

# The Method of Moderation

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## Abstract

In a risky world, a pessimist assumes the worst will happen. Someone who ignores risk altogether is an optimist. Consumption decisions are mathematically simple for both the pessimist and the optimist because both behave as if they live in a riskless world. A realist (that is, someone who wants to respond optimally to risk) faces a much more difficult problem, but (under standard conditions) will choose a level of spending somewhere between pessimist's and the optimist's. We use this fact to redefine the space in which the realist searches for optimal consumption rules. The resulting solution accurately represents the numerical consumption rule over the entire interval of feasible wealth values with remarkably few computations.

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# 1 Introduction

Solving a consumption, investment, portfolio choice, or similar intertemporal optimization problem using numerical methods generally requires the modeler to choose how to represent a policy or value function. In the stochastic case, where analytical solutions are generally not available, a common approach is to use low-order polynomial splines that exactly match the function (and maybe some derivatives) at a finite set of gridpoints, and then to assume that interpolated or extrapolated versions of that spline represent the function well at the continuous infinity of unmatched points.

This paper argues that a better approach in the standard consumption problem is to rely upon the fact that without uncertainty, the optimal consumption function has a simple analytical solution. The key insight is that, under standard assumptions, the consumer who faces an uninsurable labor income risk will consume less than a consumer with the same path for expected income but who does not perceive any uncertainty as being attached to that future income. The ‘realistic’ consumer, who *does* perceive the risks, will engage in ‘precautionary saving,’ so the perfect foresight riskless solution provides an upper bound to the solution that will actually be optimal. A lower bound is provided by the behavior of a consumer who has the subjective belief that the future level of income will be the worst that it can possibly be. This consumer, too, behaves according to the convenient analytical perfect foresight solution, but his certainty is that of a pessimist perfectly confident in his pessimism.

Using results from ?, we show how to use these upper and lower bounds to tightly constrain the shape and characteristics of the solution to problem of the ‘realist.’ Imposition of these constraints can clarify and speed the solution of the realist’s problem.

After showing how to use the method in the baseline case, we show how refine it to encompass an even tighter theoretical bound.

## 2 The Realist’s Problem

We assume that truly optimal behavior in the problem facing the consumer who understands all his risks is captured by

$$\max \mathbb{E}_t \left[ \sum_{n=0}^{T-t} \beta^n u(c_{t+n}) \right]. \quad (1)$$