Information frictions

NBER Heterogeneous-Agent Macro Workshop

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Today

So far: have assumed full information & rational expectations ("FIRE")

Today: Deviations from FIRE ("information frictions") ...

- incomplete information (e.g. noisy information, sticky information)
- deviations from rational expectations (e.g. extrapolation, cognitive discounting, level k thinking)

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Leading contender to explain key puzzles in macro & finance, e.g.

- Why does {inflation, investment, consumption} respond so sluggishly to aggregate shocks? (but not to idiosyncratic shocks?)
- Why do asset prices overreact to shocks?

Problem

- Slight problem: deviations from FIRE typically very hard to simulate on top of simple RA model
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Goal for today: Coherent framework to model *and simulate* deviations from FIRE

... not just RA, but also **HA**!

Material mostly a (not yet published) version of the approach that we have developed for [Auclert et al., 2020].

Roadmap

1. Introductory example
2. Information frictions in the sequence space
3. Examples
4. Takeaway

Introductory example

Monetary policy revisited

- Imagine we have the IKC equation for monetary policy

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + \mathbf{M} d\mathbf{Y} \tag{1}$$

where $\mathbf{M}^r \equiv \frac{\partial \mathcal{C}}{\partial r}$ and $\mathbf{M} \equiv \frac{\partial \mathcal{C}}{\partial Y}$ are Jacobians of a general household side

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  - only start responding to dr_t in period t
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- What is $d\mathbf{Y}$ then? Can we change (1) to reflect this?

- Start with the “FIRE” iMPCs ($\mathbf{M}^r$ similar)

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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- Each column s is the response of C to news shock: "output rises at date s"
- A date s news shock in our "behavioral" model has **no effect** until date s!
- What happens afterwards? Response to an **unanticipated** shock!
- We call this "Jacobian manipulation" [NB: what NPV do columns of $\mathbf{M}$ have?]

Expectations matrix

- Another way to look at this: how do agents build **expectations** about a date-s shock?
- We can define a matrix **E** that, in each column s, has the **expectations** about a date-s shock of 1. What would that look like in FIRE & behavioral model?

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- $E_{t,s} dY_s$ is then expected value of dY_s at date t

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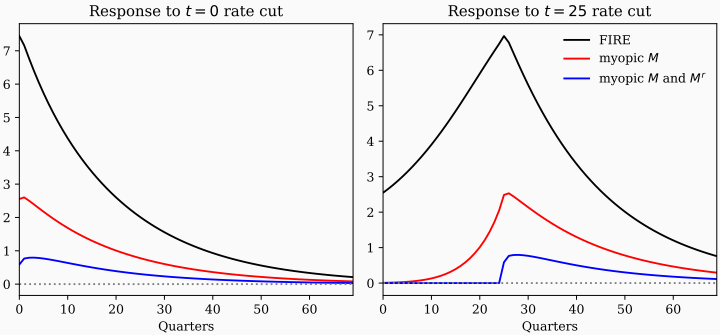

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- **Next:** Generalize this idea to much more general models of belief formation!

Some general assumptions we'll make

We will make a few implicit assumptions:

- Agents are only "behavioral" about **changes** in **aggregate** variables
    - steady state unaffected
    - not "behavioral" w.r.t. *idiosyncratic* income process
- Deviations from FIRE are **orthogonal** to idiosyncratic state
    - can relax this, but too much for today

Information frictions in the sequence space

Separable vs non-separable deviations

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- **Next:** Only focus on separable deviations. Non-separable is different.

General expectations matrix

- Consider a general $\mathbf{E} = (E_{t,s})$ matrix ...
  - entry $E_{t,s}$ captures **average** date-t expectation of unit shock at date-s
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- Will make one of these two assumptions:
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- Typical example:

$$\mathbf{E} = \begin{pmatrix} 1 & * & * & * & \cdots \\ 1 & 1 & * & * & \cdots \\ 1 & 1 & 1 & * & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \text{FIRE benchmark: } \mathbf{E} = \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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- Key: **This is a news shock** with horizon $s-\tau \Rightarrow$ like column $s-\tau$ of **M** !
- Therefore: Column s of **M** is given by

$$M_{t,s} = \sum_{\tau=0}^{\min\{t,s\}} \underbrace{\left(E_{\tau,s} - E_{\tau-1,s}\right) \cdot M_{t-\tau,s-\tau}}_{\text{date-}t\text{ effect of date-}\tau\text{ expectation revision of date-s shock}}$$

