Heterogeneous Firms: Lumpy Investment and Aggregation

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[a]Federal Reserve Board: The views expressed are my own and do not necessarily reflect those of the Board of Governors or the Federal Reserve System.

This class

- Canonical **HA household** model is super well behaved.
  - households are hit by **discrete shocks** and make **continuous choices**
  - **convex** dynamic problem, best solved by **EGM**

- What if we want to model **discrete choices**?
  - labor force participation, occupation choice
  - lumpy adjustment with fixed cost (price, investment, portfolio, ...)

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- **SSJ** can handle all of these examples and more.

- Example: canonical **HA firm** model of lumpy investment.
  - we will dicuss **SSJ implementation** & **aggregation debate**
  - based on Koby and Wolf (2020) + my own explorations (we're merging these into 1 paper)

Why is discrete choice hard?

- If agents make only discrete choices, it's easy.

- Interaction between discrete and continuous choices is the hard part.

- **Non-convexity**: FOCs are insufficient to obtain policy functions.
  - EGM does not work, more robust backward iteration is needed
  - **VFI**: easy, general, slow $\rightarrow$ my choice today
  - **EGM + upper envelope**: not so easy, requires curvature, fast $\rightarrow$ see 2022 workshop

- **Discontinuities** in policy functions.
  - SSJ relies on differentiating policies wrt aggregate inputs
  - **smoothing**: let fixed cost be a continuous random variable or add taste shocks

  $\rightarrow$ standard in many literatures, considered distasteful in some

  $\rightarrow$ payoffs are immense, be pragmatic!

Roadmap

1. Intro to investment
2. Canonical HA firm model (tutorial later today)
3. Aggregate implications of lumpiness
4. Conclusion

Intro to investment

Brief intellectual history (part 1)

- How is investment determined?

- **Neoclassical theory** of investment by Jorgenson (1963).
  - competitive firm maximizes profits, Cobb-Douglas technology, no adjustment cost

$$K \cdot r^k = \alpha Y$$

- **Q-theory** of Tobin (1969).
  - *average q* (market value of firm / replacement cost of capital) is a sufficient statistic for investment

- Synthesis by Hayashi (1982).
  - neoclassical theory + convex adjustment cost = marginal q-theory
  - conditions for marginal q = average q (linear homogeneity)

Representative firm model

- Bellman equation:

$$V_t(K_{t-1}) = \max_{K_t, I_t, N_t} F_t(K_{t-1}, N_t) - w_t N_t - I_t - \phi(I_t, K_{t-1}) + \frac{1}{1+r_t}\mathbb{E}_t[V_{t+1}(K_t)]$$
$$\text{s.t. } K_t = (1-\delta)K_{t-1} + I_t$$

- Define $W_t(K_t) = \mathbb{E}_t[V_{t+1}(K_t)]/(1+r_t)$. Marginal q is $Q_t \equiv W_t'(K_t)$.

- FOCs imply

$$\underbrace{1 + \partial_I \phi(I_t, K_{t-1})}_{\text{marginal cost of capital}} = Q_t = \underbrace{\mathbb{E}_t\left[\frac{\partial_K F_{t+1}(K_t, N_{t+1}) - \partial_K \phi(I_{t+1}, K_t) + (1-\delta)Q_{t+1}}{1+r_t}\right]}_{\text{marginal benefit of capital}}$$

Brief intellectual history (part 2)

- Neoclassical q-theory is still prominent in quantitative RANK models.
    - Christiano et al. (2005); Justiniano et al. (2010)...
    - matches well (first-order) aggregate investment dynamics (idot adjustment cost)

- What's the problem with q-theory based on **representative firm**?

