death is proximate, the probability of incurring liquidation cost also increases. The net-worth–labor-income ratio trigger bound thus increases. Indeed, at very advanced ages, the investor always rents housing services to avoid liquidation cost.

Figure 1*b* shows the fraction of wealth (the sum of net worth and current labor income) allocated to housing as a function of the beginning-of-period net-worth–labor-income ratio and the age of the investor. At a given age, the investor spends less on housing services (either renting or owning) as his net-worth–labor-income ratio increases. The investor's expenditure on housing services decreases as the investor ages. This is primarily driven by the gradual realization of the investor's earning power over his life cycle and is consistent with the permanent income hypothesis. At a low net-worth–labor-income ratio, his labor income accounts for a relatively large fraction of his spendable resources. The investor thus spends a higher fraction of his wealth on housing services. Analogously, when the investor's net-worth–labor-income ratio is high and his labor income accounts for a small fraction of his spendable resources, the investor allocates a relatively small fraction of his wealth to housing services.

On the trigger bound of owning versus renting, the investor consumes more housing services when he owns a house than when he rents. This can be explained as follows. Intratemporally, the investor will equate the marginal utility of an additional dollar spent either on housing services or on numeraire good consumption. Because renting is more expensive than owning per unit of housing services, the investor consumes less housing services relative to the numeraire good when renting than when owning. The overall pattern of the investor's numeraire good consumption (figure not shown) is very similar to the investor's housing-services consumption. In contrast to his housing-consumption behavior, on the trigger bound of owning versus renting, the investor consumes slightly more numeraire good when renting than when owning. This reflects the substitution effect of the numeraire good for housing services.

We now discuss the investor's investment decision. When the investor chooses to rent housing services for the current period, all his net worth is in the form of liquid financial assets: bonds and stocks. If the investor chooses to own a house, a fraction of his net worth is held instead in illiquid home equity. We therefore make a distinction between an investor's liquid financial portfolio (bonds and stocks) and net worth (bonds, stocks, and home equity).

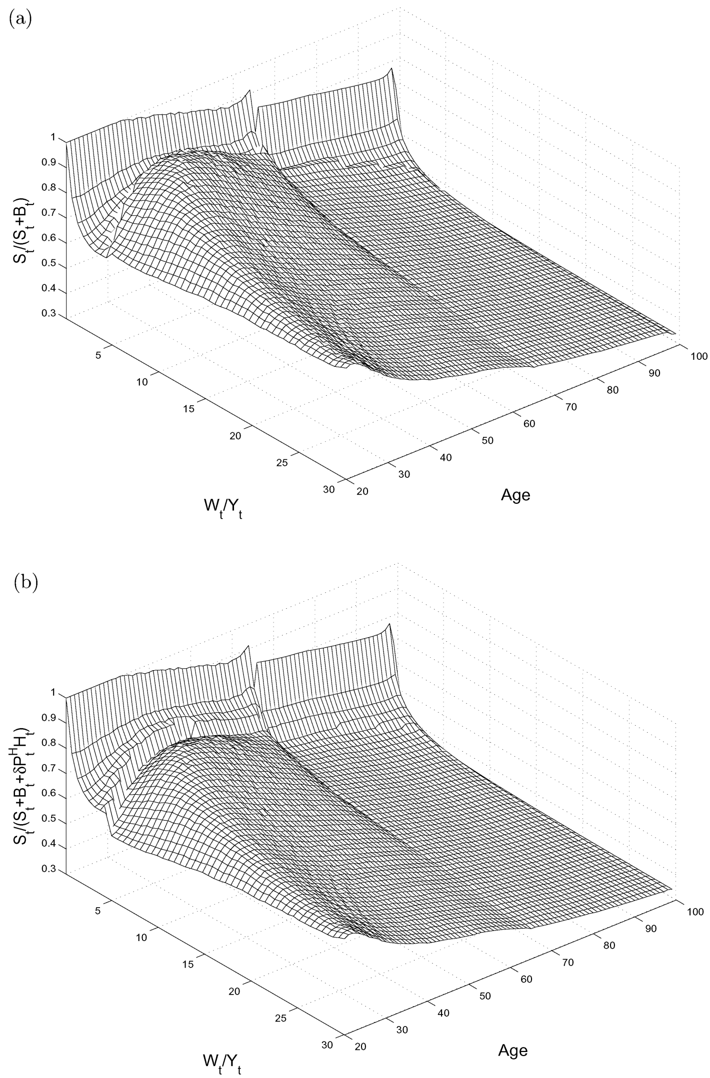

Figure 2
Optimal equity proportion in liquid financial portfolio (a) and net worth (b) as a function of the beginning-of-period net-worth–labor-income ratio (W_t/Y_t) and age for an investor *without* a housing endowment at the beginning of the period

The optimal equity proportions in the investor's liquid financial portfolio and in his net worth are shown in Figures 2*a* and 2*b*, respectively. Because the two equity proportions share similar overall features, we focus on the investor's liquid financial portfolio composition. Recall that the investor rents housing services for liquidity reasons when his beginning-of-period net-worth–labor-income ratio is low. As a renter, the investor's equity proportion can reach a very high level (close to 100%) and declines as the net-worth–labor-income ratio increases. This is consistent with the findings regarding equity proportion levels in an economy without risky housing [see Jagannathan and Kocherlakota (1996), Heaton and Lucas (2000b), and Cocco, Gomes, and Maenhout (2004), among others]. It can be attributed to the fact that the presence of labor income allows the investor to better diversify his exposure to equity risk and crowds out riskless bond holding. When the investor owns a house, his optimal equity proportion in the liquid financial portfolio is hump-shaped in both the net-worth–labor-income ratio and age. Further, the investor age at which the homeowner's equity proportion peaks is inversely related to the investor's net-worth–labor-income ratio. Two effects contribute to the above observations. First, as the investor ages or as his net-worth–labor-income ratio increases, the present value of his lifetime earnings decreases relative to his net worth. Since the investor's labor income is a close substitute for bonds, the declining present value of earnings leads the investor to increase his bond holdings and reduce his stock holdings. Second, at very young ages or at very low levels of the net-worth–labor-income ratio, homeowners are severely liquidity-constrained due to the collateral requirements associated with housing. To alleviate the liquidity concern, the investor tilts his liquid financial portfolio toward safe assets.

Interestingly, on the trigger bound of owning versus renting, the investor holds a riskier liquid financial portfolio (has a higher equity proportion) when he owns than when he rents.[12] The homeowner, however, allocates a smaller fraction of his wealth to stocks than does the renter. This is clearly demonstrated in Figures 3*a* and 3*b* in which we plot the investor's equity proportions in his liquid financial portfolio and in his net worth, respectively, when the investor is indifferent between renting versus owning a house. While the asset substitution argument explains why the homeowner would lower his equity proportion in net worth, the higher equity proportion in his liquid financial portfolio may at first glance seem puzzling. This can be explained by the *diversification* effect of owning two risky assets: Home equity and stocks. While a renter's risk exposure depends solely on his holding of risky stocks, a homeowner's risk exposure depends on his positions in both home equity and stocks. With a low return correlation between risky stocks and home equity, a homeowner reduces his stockholding in net worth but holds a riskier liquid financial

[12] Renters who derive their income predominantly from nonfinancial sources (such as labor income) and are far away from the trigger bound of becoming a homeowner may hold a higher equity proportion in their liquid portfolio than do homeowners.

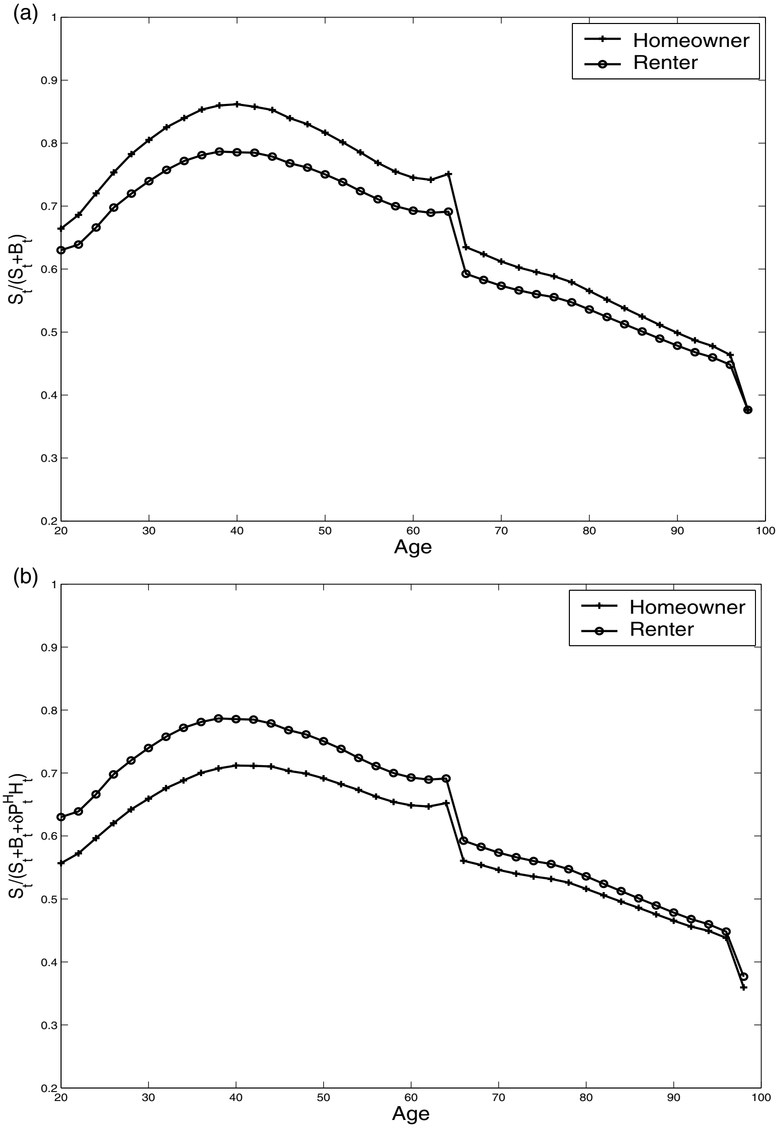

Figure 3
Optimal equity proportion in liquid financial portfolio (a) and net worth (b) as a function of age along the trigger boundary of owning versus renting for an investor without a housing endowment at the beginning of the period

portfolio. While existing studies have documented that the presence of risky home equity reduces the investor's exposure to risky stocks, our study uncovers the important buffering role of home equity for negative shocks to stock returns and labor-income growth rates.

