1. Hacer un script de python que reprodusca la tabla 2.1 pg 22 del libro de texto.
   Example 2.1
   Use a step size h to develop an approximate solution to the IVP

$$x'(t) = (1-2t)x(t), t > 0$$
$$x(0) = 1,$$

   over the interval $0 \leq t \leq 0{,}9$.
   We have purposely chosen a problem with a known exact solution

$$x(t) = exp(14 - (12 - t)^2),$$

2. Crear un script para Generar tabla 2.2 y figura 2.3 de la pg 25.
   Example 2.2
   Use Euler's method to solve the IVP

$$x'(t) = 2x(t)(1 - x(t)), t > 10,$$
$$x(10) = 15,$$

   for $10 \leq t \leq 11$ with $h = 0{,}2$.

3. Obtain the recurrence relation that enables x_{n+1} to be calculated from x_n when Euler's method is applied to the IVP $x'(t) = \lambda x(t)$, $x(0) = 1$ with $\lambda = -10$. In each of the cases $h = 1/6$ and $h = 1/12$

(a) calculate x_1 , x_2 and x_3 ,

(b) Plot the points (t_0, x_0), (t_1, x_1), (t_2, x_2), and (t_3, x_3) and compare with a sketch of the exact solution $x(t) = e^{\lambda t}$

   Comment on your results. What is the largest value of h that can be used when $\lambda = -10$ to ensure that $x_n > 0$ for all $n = 1, 2, 3, ...$?

4. This question concerns approximations to the IVP

$$x''(t) + 3x'(t) + 2x(t) = t^2, t > 0$$
$$x(0) = 1, x'(0) = 0$$

(a) Description write the above initial value problem as a first-order system and hence derive Euler's method for computing approximations to $x(t_{n+1})$ and $x'(t_{n+1})$ in terms of approximations to $x(t_n)$ and $x'(t_n)$.
   Si definimos una nueva variable como $y(t) = x'(t)$, entonces $y'(t) = x''(t)$. De esta manera obtenemos el siguiente sistema de primer orden con sus respectivas condiciones iniciales :

$$x'(t) = y(t), x(0) = 1$$
$$y'(t) = t^2 - 3y(t) - 2x(t) = t^2, y(0) = 0$$

Si consideramos las siguientes funciones

$$f(x(t_n), y(t_n), t_n) = y(t_n)$$
$$g(x(t_n), y(t_n), t_n) = (t_n)^2 - 3y(t_n) - 2x(t_n)$$

El metodo de Euler para cada ecuacion seria el siguiente:

$$x_{n+1} = x_n + hf(x(t_n), y(t_n), t_n)$$
$$y_{n-1} = y_n + hg(x(t_n), y(t_n), t_n)$$

(b) By eliminating y, show that the system

$$x'(t) = y(t) - 2x(t)$$
$$y'(t) = t^2 - y(t)$$

has the same solution x(t) as the IVP (2.18) provided that $x(0) = 1$, and that $y(0)$ is suitably chosen. What is the appropriate value of $y(0)$?

Para valores de $y(0)$ cercanos al cero se tiene buena aproximacion

(c) Apply Euler's method to the system in part (b) and give formulate for computing approximations to $x(t_{n+1})$ and $y(t_{n+1})$ in terms of approximations to $x(t_n)$ and $y(t_n)$. Si consideramos las siguientes funciones

$$f(x(t_n), y(t_n), t_n) = y(t_n) - 2x(t_n)$$
$$g(x(t_n), y(t_n), t_n) = (t_n)^2 - y(t_n)$$

El metodo de Euler para cada ecuacion seria el siguiente:

$$x_{n+1} = x_n + hf(x(t_n), y(t_n), t_n)$$
$$y_{n-1} = y_n + hg(x(t_n), y(t_n), t_n)$$

(d) Show that the approximations to $x(t_2)$ produced by the methods in (a) and (c) are identical provided both methods use the same value of h.

5. Prove that $e^x \geq 1 + x$ for all $x \geq 0$. [Hint: use the fact that $e^t \geq 1$ for all $t \geq 0$ and integrate both sides over the interval $0 \leq t \leq x$ (where $x \geq 0$.)]

Utilizando la desigualdad $e^t \geq 1$, se puede observar que las funciones de ambos lados son continuas en el intervalo $0 \leq t \leq x$. Por lo tanto la desigualdad es integrable en este mismo intervalo es decir,

$$\int_0^x e^t\, dx \geq \int_0^x 1\, dx \Longrightarrow e^x - 1 \geq x$$

Por lo tanto $e^x \geq x + 1$, Para toda $x \geq 0$