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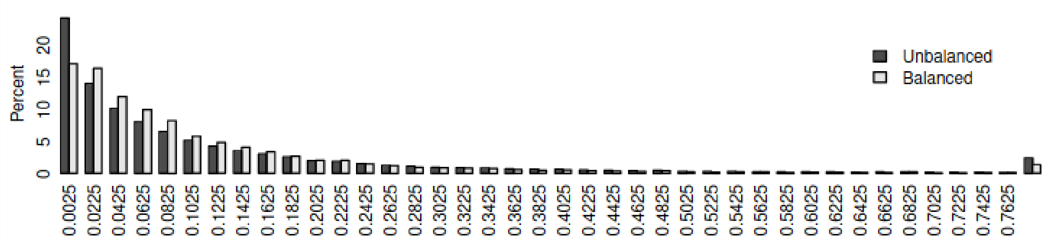
- At odds with firm-level investment, notably **lumpiness**.
    - inaction: share of firms investing near 0 in a given year
    - spikes: share of firms investing very much in a given year

Distribution of plant-level investment rates

Figure 1: Investment rate distribution (IRS data, 1998–2010). Source: Zwick and Mahon (2017)



Variable	Definition	Balanced
Average investment rate	I/K	10.4%
Inaction rate	$\|I/K\| \leq 0.01$	23.7%
Spike rate	$\|I/K\| \geq 0.2$	14.4%
Spike share of aggregate investment		24.4%

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    - spikes: share of firms investing very much in a given year
- But does lumpiness "matter" for **aggregate investment**?

Aggregation debate

- Does lumpiness "matter" for **aggregate investment** I?

- **Yes**: Caballero et al. (1995); Caballero and Engel (1999).
  - lumpiness points to presence of fixed costs, small investments are not worthwile
  - I should be more responsive when many firms invest anyway, such as expansions

$\rightarrow$ **state-depence** (nonlinearity) of aggregate investment

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  - embed HA firm block in RBC model with aggregate TFP shocks
  - when TFP is high, firms want to expand
  - but households want smooth consumption, so real wage and interest rate rise
  - and dampen the rise in share of adjusters $\rightarrow$ state-dependence largely **disappears in GE**

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- **Yet it does**: Winberry (2021)
  - vanilla RBC model gets wrong the TFP-real rate comovement
  - fix this by household habit formation $\rightarrow$ **recover** state-dependence **in GE**

Preview of results

- We'll revisit the aggregation debate wielding **sequence-space Jacobians**.

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- **1st-order irrelevance**: micro lumpiness has minimal impact on aggregate investment dynamics around steady state. Literature is not about this, but good to know.
  - vary lumpiness while fixing $\mathcal{J}^{I,r}_{0,0} \implies$ entire Jacobian is approximately identical
  - holds for all block Jacobians $\implies$ irrelevance is independent of GE closure

- Calibration of **adjustment cost** is key to state-dependence. **Identification is tricky**.
  - all papers on previous slide fall into one pitfall or another
  - targeting micro and macro moments jointly is key, can be done for firm block in isolation

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- My current take (work in progress):
  - canonical HA firm model—carefully calibrated—doesn't yield strong aggregate implications of micro lumpiness
  - issue is with the model, not necessarily with the mechanism

Canonical HA firm model

Overview

- Rooted in Abel and Eberly (1994). Quantitative macro version by Khan and Thomas (2008).

- **Starting point**: RA model with convex adjustment cost.

- **New ingredients** to match the micro data:
  - decreasing returns to scale
  - idiosyncratic productivity shocks

    $\}$ $\rightarrow$ heterogeneity in target capital, investment
  - adjustment cost includes a fixed cost $\rightarrow$ inaction, spikes

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- **Smoothing**: fixed cost ξ is continuous random variable, iid across firms and periods.

$$\phi(\xi, i, k_-) = \xi \cdot \mathbf{1}_{\{i \neq 0\}} + \frac{\varphi}{2}\left(\frac{i}{k_-}\right)^2 k_-$$

Timing

1. Firm enters period with state (z_-, k_-)
2. Draws new **productivity** z from exogenous Markov process. Usually AR(1).
3. Draws **adjustment cost** ξ from iid distribution. Usually, **uniform** on $[0, \bar{\xi}]$.
4. Chooses **investment** and produce:

$$V_t(\xi, z, k_-) = \max_{k,i,n} F(z, k_-, n) - w_t n - i - \phi(\xi, i, k_-) + W_t(z, k)$$
$$\text{s.t. } k = (1-\delta)k_- + i$$

