

# universität freiburg

## eSLIM: Using Exact Synthesis for Circuit Minimization

Franz-Xaver Reichl University of Freiburg

Friedrich Slivovsky University of Liverpool

Stefan Szeider TU Wien

### IWLS 2025 Programming Contest

Supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 508508079

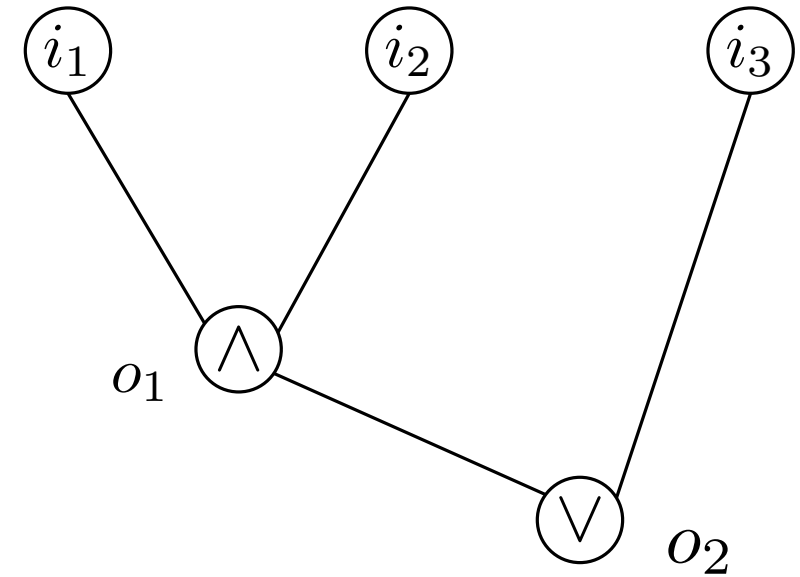
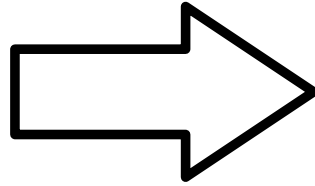


# Exact Synthesis

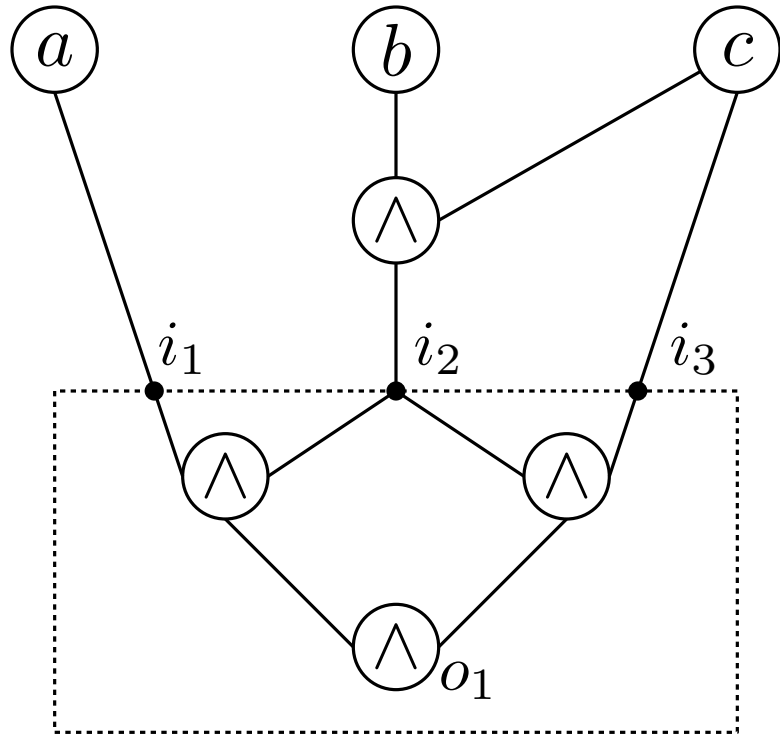
$i_1$	$i_2$	$i_3$	$o_1$	$o_2$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

# Exact Synthesis

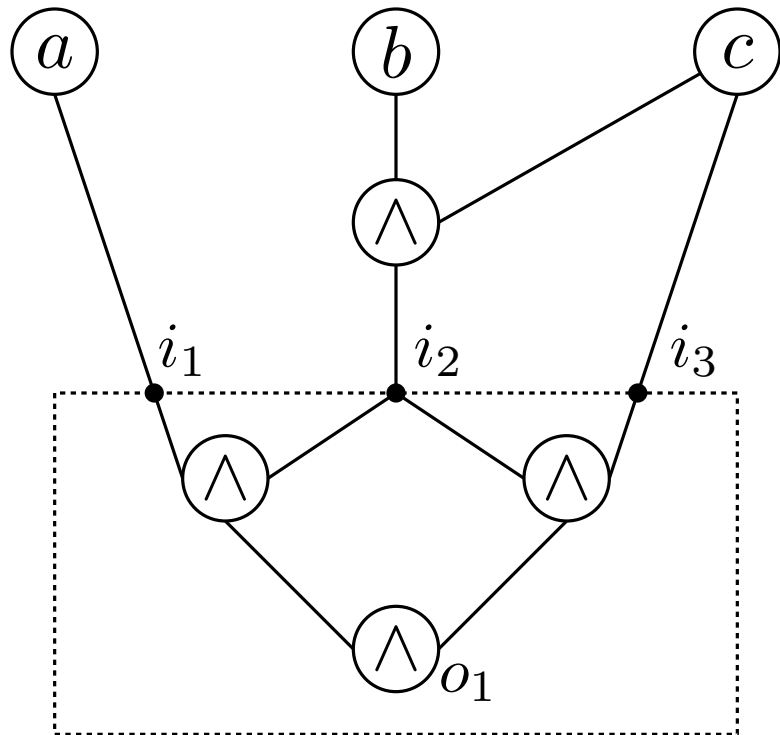
$i_1$	$i_2$	$i_3$	$o_1$	$o_2$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1



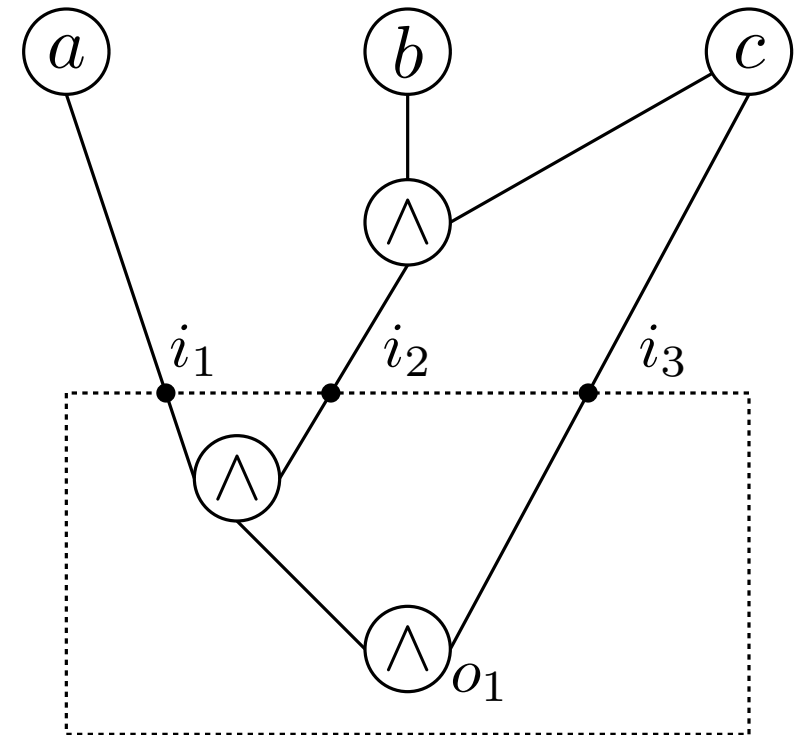
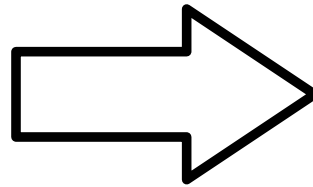
# Exact Synthesis of Subcircuits



