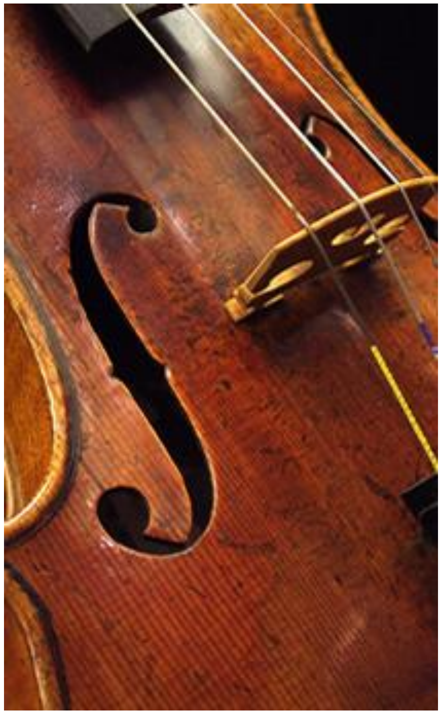

Formulas & Functions

MOTO

Whether you love or hate math class, this formula sheet will help you survive math in one piece.

Alan Ngo

Algebra – Calculus III

Arithmetic Operations

$a(b \pm c) = ab \pm ac$

$\frac{a}{b} \pm \frac{x}{y} = \frac{ay \pm bx}{by}$

$\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$

$\frac{a}{b} * \frac{x}{y} = \frac{ax}{by}$

$\frac{a/b}{x/y} = \frac{ay}{bx}$

Linear

$Ax + By = C$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y = mx + b$

$y - y_1 = m(x - x_1)$

Quadratic

$ax^2 + bx + c = 0$

$a(x-h)^2 + k = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Special Factor Rules

$x^2 - a^2 = (x-a)(x+a)$

$x^2 + a^2 = (x-ai)(x+ai)$

$x^3 \pm a^3 = (x \pm a)(x^2 \mp ab + b^2)$

Binomial Theorem

$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$

Exponents/Radicals

$x^a x^b = x^{a+b}$

$\frac{x^a}{x^b} = x^{a-b}$

$(x^a)^b = x^{ab}$

$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

$(xy)^a = x^a y^a$

$x^{\frac{a}{b}} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$

$\sqrt[a]{xy} = \sqrt[a]{x} * \sqrt[a]{y}$

$\sqrt[a]{\frac{x}{y}} = \frac{\sqrt[a]{x}}{\sqrt[a]{y}}$

Functions

$y = af(b(x-h)) + k$

$a: vertical\ stretch/shrink$

$b: horizontal\ stretch/shrink$

$h: horizontal\ shift$

$h: vertical\ shift$

Domain/Range

Interval

$x \in [a,b]$	$x \in (a,b)$	$x \in (-\infty, \infty)$
$y \in [c,d]$	$y \in (c,d)$	$y \in (-\infty, \infty)$

Set Builder

$a \le x \le b$	$a < x < b$	$\mathbb{R}$
$c \le y \le d$	$c < y < d$	$\mathbb{R}$

Logs

$log_b 1 = 0$

$log_b b = 1$

$log_b(xy) = log_b x + log_b y$

$log_b\left(\frac{x}{y}\right) = log_b x - log_b y$

$log_b x^n = n log_b x$

$log_b x = y \leftrightarrow b^y = x$

$b^{log_b x} = x$

$log_b x = \frac{log_a x}{log_a b}$

Natural Logs

$ln1 = 0$

$lne = 1$

$\ln(xy) = lnx + lny$

$\ln\left(\frac{x}{y}\right) = lnx - lny$

$lnx^n = nlnx$

$lnx = y \leftrightarrow e^y = x$

$e^{lnx} = x$

$log_b x = \frac{lnx}{lnb}$

Temp Conversions

$K = 273 + °C$

$°C = 5(°F - 32)/9$

$°F = 9°C/5 + 32$

Circle

$(x-h)^2 + (y-k)^2 = r^2$

Ellipse

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

$c^2 = a^2 - b^2$ (foci)