5. Finish period with state (z, k).

Sketch of solution (part 1)

- Labor demand and output are independent of the investment decision. Define

$$\pi_t(z, k_-) = F(z, k_-, n^*) - w_t n^*$$

- Split decision problem between **adjusters** & **non-adjusters**

$$V_t(\xi, z, k_-) = \max\left\{V_t^A(z, k_-) - \xi, V_t^N(z, k_-)\right\}$$

where

$$V_t^A(z, k_-) = \max_{i^A} \pi_t(z, k_-) - i^A - \frac{\varphi}{2}\left(\frac{i^A}{k_-}\right)^2 k_- + W_t\Big(z, (1-\delta)k_- + i^A\Big) \tag{1}$$

$$V_t^N(z, k_-) = \pi_t(z, k_-) + W_t\Big(z, (1-\delta)k_-\Big) \tag{2}$$

Sketch of solution (part 2)

- Firm invests iff fixed cost is lower than **threshold**

$$\hat{\xi}_t(z, k_-) = V_t^A(z, k_-) - V_t^N(z, k_-)$$

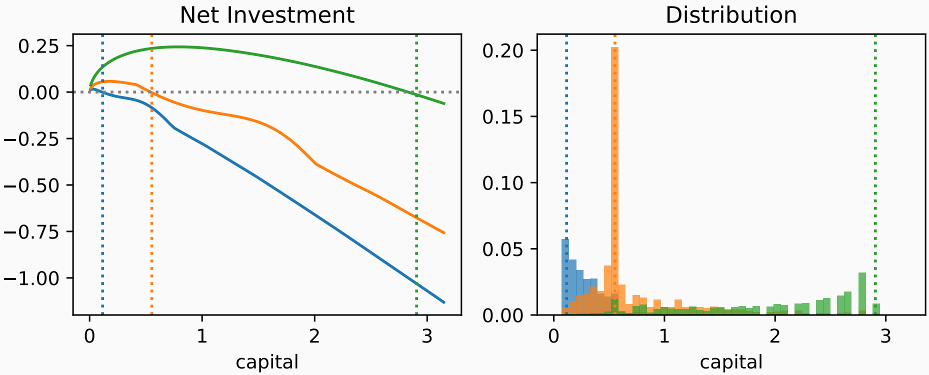
  which implies **adjustment probabilities**

$$p_t^A(z, k_-) = \Pr\left(\xi \leq \hat{\xi}_t(z, k_-)\right)$$

- **Smoothing** is achieved by aggregating over fixed cost distribution

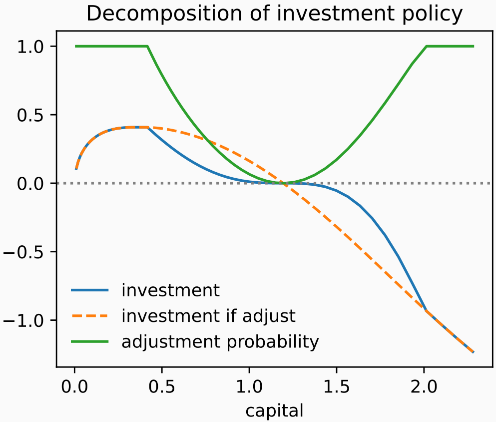
$$\begin{aligned} i_t(z, k_-) &= p_t^A(z, k_-) i_t^A(z, k_-) \\ V_t(z, k_-) &= p_t^A(z, k_-)\left(V_t^A(z, k_-) - \mathbb{E}\left[\xi \middle| \xi \leq \hat{\xi}_t(z, k_-)\right]\right) + \left(1 - p_t^A(z, k_-)\right) V_t^N(z, k_-) \end{aligned}$$

Core mechanics



- Target capital depends on productivity.
- When hit by TFP shock, move gradually to new target due to **convex cost**.

