Solution Methods for Microeconomic Dynamic Stochastic Optimization Problems

2024-03-27

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Note: The GitHub repo `SolvingMicroDSOPs repo` associated with this document contains python code that produces all results, from scratch, except for the last section on indirect inference. The numerical results have been confirmed by showing that the answers that the raw python produces correspond to the answers produced by tools available in the Econ-ARK toolkit, more specifically those in the `HARK` which has full documentation. The MSM results at the end have have been superseded by tools in the `EstimatingMicroDSOPs repo`.

Abstract

These notes describe tools for solving microeconomic dynamic stochastic optimization problems, and show how to use those tools for efficiently estimating a standard life cycle consumption/saving model using microeconomic data. No attempt is made at a systematic overview of the many possible technical choices; instead, I present a specific set of methods that have proven useful in my own work (and explain why other popular methods, such as value function iteration, are a bad idea). Paired with these notes is *Mathematica*, Matlab, and Python software that solves the problems described in the text.

Keywords Dynamic Stochastic Optimization, Method of Simulated Moments, Structural Estimation, Indirect Inference

JEL codes E21, F41

`PDF:` `https://github.com/llorracc/SolvingMicroDSOPs/blob/master/SolvingMicroDSOPs.pdf`
`Slides:` `https://github.com/econ-ark/SolvingMicroDSOPs/blob/master/SolvingMicroDSOPs-Slides.pdf`
`Web:` `https://econ-ark.github.io/SolvingMicroDSOPs`
`Code:` `https://github.com/econ-ark/SolvingMicroDSOPs/tree/master/Code`
`Archive:` `https://github.com/econ-ark/SolvingMicroDSOPs`
(Contains LaTeX code for this document and software producing figures and results)

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The notes were originally written for my Advanced Topics in Macroeconomic Theory class at Johns Hopkins University; instructors elsewhere are welcome to use them for teaching purposes. Relative to earlier drafts, this version incorporates several improvements related to new results in the paper "Theoretical Foundations of Buffer Stock Saving" (especially tools for approximating the consumption and value functions). Like the last major draft, it also builds on material in "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems" published in *Economics Letters*, available at `http://www.econ2.jhu.edu/people/ccarroll/EndogenousArchive.zip`, and by including sample code for a method of simulated moments estimation of the life cycle model *a la* Gourinchas and Parker (2002) and Cagetti (2003). Background derivations, notation, and related subjects are treated in my class notes for first year macro, available at `http://www.econ2.jhu.edu/people/ccarroll/public/lecturenotes/consumption`. I am grateful to several generations of graduate students in helping me to refine these notes, to Marc Chan for help in updating the text and software to be consistent with Carroll (2006), to Kiichi Tokuoka for drafting the section on structural estimation, to Damiano Sandri for exceptionally insightful help in revising and updating the method of simulated moments estimation section, and to Weifeng Wu and Metin Uyanik for revising to be consistent with the 'method of moderation' and other improvements. All errors are my own. This document can be cited as Carroll (2023) in the references.

Contents

1 Introduction

These lecture notes provide a gentle introduction to a particular set of solution tools for the canonical consumption-saving/portfolio allocation problem. Specifically, the notes describe and solve optimization problems for a consumer facing uninsurable idiosyncratic risk to nonfinancial income (e.g., labor or transfer income), first without and then with optimal portfolio choice,[1] with detailed intuitive discussion of the various mathematical and computational techniques that, together, speed the solution by many orders of magnitude compared to "brute force" methods. The problem is solved with and without liquidity constraints, and the infinite horizon solution is obtained as the limit of the finite horizon solution. After the basic consumption/saving problem with a deterministic interest rate is described and solved, an extension with portfolio choice between a riskless and a risky asset is also solved. Finally, a simple example shows how to use these methods (via the statistical 'method of simulated moments' ('MSM') to estimate structural parameters like the coefficient of relative risk aversion (*a la* Gourinchas and Parker (2002) and Cagetti (2003)).

2 The Problem

{sec:the-problem}

The usual analysis of dynamic stochastic programming problems packs a great many events (intertemporal choice, stochastic shocks, intertemporal returns, income growth, the taking of expectations, and more) into a single step in which the agent makes an optimal choice taking account of all of these elements. For the detailed analysis here, we will be careful to disarticulate everything that happens in the problem explicitly into separate steps so that each element can be scrutinized and understood in isolation.

We are interested in the behavior a consumer who begins period t with a certain amount of 'capital' $\mathbf{k}_t$, which is immediately rewarded by a return factor R_t with the proceeds deposited in a **b**ank account **b**alance:

$$\mathbf{b}_t = \mathbf{k}_t \mathsf{R}_t. \tag{1}$$

{eq:bLvl}

References

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[1]See Merton (1969) and Samuelson (1969) for a solution to the problem of a consumer whose only risk is rate-of-return risk on a financial asset; the combined case (both financial and nonfinancial risk) is solved below, and much more closely resembles the case with only nonfinancial risk than it does the case with only financial risk.

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