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Research on college students' physical exercise trend based on compartment model

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Abstract

As the backbone of social development, college students' level of physical exercise has always been the focus of research by experts and scholars. Most of the research methods are on the strength of literature data, questionnaire survey, mathematical statistics and comparative analysis. Based on the classification of college students and the influence and flow law of inter-class population, this paper establishes a differential equation system. By analyzing the existence and stability of the equilibrium of this system and the possible fold or backward bifurcations at the equilibrium, the quantitative analysis of college students' physical exercise trends on campus is carried out. This paper aims to improve the participation of college students in physical exercise by maximizing the number of students in the third categories. The results of theoretical proof, sensitivity analysis and numerical simulation show that in the initial stage, promoting peer-to-peer communication is the most effective measure. Secondly, when the effect of peer-to-peer interaction reaches saturation point, the way to improve physical education can achieve significant results. To fundamentally improve the enthusiasm of college students to participate in sports activities, we should start from the level of consciousness and enhance students' awareness of physical exercise from an early age.

Keywords: Compartment model; College students; Physical exercise; Fold bifurcation; Backward bifurcation; Sensitivity analysis

1. Introduction

Juvenile strength and youth strength are the most important steps to achieve China's strength. This not only refers to advanced technology, superior scientific research capabilities, strong psychological quality, but also includes a strong body and good sportsmanship that does not give up easily. However, the obesity rate and physical health of Chinese adults are not optimistic [26]. The group of college students has the characteristics of concentration and management, which is a breakthrough to effectively improve the physical quality of people. In fact, as a backbone of the development of a country with knowledge and skills, the body quality of college students has always been the focus of attention from all walks of life. The research shows that the body size, function and quality of college sports population are obviously better than non-sports population, and the mental health level of college sports population is also significantly better than non-sports population [27]. What are the factors that affect

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college students' physical exercise and how to improve their participation in physical exercise? The research on these issues is of great significance in improving the national physical quality, which is also what this paper wants to explore.

In the research on the physique of college students, how to increase the number of college students participating in physical exercise is a problem that needs to be solved now. Previous studies are based on the measurement of college students' physical fitness data, which is reflected in four aspects, respectively is the use of literature, questionnaire interviews, physical fitness tests, street surveys and other methods to study the physical fitness of college students [11,22,39], using correlation analysis and high-tech instruments to explore the influence factors of students' physique [10,24,33,35], improvement of college students' physical fitness scoring model [4,12,19,28] and the combination of machine learning algorithms to predict the results of college students' physical testing projects [7,25,30,38]. Although these studies show the physical state of college students in many aspects, they can also classify and predict the physique of college students, but they cannot reflect the dynamic process of college students' sports behavior.

In order to improve the daily physical exercise level of college students, scholars also study the behavior and motivation of daily physical exercise. Bhumika Sudha estimated the level of physical activity (PA) of North Indian professional college students by using the cross-sectional online survey [23]. Li X.X. studied the intrinsic relationship between the physical activities of college students in some colleges and universities in Changsha and the campus environment. They found that over 1/3 students did not take enough physical activity while the rest took not enough, medium or high intensity physical activity [13]. Pan W. applied the generalized estimation model to analyze the influence of fitness consciousness and fitness behavior on college students' physique. The study found that strengthening daily exercise awareness can effectively improve the physical fitness of college students [20]. However, researches on college students' sports activities are slightly insufficient. Keating XD conducted a meta-analysis on the research results of college students' sports activities, and concluded that there is a lack of multiple-level approaches for examining PA behaviors in the college student population [6].

Scholars are doing their best to analyze the physical problems of college students in China, comprehensive researches have put forward many constructive opinions on improving the physical fitness of college students. However, the real problem is always complicated. In order to discover the hidden laws of things, we need to keep experimenting to find out more realistic models. Therefore, different from traditional statistical methods, Yuan C.D. [36] applied population dynamics to human athletic activities, classifying the objects discussed in three categories. I class have no skills to participate in sports, but can develop into people who participate in the event, II class have the skills to participate in sports activities and are willing to participate in sports activities, III class have the ability to participate in sports activities but do not want to participate. Then a mathematical model is established for the competitive activities of populations fixed and non-fixed. The article made a qualitative analysis of the fixed population model, but only gives the ordinary differential equation model and the partial differential model of the non-fixed group competition activities, and there is no further qualitative analysis. On the basis of that model, Yang S.L. [32] improved the non-fixed population ordinary differential model and discussed its Hopf bifurcation. Later, Zeng Q.J. [37] based on the mathematical model established by Yang S.L., considering the time lag effect, proposed a time-lag group competitive sports activity model with time delay function. However, in the study of college students' daily physical exercise, it is too difficult to classify college students according to their skills and their willingness to participate in development activities. These articles only conduct qualitative research on models and the conclusions of the research have no practical significance, and no effective implementable measures are proposed.

In terms of the current situation of Chinese college students' physical exercise, Yang P.J. pointed out that the number of times students participate in exercise every week and the time of each exercise are low [34]. Huang W.R. combined the relevant theories of sociology and psychology, expounded how the peer intercourse finally brings out the promoting mechanism to the formation of the exercise behavior [16]. Wu Y.Q. believes, most of the college students are more active in sports awareness, but they show negative phenomena in physical exercise behavior. This may be related to the school's physical education curriculum and sports equipment. Moreover, students are more inclined to group projects such as ball games, which in particular require communication and action between people [29]. At the same time, Huang W. proposed that for the disadvantaged groups of physiology and physical defects, students should pay more attention to emotional communication and advocate mutual help among students [8]. All in all, college students do not participate in physical exercise, which is not caused by lack of time, but is related to the state of receiving media sports information and the stages of participating in physical exercise. Liu S.L. [14] demonstrated the role of physical education teachers in guiding students in the exercise process. He proposed that teachers' involvement in extracurricular independent physical exercise in the manner expected by students helps to improve students' physical fitness, special skill level and sports performance. Technical instructors, resource providers, atmosphere creators and test evaluators are the most anticipated methods for students. Yan S. [31] and others pointed out that the transactional leadership behavior of physical education teachers in colleges helps to enhance students' sense of self-efficacy in exercise. Students will actively overcome various obstacles, increase their confidence in continuous exercise, and make them interested in physical exercise, which ultimately encourages students to be willing to adhere to physical exercise. Studies found that college students participate in physical exercise mainly to relax the body and mind and reduce stress. The awareness of physical exercise mainly comes from regular classroom learning, followed by interaction between peers [15].

In summary, we found that according to different classification criteria such as the number of weekly exercises, each exercise time or the location of each exercise, the group of college students can be classified. Secondly, the willingness to exercise mainly comes from classroom teaching and peer-to-peer communication. In fact, some public policies of the government, such as holding sports events, can also promote the participation of college students in sports activities. The assumption of this article is that the government's public policy can be reflected in the university's policy at the university. The government indirectly affects college students. Yuan C.D. [36] used continuous time series model to study population dynamics of human competitive activities. Therefore, this paper classifies the group of college students by the frequency of exercise. Based on the law of the influence of physical exercise consciousness, the linear and standard influence rates are adopted respectively. Due to the long-term changes, the "enrollment rate" and "graduation rate" are added to the model. Compared with the traditional statistical methods, the dynamics method makes people better understand the global behavior from the flow pattern of college students. By combining with computer simulation and other methods, people can make a deeper and more comprehensive understanding of the changing trend of college students' physical exercise.

In Section 1, a model of the movement of sports population is established. In Sections 2 and 3, the existence, stability and possible bifurcation of the equilibrium of the model are discussed respectively. In the fourth part, the parameter sensitivity of threshold is discussed by calculating the sensitivity index, and the global sensitivity of the model is analyzed by applying the sensitivity index. In the fifth section, a summary and discussion are given.

2. Model establishment

The stages of change model [21] divides the process of human behavior into five stages of change: pre-contemplation, contemplation, preparation, action, maintenance. Among them, pre-contemplators tend to be defensive and avoid changing their thinking and behavior, they have no awareness of exercise, and at least do not want to exercise in 6 months. Contemplators recognize the importance of exercise, they are seriously thinking about changing their exercise behavior. Preparators have overcome the unknown fear and decide to take action, usually expressed as purchasing sportswear and gaining sports knowledge. Individuals in action have already changed their behavior, but these changes are unstable, and the persistence of change needs to be strengthened. Individuals in maintenance have formed a fixed behavioral pattern, which is less susceptible to interference and influence by external factors. Inspired by the stages of change model, we will divide the students into three compartments, $Little$, $Medium$ and $Completelyenough$, based on the exercise frequency. The basic demographic assumptions and epidemiological assumptions are as follows.