Hyperbola

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$

$y = \pm\left(\frac{b}{a}\right)(x-h) + k$ (asm)

$c^2 = a^2 + b^2$ (foci)

Parabola

$y = a(x-h)^2 + k$

$x = a(y-k)^2 + h$

$a = \frac{1}{4c}$ (focus)

Rational

$y = \frac{1}{f(x)}$

$f(x) \neq 0$

Pythagorean THM

$a^2 + b^2 = c^2$

Quad.	Angle°	RA°	Angle (rad)	RA	$cos\theta$	$sin\theta$	$tan\theta$	$sec\theta$	$csc\theta$	$cot\theta$
X	0	0	0	0	1	0	0	1	und	und
I	30	30	π/6	π/6	√3/2	1/2	√3/3	2/√3	2	√3
I	45	45	π/4	π/4	√2/2	√2/2	1	√2	√2	1
I	60	60	π/3	π/3	1/2	√3/2	√3	2	2/√3	√3/3
X	90	90	π/2	π/2	0	1	und	und	1	0
II	120	60	2π/3	π/3	-1/2	√3/2	-√3	-2	2/√3	-√3/3
II	135	45	3π/4	π/4	-√2/2	√2/2	-1	-√2	√2	-1
II	150	30	5π/6	π/6	-√3/2	1/2	-√3/3	-2/√3	2	-√3
X	180	0	π	0	-1	0	0	-1	und	und
III	210	30	7π6	π/6	-√3/2	-1/2	√3	-2/√3	-2	√3
III	225	45	5π/4	π/4	-√2/2	-√2/2	1	-√2	-√2	1
III	240	60	4π/3	π/3	-1/2	-√3/2	√3/3	-2	-2/√3	√3/3
X	270	90	3π/2	π/2	0	-1	und	und	-1	0
IV	300	60	5π/3	π/3	1/2	-√3/2	-√3/3	2	-2/√3	-√3/3
IV	315	45	7π/4	π/4	√2/2	-√2/2	-1	√2	-√2	-1
IV	330	30	11π/6	π/6	√3/2	-1/2	-√3	2/√3	-2	-√3
X	360	0	2π	0	1	0	0	1	und	und

Trig Identities		**Geometric**				**Polar basics**	
$sin\theta = \frac{y}{r}$	$csc\theta = \frac{r}{y}$	$A_{\square} = s^2$	$A_{rec} = lw$	$A_{\Delta} = \frac{bh}{2}$	$A_{\circ} = \pi r^2$	$x = rcos\theta$	$y = rsin\theta$
$cos\theta = \frac{x}{r}$	$sec\theta = \frac{r}{x}$	$P_{\square} = 4s$	$P_{rec} = 2l + 2w$	$P_{\Delta} = a + b + c$	$C = 2\pi r$	$r = \sqrt{x^2 + y^2}$	$\theta = arctan(\frac{y}{x})$
$tan\theta = \frac{y}{x}$	$cot\theta = \frac{x}{y}$	$V_{\square} = s^3$	$V_{prism} = Bh$	$V_{pyr} = \frac{lwh}{3}$	$V_{\circ} = \frac{\pi r^2 h}{3}$		

Identities/Trig Formulas

Distance/Midpoint	Radian / Degree	Velocity	Angular Speed	Arc length / Area of Sector
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$1° = \frac{180}{\pi} rad$	$v = \frac{s}{t}$	$v = r\acute{\omega}$	$s = r\theta$
$m = \left(\frac{(x_2 + x_1)}{2}, \frac{(y_2 + y_1)}{2}\right)$	$1\, rad = \frac{\pi}{180}°$	$v = \frac{r\theta}{t}$	$\acute{\omega} = \frac{\theta}{t}$	$\acute{A} = \frac{1}{2} r^2 \theta$

