Classical Mechanics

Summary of Newton's Laws of Motion

Newton's First Law

A body remains at rest, or moves at a constant speed in a straight line, unless acted up by a force. Any deviation from travel in a straight line is indicative of a net external force.

$$\boldsymbol{F} = m\boldsymbol{a}$$

Newton's Second Law

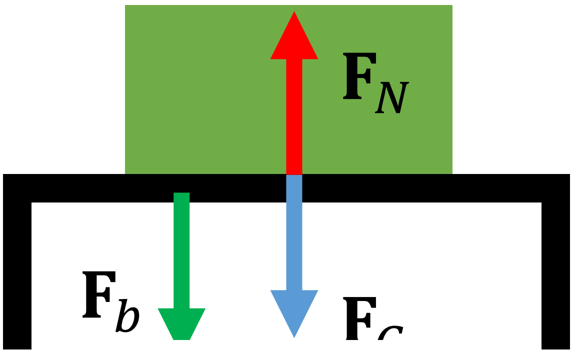
The force acting on a body is directly proportional to its rate of change of **momentum**, where momentum is equal to mass times velocity.

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a}$$

Newton's Third Law

To every action there is an equal and opposite reaction.

The box is not accelerating ($\mathbf{a} = 0$).



Box on a table

Therefore the total force on the box must be zero.

The box experiences a downward force due to gravity: $\mathbf{F}_G$.

This must be balanced by an equal force operating in the opposite direction. This is the normal force, $\mathbf{F}_N$.

The normal force is exerted on the box by the table.

Conservation of Momentum

The total momentum of a collection of interacting particles is always conserved if there is no net external force acting on them.

Work, Energy and Power

Work

Work is defined as the distance moved in opposition to a force. The amount of work done is equal to the negative of the force (**F**) times the distance travelled (**l**).

$$W = -\mathbf{F} \cdot \mathbf{l}$$

Potential Energy

The potential for an object to do work by virtue of its location, position, or state of being.

$$PE = mgh$$

Kinetic Energy

The energy associated with motion.

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Conservation of Energy

Energy is never created nor destroyed, it can only be converted from one form to another.

Example: A tennis ball with a mass of m = 58.5 g and a radius of r = 3.25 cm is thrown vertically in the air with an initial speed of v = 2.35 m s^{-1} at t = 0. At the point the ball is released (t = 0) it is at a height h_0 = 1.43 m above the ground. How high does the ball reach if we ignore the effects of air resistance?

(a) Identify the force that is acting on the ball after it is released. **Gravity**

(b) Giving your reasons, explain what would happen to the ball if the force identified in (a) did not exist.

Solution: If there was no force of gravity acting on the ball (and no air resistance) the ball would continue to travel in the same direction and at the same speed until acted on my some other force (Newton's first law of motion).

(c) Calculate the kinetic energy and the potential energy of the ball at t = 0.

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\left(58.5\ \mathrm{g}\frac{1\ \mathrm{kg}}{1000\ \mathrm{g}}\right)(2.35\ \mathrm{m\ s^{-1}})^2 = 0.1615\ \mathrm{kg\ m^2 s^{-2}}$$
$$= 0.162\ \mathrm{J}$$

$$PE = mgh = \left(58.5\ \mathrm{g}\frac{1\ \mathrm{kg}}{1000\ \mathrm{g}}\right)(9.81\ \mathrm{m\ s^{-2}})(1.43\ \mathrm{m}) = 0.8207 \mathrm{kg\ m^2 s^{-2}}$$
$$= 0.821\ \mathrm{J}$$

(d) Calculate the kinetic energy and the potential energy of the ball when it reaches its maximum height, h_{max}.

At the maximum height the speed = 0 and so KE = 0.
The total energy remains fixed so the potential energy at the maximum height is equal to the sum of the KE and PE at t = 0.