2.2 Optimal consumption and investment policies with a housing endowment

Our discussions so far have focused on the optimal policies for the investor who does not own a house at the beginning of the current period and can costlessly adjust his housing services, numeraire good consumption, and stock and bond holdings to their optimal levels. However, for an investor who owns an existing house, the value of his existing house also affects his consumption and portfolio decisions.

Figure 4*a* shows the investor's housing tenure choice at age 20 as a function of the investor's beginning-of-period net-worth–labor-income ratio and the house-value–net-worth ratio. The presence of liquidation cost leads to four different regions of actions: (1) The no-adjustment region (S) — the investor stays in the existing house, (2) the rental region (R) — the investor sells his existing house and rents housing services, (3) the upward adjustment region (U) — the investor sells his existing house and buys a larger house, and (4) the downward adjustment region (D) — the investor sells his existing house and buys a smaller house. Both the lower bound (the curve on the left) and the upper bound (the curve on the right) of the no-adjustment region decrease in the net-worth–labor-income ratio. Further, the investor chooses to rent after selling his existing house if his net-worth–labor-income ratio is low. The observed behavior can be attributed to the investor's desired housing service level without a housing endowment (see Figure 1*b*) and the effects of liquidation cost and liquidity concerns. As discussed previously, the investor's optimal housing service level is a decreasing function of the net-worth–labor-income ratio when the investor enters the current period without a housing endowment. The homeowner's no-adjustment boundaries are formed around the optimal housing service level without housing endowment. At a low net-worth–labor-income ratio, the investor is more liquidity-constrained. Upon selling his existing house, the investor rents housing services.

Liquidation cost also has a substantial impact on the investor's portfolio choices when he owns a house (Figure 4*b*).[13] While the overall life cycle features of the homeowner's equity proportions in his liquid portfolio and net worth are similar to those of the investor without a housing endowment, there are notable differences within the no-adjustment region.

Within the no-adjustment region, the investor's liquid portfolio equity proportion is U-shaped in the house-value–net-worth ratio. The finding is consistent with that of Grossman and Laroque (1990), which shows that the investor behaves in a more risk averse manner just after purchasing a new house, and in a less risk averse manner just before purchasing a new house. Higher risk tolerance thus leads to a high equity proportion in the

[13] The investor's net-worth–labor-income ratio is set at $W_t/Y_t = 3.0$.

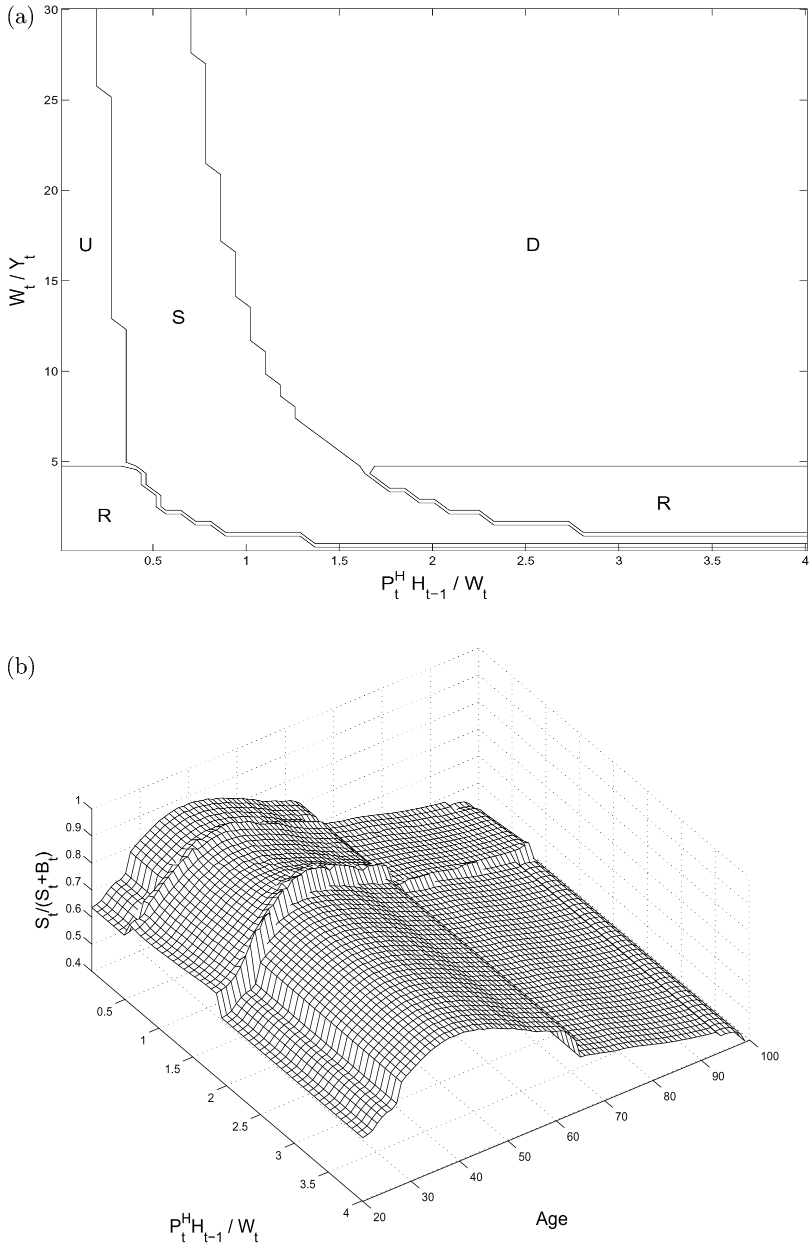

Figure 4
Trigger boundaries as a function of the house-value–net-worth ratio $(P_t^H H_{t-1}/W_t)$ for an investor at age 20 (a) and the equity proportion in liquid financial portfolio as a function of the house-value–net-worth ratio $(P_t^H H_{t-1}/W_t)$ and investor age (b)
S, stay in the existing house; R, sell the existing house and rent; U, sell the existing house and purchase a larger house; D, sell the existing house and purchase a smaller house.

investor's liquid financial portfolio near the trigger bounds. When his house-value–net-worth ratio is close to the optimal level without a housing endowment, the investor holds a safer liquid portfolio to reduce deviations from the "target" level of the house-value–net-worth ratio.

In our model, while the lower risk aversion on the trigger bounds leads to a riskier liquid portfolio, the investor's net worth equity proportion decreases in the house-value — net worth ratio (figure not shown). This is attributable to the model's collateral requirements. When the investor refrains from selling a house larger than his desired size, he holds more home equity than the optimal level without a housing endowment. As a result, the investor reduces his liquid financial asset holdings, including stocks, to finance home equity.

2.3 Welfare cost and bias in portfolio choices under alternative policy rules

We now examine the impact of housing choices on welfare and portfolio decisions under two alternative policies: Acquiring housing services only via renting, or acquiring housing services only via owning a house. We first solve numerically the optimal consumption, housing, and portfolio decisions for each of the alternative cases and then compare the resulting value functions to the value function for the baseline model. Specifically, for each point in the state space, we calculate the percentage increase in total wealth (Q_t) necessary to bring the level of the value function (indirect utility) for an alternative case to the level of the baseline model. For portfolio choices, we focus on the equity proportion in the investor's liquid portfolio on the renting-versus-owning trigger bound corresponding to the baseline model. Specifically, we compare the equity proportions in the renters' liquid portfolios in the first case, and compare the equity proportions in the homeowners' liquid portfolios in the second case. The results for the welfare calculation are shown in Figures 5*a* and 5*b* for an investor without a housing endowment at the beginning of the period ($D^o_{t-1} = 0$), while the corresponding portfolio choices are plotted in Figures 6*a* and 6*b*.

Figure 5*a* shows the welfare cost for an investor who follows the suboptimal policy of always renting housing services, as implicitly assumed in most existing studies on portfolio choices such as Heaton and Lucas (2000a) and Cocco, Gomes, and Maehout (2004). The welfare cost increases in the net-worth–labor-income ratio. This is because as an investor's net-worth–labor-income ratio increases and his liquidity concern eases, the investor benefits more from owning a house. He therefore incurs a higher welfare loss when following the suboptimal policy of always renting housing services. The welfare cost also is hump-shaped in investor age, reflecting the combined effects of liquidity constraints, the bequest motive, and the investment horizon. First, for a given net-worth–labor-income ratio, a young investor is more