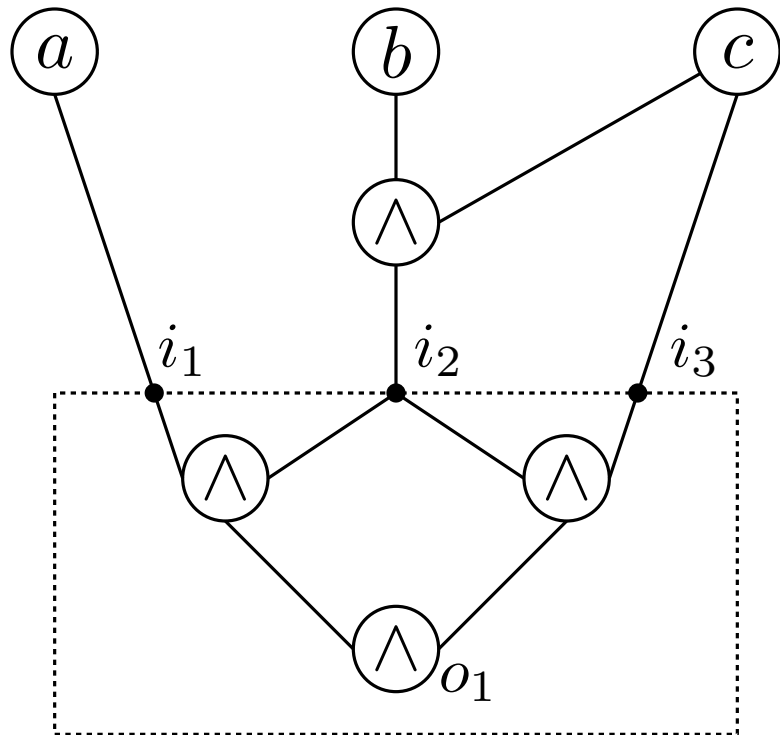
# Exact Synthesis of Subcircuits



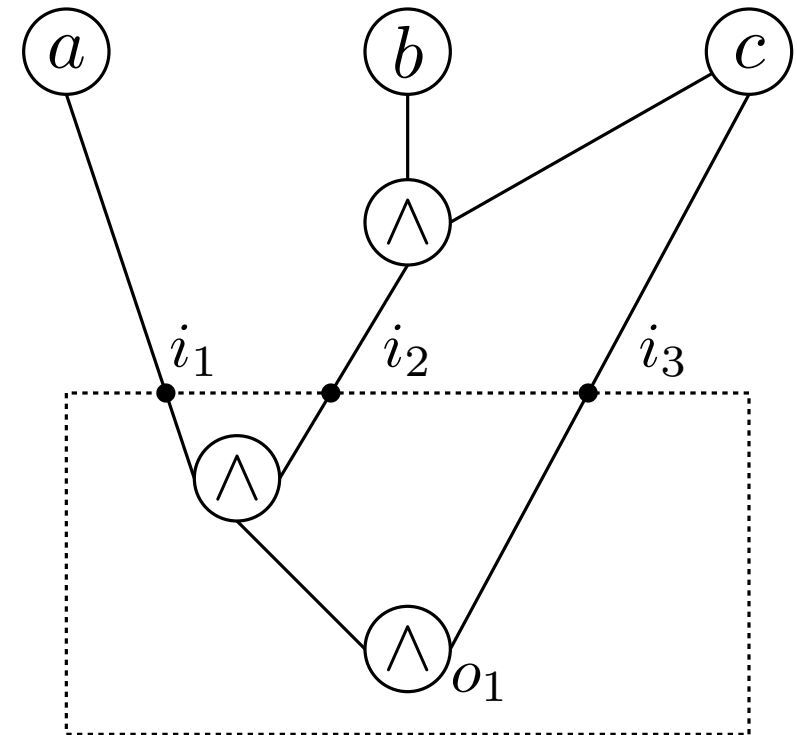
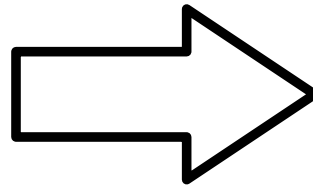
Locally optimal



# Exact Synthesis of Subcircuits

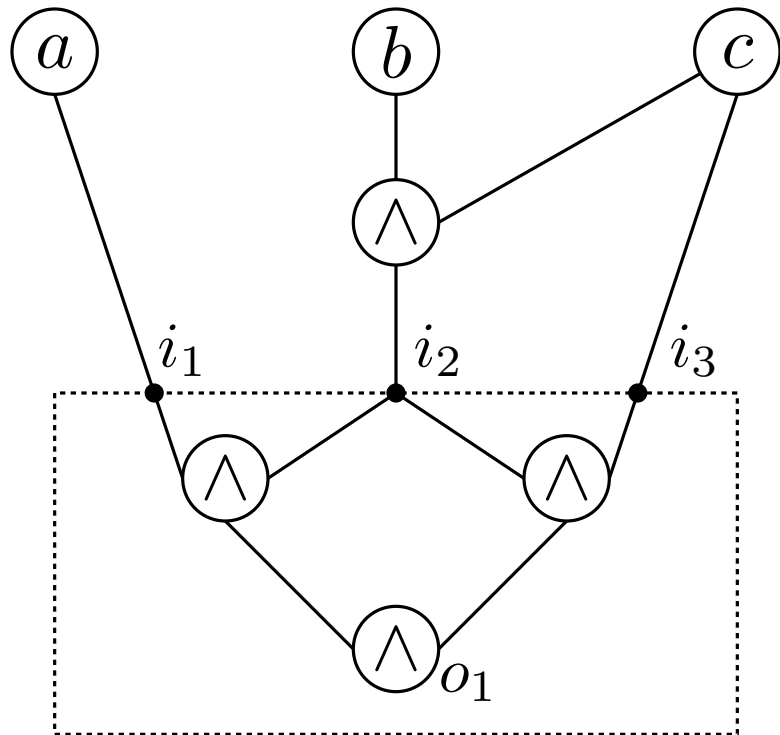


Locally optimal

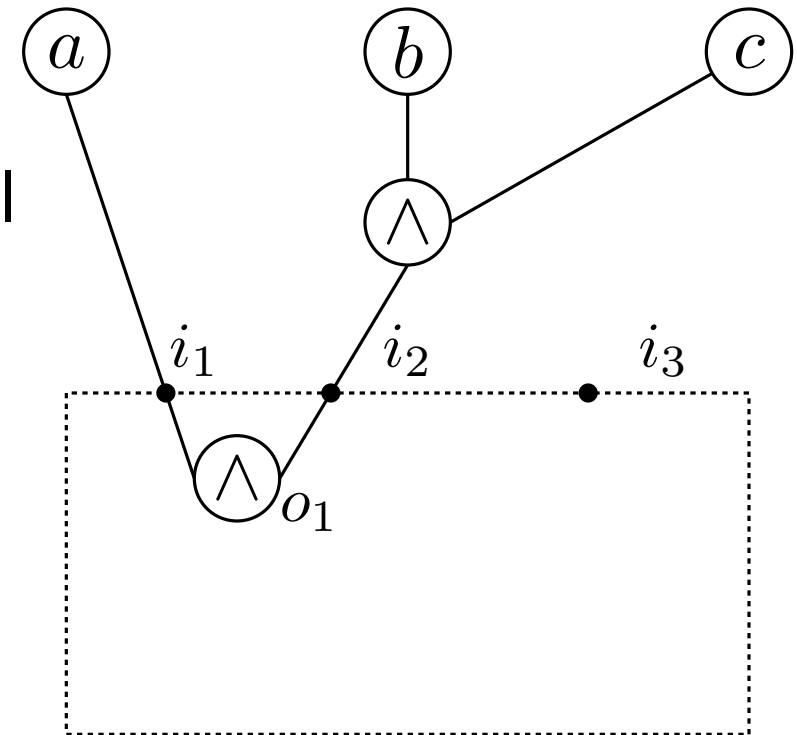
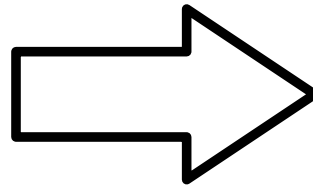


