d[1]D..1 xx

toc

Structural Estimation of **Dynamic Stochastic Optimizing Models** of Intertemporal **Choice**

- Plausible (microeconomically calibrated) uncertainty
- Life cycle or infinite horizon
- How To Add a Second Choice Variable
- Method of Simulated Moments Estimation of Parameters

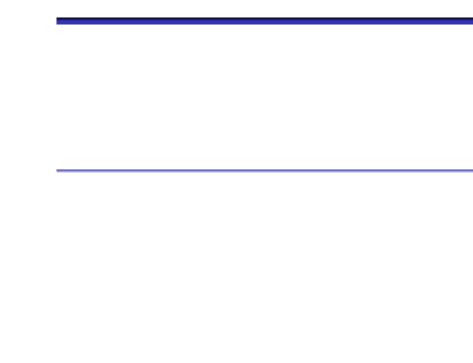
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. $\mathbf{v}_t(\mathbf{m}_t, \mathbf{p}_t) = \max_{\mathbf{c}_t} \ \mathrm{u}(\mathbf{c}_t) + \exists \mathbb{E}_t [\mathbf{v}_{t+1}(\mathbf{m}_{t+1}, \mathbf{p}_{t+1})]$

(4)

 $\mathbf{v}_t(m_t) = \max_{c_t} \ \mathbf{u}(c_t) + \beta \mathbb{E}_t[\mathcal{G}_{t+1}^{1-\rho} \mathbf{v}_{t+1}(m_{t+1})]$

• Non-Friedman (transitory/permanent) income process

e.g., AR(1)

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Define

E.g.xuse an equiprobable 7-point distribution:

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.
$$v_t'(a_t) = \beta \mathsf{R} \mathcal{G}_{t+1}^{-\rho} \left(\frac{1}{n}\right) \sum_{i=1}^n u' \left(c_{t+1}(\mathcal{R}_{t+1} a_t + \boldsymbol{\theta}_i) \right)$$

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- Define a grid of points m (indexed m[i])
- ② Use numerical rootfinder to solve $u'(c) = v_t'(m[i] v_t'(m[i])$

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Example: $\mathbf{m}_{T-1} = \{0., 1., 2., 3., 4.\}$ (solid is 'correct' soln)

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evaluations of

Numerical search for values of c_{T-1} satisfying $u'(c) = \mathfrak{v}_t'(m[i \text{ say, 6 gridpoints of } m_{T-1} \text{ may require hundreds or even thous}]$

Define vector of *end-of-period* asset values <mark>a</mark>

• For each a[j] compute $v_t'(a[j])$

ach of these $\mathfrak{v}_t{}'[j]$ corresponds to a unique c[j] via FOC:

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Principles of Approximation

Hard to approximate things that approach ∞ for relevant

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Principles of Approximation

Perfect Foresight Theory:

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Perfect foresight value function:

Perfect foresight value function:

? shows that c_t^m exists everywhere.

Define *consumed* function and its derivative as

$$\mathfrak{c}_t(a) = (\mathfrak{v}_t'(a))^{-1/2}$$

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- Construct $\mathbf{c}_t^{\ m}$ along with \mathbf{c}_t in EGM algorithm
- ② Approximate $c_t(m)$ using piecewise Hermite polynomial