Extensive vs intensive margin



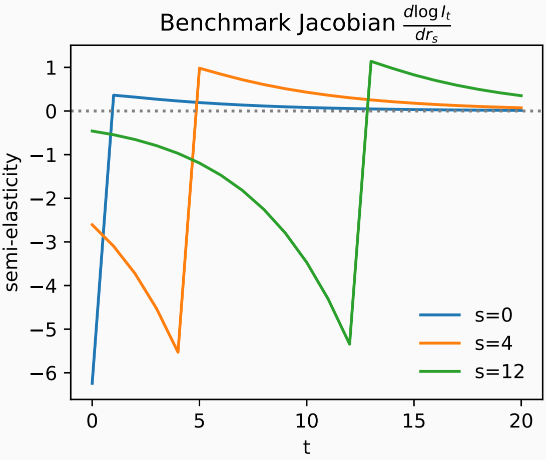
- Zoom in on middle productivity firms.
- $i(z, k_-) = p_t^A(z, k_-) i_t^A(z, k_-)$
- Adjustment probability increases with distance from target.

Aggregate implications of lumpiness

Sequence-space Jacobians

- Firm block $\{w_t, r_t\} \to \{I_t, K_t, N_t, Y_t\}$.
- Focus on **aggregate investment function** $\mathcal{I}(\{w_t, r_t\})$.
- Take a reasonable calibration and explore how adjustment costs affect $\frac{d \log I_t}{dr_s}$.
    - $\bar{\xi}$ upper bound of fixed cost distribution
    - φ quadratic cost
- Keep in mind: **recent evidence** suggests $\boxed{\dfrac{d \log I_0}{dr_0} \approx -5.}$
    - Zwick and Mahon (2017): bonus depreciation (Koby and Wolf 2020 translates it to *dr*)
    - Gormsen and Huber (2022): perceived cost of capital from conference calls
    - He et al. (2022): cost of capital policy change in China

Benchmark Jacobian



- Investment falls in anticipation of higher real rate.
- Investment rises after the shock to rebuild steady-state capital stock.

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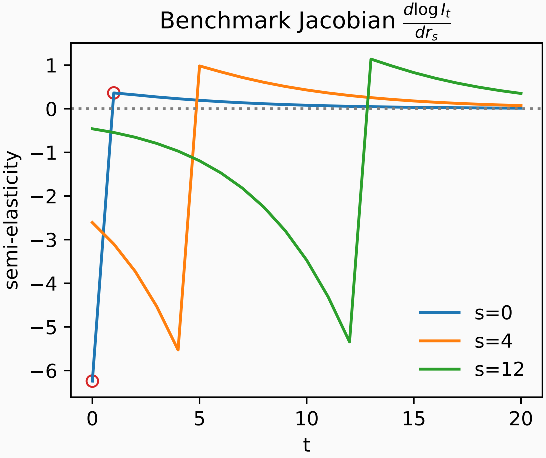

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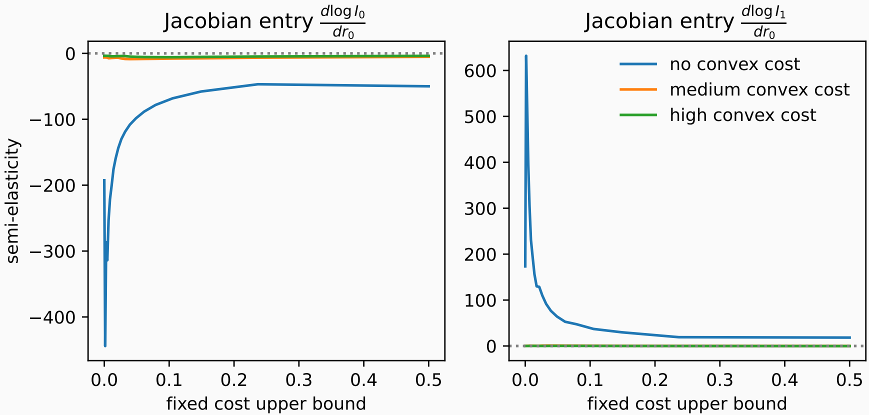
- Investment rises after the shock to rebuild steady-state capital stock.
- **Next:** vary $\bar{\xi}$ and φ.