To formulate our model, we let $L(t)$ denote the number of students of an university who do exercise less than once a week at time t. Students in pre-contemplation, contemplation and preparation only change their consciousness, and it is difficult to distinguish them from external performances. At the same time, they have a common feature that there is no exercise behavior. In addition to the professional courses, Chinese colleges and universities will have a weekly physical education course, which guarantees the frequency of freshmen training at least once a week. However, since the physical education class is a compulsory behavior, it does not reflect whether the student's attitude is active or passive. Therefore, students who do not engage in exercise activities every week or only have one physical education class per week are classified as $Little$. In the process of participating in sports class, students' attitudes toward physical exercise will change with the increase of breadth and depth of sports knowledge. Recognizing the benefits of exercise or the acquisition of sports skills will promote students' awareness of starting exercise. In this paper, we temporarily consider the influence of physical education class on the positive guidance of

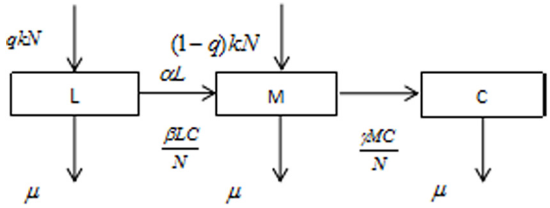


Fig. 1. Transfer diagram.

students. At the same time, in the daily interaction with classmates and friends, students who often do exercise will attract other students around them. So students in compartment $Little$ are affected by both classroom instruction and peer communication.

Let $M(t)$ denote the number of students who do exercise two or three times a week at time t. In addition to participating in the weekly physical education class, some students will also consciously and actively engage in other exercise activities. They recognize the importance of physical exercise in various channels and begin to have physical exercise behavior, but the frequency is low, and it is still at the stage of entertainment interest and unstable. Students in compartment $Medium$ is the potential group that forms the habit of sufficient physical exercise. $Medium$ corresponds to the fourth stage, action. Stable and regular physical exercise has not been formed at this stage. The influence on the surrounding companions is weak. Thus, we think the subjects belonging to $Medium$ cannot dominate the subjects of $Little$.

Let $C(t)$ denote the number of students who do exercise more than three times a week at time t. According to General Administration of Sport of China, exercising more than three times a week can be classified as a sports population. Students in compartment $Completelyenough$ have a strong sense of exercise and have good execution ability. They have developed sports behavior into a part of daily life, will not change easily and can form a greater appeal to the surrounding students.

The total population size at time t is denoted by $N(t)$, which is equal to $L(t) + M(t) + C(t)$. The meaning of parameters are explained as follows.

(1) Consider a university campus as the whole system, we are going to explore the change between the three compartments in years, and assume there is no population dynamics such as birth, death, and mobility of the population, but there will be freshmen and graduates every year, so the time unit is years. The enrollment rate and graduation rate are denoted by k and μ respectively.

(2) Before going to college, students have to take physical education class once a week in China, and this ensure that all freshmen exercise at least once a week. Some students will do some extra exercise during leisure time, assume the ratio is $1 - q$. In view of China's college entrance examination system, It can be considered that there are no or very few students who have more than three physical exercises per week. Therefore at time t, the number of students who enter the compartment $Little$ is $qkN(t)$ per unit time, the number of students who enter the compartment $Medium$ is $(1 - q)kN(t)$ per unit time.

(3) At time t, the number of students who exercised due to the influence of physical education per unit time is directly proportional to the number of students in the class, that is, the number of students removed from the compartment $Little$ in a unit of time is proportional to $L(t)$, with proportional coefficient α. Thus, the number of students enter compartment $Medium$ is $\alpha L(t)$ per unit time.

(4) Assume that the influence of peer exchange is bilinear and the flow of students between the three compartments is gradual. For students in compartment $Little$, β is the average number of adequate contacts per year with other individuals of a student of $Completelyenough$, since $L(t)/N(t)$ is the susceptible fraction of the population at time t, $\beta L(t)C(t)/N(t)$ is the average number of students of $Little$ influenced by all $Completelyenough$ per year, which implies that the contact rate β is proportional to the population size. For students in compartment $Medium$, γ is the average number of adequate contacts per year with other individuals of a student of $Completelyenough$, $\gamma M(t)C(t)/N(t)$ is the average number of students of $Medium$ influenced by all $Completelyenough$ per unit time.

Under the above four basic assumptions, we can get transfer diagram as Fig. 1.

The differential equations for the LMC model are:

$$\begin{cases} L' = qkN - \alpha L - \mu L - \frac{\beta LC}{N} \\ M' = (1-q)kN + \alpha L + \frac{\beta LC}{N} - \frac{\gamma MC}{N} - \mu M \\ C' = \frac{\gamma MC}{N} - \mu C \end{cases} \tag{1}$$

The parameters in this and other models in this paper are:
q is the proportion of freshmen without additional exercise before enrollment
k is the admission rate
μ is the graduation rate
α is the teaching influence rate
β is the influence rate of $Completelyenough$ on $Little$
γ is the influence rate of $Completelyenough$ on $Medium$

Let $x = L/N$, $y = M/N$, $z = C/N$ denote the fractions of the compartments $Little$, $Medium$ and $Completelyenough$ in the population, respectively. If the total population of the model is constant, model (1) can be transformed into the following equivalent system:

$$\begin{cases} x' = qk - \alpha x - \mu x - \beta xz \\ y' = (1-q)k + \alpha x + \beta xz - \gamma yz - \mu y \\ z' = \gamma yz - \mu z \end{cases} \tag{2}$$

If the total population of the model changes over time, due to $N' = kN - \mu N$, the input rate and output rate of the system are constant. When considering the standardized model, although the number of people changes, the proportion of each group does not change, so the change of the total population has no effect on the system.

Easy to get, the positively invariant set of system (2) is the domain $D = \{(x, y, z)|x > 0, y > 0, z \geq 0, x+y+z = 1\}$.

3. Existence of equilibrium

Theorem 1. *System (2) has a boundary equilibrium* $E_0 = (\frac{qk}{\alpha+\mu}, \frac{k}{\mu} - \frac{qk}{\alpha+\mu}, 0)$.

Proof. Obviously, there is always a boundary equilibrium for system (2).

Theorem 2. *Let* $\Delta = [(\alpha+\mu) + \beta(\frac{k}{\mu} - \frac{\mu}{\gamma})]^2 - 4\beta qk$, $R = (\frac{k}{\mu} - \frac{\mu}{\gamma})/(\frac{qk}{\alpha+\mu})$, *system (2) has*

(1) two positive equilibriums E_1, E_2 *if* $\Delta > 0$, $R < 1$ *and* $\frac{qk}{\alpha+\mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}]$,
(2) one positive equilibrium E_1 *if* $\Delta > 0$, $R > 1$,
(3) one positive equilibrium E_1 *if* $\Delta > 0$, $R = 1$, *and* $\frac{qk}{\alpha+\mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}]$,
(4) one positive equilibrium, that is, $E_1 = E_2$ *if* $\Delta = 0$ *and* $\frac{qk}{\alpha+\mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}]$,
(5) no positive equilibrium for other cases.