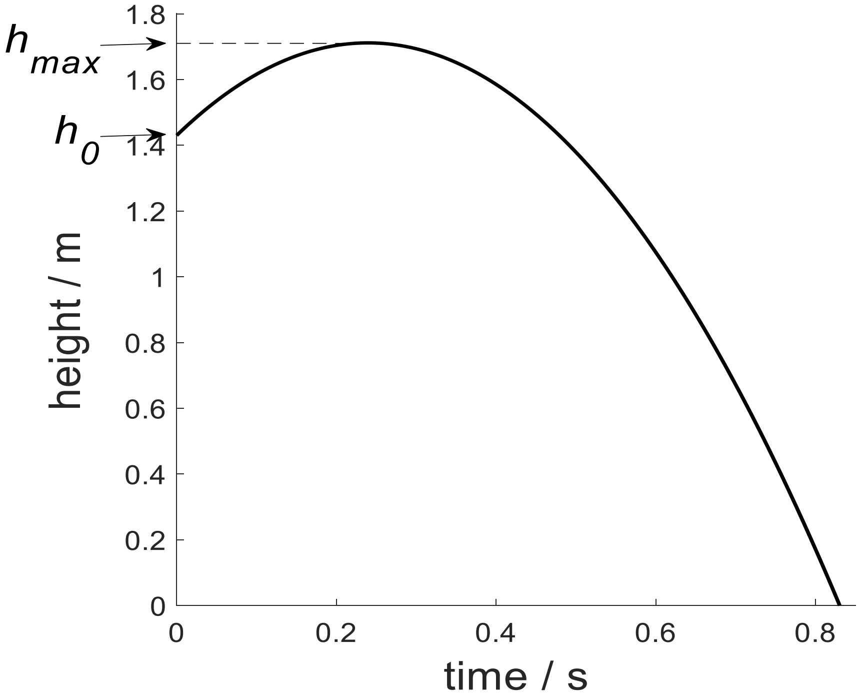
$$PE = 0.1615\ \mathrm{J} + 0.8207\ \mathrm{J} = 0.9822\ \mathrm{J} = 0.982\ \mathrm{J}$$

(e) Calculate the maximum height reached, h_{max}.

$$h_{max} = \frac{PE}{gm} = \frac{0.9822\ \mathrm{J}}{(0.0585\ \mathrm{kg})(9.81\ \mathrm{m\ s^{-2}})} = 1.7132\ \mathrm{m} = 1.71\ \mathrm{m}$$

(f) Identify the fundamental principle of classical mechanics that was used to solve this problem. **Conservation of Energy**

(g) Draw a sketch of the expected trajectory of the ball by plotting the height, h, as a function of time. Label your diagram with h_0, the initial height, h_{max}, the maximum height and h = 0.



Angular Motion (Rotation)

The most common type of non-linear motion is one that follows a circular curve. This is called **angular** motion. An example is the orbits of the electrons in Bohr's model of an atom. We define a range of parameters to describe angular motion (rotation) in the same way as linear motion (translation).

Centripetal forces

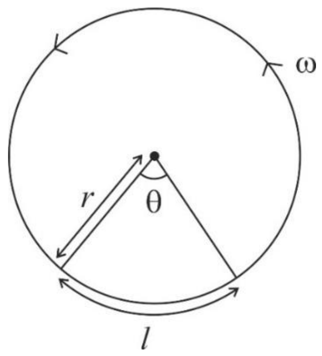
Circular motion is a deviation from linear travel and so, by Newton's first law, it requires a force. This centrally acting force is called the **centripetal force** and continually pulls the orbiting body towards a focal or pivotal point.

Angular velocity

Imagine an object moving in a circle at a constant speed. The speed of this motion can be described by the number of full rotations, or cycles, completed per second. This is angular frequency, *f*, and has units of Hz = s^{-1}.

Angular frequency can also be represented by ω which has units of radians per second (rad s^{-1}). A radian is a dimensionless unit of angle. It is formally defined to be the ratio of the length of the arc *l* subtended (spanned) by and angle θ.

$$\theta = \frac{l}{r}$$



Consider an angle of $360°$. The corresponding length of the arc is equal to the circumference of the circle: $l = 2\pi r$.

Therefore, for a rotation of $360°$, the angle, θ, in radians is:

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi$$

Therefore, there are **2π radians** in one full rotation (**360°**).

The units of angular frequency can be converted between Hz (s^{-1}) and rad s^{-1} by multiplying by 2π.

$$\omega = 2\pi\nu$$

Torque

Torque, τ, is the ability of a force to rotate a body about a particular axis. It can be thought of as the result of a 'twisting' force. The units of torque are N m. If there is more than one force acting on an object, the total torque is the sum of the individual torques due to the individual forces.

