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toc

Structural Estimation of Dynamic Stochastic Optimizing Models of Intertemporal Choice

- Efficient Solution Methods for Canonical C problem
  - CRRA utility
  - Plausible (microeconomically calibrated) uncertainty
  - Life cycle or infinite horizon
- How To Add a Second Choice Variable
- Method of Simulated Moments Estimation of Parameters

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$$\mathbf{v}_t(\mathbf{m}_t, \mathbf{p}_t) = \max_{\mathbf{c}_t} \ \mathrm{u}(\mathbf{c}_t) + \beth\mathbb{E}_t[\mathbf{v}_{t+1}(\mathbf{m}_{t+1}, \mathbf{p}_{t+1})]$$

(4)

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$$\mathrm{v}_t(m_t) = \max_{c_t} \quad \mathrm{u}(c_t) + \beta\mathbb{E}_t[\mathcal{G}_{t+1}^{1-\rho}\mathrm{v}_{t+1}(m_{t+1}$$

s.t.

Non-CRRA utility

- Non-Friedman (transitory/permanent) income process

  e.g., AR(1)

  - But micro evidence is consistent with Friedman

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Define

E.g.xuse an equiprobable 7-point distribution:

PIC

.

$$\mathfrak{v}'_t(a_t) = \beta \mathsf{R} \mathcal{G}_{t+1}^{-\rho} \left(\frac{1}{n}\right) \sum_{i=1}^{n} u'\left(\mathrm{c}_{t+1}(\mathcal{R}_{t+1} a_t + \boldsymbol{\theta}_i)\right)$$

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1. Define a grid of points $\boldsymbol{m}$ (indexed $m[i]$)
2. Use numerical rootfinder to solve $u'(c) = \mathfrak{v}_t{}'(m[i] -$

   The c that solves this becomes $c[i]$

3. Construct interpolating function $\grave{c}$ by linear interpola

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Example: $\boldsymbol{m}_{T-1} = \{0., 1., 2., 3., 4.\}$ (solid is 'correct' soln)
PIC

Numerical search for values of c_{T-1} satisfying $u'(c) = \mathfrak{v}_t{}'(m[i$
say, 6 gridpoints of $\boldsymbol{m}_{T-1}$ may require hundreds or even thous
evaluations of

Define vector of *end-of-period* asset values a

- For each $a[j]$ compute $\mathfrak{v}_t'(a[j])$

Each of these $\mathfrak{v}_t'[j]$ corresponds to a unique $c[j]$ via FOC:

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Principles of Approximation

Hard to approximate things that approach ∞ for relevant

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Perfect Foresight Theory:

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Perfect Foresight Theory:

Perfect foresight value function:

Perfect foresight value function:

? shows that $\mathrm{c}_t{}^m$ exists everywhere.

Define *consumed* function and its derivative as

.

$$\mathfrak{c}_t(a) = (\mathfrak{v}_t'(a))^{-1/\rho}$$

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1. Construct ${\mathbf{c}_t}^m$ along with $\mathbf{c}_t$ in EGM algorithm
2. Approximate $c_t(m)$ using piecewise Hermite polynomial

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Consider what happens as a_{T-1} approaches $a_{T-1} \equiv -\boldsymbol{\theta}\mathcal{R}_T{}^{-1}$

Theory says that

Theory says that

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Grid Spacing: Uniform
PIC

Grid Spacing: Same $\{a, a\}$ But Triple Exponential $e^{e^{e^{\cdots}}}$ Grow
PIC

Further improves speed and accuracy of solution

- See my talk at the conference!

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$$\mathrm{v}_{T-1}(m_{T-1}) = \max_{c_{T-1}} \quad u(c_{T-1}) + \mathbb{E}_{T-1}[\beta \mathcal{G}_T^{1-\rho} \mathrm{v}_T(m_T)]$$

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Point where constraint makes transition from binding to not is

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$$u'(m^{\#}_{T-1}) = \mathfrak{v}'_{T-1}(0.)$$

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PIC

: Constrained (solid) and Unconstrained (dashed) Consumpt

1. Construct

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PIC

: Converging $\grave{c}_{T-n}(m)$ Functions for $n = \{1, 5, 10, 15, 20\}$

Now the consumer has a choice between a risky and a saf
portfolio return is

.

$$
\begin{aligned}
\mathfrak{R}_{t+1} &= \mathsf{R}(1-\varsigma_t) + \mathbf{R}_{t+1}\varsigma_t \\
&= \mathsf{R} + (\mathbf{R}_{t+1} - \mathsf{R})\varsigma_t
\end{aligned}
$$

(24)

Now the consumer has a choice between a risky and a safe asset
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The FOC with respect to c_t now yields an Euler equation

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$$\mathrm{u}^c(c_t) = \mathbb{E}_t[\beta \mathfrak{R}_{t+1} \mathrm{u}^c(c_{t+1})].$$

(25)

while the FOC with respect to the portfolio share yields

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$$\mathbb{E}_t[m_{t+1}/m_t] = 1 \text{ if } m_t = \check{m}$$

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(26)

1. Start with coarse grid for $\boldsymbol{\theta}$ (say, 3 points)
2. Solve to convergence; call period of convergence n
3. Construct finer grid for $\boldsymbol{\theta}$ (say, 7 points)
4. Solve for period $T - n - 1$ assuming $\grave{c}_{T-n}$

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5. C

1. Start with coarse grid for $\boldsymbol{a}$ (say, 5 gridpoints)
2. Solve to convergence; call period of convergence n
3. Construct finer grid for $\boldsymbol{a}$ (say, 20 points)
4. Solve for period $T - n - 1$ assuming $\grave{c}_{T-n}$
5. Continue to ...

1. Start with coarse grid for $\boldsymbol{a}$ (say, 5 gridpoints)
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$$\mathrm{v}_t(m_t) = \max_{c_t} \quad \mathrm{u}(c_t) + \beth \mathcal{L}_{t+1} \hat{\beta}_{t+1} \mathbb{E}_t[(\mathbf{\Psi}_{t+1} \mathcal{G}_{t+1})^{1-\rho} \mathrm{v}_{t+1}(m$$

Parameterization of Uncertainty

- Probability of Death
- Demographic Adjustments to β

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: m from SCF (means (dashed) and medians (solid))

Given a set of parameter values $\{\rho, \beth\}$:

Start at age 25 with empirical *m* data

- Draw shocks using calibrated ${\sigma_{\Psi}}^2$,${\sigma_{\theta}}^2$
- Consume according to solved c_t

$\Rightarrow$ *m* distribution by age

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.

```
GapEmpiricalSimulatedMedians[ρ, ℷ]:=
[                    ConstructcFuncLife[ρ, ℷ];
```

Yields estimates of

: Estimation Results

11	ρ	11	ב
11	3.69	11	0.88
11	(0.047)	11	(0.002)
11			

PIC

: Point Estimate and Height of Minimized Function