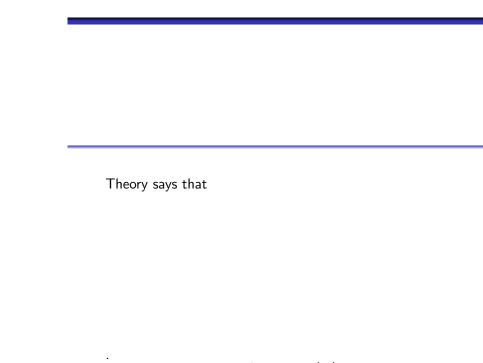
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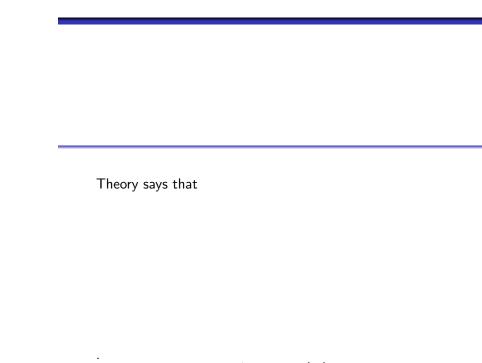
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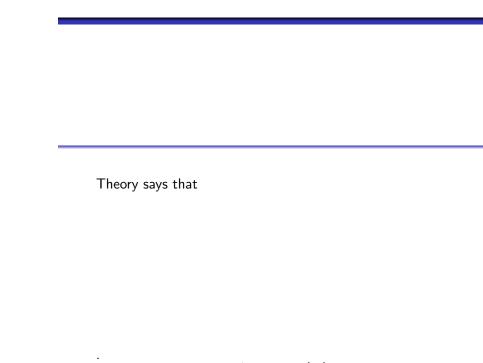
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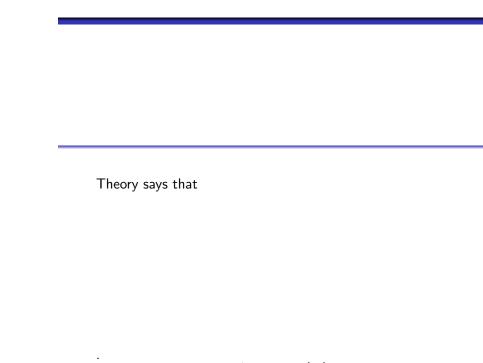
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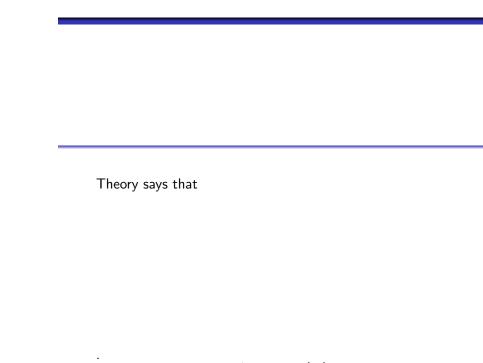
Consider what happens as a_{T-1} approaches $a_{T-1} \equiv -\boldsymbol{\theta} \mathcal{R}_T^{-1}$











Grid Spacing: Uniform PIC

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Grid Spacing: Same $\{a,a\}$ But Triple Exponential $e^{e^{e^{\cdots}}}$ Grow

Further improves speed and accuracy of solution

• See my talk at the conference!

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: Constrained (solid) and Unconstrained (dashed) Consumpt

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Construct



Construct



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: Converging $\grave{\mathbf{c}}_{T-n}(m)$ Functions for $n=\{1,5,10,15,20\}$

Now the consumer has a choice between a risky and a safe portfolio return is

$$\mathfrak{R}_{t+1} = \mathsf{R}(1 - \varsigma_t) + \mathbf{R}_{t+1}\varsigma_t$$
$$= \mathsf{R} + (\mathbf{R}_{t+1} - \mathsf{R})\varsigma_t$$

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The FOC with respect to c_t now yields an Euler equation

(25)

$$\mathbf{u}^{c}(c_{t}) = \mathbb{E}_{t}[\beta \mathfrak{R}_{t+1} \mathbf{u}^{c}(c_{t+1})].$$

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(26)

- Start with coarse grid for θ (say, 3 points)
- Solve to convergence; call period of convergence n
- 3 Construct finer grid for θ (say, 7 points)
- Solve for period T n 1 assuming c_{T-n}

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- Onstruct finer grid for a (say, 20 points)
- Solve for period T n 1 assuming c_{T-n}

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Parameterization of Uncertainty

- Probability of Death
- Demographic Adjustments to β

Parameterization of Uncertainty

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Parameterization of Uncertainty

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: m from SCF (means (dashed) and medians (solid))

- Draw shocks using calibrated $\sigma_{\Psi}^2, \sigma_{\theta}^2$
- ullet Consume according to solved c_t
- \Rightarrow m distribution by age

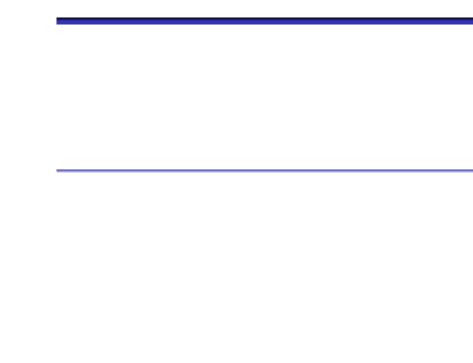
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 $\label{eq:GapEmpiricalSimulatedMedians} \begin{array}{ll} [\rho, \beth] := \\ & [& \texttt{ConstructcFuncLife}[\rho, \beth]; \end{array}$



Yields estimates of

: Estimation Results

: Point Estimate and Height of Minimized Function

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