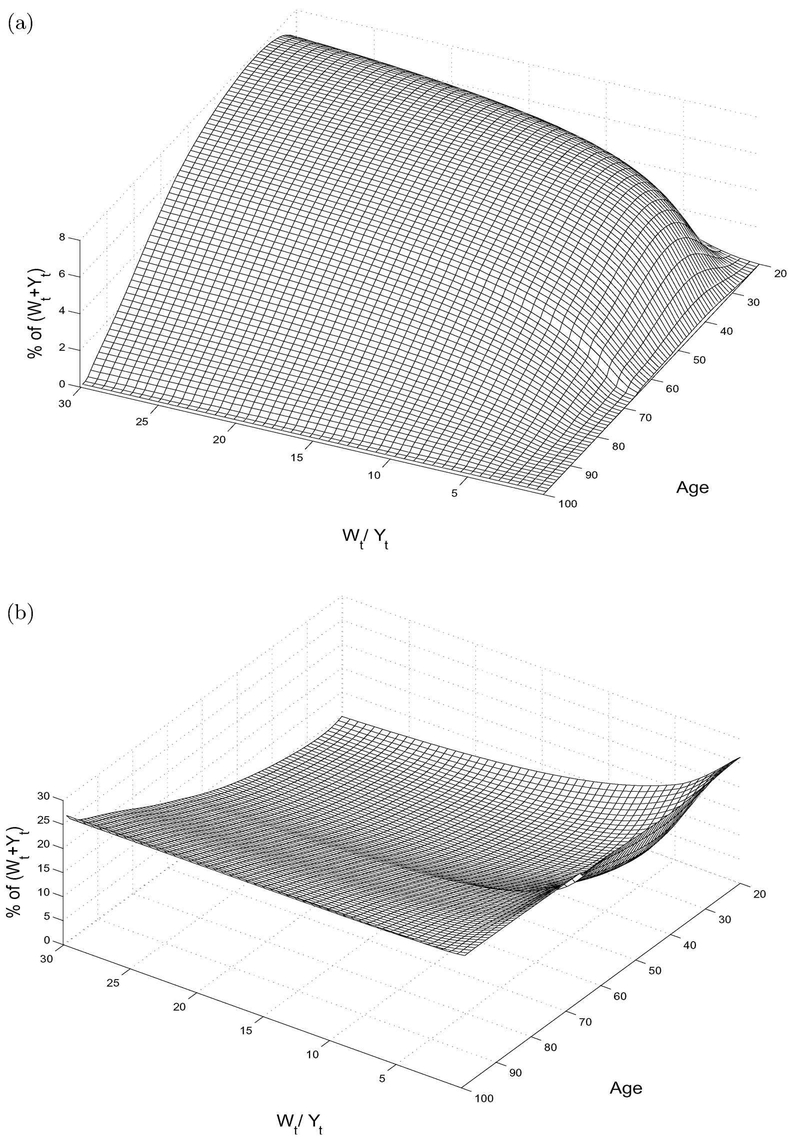

Figure 5
Welfare costs as a function of the net-worth–labor-income ratio (W_t/Y_t) and investor age under the suboptimal policy of always renting housing services (a); or acquiring housing services only from owning a house (b)

liquidity-constrained and less able to afford a house of his desired size. Therefore, following the suboptimal policy causes less welfare loss for younger investors than for older investors. Second, younger investors have a longer horizon to exploit the savings from owning a house and

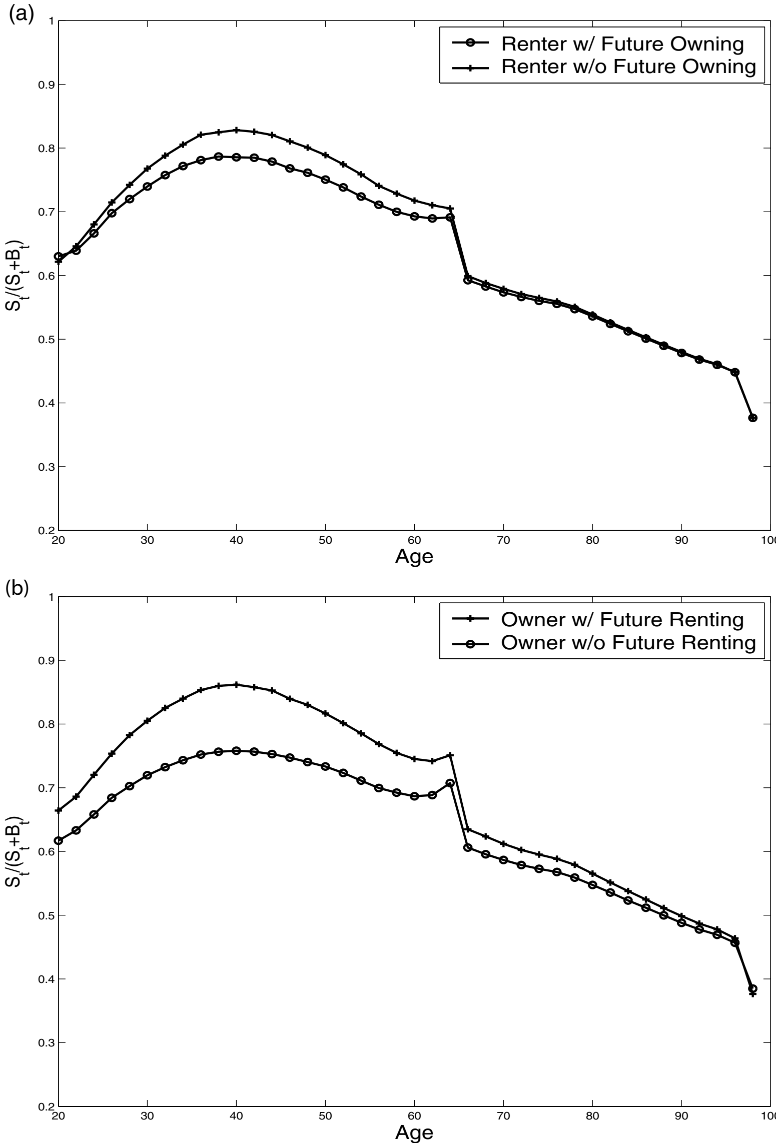

Figure 6
Portfolio choices as a function of investor age under the suboptimal policy of always renting housing services (a); or acquiring housing services only from owning a house (b)

thus incur higher welfare costs when they always rent housing services. This horizon effect thus declines in age. Further, as the investor approaches the terminal date, he also refrains from owning a house to avoid house liquidation cost at death. The bequest motive thus reinforces the horizon effect at advanced ages. Overall, the welfare cost of forgoing the opportunity to own a house is quite high for the investor with a high net worth. For example, for an investor with a net worth that is 30 times his annual labor income, the welfare cost is almost 8% of his current wealth, reflecting the large lifetime benefit of home ownership.

The investor's portfolio choice also is substantially affected if the investor follows the suboptimal policy of always renting housing services (Figure 6*a*). Specifically, relative to the optimal portfolio choice with home ownership, the equity proportion is biased upward. The bias reflects the tradeoff between earning a higher equity premium and benefiting from being a homeowner. In the baseline model, the investor holds a safer liquid portfolio while saving for initial down payment to purchase a house. Since the investor under the alternative policy will never own a house, he takes a higher risk in his liquid financial portfolio by allocating a larger fraction of his liquid financial investments to stocks. For our parameter values, the gap between the equity proportions of the optimal and suboptimal policies' liquid financial portfolios reaches a peak of 4.3% for the investor in his early forties. Our findings also imply that studies excluding home ownership market offer reasonable approximations to the optimal consumption and portfolio policies only when the investor's net worth remains very low throughout his lifetime and the investor never becomes a homeowner. Yet, the same studies usually predict that the investor accumulates a high net worth relative to his labor income, at least at some stages of his life cycle.

Since renting is the only alternative to owning, the qualitative features of welfare cost arising from always owning a house are the opposite of those arising from always renting housing services. The welfare cost decreases in the investor's net-worth–labor-income ratio and is U-shaped in the investor's age (see Figure 5*b*). Strikingly, the welfare cost of following the suboptimal policy of forgoing the opportunity to rent housing services, as in Cocco (2004), can be very large for an investor with a low net-worth–labor-income ratio or an investor approaching the terminal date. This reflects the critical role played by the house rental market in allowing investors to smooth housing consumption and avoiding house liquidation cost. For instance, for an investor with little net worth or an investor in his late nineties, the welfare cost can amount to more than 25% of his total wealth. It is intuitively clear that the investor who rents in the baseline model would be forced under the suboptimal policy to purchase a small house, which would provide far fewer services than his rental house and would likely be liquidated soon after some wealth accumulation or upon the investor's death. This finding implies that excluding the house rental market in studies of illiquid risky housing decisions may lead to distorted housing choices for investors facing severe liquidity constraints or investors at advanced ages.

The lack of rental opportunities also drastically alters the investor's portfolio choices (see Figure 6*b*). Specifically, compared to the optimal portfolio choice in an economy with a rental market, the investor's liquid portfolio stockholding is substantially biased downward. This can be explained by the excessively precautionary motive to hold bonds. Under the suboptimal policy, the investor acquires housing services only from owning a house. The homeowner thus has an incentive to hold more safe assets — bonds — in order to meet his subsequent liquidity constraints and to reduce the frequency of house turnover. For our parameter values, the equity proportion gap in the liquid financial portfolios reaches a peak of 10.3% for an investor in his early forties.

2.4 Hedging demand induced by housing price risk and the effect of moving shocks

In this section, we first examine the hedging demand for stocks induced by housing price risk when stock returns and housing returns are positively correlated. We then consider the effect of an exogenous moving shock on an investor's housing and portfolio choices.

The net-worth–labor-income ratio trigger bound of owning versus renting is shifted upward (Figure 7*a*), if stock returns and housing returns are positively correlated ($\rho_{H,S} = 0.2$), indicating the investor's need to accumulate more net worth before purchasing a house. This reflects the precautionary motive to save more in order to alleviate potential liquidity concerns caused by the positive comovement between the stock market and the housing market.

Since the investor's net-worth–labor-income ratio trigger bound is higher for a positive stock- and housing-return correlation, we plot the homeowner's liquid portfolio equity proportion on the trigger bound corresponding to a positive stock- and housing-return correlation (Figure 7*b*) and the renter's equity proportion on the trigger bound corresponding to a zero stock- and housing-return correlation (Figure 7*c*). Because the hedging demand for stocks induced by housing price risk is zero when stock returns and housing returns are uncorrelated ($\rho_{H,S} = 0$), the gap in equity proportions thus reflects the hedging demand for stocks attributable to the correlation between stock returns and housing returns. Further, since a homeowner holds a long position in housing assets, the hedging demand should reduce the homeowner's stockholding when the stock- and housing-returns are positively correlated. Indeed, compared to the baseline case, the homeowner's equity proportion is notably lower in the case with a positive stock- and housing-return correlation. For our parameter values, the homeowner's liquid portfolio equity proportion can be reduced by as much as 7.0%. Interestingly, for the renter, the hedging demand *increases* the investor's equity proportion when the stock returns and housing returns are positively correlated. For our parameter values,