Jacobian fixed cost



- Without adjustment costs, I is extremely price-elastic.
- Without convex cost, $\frac{d\log I_0}{dr_0} \approx 50$ even for very high fixed cost.

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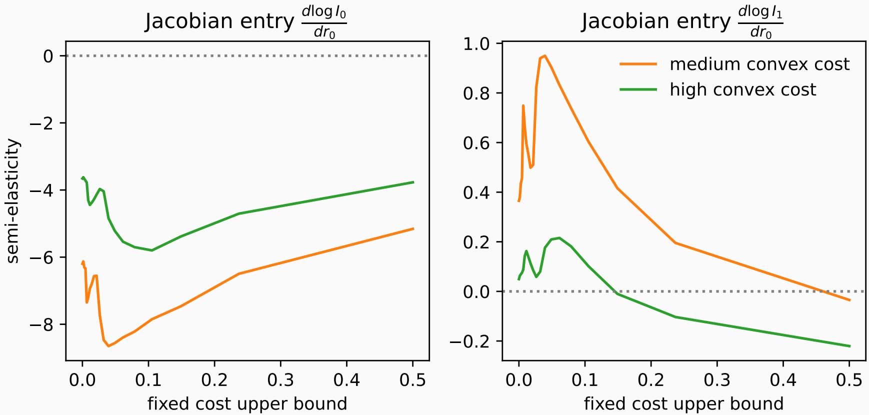

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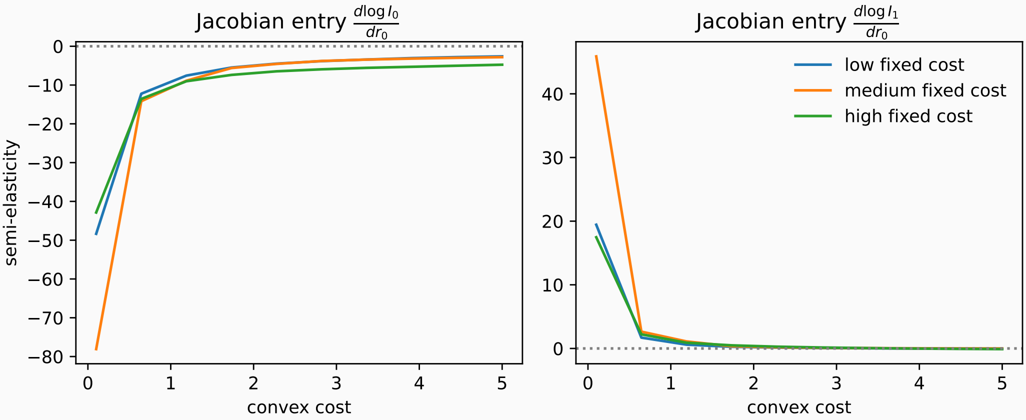
Jacobian convex cost



- Convex cost delivers straightforward dampening.

Taking stock

- Without adjustment costs, aggregate investment I is extremely price-elastic.
  - in the limit of CRS technology and no convex cost, it's infinitely elastic
  - **intuition:** without **curvature**, firm size is pinned down by demand

- Convex cost is a much more effective instrument to control elasticity of I.
  - **convex cost** affects curvature while **fixed cost** does not
  - some convex cost is necessary to get into ballpark of semi-elasticity of -5

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- Most of the classic papers have only a fixed cost.
  - fixed: Caballero and Engel (1999), Khan and Thomas (2008), Gourio and Kashyap (2007), Bachmann et al. (2013)...
  - fixed & convex: Cooper and Haltiwanger (2006); Koby and Wolf (2020); Winberry (2021)

- Do you expect heterogeneity to "matter" with almost no curvature on the firm side?