Proof. Solving the first two equations of system (2), we get

$$f(x) = a_2x^2 + a_1x + a_0, \tag{3}$$

where

$$a_2 = \beta > 0,$$

$$a_1 = -[(\alpha+\mu) + \beta(\frac{k}{\mu} - \frac{\mu}{\gamma})] < 0,$$

$$a_0 = qk > 0.$$

Let

$$\Delta = a_1^2 - 4a_2a_0 = [(\alpha+\mu) + \beta(\frac{k}{\mu} - \frac{\mu}{\gamma})]^2 - 4\beta qk.$$

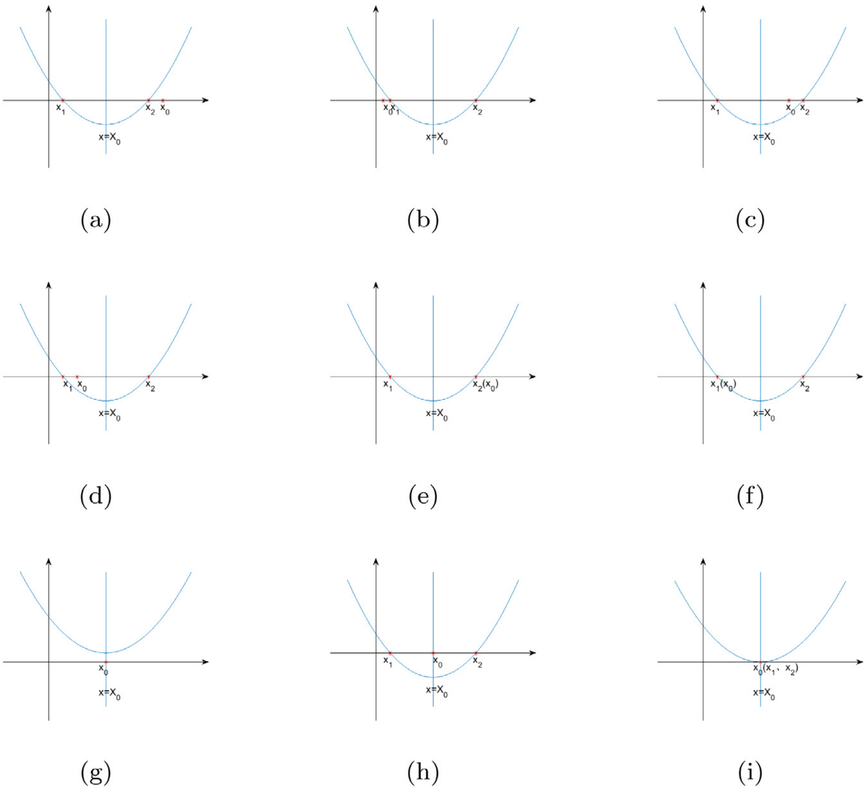


Fig. 2. Geometric image of Eq. (3).

(1) When $\Delta > 0$, Eq. (3) must have two different solutions, by the way, we have

$$f(0) = qk > 0,$$

and the symmetry axis

$$X_0 = -\frac{a_1}{2a_2} = \frac{(\alpha+\mu) + \beta(\frac{k}{\mu} - \frac{\mu}{\gamma})}{2\beta} = \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}] > 0.$$

So we can get two positive solutions x_1, x_2 in Eq. (3). Let $0 < x_1 < x_2$ and let $x_0 = \frac{qk}{\alpha+\mu}$, $g(x_0) = x_0 - (-\frac{a_1}{2a_2})$. It is necessary to judge the sign of $f(x_0)$ and the size of symmetry axis $x_0 = \frac{qk}{\alpha+\mu}$, to further study the value of the solutions; we get by calculating, $f(x_0) > 0$ is equivalent to $R < 1$, $g(x_0) > 0$ is equivalent to the straight line $x = x_0$ on the right side of the axis of symmetry, which is

$$\frac{qk}{\alpha+\mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}].$$

According to the geometric image of Eq. (3)(see Fig. 2), we can get the following results:

(a) if $f(x_0) > 0$, $g(x_0) > 0$, then $x_1, x_2 \in (0, x_0)$.
(b) if $f(x_0) > 0$, $g(x_0) < 0$, then $x_1, x_2 \in (x_0, +\infty)$.
(c) if $f(x_0) < 0$, $g(x_0) > 0$, then $x_1 \in (0, x_0)$, $x_2 \in (x_0, +\infty)$.
(d) if $f(x_0) < 0$, $g(x_0) < 0$, then $x_1 \in (0, x_0)$, $x_2 \in (x_0, +\infty)$.
(e) if $f(x_0) = 0$, $g(x_0) > 0$, then $x_1 \in (0, x_0)$, $x_2 = x_0$.
(f) if $f(x_0) = 0$, $g(x_0) < 0$, then $x_1 = x_0$, $x_2 \in (x_0, +\infty)$.
(g) if $f(x_0) > 0$, $g(x_0) = 0$, this conflicts with $\Delta > 0$.
(h) if $f(x_0) < 0$, $g(x_0) = 0$, then $x_1 \in (0, x_0)$, $x_2 \in (x_0, +\infty)$.
(i) if $f(x_0) = 0$, $g(x_0) = 0$, then $x_1 = x_2 = x_0$, where $\Delta = 0$ obviously.

(2) When $\Delta = 0$, Eq. (3) has two equal solutions. From Eq. (3), we get that $f(x_0) \geq 0$ and $x_1 = x_2 \in (0, x_0]$ if and only if $g(x_0) \geq 0$; Particularly, $x_1 = x_2 = x_0$, $f(x_0) = 0$ and $g(x_0) = 0$ are equivalent to each other. here

$$x_1 = \frac{-a_1 - \sqrt{\Delta}}{2a_2} = \frac{(\alpha + \mu) + \beta(\frac{k}{\mu} - \frac{\mu}{\gamma}) - \sqrt{\Delta}}{2\beta},$$

$$x_2 = \frac{-a_1 + \sqrt{\Delta}}{2a_2} = \frac{(\alpha + \mu) + \beta(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \sqrt{\Delta}}{2\beta}.$$

After all, we have to consider the practical meaning of the system. We can get the range of x is $(0, x_0)$ after solving the first formula of system (2) with $x_0 = \frac{qk}{\alpha+\mu}$, where k is the enrollment rate, μ is the graduation rate, which are equal in theory. Under the premise that both q and α are less than 1, x_0 is less than 1. Then the above theorem is proved.

Here

$$E_1 = (x_1, \frac{\mu}{\gamma}, \frac{k}{\mu} - \frac{\mu}{\gamma} - x_1),$$

$$E_2 = (x_2, \frac{\mu}{\gamma}, \frac{k}{\mu} - \frac{\mu}{\gamma} - x_2).$$

To understand the impact of classroom instruction on the dynamic behavior of the model (2) easily, we can rewrite Theorem 2 as follows:

Theorem 3. *Let $\alpha_1 = qk/(\frac{k}{\mu} - \frac{\mu}{\gamma}) - \mu$, $\alpha_2 = 2\sqrt{\beta qk} - \beta(\frac{k}{\mu} - \frac{\mu}{\gamma}) - \mu$, $\alpha_3 = \sqrt{\beta qk} - \mu$, and obviously there is $\alpha_1 > \alpha_2$. then the following statements are true. System (2) has*

(1) two positive equilibriums if $\alpha_2 < \alpha < \alpha_1$ with $\alpha_1 < \alpha_3$.
(2) one unique positive equilibrium if $\alpha > \alpha_1$.
(3) one unique positive equilibrium if $\alpha = \alpha_1$ with $\alpha_1 < \alpha_3$.
(4) one unique positive equilibrium if $\alpha = \alpha_2$ with $\alpha_2 < \alpha_3$.
(5) no positive equilibrium in other conditions.

Proof. Our work is mainly to prove that the conditions of Theorem 2 are equivalent to Theorem 3.

After calculation, we found

(i) $R < 1 \Leftrightarrow \alpha < \alpha_1$, which is $\frac{k}{\mu} - \frac{\mu}{\gamma} < \frac{qk}{\alpha+\mu}$,

$\Delta > 0 \Leftrightarrow \alpha > \alpha_2$, which is $\frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}] > \sqrt{\frac{qk}{\beta}} = \sqrt{\frac{qk}{\alpha+\mu} \cdot \frac{\alpha+\mu}{\beta}}$,

when $\alpha_2 < \alpha < \alpha_1$, we have

$$\frac{qk}{\alpha + \mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha + \mu}{\beta}] \Leftrightarrow \frac{qk}{\alpha + \mu} > \sqrt{\frac{qk}{\alpha + \mu} \cdot \frac{\alpha + \mu}{\beta}}$$
$$\Leftrightarrow \frac{qk}{\alpha + \mu} > \frac{\alpha + \mu}{\beta}$$

that is $\alpha < \alpha_3$. And there must be

$$\frac{\alpha + \mu}{\beta} < \frac{k}{\mu} - \frac{\mu}{\gamma} < \frac{qk}{\alpha + \mu},$$

thereby

$$\frac{\alpha + \mu}{\beta} < \sqrt{\frac{qk}{\beta}} < \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha + \mu}{\beta}] < \frac{k}{\mu} - \frac{\mu}{\gamma} < \frac{qk}{\alpha + \mu},$$

which is $\alpha_1 < \alpha_3$.