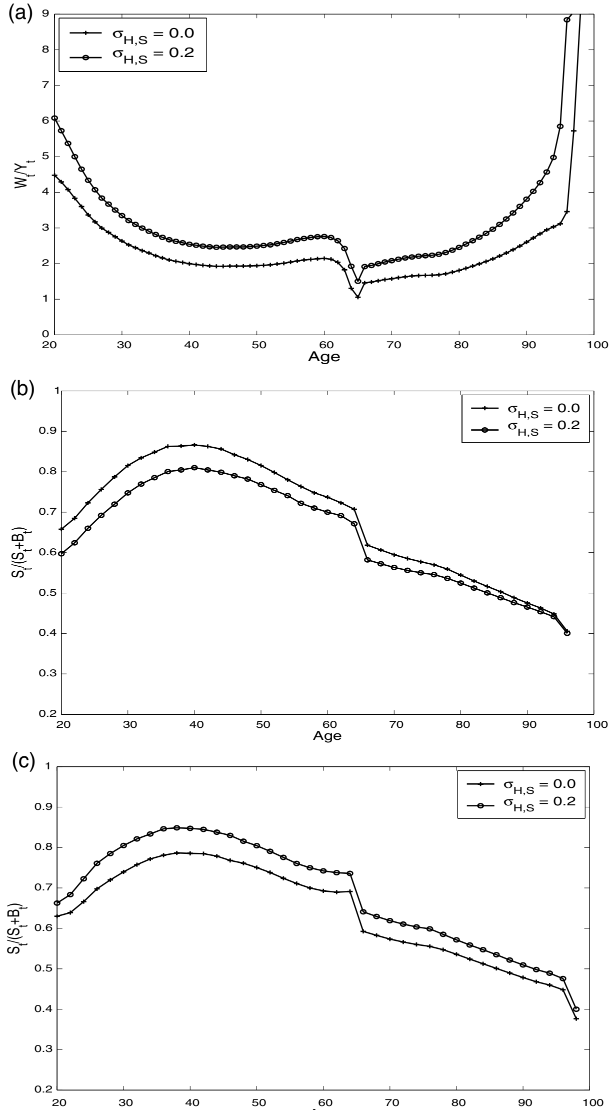

Figure 7
The net-worth–labor-income ratio trigger bound (a), the portfolio choices for homeowners (b) and renters (c) as a function of investor age when stock and housing returns are positively correlated ($\rho_{H,S} = 0.2$). Other parameter values are set at the baseline case

the renter's liquid portfolio equity proportion can be increased by as much as 6.7%. When the investor rents housing services, he is effectively taking a short position in housing assets by paying rent, which is assumed to be a fraction of the house value. When the stock returns and housing returns are positively correlated, investing in stocks helps the renter to hedge against fluctuations in his future rent and house down payment.

When facing an exogenous moving shock, investors become homeowners at higher levels of the net-worth–labor-income ratio (data not shown). Young investors, in particular, purchase a house only after they have accumulated substantially more net worth than in the case without the moving shock. This is to be expected and is caused by high housing liquidation cost and the fact that young investors have higher probabilities of moving. However, the exogenous moving shock has only a small effect on investors' portfolio choices. For both homeowners and renters, the optimal portfolios consist of slightly more equity with a positive probability of a moving shock (data not shown). This captures the reduced incentive to invest conservatively when there is a reduced home ownership benefit.

2.5 Comparative static analysis

In this section, we provide comparative static results by varying the house maintenance cost, the riskiness of the housing return, the correlation between the housing return and the labor-income growth rate, and the investor's calibrated labor-income profile. Our discussions will focus on the housing tenure decision and portfolio choices for an investor entering the current period *without* a housing endowment.

The overall qualitative features of the trigger bounds for these cases are quite similar to those of the trigger bound in the baseline case (data not shown). However, there are some notable quantitative differences. Specifically, increasing the house maintenance cost shifts the trigger bound upward for all ages, indicating that the investor accumulates more net worth before purchasing a house when the benefit of home ownership is lower. Eliminating the housing-return risk or the positive correlation between housing return and the labor-income growth rate, however, lowers the trigger bound before retirement. This is because the borrowing constraint, often binding when there is a negative housing-return shock, is now relaxed. Since zero housing risk also leads to zero correlation between housing return and labor-income growth rate, removing housing risk is more effective in inducing home ownership than eliminating a positive correlation between these factors. Unlike the cases discussed above, the effect of a changing labor-income profile on trigger bound is not monotonic. A high school graduate requires a lower net-worth–labor-income ratio to trigger home ownership at young ages (prior to 38 years), yet a higher net-worth–labor-income ratio thereafter until he reaches retirement. This reflects the difference in labor-income profiles between college and high school graduates during the working years. On one hand, at young ages, the higher growth rate of a college graduate's labor income leads to a larger desired house relative to his labor-income level, which requires more wealth on hand to meet the house collateral constraint. On the other hand, in middle age, the faster decline in the labor income of a college graduate results in a smaller desired house relative to current labor income, which explains the reversal in the trigger level of the net-worth–labor-income ratio.

For all four alternative scenarios investigated in this section, the investor's liquid portfolio equity proportions exhibit similar patterns to the baseline case (data not shown). They are all hump-shaped before retirement and steadily declining thereafter. Yet, some interesting differences emerge. First, at a higher maintenance cost, the investor stays as a renter much longer and waits until his mid-thirties to purchase a house. The investor thus holds fewer stocks in his liquid portfolio due to a renter's lack of diversification. However, once becoming a homeowner, the investor chooses a riskier liquid portfolio than the baseline case. This reflects the fact that with a smaller home ownership benefit, renting is a less onerous alternative to owning, which alleviates the impact of the negative shock to wealth and reduces the homeowner's precautionary motive to hold safe assets. Second, the elimination of housing risk or of the positive correlation between housing risk and the labor-income growth rate allows the investor to take more risk in his liquid portfolio. This reflects the fact that the investor can now hold a greater amount of stock without increasing his overall risk exposure. The effect of zero correlation is, however, much smaller than that of zero housing risk. Last, consistent with Cocco, Gomes, and Maenhout (2004), the peak of the investor's liquid portfolio equity proportion occurs much earlier for a high school graduate, reflecting the differences in labor-income profiles between investors with different educational attainments.

2.6 Simulation analysis

Given the investor's optimal decision rules defined on the state space, we can obtain time-series profiles of investors' optimal housing services, numeraire good consumption, and portfolio allocations using simulation. Specifically, we first simulate stock return, housing return, and labor-income growth rate based on a serially uncorrelated markov process with two outcomes for each variable. We then use the optimal policy rules from our state-space solution to calculate the investor's optimal numeraire good consumption, housing, and portfolio choices. Home ownership status, the net worth–labor-income ratio, and the housing-value–wealth ratio are updated each period to determine the investor's optimal decisions for the next period. The time-series profiles of the optimal decisions are generated by repeating the calculation from $t = 0$ (age 20) to $t = 80$ (age 100).

In Table 1, we summarize the results for 50,000 simulation trials (or 50,000 time-series profiles) for the baseline parameter values with a

Table 1
Summary of consumption and portfolio choices from simulation analysis

Age	Fraction owning house	Fraction selling house	Fraction moving	W_t/Y_t		$P_t^H H_t/(W_t+Y_t)$		$C_t/(W_t+Y_t)$		$S_t/(S_t+B_t)$		$S_t/(S_t+B_t+\delta P_t^H H_t)$
				Renter	Owner	Renter	Owner	Renter	Owner	Renter	Owner	Owner
20	0.000	—	0.276	0.000	—	1.340	—	0.323	—	0.994	—	—
25	0.000	—	0.205	1.622	—	1.005	—	0.241	—	0.818	—	—
30	0.034	0.131	0.135	2.419	4.168	0.814	0.604	0.195	0.145	0.815	0.849	0.729
35	0.900	0.107	0.094	2.124	3.599	0.890	0.547	0.214	0.166	0.910	0.896	0.779
40	0.999	0.144	0.075	1.483	4.608	0.943	0.449	0.226	0.141	0.939	0.884	0.792
45	1.000	0.125	0.064	—	6.407	—	0.394	—	0.107	—	0.815	0.743
50	1.000	0.110	0.054	—	8.834	—	0.349	—	0.084	—	0.732	0.676
55	1.000	0.094	0.044	—	11.792	—	0.317	—	0.071	—	0.652	0.607
60	1.000	0.085	0.038	—	15.124	—	0.293	—	0.062	—	0.582	0.546
65	1.000	0.080	0.031	—	18.456	—	0.277	—	0.054	—	0.508	0.478
70	1.000	0.054	0.031	—	30.135	—	0.279	—	0.054	—	0.487	0.459
75	1.000	0.052	0.031	—	28.970	—	0.276	—	0.054	—	0.467	0.440
80	1.000	0.049	0.031	—	27.933	—	0.269	—	0.053	—	0.447	0.421
85	1.000	0.045	0.031	—	27.039	—	0.260	—	0.053	—	0.429	0.405
90	1.000	0.044	0.031	—	26.313	—	0.249	—	0.053	—	0.413	0.391
95	1.000	0.041	0.031	—	25.745	—	0.236	—	0.053	—	0.397	0.378
99	0.963	0.048	0.031	25.212	25.468	0.214	0.226	0.051	0.052	0.358	0.368	0.351

Source: U.S. Census Bureau (2003).

This table reports the summary of 50,000 simulations of fraction of investors owning houses, fraction selling their houses, fraction moving due to exogenous reasons, and average net-worth–labor-income ratio (W_t/Y_t), housing-value–wealth ratio ($P_t^H H_t/(W_t+Y_t)$), consumption–wealth ratio ($C_t/(W_t+Y_t)$), stock proportion in liquid financial portfolio ($S_t/(S_t+B_t)$), and stock proportion in net worth ($S_t/(S_t+B_t+\delta P_t^H H_t)$). The results are for the baseline parameter values with exogenous moving shocks calibrated to the intercounty migration rates for people with college degrees as reported in the 2001 Current Population Survey.

positive exogenous moving shock calibrated to the intercounty migration rates of college graduates.[14] The simulation begins with renters with zero initial net worth, so it depicts the time-series profiles of typical college graduates freshly out of school and starting to receive labor income. In the table, we show the average age profiles for the fraction of investors owning and selling their houses, the fraction of investors experiencing exogenous moving shocks, the net-worth–labor-income ratio, house value and numeraire good consumption as a proportion of total wealth (net worth plus labor income), and equity proportions in both the liquid portfolio and net worth. We further separate the decisions of homeowners from renters whenever it is feasible.