Not limited to equivalent replacements  
→ Make use of don't cares

# Exact Synthesis of Subcircuits

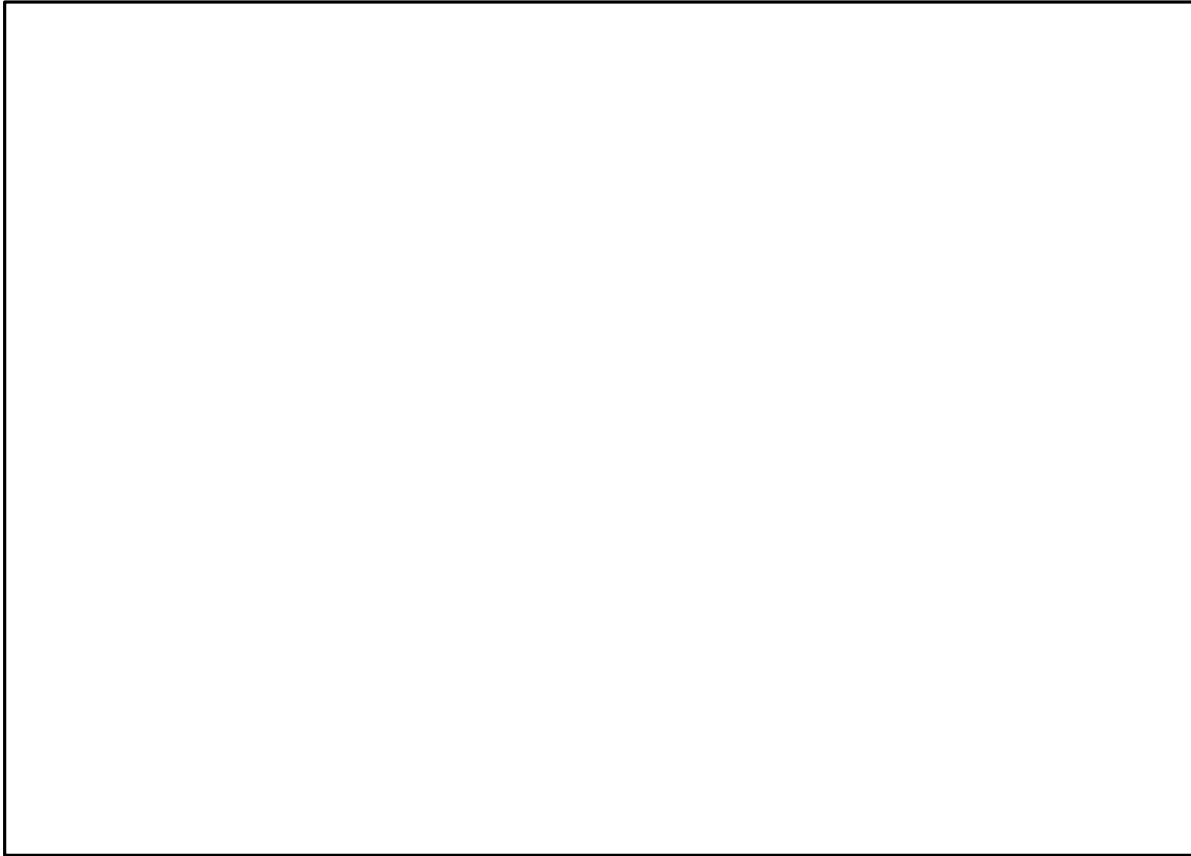


Globally optimal



Not limited to equivalent replacements  
→ Make use of don't cares

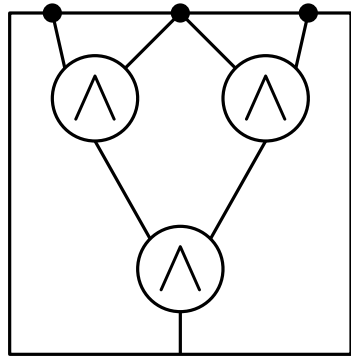
# Workflow



1. Initial circuit

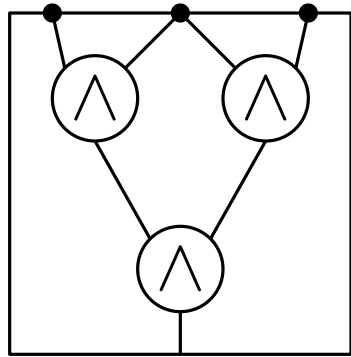


# Workflow

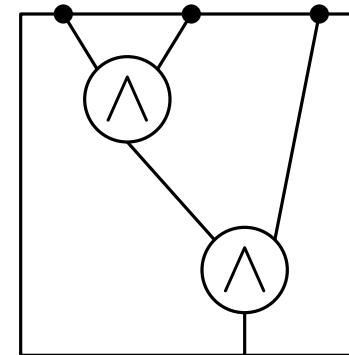


1. Initial circuit
2. Select subcircuit

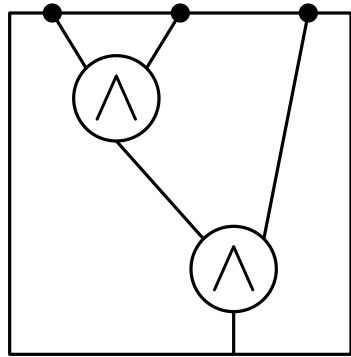
# Workflow



1. Initial circuit
2. Select subcircuit
3. Synthesise replacement

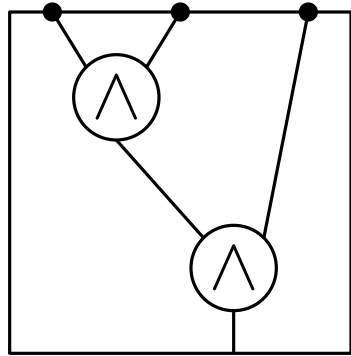