Pythagorean Identities	Even/Odd Identities	Power Reducers	Sum/Difference Identities
$\sin^2 x + \cos^2 x = 1$	$sin(-x) = -sinx$	$sin^2 x = \frac{1-cos2x}{2}$	$\sin(x \pm y) = sinx\, cosy \pm cosx\, siny$
$\tan^2 x + 1 = \sec^2 x$	$cos(-x) = cosx$	$cos^2 x = \frac{1+cos2x}{2}$	$\cos(x \pm y) = cosx\, cosy \mp sinx\, siny$
$1 + \cot^2 x = \csc^2 x$	$tan(-x) = -tanx$	$tan^2 x = \frac{1-cos2x}{1+cos2x}$	$\tan(x \pm y) = \frac{tanx \pm tany}{1 \mp tanx\, tany}$

Double Angle Identities	Half Angles	Co-function Identities	Reciprocal Identities
$sin2x = 2sinx\, cosx$	$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{2}}$	$sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$	$sinx = \frac{1}{cscx}$
$cos2x = \cos^2 x - \sin^2 x$	$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+cosx}{2}}$	$cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$	$cos = \frac{1}{secx}$
$cos2x = 1 - 2sin^2 x$	$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{1+cosx}}$	$tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$	$tanx = \frac{1}{cotx}$
$cos2x = 2cos^2 x - 1$	$\tan\left(\frac{x}{2}\right) = \frac{sinx}{1+cosx}$	$cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$	$secx = \frac{1}{cosx}$
$tan2x = \frac{2tanx}{1-\tan^2 x}$	$\tan\left(\frac{x}{2}\right) = \frac{(1-cosx)}{sinx}$	$sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$	$cscx = \frac{1}{sinx}$
$tan2x = \frac{sin2x}{cos2x}$	$\tan\left(\frac{x}{2}\right) = \frac{sin\left(\frac{x}{2}\right)}{cos\left(\frac{x}{2}\right)}$	$csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$	$cotx = \frac{1}{tanx}$

Laws of Sines/Cosines

			Semi perimeter/Area
$\frac{sinA}{a} = \frac{sinB}{b}$	$A = \frac{ab}{2} sinC$	$c^2 = a^2 + b^2 - 2ab * cosC$	
$\frac{sinB}{b} = \frac{sinC}{c}$	$A = \frac{ac}{2} sinB$	$b^2 = a^2 + c^2 - 2ac * cosB$	$A = \sqrt{s(s-a)(s-b)(s-c)}$
$\frac{sinA}{a} = \frac{sinC}{c}$	$A = \frac{bc}{2} sinA$	$a^2 = b^2 + c^2 - 2bc * cosA$	$s = \frac{a+b+c}{2}$

Standard Derivatives

Basic Rules	Trig	Inverse Trig	Exponents	Logs
$\frac{d}{dx}k = 0$	$\frac{d}{dx}sinx = cosx$	$\frac{d}{dx}arcsinx = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}lnx = \frac{1}{x}$
$\frac{d}{dx}(ku) = ku'$	$\frac{d}{dx}cosx = -sinx$	$\frac{d}{dx}arccosx = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}b^x = b^x lnb$	$\frac{d}{dx}log_b x = \frac{1}{xlnb}$
$\frac{d}{dx}(u \pm v) = u' \pm v'$	$\frac{d}{dx}tanx = \sec^2 x$	$\frac{d}{dx}arctanx = \frac{1}{x^2+1}$	**Definition**	**LH's Rules**
$\frac{d}{dx}(uv) = uv' + vu'$	$\frac{d}{dx}cotx = -\csc^2 x$	$\frac{d}{dx}arccotx = \frac{-1}{x^2+1}$	$\frac{d}{dx}f = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	$\lim_{x\to\infty}\frac{u}{v} = \lim_{x\to\infty}\frac{u\prime}{v\prime}$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu'-uv'}{v^2}$	$\frac{d}{dx}secx = secx\ tanx$	$\frac{d}{dx}arcsecx = \frac{1}{x\sqrt{x^2-1}}$	$\frac{d}{dx}f = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$	$\lim_{x\to 0}\frac{u}{v} = \lim_{x\to 0}\frac{u\prime}{v\prime}$
$\frac{d}{dx}x^n = nx^{n-1}$	$\frac{d}{dx}cscx = -cscx\ cotx$	$\frac{d}{dx}arccscx = \frac{-1}{x\sqrt{x^2-1}}$		

Chain Rules/Misc.