Investors purchase houses early in their lifetime after they accumulate enough net worth to take advantage of home ownership. The fraction of investors owning their first home increases with investor age until the mid-forties, when almost all investors have become homeowners. After age 95, however, the mortality rates are so high that upon selling their house investors may choose to move into a rental home to prepare for bequeathing their wealth. The house-selling rate among existing homeowners is in general very low, reflecting the deterrent effect of liquidation cost on home-selling decisions. Before age 35, the home-selling rate largely tracks the probability of incurring the exogenous moving shock and declines with age. The rate then picks up and reaches a peak level of 14.4% at age 40, as more homeowners adjust their housing service levels. The selling rate declines thereafter, reflecting a decrease in the probability of exogenous moving as well as the fact that at this stage of their lives investors are more capable of purchasing a house matching their lifetime consumption level and hence have no need to make frequent adjustments. The cumulative house turnover is about 5.06, implying that an investor sells his house about five times during his lifetime.

The average housing-value–wealth ratio exhibits a U-shape for renters between age 20 and the early forties, but it steadily declines for homeowners as they age. This is attributed to the time-series behavior of the investor's net-worth–labor-income ratio and the fact that investors with a high net-worth–income ratio become homeowners earlier. As an investor ages, his human capital is gradually realized and saved in the form of liquid financial assets. This leads to the initial decline in the housing-value–wealth ratio. However, a young renter who experiences a sequence of positive shocks to stock returns and labor-income growth rates will soon accumulate enough net worth to become a homeowner. So the endogenous housing tenure choice will reduce the average net

[14] We choose to report the simulation results for the case with a positive probability of exogenous moving so that we can better match the patterns of home ownership and house-selling rates observed in the data in our empirical analysis.

worth–labor-income ratio for the renters left behind. The lower net worth–labor-income ratio in turn increases the renters' average housing-related expenditures. After age 40, most investors become homeowners and enter their prime time of saving. As they accumulate more net worth, the housing-value–net-worth ratio declines monotonically. For the same reason, the average fraction of total wealth allocated to numeraire good consumption is U-shaped in age for young renters and decreasing in age for homeowners. When renting, investors spend on average 32.3% of their current wealth on numeraire good consumption at age 20. The percentage then decreases to 19.5% by the time the investors reach age 30. It then goes back up to about 22.6%. When owning, investors allocate 16.6% of their total wealth to numeraire good consumption at age 35, and the proportion gradually reduces to 5.4% at age 65 and beyond. On average, renters spend a higher fraction of their current wealth on numeraire good consumption than homeowners, primarily because of their lower average net-worth–labor-income ratio.

The hump-shaped average net-worth–labor-income ratio for young renters leads to a U-shaped portfolio allocation pattern (Table 1). The investor holds a high equity proportion of 99.4% at age 20 due to the high present value of human capital at this age. As the investor saves his labor income in the form of financial assets and approaches the trigger bound of becoming a homeowner, his equity proportion declines. Saving for a housing down payment is another reason for a young investor to tilt his savings toward safer assets. Once a renter successfully accumulates enough wealth, he becomes a homeowner and lowers the equity proportion in his net worth by substituting risky housing assets for risky stocks to control the overall risk exposure. In the meantime, he also raises the equity proportion in his liquid financial portfolio to take advantage of the diversification benefit from the low correlation between housing returns and stock returns. As the investor ages, the equity proportions in both his liquid portfolio and his net worth decline rapidly as he realizes his earning ability and saves more in the form of financial assets. Among investors over age 35 who are still unable to accumulate enough net worth to buy a house, the average stock investment increases due to their higher earnings potential (relative to their wealth on hand). From age 35 to 40, the average equity proportion in the renter's liquid portfolio is higher than that of homeowners in the same age group.

3. Empirical Analysis of Household Portfolio Choices

In this section, we empirically investigate households' portfolio choices in the presence of risky housing using panel data. We focus on a household's stock investment as a proportion of either the household's net worth or its liquid financial asset holdings. We examine how a household's stock investment depends upon the state variables identified in our theoretical analysis, including the housing tenure choice, house-value–net-worth ratio, age of the head of the household, net-worth–income ratio, and other variables previously shown in the literature to affect portfolio decisions, such as total net worth, mortgage–net-worth ratio, ownership of business and other real estate assets, and number of children in the household. We also include annual dummy variables to allow for possible time-varying effects.

3.1 Estimation method

An important feature of household portfolio choices is that a large fraction of households do not own stocks. This leads to the classic sample selection problem. In a cross-sectional data case, the sample selection issue has been extensively investigated [see Powell (1994)]. However, the sample selection problem in a panel or longitudinal data case is more challenging, and the appropriate econometric methodology is less well-developed due to the concurrent unobserved heterogeneity or "fixed effects" in both the sample selection equation and the main equations of interest.[15] In this case, the dependent variable in the main equation (equity proportion in our case) is a *nonlinear* function of the variables and unobserved individual effects that explain stock market participation. As a result, the "fixed effects" cannot be easily eliminated by the standard differencing approach widely adopted in linear panel data models. Failure to account for such individual-specific effects may result in biased and inconsistent estimates of the parameters of interest.

In the following analysis, we use the two-step estimation procedure proposed by Kyriazidou (1997). In the first step, the coefficients of the stock market participation (selection) equation are consistently estimated. The estimates are then used to construct a kernel weight to consistently estimate the equity proportion equation. Specifically, we consider the following model:

$$y^*_{it} = x'_{it}\beta + \alpha_i + \epsilon_{it}, \tag{13}$$

$$d_{it} = 1\{z'_{it}\gamma + \eta_i - u_{it}\}, \tag{14}$$

$$y_{it} = d_{it}y^*_{it}, \tag{15}$$

where subscript i, $i = 1, \ldots, n$, represents the individual and subscript t, $t = 1, \ldots, T$, denotes the time period. y^*_{it} is individual i's desired

[15] In the case of asset allocation, unobserved individual-specific characteristics, such as the investor's risk tolerance, can affect both the stock market participation decision and the asset allocation decision conditional on participation.

stockholding at time period t, x_{it} and z_{it} are vectors of explanatory variables with possibly common elements, and d_{it} is an indicator function that takes the value of 1 if individual i owns stocks at time t and zero otherwise. α_i and η_i are unobserved time-invariant individual-specific effects in the equity proportion equation and stock market participation equation, respectively. Both α_i and η_i can be correlated with x_{it} or z_{it}. ϵ_{it} and u_{it} are unobserved disturbances, which can also be correlated with each other. The desired equity proportion (y_{it}^*) is only observed when the investor participates in the stock market.[16]

In our model, it is possible to obtain a consistent estimate for γ in the sample selection equation by applying the conditional logit model to the individuals that change participation status between time periods.[17] The estimation of coefficients in the equity proportion equation, β, poses two additional problems: The unobserved individual-specific effect α_i and the sample selection effect represented by Equation (15). To solve the first problem, one can take the time difference on observations for which $d_{i1} = d_{i2} = 1$. This will eliminate the effect of α_i from Equation (13). However, applying the standard ordinary least square estimation to the differenced subsample will yield inconsistent estimates for β, since simple differencing cannot simultaneously eliminate the sample selection bias. This can be seen from the regression function for the time-differenced subsample below. Denote $\xi_i = \{z_{i1}, z_{i2}, x_{i1}, x_{i2}, \alpha_i, \eta_i\}$. We have

$$E(y_{i1} - y_{i2} | d_{i1} = d_{i2} = 1, \xi_i) = (x_{i1} - x_{i2})\beta + E(\epsilon_{i1} - \epsilon_{i2} | d_{i1} = d_{i2} = 1, \xi_i). \quad (16)$$

In general, there is no reason to expect that $E(\epsilon_{i1} - \epsilon_{i2} \mid d_{i1} = d_{i2} = 1, \xi_i) = 0$. Denote $\lambda_{it} \equiv E(\epsilon_{it} \mid d_{i1} = d_{i2} = 1, \xi_i)$. λ_{it} is referred to as the "sample selection effect" in the literature. In our case, the sample selection effect depends not only on the partially unobservable conditioning vector ξ_i, but also on the generally unknown joint conditional distribution of (ϵ_{i1}, ϵ_{i2}, u_{i1}, u_{i2}), which can differ across individuals and over time. However, under the conditional exchangeability assumption that $(\epsilon_{i1}, \epsilon_{i2}, u_{i1}, u_{i2})$ and $(\epsilon_{i2}, \epsilon_{i1}, u_{i2}, u_{i1})$ are identically distributed conditional on ξ_i, the sample selection effect would be the same between two time periods for individual i with $z_{i1}'\gamma = z_{i2}'\gamma$. This suggests that we can estimate the equity proportion equation consistently by OLS based on a subsample that consists of stockholders with $z_{i1}'\gamma = z_{i2}'\gamma$. Let $\hat{\gamma}_n$ be the consistent estimate for γ using the conditional logit. Denote $\Delta x_i = x_{i1} - x_{i2}$, $\Delta y_i = y_{i1} - y_{i2}$, and

[16] In a recent article, Kullmann and Siegel (2003) examine the portfolio choice of households as a function of their exposure to real estate risk. However, they assume that the fixed effects appear in the stockholding equation but not in the participation equation. Further, they assume that the conditional error in the stockholding equation is a linear function of the error term in the participation equation. Violation of these assumptions may lead to biased and inconsistent estimates for both equations.

[17] For expositional purposes, in our discussion below, we use subscript 1 and 2 to represent two adjacent time periods in which the investor is a stockholder.

$\Delta z_i \hat{\gamma}_n = z_{i1}\hat{\gamma}_n - z_{i2}\hat{\gamma}_n$. The coefficients in the equity proportion equation then can be estimated as

$$\hat{\beta}_n = \left[\sum_{i=1}^{n} \hat{\psi}_{in} \Delta x_i' \Delta y_i d_{i1} d_{i2}\right]^{-1} \left[\sum_{i=1}^{n} \hat{\psi}_{in} \Delta x_i' \Delta y_i d_{i1} d_{i2}\right], \qquad (17)$$

where $\hat{\psi}_{in}$ is a weight that declines to zero as the magnitude of the difference $|z_{i1}\hat{\gamma}_n - z_{i2}\hat{\gamma}_n|$ increases. $\hat{\psi}_{in}$ can be represented as follows:

$$\hat{\psi}_{in} \equiv \frac{1}{h_n} K\left(\frac{\Delta z_i \hat{\gamma}_n}{h_n}\right), \qquad (18)$$

where $K(\cdot)$ is a kernel density function and $h_n = hn^{-1/[2(r+1)+1]}$ is a sequence of bandwidth that approaches zero as $n \to \infty$, where r is the degree of continuous differentiability of $K(\cdot)$.