Moment of inertia

The moment of inertial, *I*, of a rigid body provides a measure of the extent to which the body resists changing its state of rotational motion. It is measured in units of kg m^2. It depends on the distribution of mass within the rigid body.

Torque and moment of inertia are related through the rotational equivalent of Newton's second law for rotation, where α is **angular acceleration**.

$$\tau = I\alpha$$

Rotational kinetic energy

The kinetic energy associated with rotation is defined with the following equation.

$$KE_r = \frac{1}{2}I\omega^2$$

Rotational kinetic energy is measured in Joules. The total kinetic energy of a rigid body is the sum of the rotational and translational kinetic energy.

Angular momentum

The angular momentum, L, of a rigid body with a moment of inertia, I, rotating with angular velocity, ω, is

$$L = I\omega$$

This is the rotational analogue of linear momentum: $p = mv$.

The units of angular momentum are kg m^2 s^{-1}

As with translational motion, in the absence of an external torque, the angular momentum of a rotating rigid body is conserved.

For a system of many rigid bodies and/or particles, the total angular momentum about any axis is the sum of the individual angular momenta and the conservation of angular momentum applies.

Note: Angular momentum and torque are vector quantities. Their direction is always along the axis of rotation. Therefore, we do not need to consider their vector properties explicitly but we do need to be careful to ensure that we use the correct sign.

Table 5. Comparison of properties of linear vs. angular motion

Linear (Translation)			Angular (Rotation)		
Quantity	**Symbol**	**Units**	**Quantity**	**Symbol**	**Units**
velocity	$\mathbf{v} = \frac{d\mathbf{r}}{dt}$	m s^{-1}	Angular frequency	ω	rad s^{-1}

Linear (Translation)			Angular (Rotation)		
Quantity	**Symbol**	**Units**	**Quantity**	**Symbol**	**Units**
acceleration	$\mathbf{a} = \frac{d\mathbf{v}}{dt}$	m s^{-2}	angular acceleration	α	rad s^{-2}
mass	m	kg	moment of inertia	I	kg m^2
momentum	$\mathbf{p} = m\mathbf{v}$	kg m s^{-1}	angular momentum	$L = I\omega$	kg m^2 s^{-1}
kinetic energy (translation)	$KE_t = \frac{1}{2}mv^2$	J	kinetic energy (rotation)	$KE_r = \frac{1}{2}I\omega^2$	J
force	$\mathbf{F} = m\mathbf{a}$	N	torque	$\tau = I\alpha$	N m

Electrostatics

Electrostatics is the study of the interactions of charged particles. This is a key concept in Chemistry that underpins a wide range of phenomena from the atom structure to molecular properties like solubility.

Fundamental properties of electric charge:

- There are two types of charge: positive (+) and negative (-)
- Charges of opposite sign attract and charges of the same sign repel.
- Charge is never created or destroyed – it is *conserved*.
- Charge always comes in integer multiples of a basic unit – it is *quantised*.

Fundamental unit of charge is conventionally denoted by *e* and has units of Coulombs (C).

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$

The charge of a single electron is –*e* and the charge of a single proton is +*e*.

Coulomb's Law (Electrostatic Force)

The magnitude of the **electrostatic force**, F, between two charged particles, q_1 and q_2, is proportional to the product of their charges and inversely proportional to the square of their separation, r.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

The **permittivity of a medium** (ε) is the ability of a substance to store electrical energy in an electric field. The permittivity of a medium is related to the vacuum permittivity via its **electric susceptibility** (χ).

$$\varepsilon = (1 - \chi)\varepsilon_0$$

Where the vacuum permittivity is:

$$\varepsilon_0 = 8.85\ x\ 10^{-12}\ F\ m^{-1}$$

Farads (F) are the unit of capacitance, which is the ability of a medium to store charge.

The pre-factor in Coulomb's law can be combined into the electric force constant (Coulomb's constant), k_e.

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.998\ \times 10^{9}\ \mathrm{N\ \ m^2C^{-2}}$$

Note: this constant includes the vacuum permittivity and so applies to the case of the electrostatic force in a vacuum.

Electric field

The '*field* of a force' is a concept associated with forces that 'act-at-a-distance'.

The field is a normalised (or scaled) version of the force.

The electric field, E, is the electrostatic force per unit charge.