Micro moments

- Many combinations of fixed & convex cost can match the elasticity of aggregate I.
- Turn to micro data (**lumpiness**) to pin down the right combination.
- Recall that inaction rate is $\Pr\left(\left|\frac{i}{k_-}\right| \leq 0.01\right)$ and spike rate is $\Pr\left(\left|\frac{i}{k_-}\right| \geq 0.2\right)$.
- Strategies to identifying fixed cost.
  - aggregate (sectoral) time series: Caballero and Engel (1999); Bachmann et al. (2013)
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  - inaction rate: Khan and Thomas (2008)
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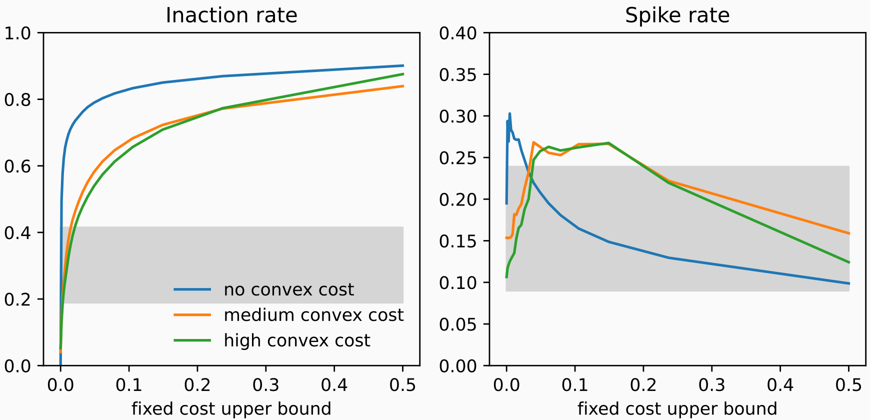
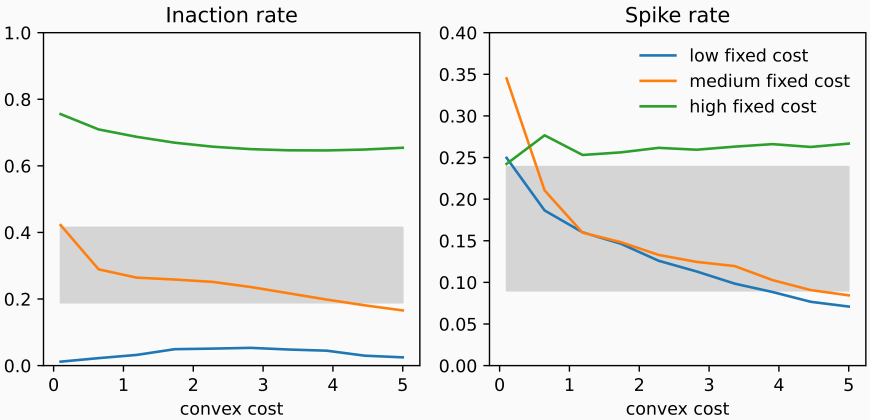
- Claim: inaction rate is the most useful lumpiness metric in this model.

Micro moments fixed cost



- Gray area shows observed range (1998–2010) from Zwick and Mahon (2017).

Micro moments convex cost



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Taking stock

- Lumpiness is often measured by either inaction rate or spike rate.
- Weak identification of fixed cost from spike rate.
- Inaction rate calls for small fixed cost.
    - higher than in Khan and Thomas (2008)
    - much smaller than in Bachmann et al. (2013); Winberry (2021)

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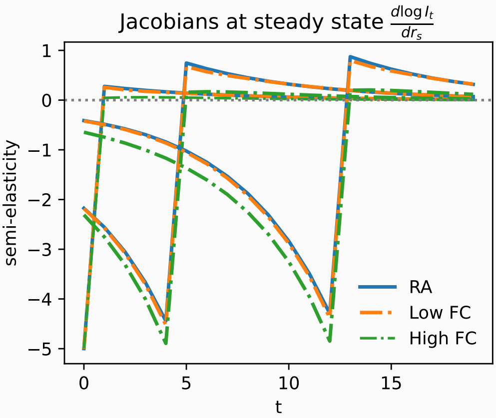
- **Next**: inaction is the key statistic for the aggregate implications of lumpiness.
  - mechanism relies on variation in share of adjusters
  - for realistic inaction (small fixed cost), aggregate effects are small