The proposed two-step estimate is, however, asymptotically biased. To correct the bias, Kyriazidou (1997) proposes that we re-estimate β using a slow bandwidth $h_{n,\delta} = n^{-\delta/[2(r+1)+1]}$, where $0 < \delta < 1$. Denote the alternative β estimate as $\hat{\beta}_{n,\delta}$. The bias-corrected estimator of β is given by

$$\hat{\hat{\beta}}_{n,\delta} = \frac{\hat{\beta}_n - n^{-(1-\delta)(r+1)/[2(r+1)+1]}\hat{\beta}_{n,\delta}}{1 - n^{-(1-\delta)(r+1)/[2(r+1)+1]}}. \qquad (19)$$

As shown in Kyriazidou (1997), the proposed bias-corrected estimator for β is consistent and asymptotically normally distributed.[18]

3.2 Data and descriptive statistics

We apply the estimation method discussed above to the PSID data from 1984 to 2001, along with the PSID Wealth Supplements for the years 1984, 1989, 1994, 1999, and 2001. PSID provides detailed information on respondents' income, housing, and family structure on an annual basis. However, detailed information on these households' financial asset holdings and net worth is collected much less frequently in the PSID Wealth Supplements. Thus, our empirical investigation is based on the five survey years corresponding to the PSID Wealth Supplements.[19]

---

[18] In our application, we set the constant $h = 1$ and $\delta = 0.1$ as in the base case of Kyriazidou (1997) and use a third order kernel function ($r = 3$) given by

$$K(v) = 1.1\exp(-v^2/2) - 0.1\exp(-v^2/22)(1/\sqrt{11}).$$

The corresponding bandwidth h_n is set at $h_n = n^{-1/9}$. We have experimented with other kernel functions such as a standard normal function and the kernel function with a higher order of continuous differentiability. The results are similar and are not reported in the article.

[19] Households in the poverty sample and the Latino sample are excluded to keep the sample representative. Households with negative wealth as well as those observations in the top or bottom 1% for any of the key variables are removed to reduce potential bias caused by outliers. For detailed discussion on the data, we refer readers to Vissing-Jørgensen (2002) and Kullmann and Siegel (2003).

Table 2 presents the sample averages for key variables across different age groups and various survey years. These variables include percentage of stock ownership, equity proportions, percentage of home ownership, home value as a fraction of homeowner's total net worth, mortgage–net-worth ratio, households' net-worth and income levels (in 1984 dollars), net-worth–income ratio, age of the head of the household, number of children under age 18 living in the household, percentage of households owning a business, and percentage of households owning other real estate assets. To highlight the contrast between renters and homeowners, we report the summary statistics for renters and homeowners separately whenever possible. For the investors' portfolio compositions, we use two different measures: (1) Stockholding as a fraction of household net worth, and (2) stockholding as a fraction of household liquid financial assets.[20]

Stock ownership exhibits a strong hump shape in age, with increasing household stock ownership until investors reach their fifties and declining stock ownership as they enter retirement and become more dependent upon cash-generating assets. The percentage of stock ownership among homeowners is much higher than the percentage among renters. It also exhibits strong time variation. Stock ownership increases drastically from 16.1% for renters and 32.4% for homeowners in 1984 to 24% for renters and 45% for homeowners in 1994, and then stabilizes for 1999 and 2001. Consistent with the prediction of our model, equity proportions initially increase in age for both groups. While the model predicts that equity proportions gradually decrease once young investors overcome initial liquidity constraints, the observed equity proportions do not exhibit any obvious trend after peaking between ages 45 and 55. Overall, the observed equity proportions in the data are lower than the predicted values in our simulation analysis, particularly for young investors. Similar to the time-varying pattern in stock market participation, the equity proportions under both measures experience significant increases from 1984 to 1999 and decline slightly afterward.

Consistent with the prediction of our model, home ownership increases in the age of the head of the household until retirement. At advanced ages (75 years and beyond), the percentage of home ownership declines. Over the sample period, the percentage of home ownership shows a slight increase from 72.7% in 1984 to 76.7% in 2001. The observed percentage of home ownership is in general lower than the value in our simulation analysis even after incorporating realistic moving shocks. This indicates that the actual economic frictions to home ownership are stronger than those explicitly modeled in our framework. The average home-value–net-worth

[20] Financial assets include checking and savings accounts, money market funds, CDs, savings bonds, Treasury bills, stocks, mutual funds, investment trusts, and bonds and stocks held in IRAs, as well as other financial assets such as life insurance policies.

Table 2
Summary statistics for housing and portfolio choices from the Panel Study of Income Dynamics (PSID) data, 1984–2001

		Age groups						Year				
Variables		<35	35–45	45–55	55–65	65–75	>75	1984	1989	1994	1999	2001
OWNSTOCK		0.2495	0.3944	0.4688	0.4600	0.4019	0.3049	0.2794	0.3277	0.3960	0.4013	0.4285
OWNSTOCK	Renter	0.1969	0.2417	0.2756	0.2642	0.2083	0.2286	0.1608	0.2215	0.2396	0.2250	0.2554
	Owner	0.2996	0.4355	0.5008	0.4850	0.4229	0.3269	0.3240	0.3640	0.4504	0.4555	0.4811
S/NW	Renter	0.4409	0.4259	0.4714	0.4128	0.4157	0.5434	0.2900	0.2829	0.4200	0.5565	0.6096
	Owner	0.1617	0.2067	0.2351	0.2271	0.2549	0.2566	0.1296	0.1414	0.2267	0.2722	0.2667
S/SB	Renter	0.4474	0.5318	0.5606	0.5398	0.5242	0.5810	0.3852	0.4076	0.5268	0.5743	0.5578
	Owner	0.4617	0.5158	0.5546	0.5264	0.5495	0.5232	0.4172	0.4151	0.5383	0.5849	0.5851
OWNHOUSE		0.5117	0.7880	0.8582	0.8870	0.9019	0.7761	0.7266	0.7452	0.7417	0.7649	0.7672
PH/NW		2.1015	1.5223	1.1288	0.7588	0.6290	0.6276	1.0436	1.1564	1.1528	1.3735	1.4703
MORT/NW		1.4839	0.9348	0.5663	0.2346	0.0959	0.0644	0.4546	0.5967	0.5949	0.8199	0.8786
NETWORTH ($100K)	Renter	0.1581	0.2574	0.3833	0.5771	0.4038	0.4649	0.1844	0.2233	0.2759	0.2635	0.3348
	Owner	0.5882	1.0990	1.5461	1.7729	1.9119	1.5228	1.0579	1.2282	1.3118	1.4602	1.5251
INCOME ($100K)	Renter	0.2155	0.2545	0.2633	0.2357	0.1479	0.1054	0.2008	0.2238	0.2044	0.2287	0.2373
	Owner	0.3142	0.3874	0.4070	0.3260	0.2284	0.1461	0.2894	0.3292	0.3257	0.3454	0.3487
NW/INC	Renter	0.8983	1.0959	1.5502	2.2670	2.5653	5.1170	1.0048	1.2523	1.8647	1.5325	1.6222
	Owner	2.2727	3.4232	4.6423	6.8697	9.7162	12.192	4.5961	4.3790	6.1475	5.7744	6.2095
AGEHEAD	Renter	26.866	38.986	48.670	58.976	69.464	83.356	38.178	39.197	40.199	39.241	39.950
	Owner	29.542	39.474	49.154	59.260	69.378	80.632	48.812	49.171	49.559	50.403	50.409
NUMOFKIDS	Renter	0.6799	1.1254	0.5968	0.1415	0.0357	0.0063	0.6772	0.6943	0.6703	0.6452	0.6127
	Owner	1.2106	1.5791	0.6890	0.1454	0.0654	0.0183	0.8698	0.8419	0.8190	0.7789	0.7369
OWNBUSI	Renter	0.0672	0.0932	0.1139	0.0708	0.0298	0.0254	0.0755	0.0777	0.0813	0.0705	0.0600
	Owner	0.1518	0.2092	0.2221	0.1749	0.1250	0.0971	0.1687	0.1953	0.1715	0.1752	0.1743
OWNOTHRE	Renter	0.0565	0.1243	0.1595	0.1745	0.0952	0.0762	0.1084	0.1192	0.0967	0.0705	0.0600
	Owner	0.1358	0.2095	0.2794	0.3197	0.3083	0.2216	0.2644	0.2613	0.2426	0.2230	0.2132
No. of obs.		4389	4100	3095	1876	1712	1407	3003	3030	3523	3441	3582

This table reports percentage of stock ownership (OWNSTOCK), average equity proportion in net worth (S/NW) and liquid financial assets (S/SB), percentage of home ownership (OWNHOUSE), average house-value–net-worth ratio (PH/NW), average mortage — net worth ratio (MORT/NW), household net worth (NETWORTH, and income (INCOME) in 1984 dollar, net worth — income ratio (NW/INC), age of the head of household (AGEHEAD), number of children under age 18 living in the household (NUMOFKIDS), percentage of households owning a business (OWNBUSI), and percentage of households owning other real estate assets (OWNOTHRE), across age groups and across various years.

ratio is, in general, higher than our simulation result and increased by more than 40% from 1984 to 2001. The increase is probably a result of various government-sponsored programs over the last couple of decades encouraging home ownership. Consistent with our simulation result, home value as a percentage of household net worth decreases in age, reflecting the fact that households accumulate more net worth as they age. For households in the 35-and-younger age group, home value is slightly more than twice a household's net worth. Meanwhile, for the 75-and-beyond age group, home value accounts for, on average, only 63% of a household's net worth. The increases in home ownership and in the house-value–net-worth ratio over time are accompanied by increases in the mortgage–net-worth ratio, consistent with the notion that a greater number of low-net-worth households become homeowners with the availability of low down payment mortgages. The average mortgage balance as a fraction of total net worth decreases in age, consistent with the conventional wisdom that a homeowner pays down his mortgage and accumulates home equity over time.