Therefore, the electric field of charge q_1 is:

$$E = \frac{F}{q_2} = \frac{1}{4\pi\varepsilon}\frac{q_1}{r^2}$$

The electric field has units of N C^{-1}.

The electric field is a vector and acts in the same direction as the electrostatic force: either towards (negative) or away from (positive) q_1 along r.

Example: Hydrogen Atom

(a) Calculate the electrostatic force between the proton and electron in a hydrogen atom, assuming the electron is located at the bohr radius, a_0.

(b) Calculate the electric field of the proton at the bohr radius.

Electrostatic potential (Coulomb potential)

The electrostatic potential at a point in an electric field is defined as the work done in bringing a unit positive charge from infinity to that point. The potential is a **scalar property**.

$$V = \frac{1}{4\pi\varepsilon}\frac{q_1}{r}$$

The units of electrostatic potential is Volts (V).

This corresponds to: 1 V = 1 J C^{-1} = 1 kg m^2 s^{-3} A^{-1}

Electrostatic potential energy

Potential energy is stored energy with respect to an electrostatic force.

Potential energy is equal to the work required to move a charge against an electric field.

This is similar to gravitation potential energy which is equal to the work required to move and object against a gravitational field.

For two point charges, q_1 and q_2, separated by r, the electrostatic potential energy is:

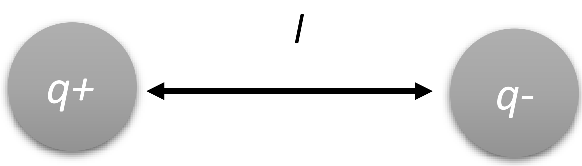
$$U = q_2 V = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r}$$

The units of potential energy is Joules (J).

Practical Application: Molecular Interactions

Electric dipole moment: charge separation in a polar molecule generates an electric dipole moment.

e.g. $\mathrm{H}^{\delta+}\mathrm{Cl}^{\delta-}$



Electric current and circuits

In contrast to electrostatics, which deals with interactions between charged particles that are fixed in space (i.e. static), electricity and electric circuits deal with the movement of electric charge. The basic principles of electricity and electric circuits are important in Chemistry topics like **Electrochemistry** and also for analytical and spectroscopy methods like **Mass Spectrometry** and **Nuclear Magnetic Resonance (NMR)**, where the instrumentation and detectors involve circuits and moving charges.

Electric current, I, is a measure of the amount of charge, Q, that passes a specific point per unit time. It is measured in Amperes (A), one of the base SI units.

$$I = \frac{dQ}{dt}$$

Electrical resistance, R, is a measure of the opposition to the flow of electric current. It is measured in Ohms (Ω).

Electric potential difference, also known as the **voltage**, V, is the difference in electric potential between two points. It is measured in units of volts. It can also be defined as the different in potential energy per unit of charge.

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the conductor. The constant of proportionality is the resistance of the conductor.

$$V = IR$$

The **electric power** (watts) is the work done to produce the current, I, in opposition to the electric potential difference, V.

$$P = IV$$

Example: A hair dryer is designed to generate a heat output of P = 1500 W by passing an electric current, I, through a heating element.

(a) Calculate the current, I, required to generate P = 1500 W, assuming that the hair dryer was designed to use mains electricity in the USA, which supplies an electric potential difference of V = 120 V, and that the power output comes purely from resistive heating.

Solution: Power (the energy output per unit time) is equal to the current (flow of charge per unit time) times the electric potential difference (often called the “voltage”).

$$I = \frac{P}{V} = \frac{1500\ \mathrm{W}}{120\ \mathrm{V}} = 12.5\ \mathrm{A}$$

(b) Using Ohm’s law, calculate the resistance of the heating element inside the hair dryer.

$$R = \frac{V}{I} = \frac{120\ \mathrm{V}}{12.5\ \mathrm{A}} = 9.60\ \Omega$$

(c) Calculate the power output if the same hair drier was used in the UK, where the mains electricity supplies an electric potential difference of V = 230 V. Comment on the result.

$$I = \frac{V}{R} = \frac{230\ \mathrm{V}}{9.60\ \Omega} = 24.0\ \mathrm{A}$$

$$P = IV = (24.0\ \mathrm{A})(230\ \mathrm{V}) = 5520\ \mathrm{W} = 5.52\ \mathrm{kW}$$