Two calibrations

	Data	Low FC	High FC
Fixed cost upper bound		0.0035	1.1
Convex cost		1.95	1.7
Inaction rate	0.24	0.21	0.82
Spike rate	0.14	0.17	0.17
Aggregate inv. elast.	-5	-5	-5

- Massively different fixed cost distributions $\Longrightarrow$ reflected in inaction rate.
- Same spike rate, same aggregate semi-elasticity.

Jacobians around steady state

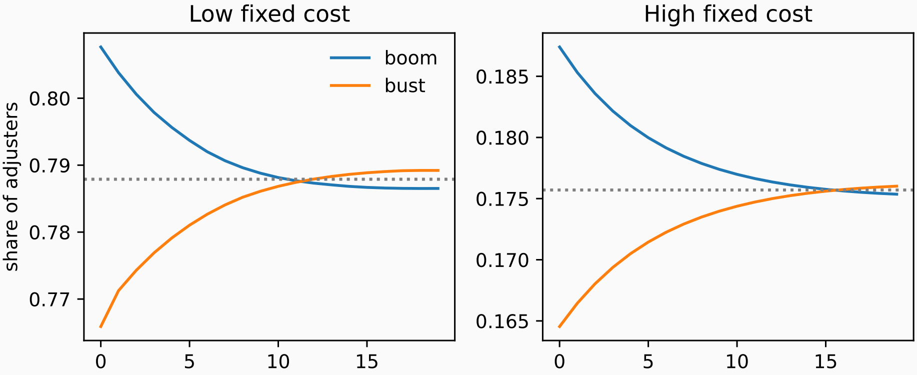


- Low FC model implies same aggregate dynamics as RA model.
- High FC model implies visibly, but not impressively, more anticipation.
- What about state dependence?

State-dependent Jacobians

- Different approaches to quantifying nonlinearities.
  - Khan and Thomas (2008) look at skewness and kurtosis in simulated data.
  - Winberry (2021) compares impulse responses after a history of bad vs good TFP shocks.
- We will compute sequence-space Jacobians around large TFP shocks.
  - **caveat:** can't use fake news algorithm—why?
- TFP shock: AR(1) with $\rho = 0.9$, size such that output rises by 5% in low FC model.
  - output fell by 4.3% in the GFC

Share of adjusters



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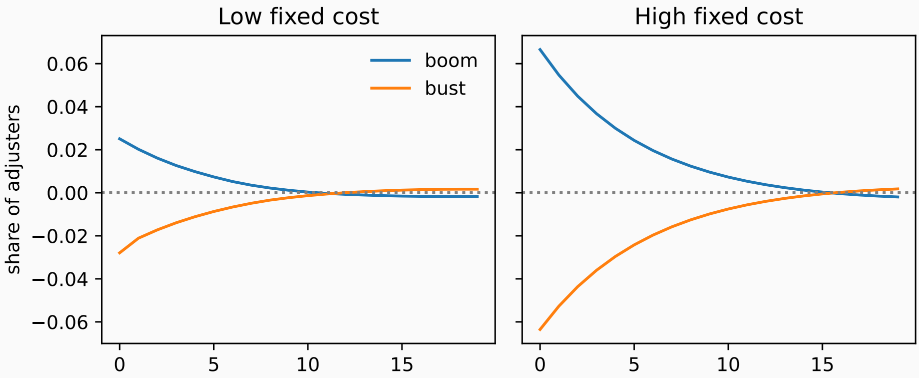
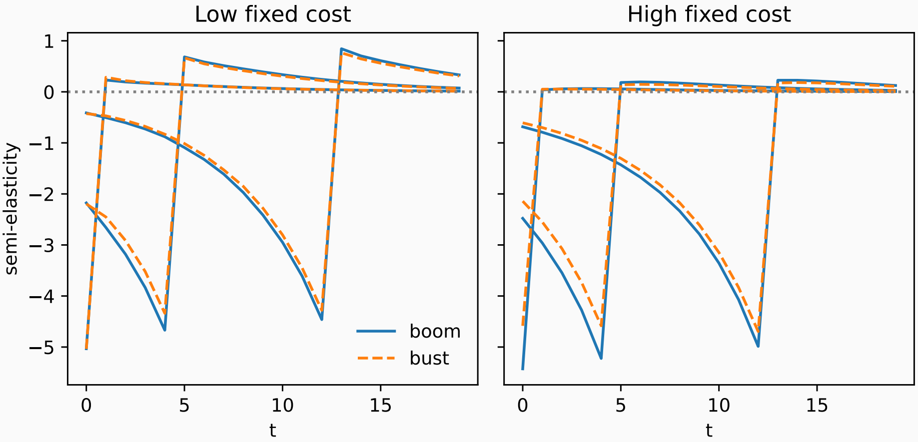