# Workflow



1. Initial circuit
2. Select subcircuit
3. Synthesise replacement
4. Insert Replacement

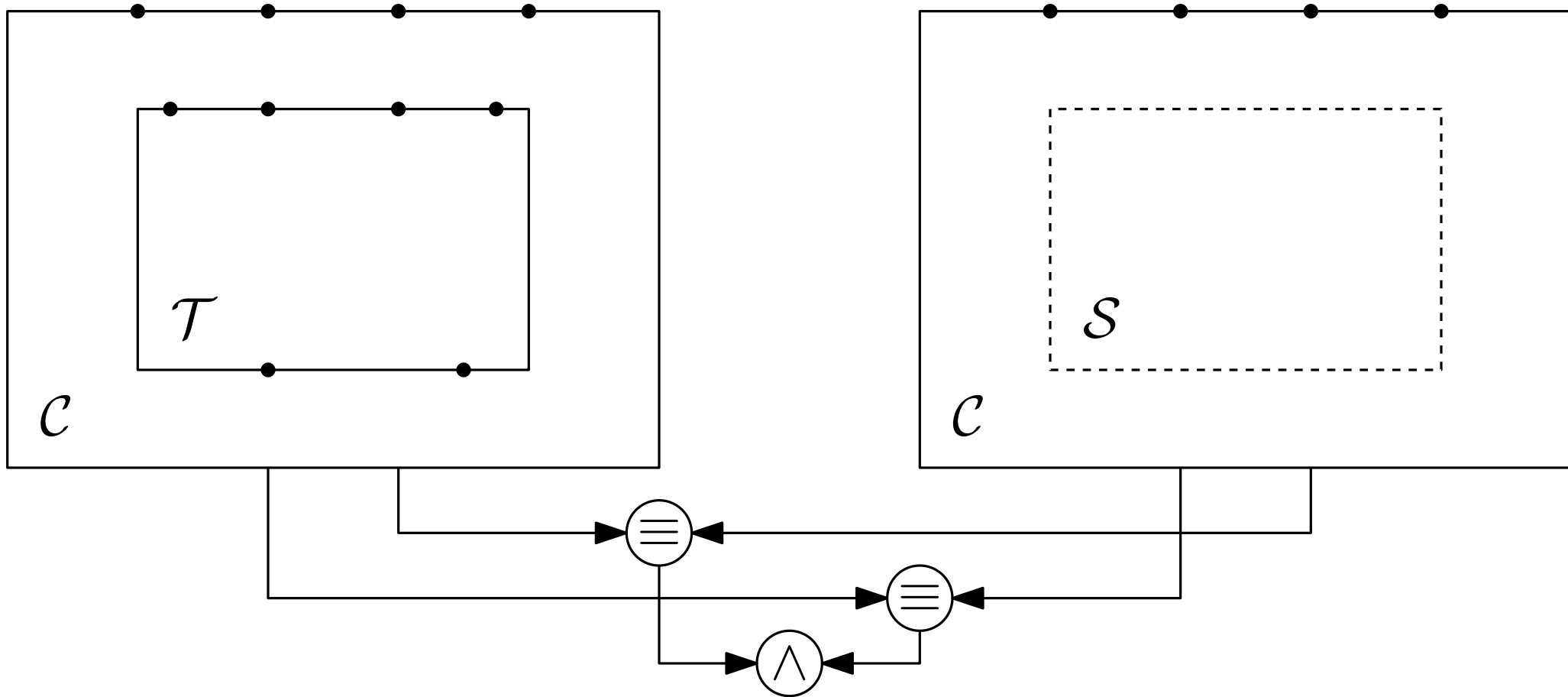
# Workflow



1. Initial circuit
2. Select subcircuit
3. Synthesise replacement
4. Insert Replacement
5. Repeat

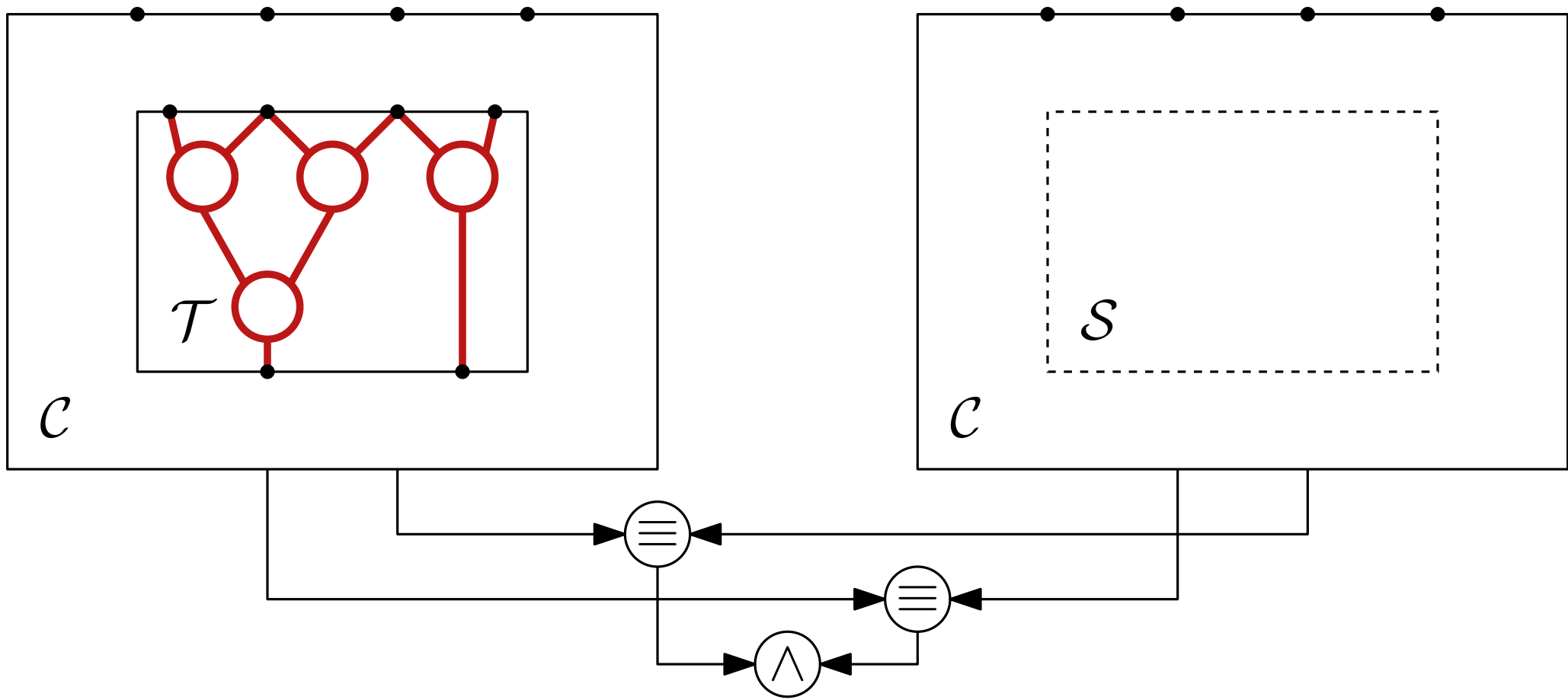
# QBF-based Synthesis

$$\exists \mathcal{T} \forall I \exists G. \mathcal{C}[\mathcal{S} \leftarrow \mathcal{T}] \equiv \mathcal{C}$$



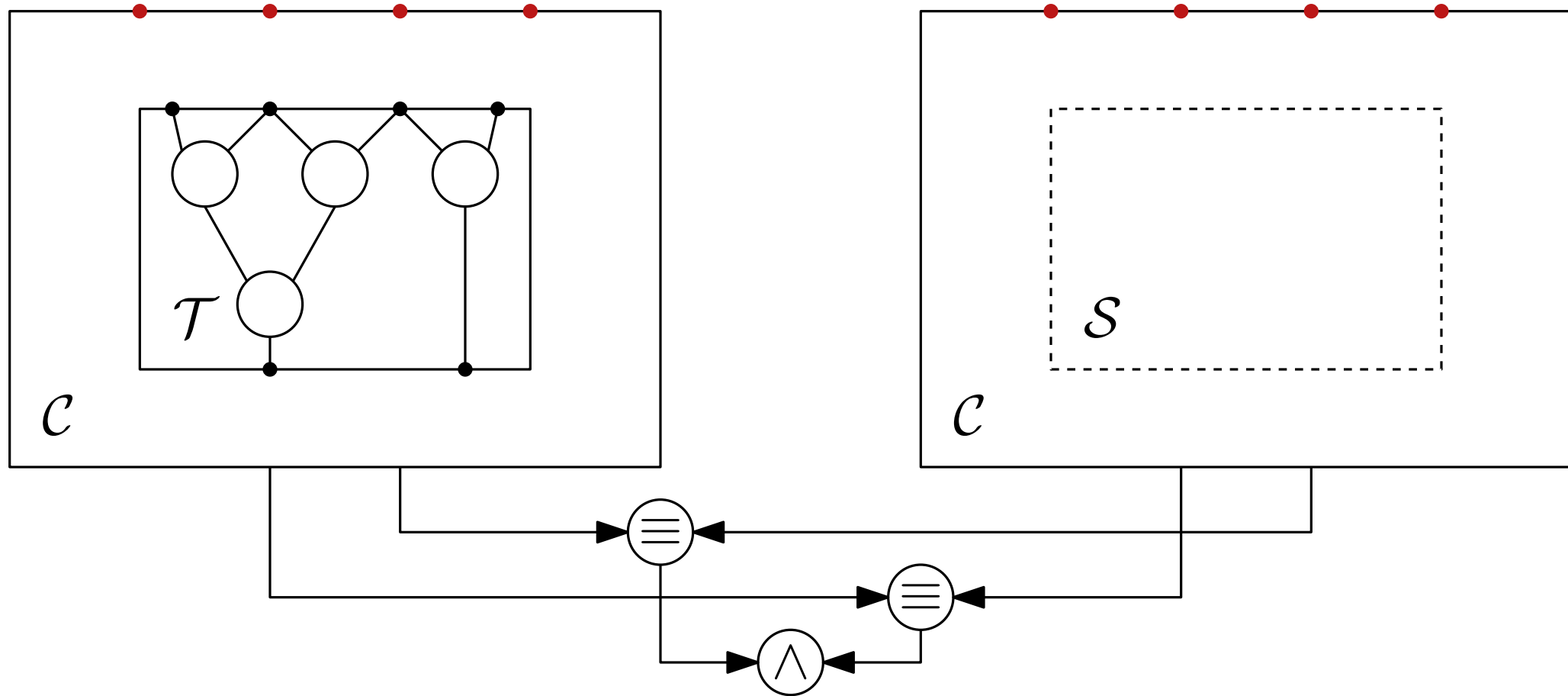
# QBF-based Synthesis

$$\exists \mathcal{T} \forall I \exists G. \mathcal{C}[S \leftarrow \mathcal{T}] \equiv \mathcal{C}$$