In fact, if

$$\frac{k}{\mu} - \frac{\mu}{\gamma} < \frac{\alpha + \mu}{\beta} < \frac{qk}{\alpha + \mu},$$

then

$$\frac{1}{2}[(\frac{k}{\mu}-\frac{\mu}{\gamma})+\frac{\alpha+\mu}{\beta}]<\frac{\alpha+\mu}{\beta}<\sqrt{\frac{qk}{\beta}},$$

This contradicts $\Delta>0$.

(ii) $R>1\Leftrightarrow\alpha>\alpha_1$. According to $\alpha_1>\alpha_2$ we know that there must be $\alpha>\alpha_2$, which is $\Delta>0$.

(iii) $R=1\Leftrightarrow\alpha=\alpha_1$. Same as above, there must be $\Delta>0$.

Also

$$\frac{qk}{\alpha+\mu}>\frac{1}{2}[(\frac{k}{\mu}-\frac{\mu}{\gamma})+\frac{\alpha+\mu}{\beta}]\Leftrightarrow\frac{qk}{\alpha+\mu}>\frac{1}{2}(\frac{qk}{\alpha+\mu}+\frac{\alpha+\mu}{\beta})\Leftrightarrow\frac{qk}{\alpha+\mu}>\frac{\alpha+\mu}{\beta},$$

which is $\alpha<\alpha_3$.

(iv) $\Delta=0\Leftrightarrow\alpha=\alpha_2$, which is $\frac{1}{2}[(\frac{k}{\mu}-\frac{\mu}{\gamma})+\frac{\alpha+\mu}{\beta}]=\sqrt{\frac{qk}{\beta}}$, at this time

$$\frac{qk}{\alpha+\mu}>\frac{1}{2}[(\frac{k}{\mu}-\frac{\mu}{\gamma})+\frac{\alpha+\mu}{\beta}]\Leftrightarrow\frac{qk}{\alpha+\mu}>\sqrt{\frac{qk}{\beta}}\Leftrightarrow\frac{qk}{\alpha+\mu}>\frac{\alpha+\mu}{\beta},$$

which is $\alpha<\alpha_3$.

Summarizing the above reasoning, we get

(i) $\Delta>0$, $R<1$, $\frac{qk}{\alpha+\mu}>\frac{1}{2}[(\frac{k}{\mu}-\frac{\mu}{\gamma})+\frac{\alpha+\mu}{\beta}]$ is equivalent to $\alpha_2<\alpha<\alpha_1$ with $\alpha_1<\alpha_3$.

(ii) $\Delta>0$, $R>1$ is equivalent to $\alpha>\alpha_1$.

(iii) $\Delta>0$, $R=1$, $\frac{qk}{\alpha+\mu}>\frac{1}{2}[(\frac{k}{\mu}-\frac{\mu}{\gamma})+\frac{\alpha+\mu}{\beta}]$ is equivalent to $\alpha=\alpha_1$ with $\alpha_1<\alpha_3$.

(iv) $\Delta=0$, $\frac{qk}{\alpha+\mu}>\frac{1}{2}[(\frac{k}{\mu}-\frac{\mu}{\gamma})+\frac{\alpha+\mu}{\beta}]$ is equivalent to $\alpha=\alpha_2$ with $\alpha_2<\alpha_3$.

As can be seen from Theorem 3, the teaching influence rate α has a great influence on the existence of the equilibrium of system (2). Thus, the change in the impact of classroom education on students will lead to changes in the balance of the entire system.

4. Stability analysis

4.1. The local stability of boundary equilibrium

Theorem 4. *If $R<1$, the Boundary equilibrium E_0 is locally asymptotically stable.*

Proof. At a point $E(x,y,z)$ of system (2), the Jacobi matrix is

$$J=\begin{pmatrix}-\alpha-\mu-\beta z & 0 & -\beta x\\ \alpha+\beta z & -\gamma z-\mu & \beta x-\gamma y\\ 0 & \gamma z & \gamma y-\mu\end{pmatrix}. \tag{4}$$

Thus at the boundary equilibrium E_0, we have

$$J_0=\begin{pmatrix}-\alpha-\mu & 0 & -\beta x_0\\ \alpha & -\mu & \beta x_0-\gamma y_0\\ 0 & 0 & \gamma y_0-\mu\end{pmatrix}. \tag{5}$$

Note that the matrix J_0 has three eigenvalues, of which $\lambda_1=-\mu$, $\lambda_2=-\alpha-\mu$, and $\lambda_3=\lambda y_0-\mu=\gamma(\frac{k}{\mu}-\dfrac{qk}{\alpha+\mu})-\mu$. It is obvious that the three eigenvalues are negative if $R<1$, which means $\lambda_3<0$. According to $Routh$–$Hurwitz$ criterion, we get that E_0 is locally asymptotically stable.

4.2. The local stability of positive equilibrium

Theorem 5. *The positive equilibrium E_1 is locally asymptotically stable, E_2 is unstable if they exist and are not equal to each other. Specially, system (2) undergoes a backward bifurcation when $E_1=E_2$.*

Proof. Assume $E^* = (x^*, y^*, z^*)$ is the positive equilibrium, and the Jacobi matrix at E^* is

$$J^* = \begin{pmatrix} -\alpha - \mu - \beta z^* & 0 & -\beta x^* \\ \alpha + \beta z^* & -\gamma z^* - \mu & \beta x^* - \mu \\ 0 & \gamma z^* & 0 \end{pmatrix}. \tag{6}$$

And the characteristic equation is as follows

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \tag{7}$$

where

$$\begin{aligned} b_1 &= \gamma z^* + \mu + \alpha + \mu + \beta z^*, \\ b_2 &= (\gamma z^* + \mu)(\alpha + \mu + \beta z^*) - (\beta x^* - \mu)\gamma z^*, \\ b_3 &= \mu\gamma z^*(\alpha + \mu + \beta z^* - \beta x^*). \end{aligned}$$

When $\Delta > 0$, system (2) has two different positive equilibriums E_1 and E_2, After calculating, we knew that $b_1 > 0, b_2 > 0, b_3 > 0, b_1 b_2 - b_3 > 0$ at x_1, but $b_3 < 0$ at x_2. So, according to the *Routh–Hurwitz* Criterion, the positive equilibrium E_1 is locally asymptotically stable and E_2 is unstable.

When $\Delta = 0$, that is $E_1 = E_2$, we get $b_3 = 0$ by calculating. Then the characteristic equation (7) is equivalent to

$$\lambda(\lambda^2 + b_1\lambda + b_2) = 0, \tag{8}$$

Obviously, Eq. (8) has three eigenvalues and one of them is zero, note as $\lambda_1 = 0$. Meanwhile, there is $b_1 > 0, b_2 > 0, \Delta = b_1^2 - 4b_2 > 0$, which means the other two eigenvalues of the characteristic equation exist and are both negative, and this is to say fold bifurcation condition is satisfied.

4.3. Fold bifurcation

Suppose the system

$$x' = f(x, \alpha), x \in R^1, \alpha \in R^1$$

with a smooth f has the equilibrium $x = 0$ at $\alpha = 0$ with $\lambda = f_x(0, 0) = 0$. Expand $f(x, \alpha)$ as a Taylor series with respect to x at $x = 0$:

$$f(x, \alpha) = f_0(\alpha) + f_1(\alpha)x + f_2(\alpha)x^2 + O(x^3)$$

If $f_0(0) = f(0, 0) = 0$ and $f_1(0) = f_x(0, 0) = 0$, fold bifurcation condition is satisfied [9].