- Share of adjusters **relative to ss** fluctuates more in high FC calibration.

State-dependent Jacobians



- Variation in share of adjusters drives sensitivity to additional shocks.
  - high FC: semi-elasticity to *r* shock is 5.4 in boom vs 4.6 in bust

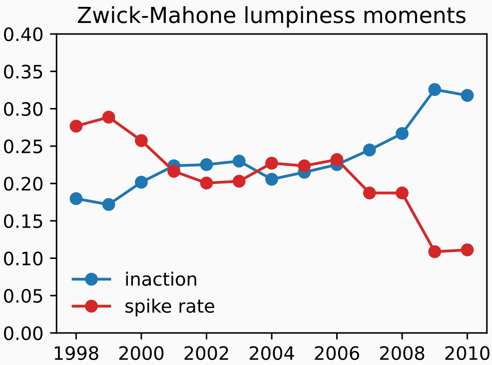
Taking stock

- State-dependence of aggregate investment relies on **variation in share of adjusters**.
- Easier to achieve if share of adjusters is low to begin with. But that's at odds with moderate **inaction rate** in micro data.

Taking stock

- State-dependence of aggregate investment relies on **variation in share of adjusters**.
- Easier to achieve if share of adjusters is low to begin with. But that's at odds with moderate **inaction rate** in micro data.
- Should we take the empirical inaction rate so seriously?
  - Zwick and Mahon (2017) have most comprehensive sample (IRS) but don't observe negative investment, so the 23.7% they report on avg is an upper bound
  - Cooper and Haltiwanger (2006) find 8% for manufacturing firms; probably a lower bound
  - Bachmann et al. (2013): fuzzy mapping between productive units in data / model
- Calibrating model to 80%+ inaction rate is hard to defend. Why don't we look at **cyclicality of inaction rate** in the data?
  - main idea of Gourio and Kashyap (2007), now we have better data

Cyclical inaction



- Between 2007 and 2009, inaction rate rose by 33%.
- About 5$\times$ more than in our simple PE experiment with similar fall in output.
- Suggests that state-dependence mechanism may have bite after all.

Conclusion

Conclusion

- We discussed the emergence of the canonical **HA firm model**.
- Model can account for **micro lumpiness** & **macro state-dependence** of investment that RA model cannot.
- Non-trivial aggregate implications hinge on calibration that's unrealistic in light of modern evidence.
    - substantial 1^{th}-order effects are not even on the table
    - for state-dependence, inaction rate is better measure of lumpiness than spike rate

  $\rightarrow$ stronger identification & more relevant for mechanism
- Literature emphasized role of GE price adjustments.
    - small changes in prices have large effect only if investment demand is very elastic
    - modern firm-level evidence rejects this view
    - model needs convex cost to reach semi-elasticity $\approx$ -5

Questions?

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