Household net worth exhibits a hump shape in age and reaches a peak mean value of about $58,000 for renters between ages 55 and 65 and a peak mean value of about $191,000 for homeowners between ages 65 and 75. Household income also demonstrates a similar hump-shaped pattern, peaking between ages 45 and 55. Over the sample period, both the net worth and income exhibit a slight upward trend, with the largest increase occurring between 1984 and 1989. Consistent with the prediction of our model, the net-worth–income ratio increases in age, reflecting wealth accumulation as the investor ages. Yet, the actual level of savings is lower than our simulation results would suggest.

In all sample years, homeowners on average are about 10 years older and have more children under age 18 living in the household than do renters. The percentages of households owning a business or other real estate assets are both hump-shaped in age. While the fraction of households owning a business stayed relatively stable over the sample period, the percentage of households owning other real estate assets steadily declined from 10.8% in 1984 to 6% in 2001 for renters and from 26.4 to 21.3% for homeowners over the same time period.

3.3 Estimation results

In Table 3, we present the conditional logit estimates for stock market participation decisions [Equation (14)] for renters and homeowners. We find that net worth and income have positive effects on investors' stock market participation decisions but at a decreasing rate. This is reflected by their positive linear and negative quadratic coefficients for both renters and homeowners. Age also has a positive effect at a decreasing rate on homeowners' stock market participation decisions, but the effect is

Table 3
Stock market participation decision for renters and homeowners

	Renter		Homeowner	
Variables	Coeff.	SE	Coeff.	SE
NETWORTH ($100K)	2.127	0.560***	0.707	0.100***
NETWORTH^2	−0.390	0.116***	−0.047	0.009***
INCOME ($100K)	6.458	2.081***	1.679	0.696**
INCOME^2	−4.573	1.907**	−0.910	0.569
AGEHEAD/10	2.147	3.439	1.560	0.809*
AGEHEAD^2/100	−0.081	0.063	−0.095	0.024***
NUMOFKIDS	0.067	0.166	0.071	0.048
OWNBUSI	−0.102	0.427	−0.504	0.129***
OWNOTHRE	1.243	0.490**	−0.231	0.108**
PH/NW			−1.902	0.238***
PH/NW^2			0.193	0.035***
MORT/NW			1.300	0.249***
MORT/NW^2			−0.215	0.051***
YEAR89	−0.346	1.746	−0.296	0.409
YEAR94	−0.436	3.479	−0.053	0.805
YEAR99	−1.088	5.124	−0.433	1.189
YEAR01	−0.856	5.836	−0.397	1.355

This table reports results estimated using a conditional logit model for the stock market participation decisions of renters and homeowners based on the Panel Study of Income Dynamics (PSID) data. See Table 2 for variable definitions. *, significant at 10% level; **, significant at 5% level; ***, significant at 1% level.

insignificant for renters. For both renters and homeowners, the number of children and year dummies do not show any significant effects on the stock market participation decision.

Owning a business or other real estate assets has a significant negative effect on a homeowner's stock market participation decision. For a renter, however, the effect of owning a business is insignificant, while the effect of owning other real estate assets (for investment purposes) is significant yet positive. This reflects the tradeoff between the diversification benefit and the benefit of holding liquid financial assets. On one hand, the diversification benefit from investing in a business or other real estate assets implies a higher likelihood of stock market participation among owners of these assets. On the other hand, investing in these relatively illiquid assets leaves a lower level of liquid wealth available for investment in stocks, which can be insufficient to overcome the initial cost to enter the stock market.[21] The net impact of these variables on stock market participation depends on which effect dominates. For renters, the effect of diversification benefit seems to outweigh that of the reduction in the size of liquid assets. For homeowners who have already invested heavily in home equity, however,

[21] Gomes and Michaelides (2004) demonstrate that a small participation cost may be sufficient to deter an investor from entering the stock market.

owning a business or other real estate assets reduces the total available liquid funds to such a low level that they find it inefficient to invest in stocks.

All housing-related variables have a significant impact on homeowners' stock market participation decisions. Specifically, the home-value–net-worth ratio affects stock market participation negatively but at a decreasing rate.[22] This is consistent with the participation cost explanation: A higher home-value–net-worth ratio implies less liquid wealth is available for equity investments. Interestingly, the outstanding mortgage–net-worth ratio has a positive effect on homeowners' stock market participation decisions. This can be attributed to the effects of house liquidation cost and refinancing charges. Consider two households with identical house values and identical levels of net worth but different outstanding mortgage balances. The household with a higher mortgage–net-worth ratio holds more wealth in liquid form and less in illiquid home equity. In the event of a large negative shock to stock returns, the household with more wealth in liquid form can better weather the shock without having to incur refinancing charges or selling costs to access home equity.

For homeowners, a high net-worth–income ratio is associated with higher equity proportions in both their net worth and liquid financial assets (Table 4). Recall that our model predicts that the homeowner's equity proportions are hump-shaped in the net-worth–income ratio. Our empirical results suggest that most homeowners lie on the upward sloping segment of the equity proportion and net-worth–income ratio relationship. The effect of the net-worth–income ratio is, however, insignificant for renters. Income has no significant effect on either renters' or homeowners' equity proportions in either net worth or liquid financial assets. This is consistent with the specification in our theoretical model and suggests that level variables such as net worth and income affect asset allocations mostly through the market participation decision.

Both equity proportion measures increase significantly as a renter ages. While a homeowner's net worth equity proportion is hump-shaped in age, his liquid financial assets equity proportion decreases in age. While these results are largely consistent with our findings based on state space and simulation analysis, there are also some notable differences. Empirically, investors (particularly renters and young homeowners) seem to stay within the upward-sloping portion of the age–equity proportion relation for an extended period of time, that is, their equity proportions peak much later. For instance, based on our estimation, the net worth equity proportion reaches its peak value at age 43.3 for homeowners. This suggests that

[22] The estimated coefficients imply that the impact reaches a minimum at the house-value–net-worth ratio of 4.9, implying that the vast majority of households are on the declining portion of the quadratic curve.

Table 4
Equity proportion for renters and homeowners

	Renter				Homeowner			
	S/NW		S/SB		S/NW		S/SB	
Variables	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
NW/INC/10	0.268	0.315	0.231	0.231	0.035	0.017**	0.055	0.027**
$NW/INC^2/100$	−0.034	0.033	−0.025	0.023	−0.004	0.002*	−0.006	0.003*
INCOME ($100K)	0.171	0.396	−0.172	0.280	0.008	0.020	−0.013	0.031
$INCOME^2$	−0.070	0.062	−0.010	0.052	−0.001	0.001	0.001	0.002
AGEHEAD/10	0.073	1.614	0.994	1.237	0.234	0.133*	−0.190	0.230
$AGEHEAD^2/100$	0.094	0.045**	0.076	0.026***	−0.027	0.004***	−0.039	0.006***
NUMOFKIDS	0.078	0.069	0.182	0.071***	0.013	0.009	0.023	0.016
OWNBUSI	0.137	0.164	0.150	0.196	−0.062	0.019***	−0.034	0.028
OWNOTHRE	−0.651	0.247**	−0.702	0.148***	−0.037	0.013***	0.031	0.023
PH/NW					−0.426	0.060***	−0.090	0.104
PH/NW^2					0.024	0.022	0.025	0.034
MORT/NW					0.472	0.061***	0.155	0.097
$MORT/NW^2$					−0.074	0.033***	−0.094	0.047**
YEAR89	−0.504	0.711	−0.791	0.589	0.044	0.065	0.320	0.115***
YEAR94	−0.701	1.443	−1.468	1.184	0.169	0.129	0.805	0.230***
YEAR99	−1.152	2.083	−2.259	1.791	0.251	0.191	1.180	0.340***
YEAR01	−1.151	2.357	−2.506	2.033	0.279	0.217	1.321	0.387***

This table reports results estimated using the second-stage equity proportion equation of a panel data sample selection model based on the Panel Study of Income Dynamics (PSID) data, for renters and homeowners, respectively. See Table 2 for variable definitions. *, significant at 10% level; **, significant at 5% level; ***, significant at 1% level.

more severe liquidity constraints exist in the real world than in our model. Further, the number of children under age 18 in the family does not significantly affect the equity proportion in total net worth. However, it increases liquid financial portfolio equity proportion for renters, indicating a greater demand for assets with higher returns to finance the children's future college education.

While owning a business crowds out stocks in homeowners' total net worth, it increases equity proportions for renters because of the diversification benefits. However, the latter is not statistically significant. Owning other real estate assets significantly reduces stock investments in the total net worth of both renters and homeowners, reflecting the substitution effect in which other real estate investments take the place of risky stocks.

The "crowding out" effect is also reflected in the negative coefficient of the home-value–net-worth ratio in the equation for the homeowners' net worth equity proportion. The same ratio has an insignificant effect on the homeowner's liquid financial portfolio equity proportion, a result weakly consistent with the findings of our numerical analysis. Similar to its effect on stock market participation, a higher mortgage–net-worth ratio leads to higher equity proportions in both the liquid financial portfolio and the total net worth for homeowners, reflecting their increased tolerance for stock market shocks when a larger fraction of their wealth is in liquid form.

Finally, while year dummies have positive but insignificant effects on homeowners' and renters' net worth equity proportions, the equity proportion in the homeowners' liquid financial portfolio has, on average, increased significantly over time. The increase coincides with the relatively strong performance of the stock markets over the sample time period. This result points to the need to include portfolio rebalancing costs and/or time-varying expected returns in order to match the model to the empirically observed equity proportions in households' liquid portfolios.

4. Conclusion

In this article, we analyzed the optimal dynamic consumption, housing, and portfolio decisions for an investor who receives stochastic labor income and also faces substantial housing risk, collateral requirements, and liquidation cost. Our results indicate that the investor optimally chooses to rent housing services when he faces severe liquidity constraints and/or a high mortality rate. However, when the liquidity constraint is relaxed the investor chooses owning a house over renting to benefit from home ownership.

Home ownership has a significant impact on the investor's portfolio decision. When indifferent between renting and owning a house, the investor who owns a house substitutes home equity for risky stocks in his net worth, yet increases the equity proportion in his liquid financial portfolio to take advantage of the diversification benefit afforded by the low correlation between stock returns and housing returns. Following the suboptimal policy of either always renting housing services or only owning a house can lead to large welfare losses and drastically alter the investor's portfolio choices. Liquidation cost creates a no-adjustment region for housing services, which in turn influences the investor's portfolio decisions. When close to the boundaries of the no-adjustment region, the investor holds a higher equity proportion in his liquid portfolio in order to achieve the optimal risk-return tradeoff.