Theorem 6. *If $R = 1$ and $\alpha \neq \alpha_3$ hold, then system (2) undergoes a fold bifurcation at the boundary equilibrium E_0.*

Proof. Let $x_1 = x - \frac{qk}{\alpha+\mu}, y_1 = y - (\frac{k}{\mu} - \frac{qk}{\alpha+\mu}), z_1 = z$. then system (2) is equivalent to the following system when $R = 1$:

$$\begin{cases} x_1' = -(\alpha + \mu)x_1 - \beta x_1 z_1 - \beta \frac{qk}{\alpha+\mu} z_1 \\ y_1' = \alpha x_1 + \beta x_1 z_1 + \beta \frac{qk}{\alpha+\mu} z_1 - \gamma y_1 z_1 - \mu z_1 - \mu y_1 \\ z_1' = \gamma y_1 z_1 \end{cases} \tag{9}$$

We can get the following equation by rewriting the system above

$$x' = Ax + \frac{1}{2}B(x, x) + O(\|x\|^3), x \in R^3$$

where

$$A = \begin{pmatrix} -(\alpha + \mu) & 0 & -\frac{\beta qk}{\alpha+\mu} \\ \alpha & -\mu & \frac{\beta qk}{\alpha+\mu} - \mu \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

$$B(x,x)=\begin{pmatrix}-\beta x_1x_3-\beta x_3x_1\\ \beta x_1x_3+\beta x_3x_1-\gamma x_2x_3-\gamma x_3x_2\\ \gamma x_2x_3+\gamma x_3x_2\end{pmatrix}. \tag{11}$$

$B(x,y)$ is multilinear function, which can be calculated as

$$B_i(x,y)=\sum_{j,k=1}^{n}\frac{\partial^2F_i(\xi)}{\partial\xi_j\partial\xi_k}|_{\xi=0}x_jy_k,\qquad i=1,2,\dots,n.$$

Then we get the feature vector, $q=(\frac{\beta qk}{(\alpha+\mu)^2-\beta qk}\quad 1\quad -\frac{(\alpha+\mu)^2}{(\alpha+\mu)^2-\beta qk})^T$, for zero eigenvalue by calculating $Aq=0$, where $(\alpha+\mu)^2-\beta qk\neq 0$, and that is $\alpha\neq\alpha_3$. Also, we can get the accompanying feature vector as $p=(0\quad 0\quad 1)^T$ by calculating $A^Tp=0$.

According to the Topological normal form for the fold bifurcation [9], the system's limiting equation can be obtained as follows

$$x'=\frac{1}{2}\sigma x^2+\frac{1}{6}(\delta-3\langle p,B(q,A^{INV}a)\rangle)x^3+O(x^4),$$

among them

$$\sigma=\langle p,B(q,q)\rangle,\delta=\langle p,C(q,q,q)\rangle,a=B(q,q)-\langle p,B(q,q)\rangle q.$$

After calculation, we get

$$\sigma=\langle p,B(q,q)\rangle=-2\gamma\frac{(\alpha+\mu)^2}{(\alpha+\mu)^2-\beta qk}.$$

And we can find that $\sigma\neq 0$ if $\alpha\neq\alpha_3$, yet we do not know σ is positive or negative, system above is locally topologically equivalent near the origin to the following normal forms

$$x'=\alpha\pm x^2.$$

So system (2) undergoes a fold bifurcation at E_0 if $R=1$ and $\alpha\neq\alpha_3$.

Theorem 7. *When $\Delta=0$, system (2) undergoes a fold bifurcation at the positive equilibrium E_1(or E_2).*

Proof. When $\Delta=0$, the positive equilibrium E_1(or E_2) can be noted as

$$E^*=(x^*,y^*,z^*)=(\frac{1}{2}(\frac{\alpha+\mu}{\beta}+\frac{k}{\mu}-\frac{\mu}{\gamma}),\frac{\mu}{\gamma},\frac{1}{2}(\frac{k}{\mu}-\frac{\mu}{\gamma}-\frac{\alpha+\mu}{\beta})).$$

Let $x_1^*=x-x^*,x_2^*=y-y^*,x_3^*=z-z^*$, then rewrite system (2) into the following form

$$x'=Ax+\frac{1}{2}B(x,x)+O(\|x\|^3),$$

among them

$$A=\begin{pmatrix}-(\alpha+\mu+\beta z^*) & 0 & -\beta x^*\\ \alpha+\beta z^* & -\gamma z^*-\mu & \beta x^*-\mu\\ 0 & \gamma z^* & 0\end{pmatrix}. \tag{12}$$

$$B(x,x)=\begin{pmatrix}-\beta x_1x_3-\beta x_3x_1\\ \beta x_1x_3+\beta x_3x_1-\gamma x_2x_3-\gamma x_3x_2\\ \gamma x_2x_3+\gamma x_3x_2\end{pmatrix}. \tag{13}$$

Similarly, the feature vector corresponding to the zero eigenvalue is $q=(1\quad 0\quad -1)^T$, the accompanying feature vector is $p=(1-\frac{\mu}{\beta x^*}\quad 1\quad 1+\frac{\mu}{\gamma z^*})^T$.

According to the Topological normal form for the fold bifurcation [9], the limiting equation of the system is $x'=\frac{1}{2}\sigma x^2+O(\|x\|^3)$, among them $\sigma=\langle p,B(q,q)\rangle=-\frac{2\mu}{x^*}\neq 0$, which is to say that the system is locally topologically equivalent near the origin to $x'=\alpha-x^2$. So system (2) undergoes a fold bifurcation at E_1(or E_2) when $\Delta=0$.

4.4. Backward branch

Let γ be the bifurcation parameter. Introducing $x = x_1, y = x_2, z = x_3$, the system (2) becomes

$$\begin{cases} x_1' = qk - \alpha x_1 - \mu x_1 - \beta x_1 x_3 \triangleq f_1(x,\gamma) \\ x_2' = (1-q)k + \alpha x_1 + \beta x_1 x_3 - \gamma x_2 x_3 - \mu x_2 \triangleq f_2(x,\gamma) \\ x_3' = \gamma x_2 x_3 - \mu x_3 \triangleq f_3(x,\gamma) \end{cases} \tag{14}$$

with $R = 1$ corresponding to $\gamma = \gamma^* = \frac{\mu}{\frac{k}{\mu} - \frac{qk}{\alpha+\mu}}$. The boundary equilibrium is $E_0 = (\frac{qk}{\alpha+\mu}, \frac{k}{\mu} - \frac{qk}{\alpha+\mu}, 0)$. So the Jacobi matrix of system (2) around the boundary equilibrium E_0 when $\gamma = \gamma^*$ is

$$J(E_0,\gamma^*) = \begin{pmatrix} -(\alpha+\mu) & 0 & -\frac{\beta qk}{\alpha+\mu} \\ \alpha & -\mu & \frac{\beta qk}{\alpha+\mu} - \mu \\ 0 & 0 & 0 \end{pmatrix}. \tag{15}$$

It is obvious that zero is a simple eigenvalue of $J(E_0,\gamma^*)$ and all other eigenvalues of $J(E_0,\gamma^*)$ are negative.

According to the calculation of Section 4.3, a right eigenvector associated with zero is $\omega = (\omega_1, \omega_2, \omega_3)^T = (\frac{\beta qk}{(\alpha+\mu)^2 - \beta qk} \quad 1 \quad -\frac{(\alpha+\mu)^2}{(\alpha+\mu)^2-\beta qk})^T$, and the left eigenvector is $\nu = (\nu_1, \nu_2, \nu_3)^T = (0 \quad 0 \quad 1)^T$. Algebraic calculations show that

$$\frac{\partial^2 f_1(E_0,\gamma^*)}{\partial x_1 \partial x_3} = \frac{\partial^2 f_1(E_0,\gamma^*)}{\partial x_3 \partial x_1} = -\beta, \frac{\partial^2 f_2(E_0,\gamma^*)}{\partial x_1 \partial x_3} = \frac{\partial^2 f_2(E_0,\gamma^*)}{\partial x_3 \partial x_1} = \beta,$$

$$\frac{\partial^2 f_2(E_0,\gamma^*)}{\partial x_2 \partial x_3} = \frac{\partial^2 f_2(E_0,\gamma^*)}{\partial x_3 \partial x_2} = -\gamma, \frac{\partial^2 f_3(E_0,\gamma^*)}{\partial x_2 \partial x_3} = \frac{\partial^2 f_3(E_0,\gamma^*)}{\partial x_3 \partial x_2} = \gamma,$$

$$\frac{\partial^2 f_2(E_0,\gamma^*)}{\partial x_3 \partial \gamma} = -\frac{k}{\mu} + \frac{qk}{\alpha+\mu}, \frac{\partial^2 f_3(E_0,\gamma^*)}{\partial x_3 \partial \gamma} = \frac{k}{\mu} - \frac{qk}{\alpha+\mu}.$$

The rest of the second derivatives appearing in the formula for a and b in following equations are all zero. Hence,

$$\begin{aligned} a &= \sum_{k,i,j=1}^{3} \nu_k \omega_i \omega_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(E_0,\gamma^*) = \nu_3 \omega_2 \omega_3 \frac{\partial^2 f_3}{\partial x_2 \partial x_3}(E_0,\gamma^*) + \nu_3 \omega_3 \omega_2 \frac{\partial^2 f_3}{\partial x_3 \partial x_2}(E_0,\gamma^*) \\ &= -\frac{2\gamma(\alpha+\mu)^2}{(\alpha+\mu)^2 - \beta qk}, \end{aligned}$$

$$\begin{aligned} b &= \sum_{k,i=1}^{3} \nu_k \omega_i \frac{\partial^2 f_k}{\partial x_i \partial \gamma}(E_0,\gamma^*) = \nu_3 \omega_3 \frac{\partial^2 f_3}{\partial x_3 \partial \gamma}(E_0,\gamma^*) \\ &= -\frac{(\alpha+\mu)^2}{(\alpha+\mu)^2 - \beta qk}(\frac{k}{\mu} - \frac{qk}{\alpha+\mu}). \end{aligned}$$

We can get $a > 0, b > 0$ easily when $(\alpha+\mu)^2 - \beta qk < 0$, which is $\alpha < \alpha_3$.

