HW3. $P_j(x) = u_{j-1}\, a_{-1}(x) + u_j\, a_0(x) + u_{j+1}\, a_1(x)$

$$a_{-1}(x) = \frac{(x - x_j)(x - x_{j+1})}{2h^2} \qquad a_1(x) = \frac{(x - x_{j-1})(x - x_j)}{2h^2}$$

$$a_0(x) = -\frac{(x - x_{j-1})(x - x_{j+1})}{h^2}$$

$$P_j'(x) = u_{j-1}\, a_{-1}'(x) + u_j\, a_0'(x) + u_{j+1}\, a_1'(x)$$

$$= u_{j-1}\left[\frac{2x - (x_{j+1} + x_j)}{2h^2}\right] + u_j\left[\frac{x_{j-1} + x_{j+1} - 2x}{h^2}\right]$$

$$+ u_{j+1}\left[\frac{2x - (x_{j-1} + x_j)}{2h^2}\right]$$

Evaluando en $x = x_j$

$$P_j'(x_j) = u_{j-1}\left[\frac{x_j - x_{j+1}}{2h^2}\right] + u_j\left[\frac{(x_{j-1} - x_j) + (x_{j+1} - x_j)}{h^2}\right]$$

$$+ u_{j+1}\left[\frac{x_j - x_{j-1}}{2h^2}\right]$$

$x_j - x_{j+1} = -h$; $x_{j-1} - x_j = -h$; $x_j - x_{j-1} = h$; $x_{j+1} - x_j = h$

$$\Rightarrow P_j'(x_j) = u_{j-1}\left[\frac{-h}{2h^2}\right] + u_j\left[\frac{-h + h}{h^2}\right] + u_{j+1}\left[\frac{h}{2h^2}\right]$$

$$\Rightarrow P_j'(x_j) = \frac{u_{j+1} - u_{j-1}}{2h}$$

Si: $P_j'(x_j) = w_j \Rightarrow w_j = \frac{u_{j+1} - u_{j-1}}{2h}$