Detailed empirical analysis of households' portfolio choices with risky housing assets using the PSID data shows that renters and homeowners' portfolio choices have different determinants and react differently to key variables such as the net-worth–income ratio and investor age. This empirical evidence lends some support to the predictions of our model based on both policy rules and the simulation analysis. Possible future extensions include incorporating refinancing costs, as in Campbell and Cocco (2003), and endogenizing the cost differential between renting and owning on the basis of moral hazard and/or adverse selection.

Appendix

We simplify the investor's optimization problem by normalizing the investor's continuous choice variables by his wealth Q_t. Let $c_t = C_t/Q_t$ be the consumption–wealth ratio; $h_t = P_t^H H_t/Q_t$ the house value–wealth ratio; $b_t = B_t/Q_t$ the fraction of the investor's wealth invested in bonds after trading; and $s_t = S_t/Q_t$ the fraction of wealth allocated to stocks after trading. By assuming the Cobb–Douglas utility function and proportional housing maintenance and liquidation costs, we ensure that the numeraire good consumption, housing service, and portfolio rules, $\{c_t, h_t, b_t, s_t\}$, are independent of investor's wealth level, Q_t. With the above normalization, the relevant state variables for the investor's problem can be written as $x_t = \{D_{t-1}^o, D_t^m, w_t, \bar{h}_{t-1}\}$, where $w_t = W_t/Y_t$ is the investor's net-worth–labor-income ratio and $\bar{h}_{t-1} = P_t^H H_{t-1}/W_t$ is the homeowner's beginning-of-period house-value–net-worth ratio. Let $y_t = Y_t/Q_t$ be the investor's labor-income–wealth ratio, and let $\tilde{g}_{t+1} = \Delta \log Y_{t+1} = f(t+1) + \epsilon_{t+1}$ be the real labor-income growth rate. We can write the real (gross) growth rate of the investor's wealth $q_{t+1} = Q_{t+1}/Q_t$ as follows:

$$q_{t+1} = b_t R_f + s_t \tilde{R}_{t+1}^S + D_t^o h_t [\tilde{R}_{t+1}^H (1-\phi) - (1-\delta) R_f] + y_t \exp\{\tilde{g}_{t+1}\}. \quad (20)$$

Similarly, the budget constraint in Equation (7) can be written as:

$$\begin{aligned} 1 = {} & c_t + b_t + s_t + \alpha h_t[(1-D_{t-1}^o)(1-D_t^o) + D_{t-1}^o D_t^m (1-D_t^o) + D_{t-1}^o (1-D_t^m) D_t^s (1-D_t^o)] \\ & + (\psi + \delta) h_t [(1-D_{t-1}^o) D_t^o + D_{t-1}^o D_0^m D_t^o + D_{t-1}^o (1-D_0^m) D_t^s D_t^o] \\ & + (\psi + \delta - \phi) \bar{h}_{t-1} w/(1+w) [D_{t-1}^o (1-D_t^m)(1-D_t^s)], \quad t = 0, \ldots, T \end{aligned} \quad (21)$$

Defining $v_t(x_t) = V_t(X_t)/[(Q_t/P_t^{H^\omega})^{1-\gamma}]$ to be the normalized value function, the investor's problem can be restated as follows:

$$\begin{aligned} v_t(x_t) = \max_{a(t)} \Bigg\{ & \lambda_t \left[\frac{(c_t^{1-\omega} h_t^\omega)^{1-\gamma}}{(1-\gamma)} + \beta E_t \left[v_{t+1}(x_{t+1}) q_{t+1}^{(1-\gamma)} (\tilde{R}_{t+1}^H)^{\omega(\gamma-1)} \right] \right] \\ & + (1-\lambda_t) \frac{\beta(1-\beta^L)[A_L \omega^\omega (1-\omega)^{1-\omega}]^{1-\gamma}}{(1-\beta)(1-\gamma)(\alpha)^{\omega(1-\gamma)}} \Bigg\}, \quad t = 0, \ldots, T \end{aligned} \quad (22)$$

s.t.

$$c_t > 0, \quad h_t > 0, \quad b_t \geq 0, \quad s_t \geq 0 \quad (23)$$

and Equations (20) and (21).

The above problem can be solved numerically using backward recursion. Depending on the investor's house ownership status and the realization of the moving shock at the beginning of the period, we can further specify the state and choice variables as follows. If the investor rents his housing services in the previous period ($D_{t-1}^o = 0$) or if he is a homeowner who experiences a moving shock ($D_{t-1}^o = D_t^m = 1$),

$$x_t \equiv \{w_t\} \quad \text{and} \quad a_t \equiv \{c_t, h_t, b_t, s_t, D_t^o\}.$$

If the investor owns his residence in the previous period and does not have to move for exogenous reasons ($D_{t-1}^o = 1$ and $D_t^m = 0$),

$$x_t \equiv \{w_t, \bar{h}_{t-1}\} \quad \text{and} \quad a_t \equiv \{c_t, h_t, b_t, s_t, D_t^o, D_t^s\}.$$

To solve the above problem, we discretize the net-worth–labor-income ratio, $w = W_t/Y_t$, into a grid of 2000 over the interval [0, 50], and the house-value–net-worth ratio, $\bar{h}_{t-1} = P_t^H H_{t-1}/W_t$, into a grid of 500 over the interval [0, 4], respectively. At the terminal date T, the investor's value function takes the value of the bequest function and is a constant:

$$v_T(x_T) = \frac{\beta(1-\beta^L)[A_L\omega^\omega(1-\omega)^{1-\omega}]^{1-\gamma}}{(1-\beta)(1-\gamma)(\alpha)^{\omega(1-\gamma)}} \tag{24}$$

at all points in the state space. The value function at date T is then used to solve for the optimal decision rules for all points on the state space grid at date $T-1$. Linear interpolation is used to calculate the value function for points in the state space that lie between grid points. The procedure is repeated recursively for each time period until the solution for date $t=0$ is found.

References

Anderson, R. N., 2001, "United State Life Tables 1998," *National Vital Statistics Reports*, 48, 1–40.

Campbell, J., and J. F. Cocco, 2003, "Household Risk Management and Optimal Mortgage Choice," *Quarterly Journal of Economics*, 118, 1149–1194.

Carroll, C. D., and W. E. Dunn, 1997, "Unemployment Expectations, Jumping (S,s) Triggers, and Household Balance Sheets," *NBER Macroeconomics Annual*, 165–229, MIT Press, Cambridge, MA.

Claus, J., and J. Thomas, 2001, "Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets," *Journal of Finance* 56, 1629–1666.

Cocco, J. F., 2004, "Portfolio Choice in the Presence of Housing," *Review of Financial Studies*, forthcoming.

Cocco, J. F., F. J. Gomes, and P. J. Maenhout, 2004, "Consumption and Portfolio Choice over the Life-Cycle," *Review of Financial Studies*, forthcoming.

Damgaard, A., B. Fuglsbjerg, and C. Munk, 2003, "Optimal Consumption and Investment Strategies with a Perishable and an Indivisible Durable Consumption Good," *Journal of Economic Dynamics and Control*, 28, 209–253.

Dammon, R. M., C. S. Spatt, and H. H. Zhang, 2001, "Optimal Consumption and Investment with Capital Gains Taxes," *Review of Financial Studies*, 14, 583–616.

Davis, S. J., and P. Willen, 2000, "Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice," working paper, Graduate School of Business, Univeristy of Chicago.

Deng, Y., J. M. Quigley, and R. Van Order, 2000, "Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options," *Econometrica*, 68, 275–308.

Fama, E. F., and K. R. French, 2002, "The Equity Premium," *Journal of Finance*, 57, 637–659.

Flavin, M., and T. Yamashita, 2002, "Owner-Occupied Housing and the Composition of the Household Portfolio over the Life Cycle," *American Economic Review*, 92, 345–362.

Goetzmann, W. N., and M. Spiegel, 2000, "The Policy Implication of Portfolio Choice in Underserved Mortgage Markets," working paper, Yale University.

Gomes, F., and A. Michaelides, 2004, "Optimal Life-Cycle Asset allocation: Understanding the Empirical Evidence," *Journal of Finance*, forthcoming.

Grossman, S. J., and G. Laroque, 1990, "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods," *Econometrica*, 58, 25–51.

Heaton, J., and D. Lucas, 2000a, "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk", *Journal of Finance*, 55, 1163–1198.

Heaton, J., and D. Lucas, 2000b, “Portfolio Choice in the Presence of Background Risk”, *Economic Journal*, 110, 1–26.

Hu, X., 2002, “Portfolio Choice for Homeowners,” working paper, Northwestern University.

Jagannathan, R., and N. R. Kocherlakota, 1996, “Why Should Older People Invest Less in Stocks Than Younger People?” *Federal Reserve Bank of Minneapolis Quarterly Review*, 20, 11–23.

Kullmann, C., and S. Siegel, 2003, “Real Estate and its Role in Household Portfolio Choice,” working paper, University of British Columbia.

Kyriazidou, E., 1997, “Estimation of a Panel Data Sample Selection Model,” *Econometrica*, 65, 1335–1364.

Powell, J. L., 1994, “Estimation of Semiparametric Models,” *Handbook of Econometrics*, 4, 2444–2521.

U.S. Department of Labor, Bureau of Labor Statistics, 2003, Consumer Expenditures in 2001, Report 966.

U.S. Census Bureau, Population Division, 2003, Geographic Mobility: March 2000–March 2001.

U.S. Census Bureau, Housing and Household Economic Statistics Division, 1992, 1991 Residential Finance Survey.

Viceira, L. M., 2001, “Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income,” *Journal of Finance*, 56, 433–470.

Vissing-Jogensen, A., 2002, “Towards an Explanation of Household Portfolio Heterogeneity: Nonfinancial Income and Participation Cost Structures,” working paper, University of Chicago.

Yao, R., and H. H. Zhang, 2004, “Optimal Life-Cycle Asset allocation with Housing as a Collateral,” working paper, Baruch College and University of North Carolina.