In summary, according to (i) and (ii) of Theorem 4.1 of Castillo-Chavez [3], we have the following results.

Theorem 8. *When $R = 1$, system (2) undergoes a backward bifurcation if $\alpha < \alpha_3$, and has no bifurcation if $\alpha > \alpha_3$.*

4.5. Global stability of equilibrium

Theorem 9. *System (2) has no periodic solution.*

Proof. Since system (2) has invariant set $\Omega = \{(x, y, z)|x > 0, y > 0, z \geq 0, x + y + z = 1\}$, it is easy to get that the boundary curve of Ω cannot be a periodic solution of system (2). Assuming $\varphi(t) = \{x(t), y(t), z(t)\}$ is a periodic solution of system (2), the plane region enclosed by trajectory of $\varphi(t)$ is Π which is inside of the domain

Ω. Let f_1, f_2, f_3 represent the three equations on the right of system (2), $\vec{f} = (f_1 \quad f_2 \quad f_3)^T$($T$ denotes transpose). Let $g(x, y, z) = (\frac{1}{xyz})\vec{r} \times \vec{f}$ where $\vec{r} = (x \quad y \quad z)^T$. Thus $\vec{g} \cdot \vec{f} = 0$.

Let

$$\vec{g} = (g_1, g_2, g_3) = \frac{1}{xyz}(yf_3 - zf_2, zf_1 - xf_3, xf_2 - yf_1),$$

$$curlg = (\frac{\partial g_3}{\partial y} - \frac{\partial g_2}{\partial z}, \frac{\partial g_1}{\partial z} - \frac{\partial g_3}{\partial x}, \frac{\partial g_2}{\partial x} - \frac{\partial g_1}{\partial y}),$$

We get inside Ω

$$curlg \cdot (1, 1, 1)^T = \frac{\partial g_3}{\partial y} - \frac{\partial g_2}{\partial z} + \frac{\partial g_1}{\partial z} - \frac{\partial g_3}{\partial x} + \frac{\partial g_2}{\partial x} - \frac{\partial g_1}{\partial y} < 0.$$

Assume the direction of the plane region Π upward, the direction of the trajectory of $\varphi(t)$ and Π conform to the right-hand rule. Due to vector $(1, 1, 1)$ is the normal vector of plane region Π, according to the general *Bendixson–Dulac* criterion [2], we can get that system (2) has no periodic solution.

Combining Theorems 1, 2, 6, 8, and 9, we get

Theorem 10. *The following conclusions are established for system (2)*

(i) The boundary equilibrium E_0 is globally asymptotically stable if $R < 1$.
(ii) The positive equilibrium E_1 is globally asymptotically stable, E_2 is unstable if $R < 1$, $\Delta > 0$ and $\frac{qk}{\alpha+\mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}]$.
(iii) The positive equilibrium E_1 is globally asymptotically stable if $R > 1$, $\Delta > 0$.
(iv) System (2) undergoes a fold bifurcation at E_0 if $R = 1$ and $\alpha \neq \alpha_3$.
(v) The positive equilibrium E_1 is globally asymptotically stable, if $R = 1$, $\Delta > 0$, and $\frac{qk}{\alpha+\mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}]$.
(vi) System (2) has a backward bifurcation if $R = 1$ and $\alpha < \alpha_3$.
(vii) System (2) undergoes a fold bifurcation at E_3 if $\Delta = 0$, $\frac{qk}{\alpha+\mu} > \frac{1}{2}[(\frac{k}{\mu} - \frac{\mu}{\gamma}) + \frac{\alpha+\mu}{\beta}]$.

5. Numerical simulation and sensitivity analysis

5.1. Numerical simulation

Several numerical examples are simulated with various values for the teaching influence rate, α, with fixed values for the proportion of freshmen without additional exercise before enrollment, q, the admission rate constant, k, the graduation rate constant, μ, the influence rate of *Completelyenough* on *Little*, β, the influence rate of *Completelyenough* on *Medium*, γ, and the initial values of three compartments.

(I) The fixed initial values are $x(0) = 0.6, y(0) = 0.3, z(0) = 0.1$, and the other parameters are $\beta = 0.9, \gamma = 0.5, q = 0.8, k = \mu = 0.25$.
After calculation, we get $\alpha_1 = 0.15, \alpha_2 \doteq 0.1485, \alpha_3 = 0.1743$.
(II) Change the initial value with fixed parameters $\beta = 0.9, \gamma = 0.5, q = 0.8, k = \mu = 0.25$, and at this time we have $R = 1$.

In Fig. 3, "+" in each figure indicates the number of students in compartment *Little*, "*" indicates the number of students in compartment *Medium*, "–" indicates the number of students in compartment *Completelyenough*. It can be seen that when the existence condition of the positive equilibrium is satisfied with α, the system tends to a non-zero equilibrium state, and when the conditions of the positive equilibrium are not satisfied, the system tends to a boundary equilibrium.

5.2. Local sensitivity analysis of R

In determining how best to increase the number of students in category *Completelyenough*, it is necessary to know the relative importance of the different factors responsible for its transmission. The existence of equilibrium is