(Here convention is $E_{-1,s} = 0$)

$$\mathbf{E} = \begin{pmatrix} 1 & 0.3 & 0.2 & 0.1 & \cdots \\ 1 & 1 & 0.5 & 0.3 & \cdots \\ 1 & 1 & 1 & 0.6 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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- Contribution:

$$M_{t,2} = \ldots + (0.5 - 0.2) \cdot M_{t-1,1} + \ldots$$

Two special cases

$$M_{t,s} = \sum_{\tau=0}^{\min\{t,s\}} (E_{\tau,s} - E_{\tau-1,s}) \cdot M_{t-\tau,s-\tau}$$

- FIRE $E_{t,s} = 1 \Rightarrow$ only $\tau = 0$ term survives since $E_{-1,s} = 0 \Rightarrow M_{t,s} = M_{t,s}$

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- No-foresight example from above: $E_{t,s} = 0$ for all $t < s$. This implies only $\tau = s$ term can ever be positive
  - $\rightarrow$ $M_{t,s} = 0$ whenever $t < s$
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  Exactly our matrix from before!
- Side remark: We can write $M_{t,s}$ also in terms of the fake news matrix:

$$M_{t,s} = \sum_{\tau=0}^{\min\{t,s\}} E_{\tau,s} \cdot \mathcal{F}_{t-\tau,s-\tau}$$

Examples

- Next, we'll walk through examples from the literature
- For each, there is an **E** and an **M**

Examples

- [Mankiw and Reis, 2002] proposed an information-based microfoundation of nominal rigidities
- Consider a mass 1 of price setters, who, ideally, would like to set their price equal to some markup over marginal cost

$$\log P_{it} = \log \mu + \log MC_t \qquad \text{where } MC_t \text{ is stochastic}$$

- Idea: Only random fraction $1 - \theta$ of price setters receive latest information in any given period
- This is called "sticky information" model. In limit case where $\theta = 0$, this boils down to flexible prices

$$\log P_t = \log \mu + \log MC_t$$

(1) Nesting sticky information

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- This allows to solve $d\log P_t$ for **arbitrary** shocks to marginal cost $d\log MC_t$!

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- See [Auclert et al., 2020] for details + application of this idea to general equilibrium

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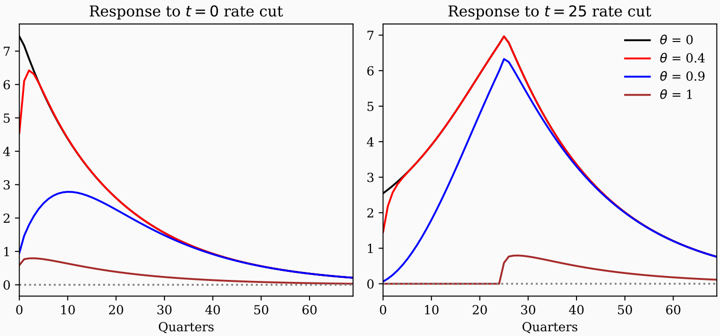

- Intermediate θ generates strong hump shape
- Part of the reason is endogenous: when $d\mathbf{Y}$ is smaller initially $\Rightarrow$ $d\mathbf{C}$ falls too

(3) Dispersed information

- These models assume there is lots of heterogeneity in learning: Some learn it all immediately, others much later. What if instead all agents learn equally quickly?
- To motivate this, let's think of dY_s stemming from an $MA(\infty)$ process

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  2. about an **endogenous** process: agents get signals about $\widetilde{dY}_t$
- 2 is harder! (Why?) Do 1 for now.

(3) Dispersed information about innovation

- Assume each agent i receives signals about current + past innovation

$$s_{jt}^{(i)} = \epsilon_{t-j} + \nu_{jt}^{(i)}$$

where $\nu_{jt}^{(i)} \sim \mathcal{N}\left(0, \tau_j^{-1}\right)$ iid. Allows for arbitrary precisions τ_j.

- Imagine we hit this economy with a one time shock $\epsilon_0 = 1$ at date 0.
- How does agents' average expectations evolve? Bayesian updating:

$$\overline{\mathbb{E}}_t \epsilon_0 = \frac{\sum_{j=0}^{t} \tau_j}{\tau_\epsilon + \sum_{j=0}^{t} \tau_j} \equiv 1 - \theta_t$$

- See appendix of [Auclert et al., 2020] for this model. See appendix of [Angeletos and Huo, 2021] for a related one.