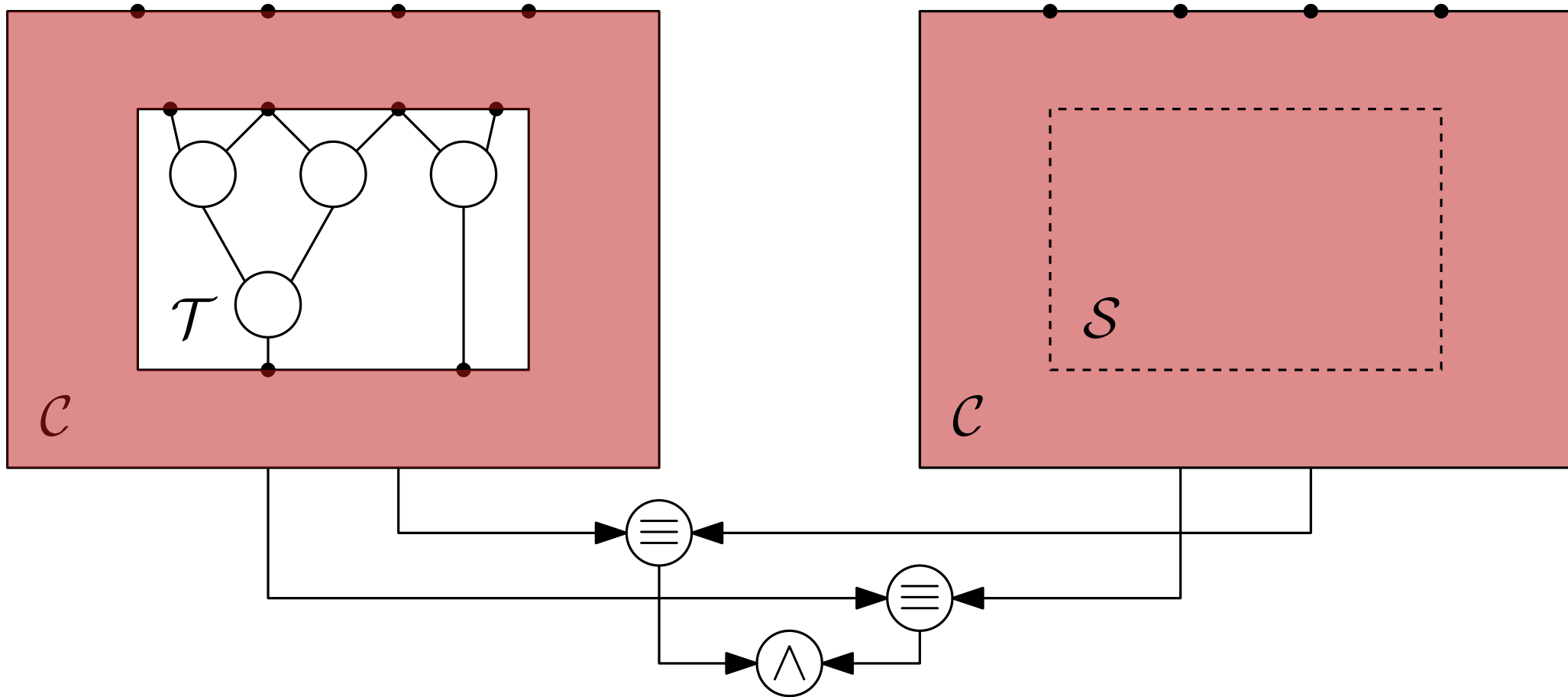
# QBF-based Synthesis

$$\exists \mathcal{T} \forall I \exists G. \mathcal{C}[S \leftarrow \mathcal{T}] \equiv \mathcal{C}$$



# QBF-based Synthesis

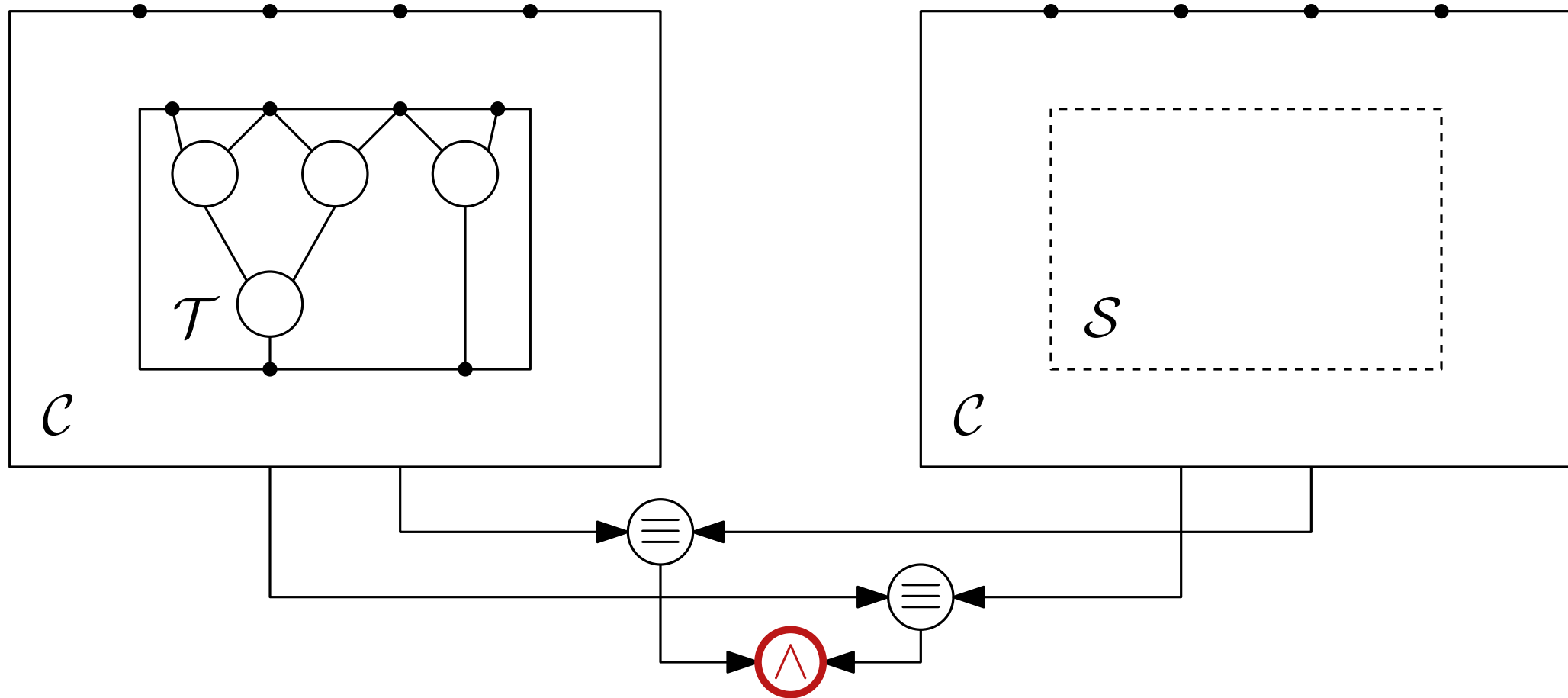
$$\exists \mathcal{T} \forall I \exists G. \mathcal{C}[S \leftarrow \mathcal{T}] \equiv \mathcal{C}$$





# QBF-based Synthesis

$$\exists \mathcal{T} \forall I \exists G. \mathcal{C}[\mathcal{S} \leftarrow \mathcal{T}] \equiv \mathcal{C}$$



# Boolean Relations

- Assign inputs to permitted outputs

# Boolean Relations

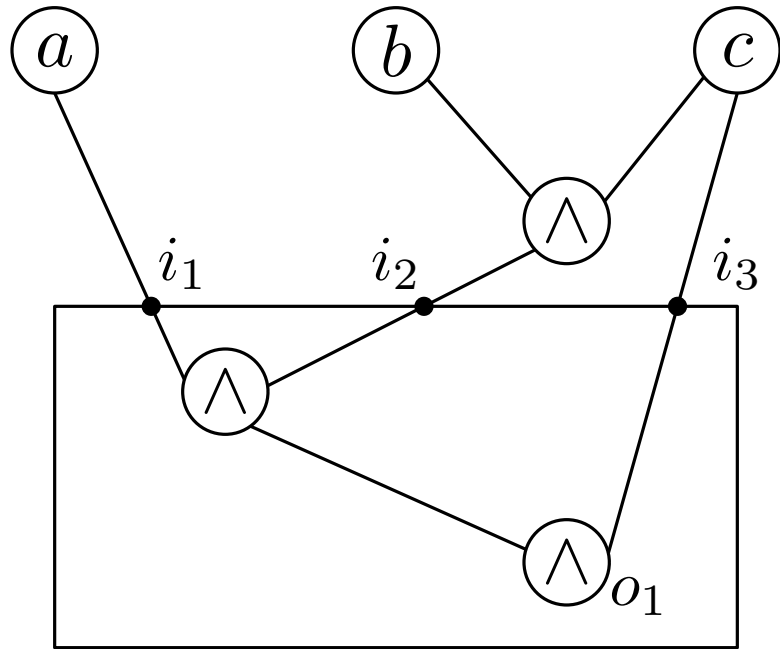
- Assign inputs to permitted outputs
- $R : \{0, 1\}^{\text{in}(\mathcal{S})} \rightarrow (\mathcal{P}(\{0, 1\}^{\text{out}(\mathcal{S})}) \setminus \emptyset)$

# Boolean Relations

- Assign inputs to permitted outputs
- $R : \{0, 1\}^{\text{in}(\mathcal{S})} \rightarrow (\mathcal{P}(\{0, 1\}^{\text{out}(\mathcal{S})}) \setminus \{\emptyset\})$
- Allow to represent don't cares