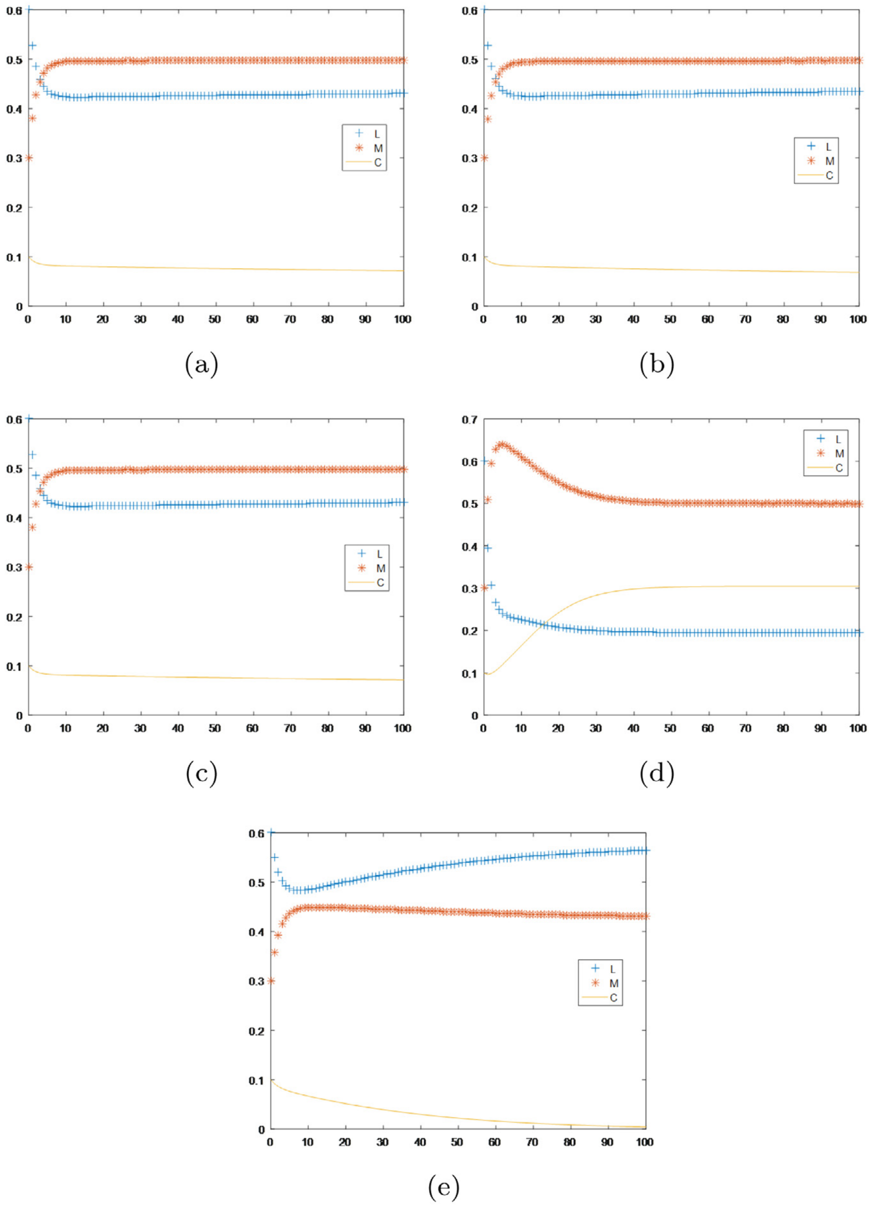


Fig. 3. The time series and orbits of system (2) with (a) $\alpha = 0.149$ which is $\alpha_2 < \alpha < \alpha_1 < \alpha_3$; (b) $\alpha = 0.1485$ which is $\alpha = \alpha_2 < \alpha_3$; (c) $\alpha = 0.15$ which is $\alpha = \alpha_1 < \alpha_3$; (d) $\alpha = 0.5$ which is $\alpha > \alpha_1$; (e) $\alpha = 0.1$ which is $\alpha < \alpha_2$.

directly related to threshold R, hence sensitivity and uncertainty analysis of the dependence of the threshold on the parameters of the model give a comparison of various intervention strategies. Here, we have computed normalized sensitive index for threshold R with respect to three parameters such as teaching influence rate α, proportion of freshmen without additional exercise before enrollment, q, the influence rate of $Completelyenough$ on $Medium$, γ. Given that the change of the number of enrollment and graduates in the university is very small, we assume the annual enrollment rate k and graduation rate μ of a university are equal, which means $k = \mu = 0.25$. According to the equation of the positive equilibrium, we have $\gamma > 0.25$.

The local sensitivity index [5] of R, that depends differentially on parameter μ is defined as

$$\frac{ER}{E\mu} = \frac{\partial R}{\partial \mu} \times \frac{\mu}{R}.$$

The sensitivity index of R with respect to γ, α with q is derived and it is given by

$$\frac{ER}{E\gamma} = \frac{\partial R}{\partial \gamma} \times \frac{\gamma}{R} = \frac{k\gamma}{k\gamma - \mu^2} - 1 = \frac{\gamma}{\gamma - 0.25} - 1,$$

$$\frac{ER}{E\alpha} = \frac{\partial R}{\partial \alpha} \times \frac{\alpha}{R} = \frac{\alpha}{\alpha + \mu} = \frac{\alpha}{\alpha + 0.25},$$

$$\frac{ER}{Eq} = \frac{\partial R}{\partial q} \times \frac{q}{R} = -1.$$

After further analysis, we can see that

$$\begin{cases} \frac{ER}{E\gamma} < 0 \qquad when \quad 0 < \gamma < 0.25 \\ \frac{ER}{E\gamma} > 1 \qquad when \quad 0.25 < \gamma < 0.5 \\ 0 < \frac{ER}{E\gamma} < 1 \qquad when \quad 0.5 < \gamma < 1 \\ \lim\limits_{\gamma \to 0.25} |\frac{ER}{E\gamma}| = +\infty \\ 0 < \frac{ER}{E\alpha} < 1 \qquad when \quad 0 < \alpha < 1 \\ |\frac{ER}{Eq}| = 1 \qquad when \quad 0 < q < 1 \end{cases} \tag{16}$$

The analysis shows that γ has a complicated effect on R. First of all, when $0 < \gamma < 0.25$, as the value of γ increases, R becomes smaller and negative. Based on assumptions, students of compartment $Completelyenough$ are mainly from $Medium$, in order to get a positive equilibrium, the number of students entering $Completelyenough$ should be greater than the number of students leaving in unit time at time t, that is, the influence rate of $Completelyenough$ on $Medium$, γ, should be greater than 0.25. And when γ approaching 0.25 from the left, $\frac{ER}{E\gamma}$ tends to be infinite, when γ approaching 0.25 from the right, $\frac{ER}{E\gamma}$ tends to negative infinity. Secondly, when $0.25 < \gamma < 0.5$, as γ increases, although $\frac{ER}{E\gamma}$ becomes smaller and smaller, its value is always greater than 1, at this point, γ has a very strong influence on R. Finally, when $0.5 < \gamma < 1$, as γ increases, $\frac{ER}{E\gamma}$ becomes smaller and smaller, and its value is less than 1. When $0 < \alpha < 1$, $\frac{ER}{E\alpha}$ increases with α and is less than 1. When $0 < q < 1$, the value of $\frac{ER}{Eq}$ is always -1.

According to Fig. 5, we can get the following conclusions: there would be no positive equilibrium if the influence rate of $Completelyenough$ on $Medium$, γ, is less than the graduation rate constant, μ. Noteworthy, increasing γ can significantly increase the value of R when γ is small, thereby changing the state of the equilibrium. As γ gets bigger l, such as more than $\frac{(1+\sqrt{2})}{4}$, improving the teaching influence rate α can have a greater effect.

5.3. Sensitivity analysis of the compartment population

In order to determine how sensitive the system of equations is to changes in parameter values, a global analysis was conducted using a Latin Hypercube Sampling (LHS) combined with Partial Rank Correlation Coefficient (PRCC) analysis. Similar to the Partial Correlation Coefficient, the PRCC value measures the correlation of the ranked ordering of the variable values rather than the variable values themselves, while discounting the effects of the other parameters on the output. PRCC measures the non-linear but monotonic relationship between two variables, which provides a measure of monotonicity between parameters [17]. LHS is a hierarchical Monte Carlo sampling method [17,18], which divides the range of each parameter into n equal intervals according to the obeyed distribution function of the parameters, and randomly samples one sample from each interval. The combination of LHS and PRCC makes this technique more efficient [1]. PRCC values range from -1 to 1, with an absolute value of PRCC close to 1 indicating the parameter has a strong impact on the model output and the sign indicating whether the correlation is positive or negative. The p-value of the PRCC indicates the significance of the value, and it is significant if the p-value is less than 0.05.

Due to the change of the enrollment rate constant k and the graduation rate constant μ that is small and difficult to be manipulated, we only consider the proportion of freshmen without additional exercise before enrollment, q, the teaching influence rate, α, the influence rate of $Completelyenough$ on $Little$, β, and the influence rate of $Completelyenough$ on $Medium$, γ. PRCC was calculated between each of the 4 input parameters and two output

Table 1
Parameters ranges and baseline values used in LHS/PRCC analysis.

Parameter	Baseline value	Variance	Distribution	Range
q	0.8	0.01	Normal	(0,1)
α	0.5	0.01	Normal	(0,1)
β	0.9	0.01	Normal	(0,1)
γ	0.5	0.01	Normal	(0,1)
Dummy	1	0.01	Normal	(1,10)

Table 2
Result of sensitivity analysis.

Rank	Little(L(t))			Completelyenough(C(t))		
	Parameter	PRCC	P-value	Parameter	PRCC	P-value
1	q	0.9377	0	γ	0.9917	0
2	α	−0.8962	0	q	−0.8842	9.9762e−306
3	γ	−0.8883	0	α	0.8278	2.3072e−224
4	β	−0.5407	2.9144e−83	β	0.3884	9.5195e−44

variables, $L(t)$ and $C(t)$. Set sample size $n = 1000$. Since there is no precedent for applying the compartments model to college students' physical exercise in China, there is very little relevant information. Therefore, our paper is mainly to theoretically explain the feasibility of this work. Due to lack of information about the underlying distributions of the parameters in our model, we assign a uniform distribution to every parameter. Since the parameters involved in the system in this paper have not been exactly estimated, we have taken some values for each parameter to facilitate the simulation. The distribution value range of each parameter along with baseline values is shown in Table 1.

The LHS/PRCC analysis results in Table 2 and Fig. 6 indicate that regardless of whether it is for the students of warehouse $Little$ or $Completelyenough$, the impact of each parameter on its final size is very significant ($p < 0.01$). The number of students in the two compartments is all sensitive to changes in the parameters q, α and γ. The PRCC results for the $Little$ show that q has the highest influence on the results, followed by α and γ, at the same time, γ has the highest influence on the results of $Completelyenough$, followed by α and q. Table 2 shows that the ranking of relative importance of parameter will be differently ordered among different output variables. For example, the effect of γ to $L(t)$ is relatively small, but highest on $C(t)$. And the sign of parameters is different with different output variables. Fig. 6 shows that increasing the parameter q will decrease the value of $C(t)$ while increasing the value of $L(t)$, and increasing the parameter α, β, γ will decrease the value of $L(t)$ while increasing the value of $C(t)$.