(3) Dispersed information cont'd

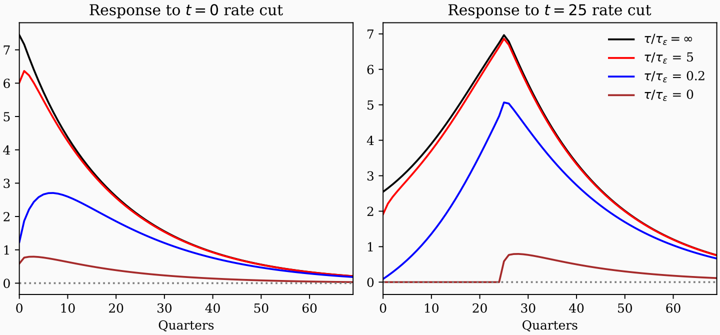
- Given θ_t this almost looks like sticky information / expectations!

$$\mathbf{E} = \begin{pmatrix} 1 & 1-\theta_0 & 1-\theta_0 & 1-\theta_0 & \cdots \\ 1 & 1 & 1-\theta_1 & 1-\theta_1 & \cdots \\ 1 & 1 & 1 & 1-\theta_2 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- In fact, for a given sequence of τ_j, can replicate sticky information / expectations
    - intuition: only **average expectation** matters to first order
    - **Heterogeneity** of who has what information does not matter!

(3) Dispersed info plot

- Plot similar to sticky expectations, but a bit less hump-shaped



(4) Cognitive discounting

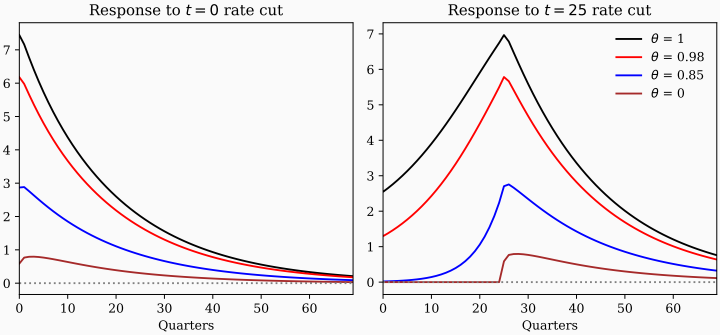
- [Gabaix, 2020] introduces **cognitive discounting**
- Main idea: agents respond to a shock that hits in h periods as if shock size was dampened by θ^h
- This is equivalent to assuming agents expect shock size θ^h of unit shock. Hence:

$$\mathbf{E} = \begin{pmatrix} 1 & \theta & \theta^2 & \theta^3 & \cdots \\ 1 & 1 & \theta & \theta^2 & \cdots \\ 1 & 1 & 1 & \theta & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

  Conceptually different from dispersed info / sticky info: Dampening relative to diagonal, not relative to first period!

(4) Cognitive discounting - plots

- Doesn't generate humps, but dampens forward guidance very strongly



(5) Level k thinking

- [Farhi and Werning, 2019] is the first paper combining HA + deviations from FIRE.
- They use **level k thinking:** (explained in context of our introductory economy)
  - $k = 1$: all agents believe output is at steady state
  - $k = 2$: all agents believe *all other* agents are have level $k = 1$
  - $k = 3$: al agents believe all other agents have level $k = 2$, ... etc

- Level $k = 1$ is easily handled. In fact, that was our intro example:

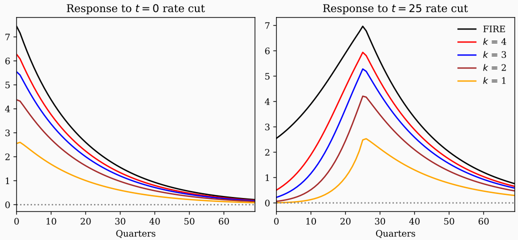
$$\mathbf{E} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ 1 & 1 & 0 & 0 & \cdots \\ 1 & 1 & 1 & 0 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \mathbf{M}^{(1)} = \begin{pmatrix} M_{00} & 0 & 0 & 0 & \cdots \\ M_{10} & M_{00} & 0 & 0 & \cdots \\ M_{20} & M_{10} & M_{00} & 0 & \cdots \\ M_{30} & M_{20} & M_{10} & M_{00} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where (1) indicates $k = 1$. IKC is then simply:

$$d\mathbf{Y}^{(1)} = \mathbf{M}^r dr + \mathbf{M}^{(1)} \cdot d\mathbf{Y}^{(1)}$$

- What about $k > 1$? Solve recursively:

$$d\mathbf{Y}^{(k+1)} = \underbrace{\mathbf{M}^r d\mathbf{r} + \mathbf{M} d\mathbf{Y}^{(k)}}_{\text{other agents are expected to behave according to level } k} + \underbrace{\mathbf{M}^{(1)} \cdot \left(d\mathbf{Y}^{(k+1)} - d\mathbf{Y}^{(k)}\right)}_{\text{...but everyone is unaware that economy may deviate from level } k}$$



Takeaway

Conclusion

- Information rigidities can be nested quite nicely in the sequence space
- This not just gives us a straightforward way of simulating them for RA models, but allows us to apply it to HA models equally well!

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