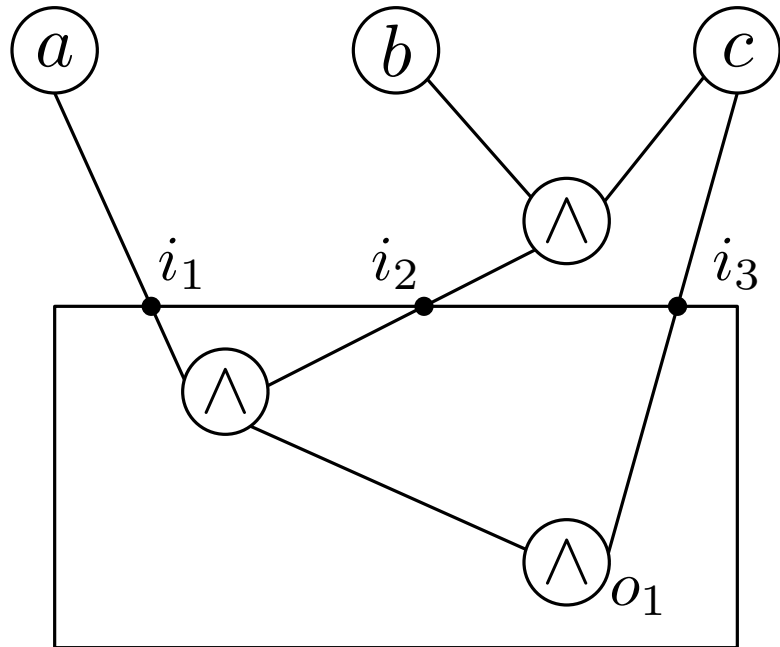
# Boolean Relations

- Assign inputs to permitted outputs
- $R : \{0, 1\}^{\text{in}(\mathcal{S})} \rightarrow (\mathcal{P}(\{0, 1\}^{\text{out}(\mathcal{S})}) \setminus \{\emptyset\})$
- Allow to represent don't cares



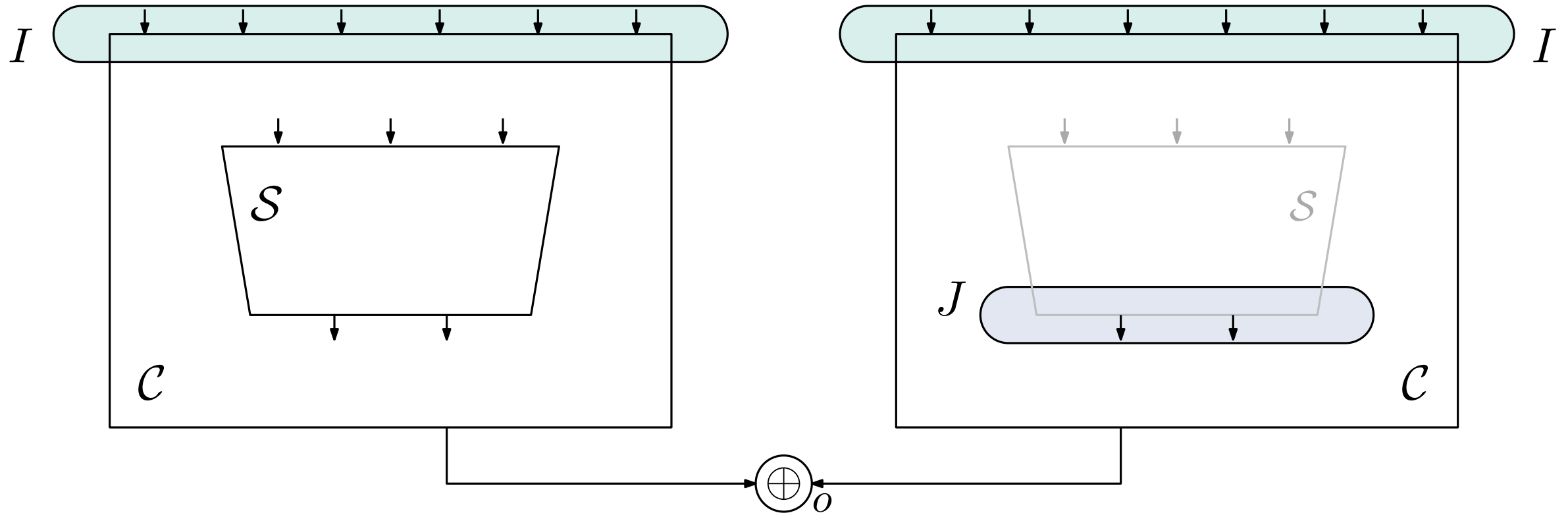
# Boolean Relations

- Assign inputs to permitted outputs
- $R : \{0, 1\}^{\text{in}(\mathcal{S})} \rightarrow (\mathcal{P}(\{0, 1\}^{\text{out}(\mathcal{S})}) \setminus \{\emptyset\})$
- Allow to represent don't cares



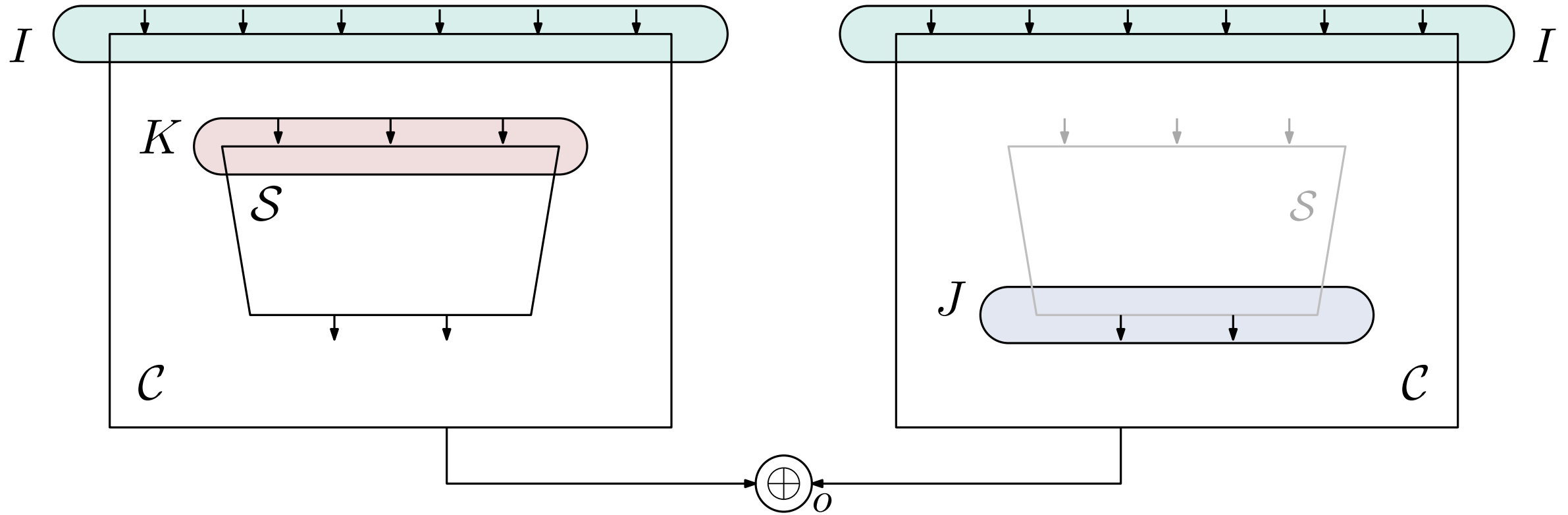
$i_1$	$i_2$	$i_3$	
0	0	0	$\{\neg o_1\}$
0	0	1	$\{\neg o_1\}$
0	1	0	$\{o_1, \neg o_1\}$
0	1	1	$\{\neg o_1\}$
1	0	0	$\{\neg o_1\}$
1	0	1	$\{\neg o_1\}$
1	1	0	$\{o_1, \neg o_1\}$
1	1	1	$\{o_1\}$