Figs. 7 and 8 show the scatter plots of the outcome variables, $L(t)$ and $C(t)$, against each parameter in order to verify that the monotonicity condition holds. The y-axis represents the output variable residuals, and the x-axis represents the residuals of each of the four input parameters. It can be seen from the figure that there is a very obvious linear monotonic relationship between the input parameters and the output results.

6. Discussion

Based on the dynamics principle of compartment model, this paper quantitatively analyzes how the trend of college students' participation in physical exercise changed. A dynamic system was established based on basic demographic and kinetic hypothesis. According to the definition of equilibrium of the ordinary differential equation system, when the boundary equilibrium of the system exists, the proportion of students in compartment $Little$ to the total number is $\frac{qk}{\alpha+\mu}$, the proportion of students in compartment $Medium$ to the total number is $\frac{k}{\mu} - \frac{qk}{\alpha+\mu}$, and there are no students in compartment $Completelyenough$. When the positive equilibrium of the system exists, all three compartments will exist. Therefore, in order to maximize the proportion of students in the compartment $Completelyenough$, the system needs to reach a positive balance.

If other conditions remain unchanged, when $\alpha = \alpha_2 \leq \alpha_3$, that is, teaching influence rate α, is equal to α_2 with $\alpha_2 \leq \alpha_3$, the number of students in the three compartments will tend to be in a non-zero equilibrium; when

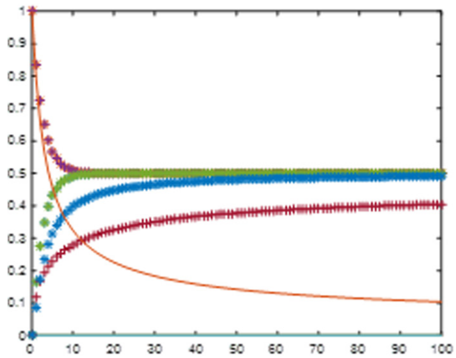


Fig. 4. Time series diagrams with different initial values.

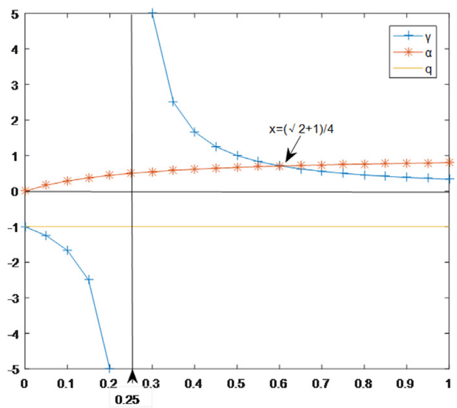


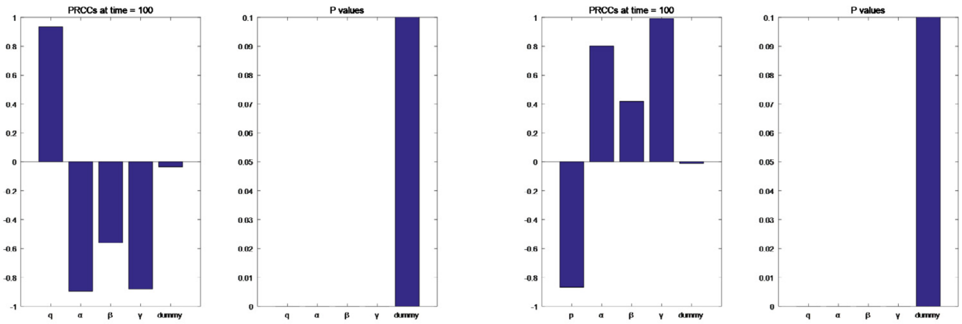
Fig. 5. Threshold $R's$ sensitivity analysis of parameters.

$\alpha_2 < \alpha < \alpha_1 < \alpha_3$, the number of students in various compartments will fluctuate in the early stage, but will eventually stabilize in a non-zero state; if $\alpha = \alpha_1 < \alpha_3$ or $\alpha > \alpha_1$, the number of students in various categories will also stabilize gradually. These states indicate that all three types of students in the college student group will exist.

Considering the continuous-time system dependent on parameters, when the single characteristic value of the system approaches zero with the change of parameters, the hyperbolic property of the system equilibrium will be destroyed.

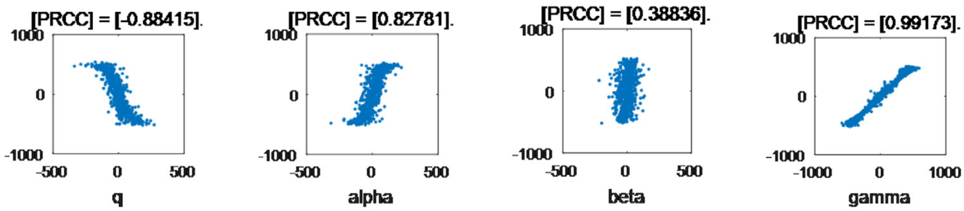
In the system, the fold branching or backward branching occurs at $R = 1$, indicating that the model may branch, and stable boundary equilibrium and positive equilibrium coexist at the time. This means that even if $\alpha_1 < \alpha_3$, but due to the occurrence of backward branching, the system may still have a stable boundary equilibrium, and the threshold will not be sufficient to describe the final stable state of the LMR system. We should pay attention to the change of the initial value. As shown in Fig. 4, the value of the parameter at this time is set as exactly $R = 1$ and $\alpha_1 < \alpha_3$, and when the given initial value is different, two equilibriums will appear in the model, one of which is the boundary equilibrium. That is, under different initial conditions, there may be all three types of students, or there may end up being only two types ($Little$ and $Medium$) of students.

Based on theoretical analysis, numerical simulation and sensitivity analysis, we found that the proportion of freshmen without additional exercise before enrollment, q, the teaching influence rate, α, the influence rate of $Completelyenough$ on $Little$, β, and the influence rate of $Completelyenough$ on $Medium$, γ, will all have an impact on the final stable state of the system, in which the strength of influence is q, α, γ in descending order. This paper puts forward the following suggestions in order to provide references for improving the physical exercise level of college students.



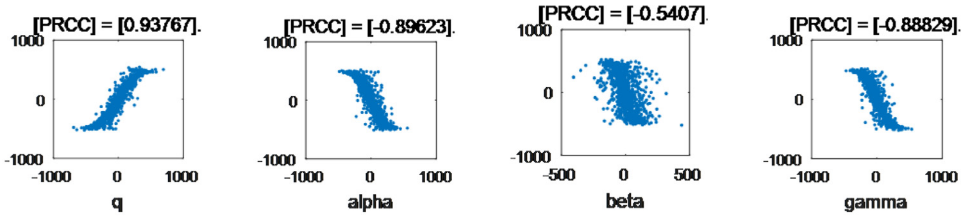
(a) (b)

Fig. 6. Graphical representation of LHS/PRCC sensitivity analysis results. (a) outcome: students in warehouse *Little*. (b) outcome: students in warehouse *Completelyenough*.



(a) (b) (c) (d)

Fig. 7. Partial rank scatter plots of the ranks for the $L(t)$ and each of the four input parameters from 1000 Monte Carlo simulations.



(a) (b) (c) (d)

Fig. 8. Partial rank scatter plots of the ranks for the $C(t)$ and each of the four input parameters from 1000 Monte Carlo simulations.

1. Tracing back to the source, developing awareness from a young age: In the process of growing up, although the schoolwork is heavy, the importance of physical exercise should not be neglected. Expanding the breadth and depth of sports propaganda in primary and secondary schools will help to fundamentally improve the awareness of physical exercise for college students;
2. Enhance the efficiency of class: The traditional sports classrooms that are uniform and boring have gradually separated from the times. Only those who are closer to life and more novel can stimulate the nerves of students, so that they can actively absorb physical exercise knowledge;
3. Promote sports exchanges: College students can reach out to students from all corners of the country. By holding sports events or community exchanges, activities can increase the probability of communication among students, and make peer interactions play a role in promoting students' physical exercise.

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