# Computing Boolean Relations



Find assignments for  $I$  and  $J$  such that  $o$  is true

# Computing Boolean Relations

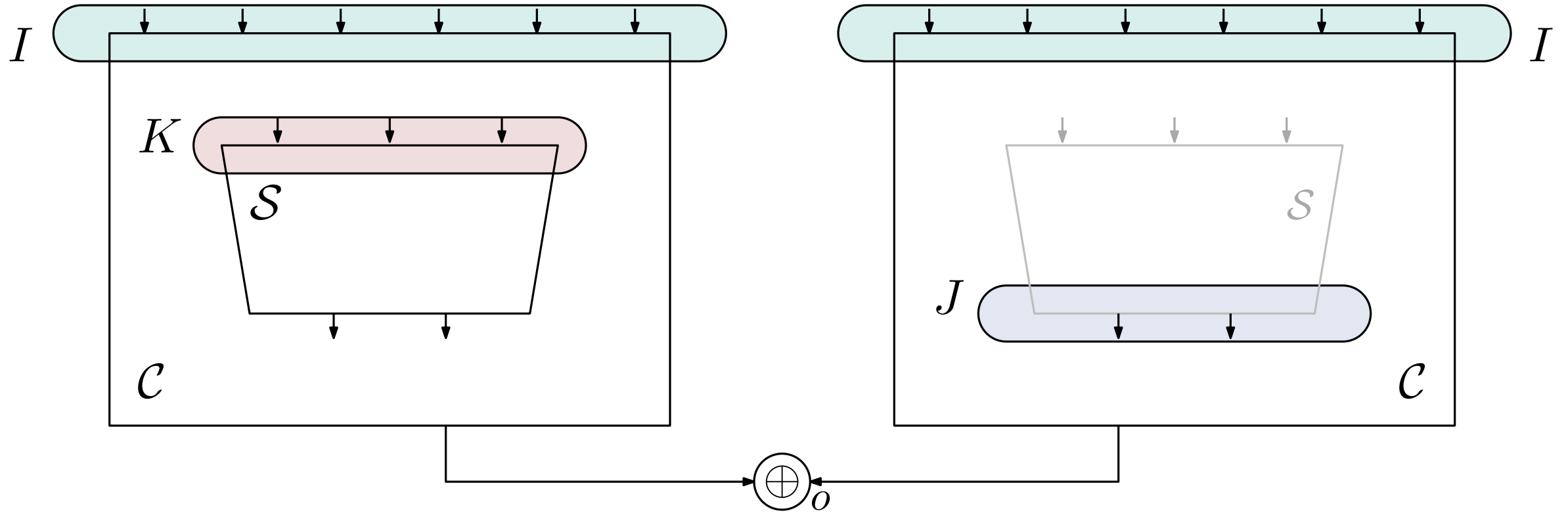


Find assignments for  $I$  and  $J$  such that  $o$  is true

Under the assignment  $K$ , the subcircuit  $S$  must not attain  $J$



# Computing Boolean Relations



Find assignments for  $I$  and  $J$  such that  $o$  is true

Under the assignment  $K$ , the subcircuit  $S$  must not attain  $J$

Iteratively compute assignments  $I, J, K$  by assumption based SAT solving

# SAT-based Synthesis

## Exact Synthesis

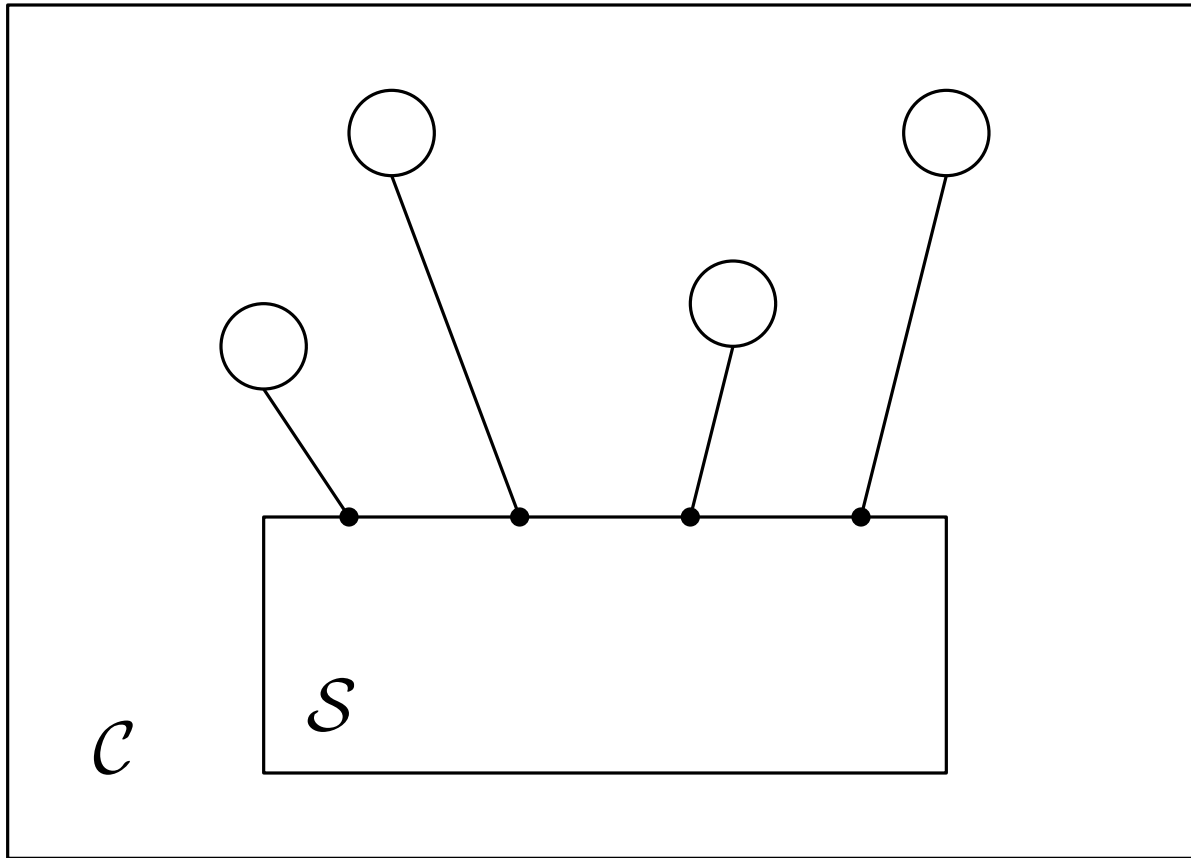
$i_1$	$i_2$	$i_3$	
0	0	0	$\neg o_1 \wedge \neg o_2$
0	0	1	$\neg o_1 \wedge o_2$
0	1	0	$\neg o_1 \wedge \neg o_2$
0	1	1	$\neg o_1 \wedge o_2$
1	0	0	$\neg o_1 \wedge \neg o_2$
1	0	1	$\neg o_1 \wedge o_2$
1	1	0	$o_1 \wedge o_2$
1	1	1	$o_1 \wedge o_2$

# SAT-based Synthesis

Synthesising a relation

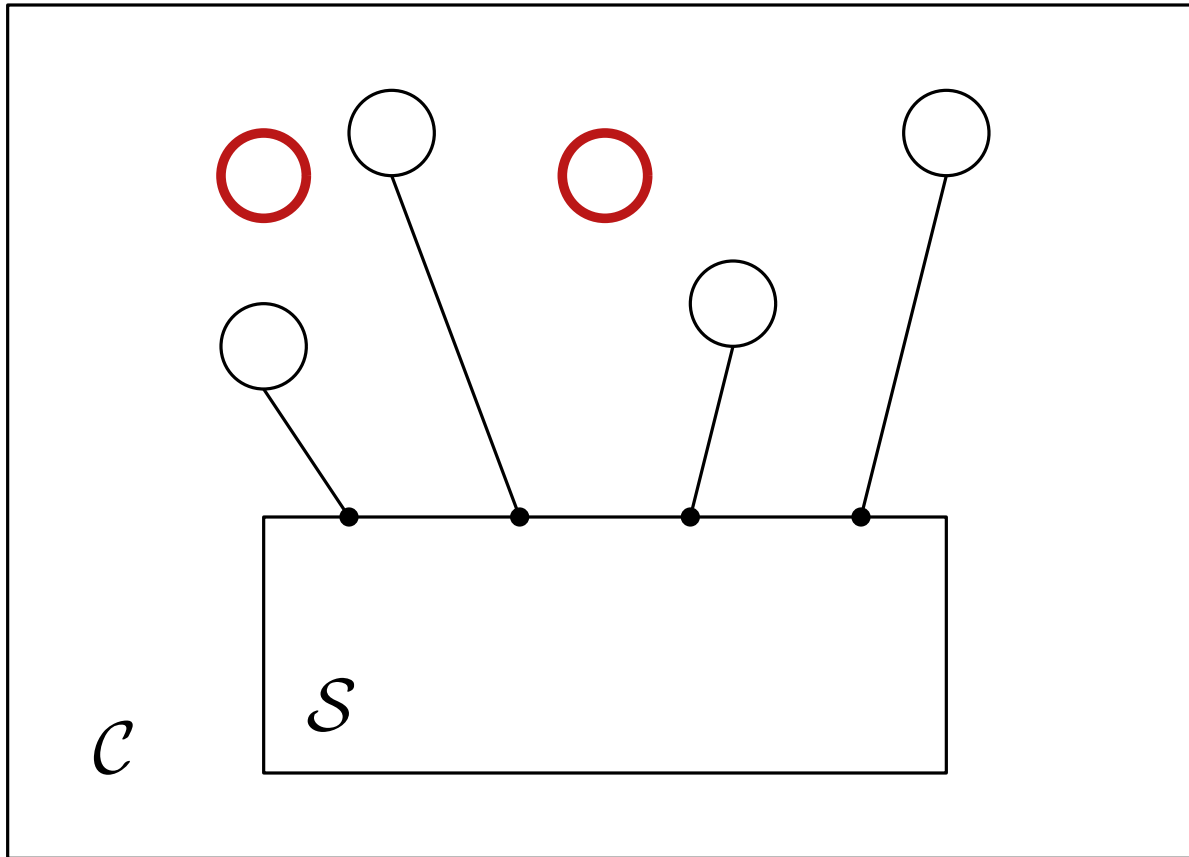
$i_1$	$i_2$	$i_3$	
0	0	0	$\neg o_1$
0	0	1	$(\neg o_1 \vee \neg o_2) \wedge (o_1 \vee o_2)$
0	1	0	$\neg o_1 \vee \neg o_2$
0	1	1	$o_2$
1	0	0	$(\neg o_1 \vee o_2) \wedge (o_1 \vee \neg o_2)$
1	0	1	$(\neg o_1 \vee \neg o_2) \wedge (o_1 \vee o_2)$
1	1	0	$\neg o_1 \vee \neg o_2$
1	1	1	$o_1$

# Additional Inputs



$\mathcal{C}$  can contain nodes with functionality used in  $\mathcal{S}$

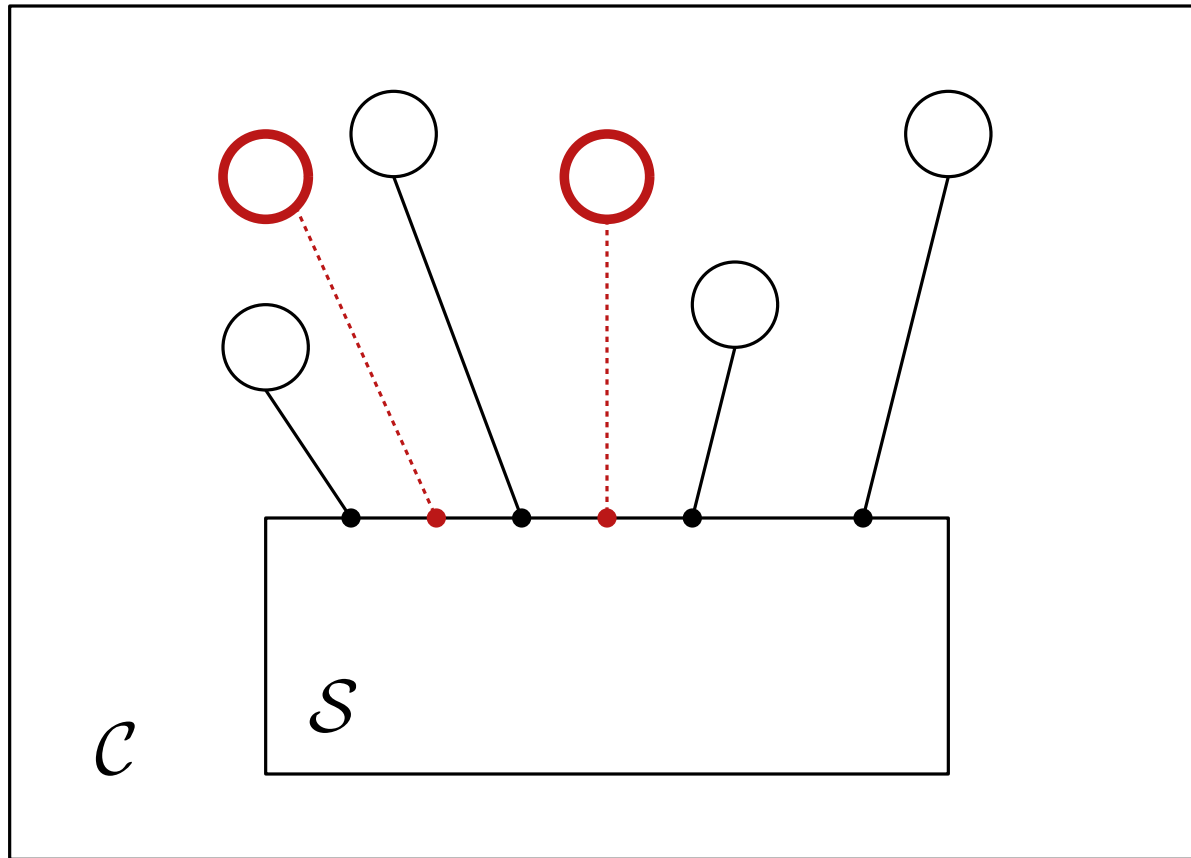
# Additional Inputs



$\mathcal{C}$  can contain nodes with functionality used in  $\mathcal{S}$

Select additional nodes

# Additional Inputs

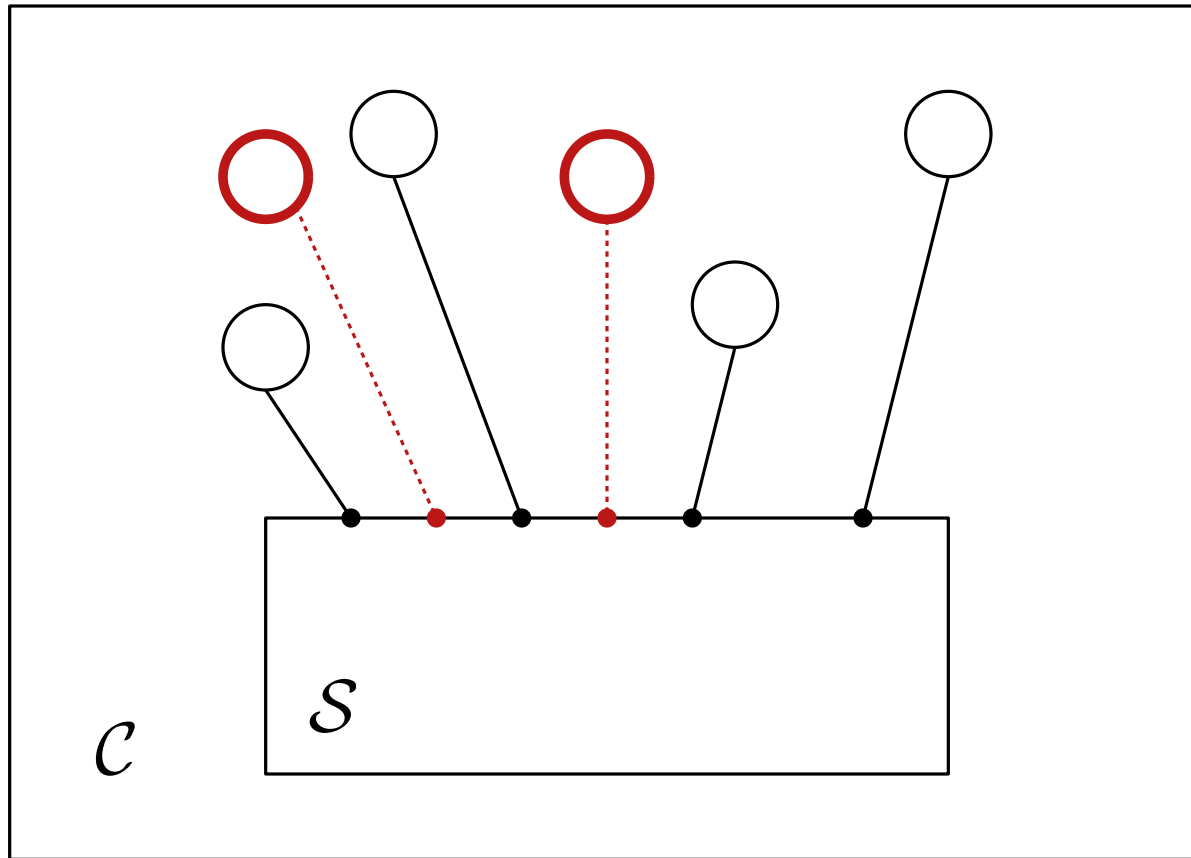


$\mathcal{C}$  can contain nodes with functionality used in  $\mathcal{S}$

Select additional nodes

Allow these nodes as inputs when synthesizing replacement

# Additional Inputs



$\mathcal{C}$  can contain nodes with functionality used in  $\mathcal{S}$

Select additional nodes

Allow these nodes as inputs when synthesizing replacement

Works better with QBF-based synthesis

# Availability

- Implemented in our tool eSLIM

`https://github.com/fxreichl/eSLIM`



# Availability

- Implemented in our tool eSLIM

`https://github.com/fxreichl/eSLIM`

- Integrated within ABC

Available with the `&eslim` command

Currently limited to SAT-based approach