Basic Rules	Trig	Inverse Trig	Exponents	Logs
$\frac{d}{dx}u(v(x)) = u'(v) * v'(x)$	$\frac{d}{dx}sinu = u'cosu$	$\frac{d}{dx}arcsinu = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}e^u = u'e^u$	$\frac{d}{dx}lnu = \frac{u'}{u}$
$\frac{d}{dx}u^n = u' * nu^{n-1}$	$\frac{d}{dx}cosu = -u'sinu$	$\frac{d}{dx}arccosu = \frac{-u'}{\sqrt{1-u^2}}$	$\frac{d}{dx}b^u = u'b^u lnb$	$\frac{d}{dx}log_b u = \frac{u'}{ulnb}$
Linear Approx./Differential	$\frac{d}{dx}tanu = u'\sec^2 u$	$\frac{d}{dx}arctanu = \frac{u'}{u^2+1}$	**Polar**	
$L(x) = f'(a)(x - a) + f(a)$	$\frac{d}{dx}cotu = -u'\csc^2 u$	$\frac{d}{dx}arccotu = \frac{-u'}{u^2+1}$	$\frac{dy}{dx} = \frac{\left[\frac{dr}{d\theta}sin\theta + rcos\theta\right]}{\left[\frac{dr}{d\theta}cos\theta - rsin\theta\right]}$	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$
$\Delta y = f(x+h) - f(x)$	$\frac{d}{dx}secu = u'secu\ tanu$	$\frac{d}{dx}arcsecu = \frac{u'}{u\sqrt{u^2-1}}$		
$dy = f'(x)\,dx$	$\frac{d}{dx}cscu = -u'cscu\ cotu$	$\frac{d}{dx}arccscu = \frac{-u'}{u\sqrt{u^2-1}}$		

Special Chain Rules

Function-Power			Implicit/Logarithmic Differentiation
$\frac{d}{dx}sin^n u = nu'sin^{n-1}u\ cosu$	$\frac{d}{dx}\sec^n u = nu'sec^n u\ tanu$	$\frac{d}{dx}e^{u^n} = nu'u^{n-1}e^{u^n}$	$[f(x) + f(y) = a] \rightarrow f'(x) + \frac{dy}{dx}f'(y) = 0$
$\frac{d}{dx}\cos^n u = -nu'cos^{n-1}u\ sinu$	$\frac{d}{dx}\csc^n u = -nu'csc^n u\ cotu$	$\frac{d}{dx}b^{u^n} = nu'u^{n-1}b^{u^n}\ln b$	$\frac{d}{dx}x^x = x^x(\ln(x) + 1)$
$\frac{d}{dx}\tan^n u = nu'tan^{n-1}u\sec^2 u$	$\frac{d}{dx}\cot^n u = -nu'cot^{n-1}u\csc^2 u$	$\frac{d}{dx}ln^n u = \frac{nu\prime ln^{n-1}u}{u}$	$\frac{d}{dx}\left[\frac{(x\pm a)^n}{(x\pm b)^m}\right] = \left[\frac{(x\pm a)^n}{(x\pm b)^m}\right]\left[\frac{n}{(x\pm a)} - \frac{m}{(x\pm b)}\right]$

Basic Integrals	**Integration By Parts**	**Trig Sub Rules**
$\int k\,dx = kx + c$	$\int u\,dv = uv - \int v\,du$ (*Remember LIPTE*)	$\sqrt{a^2 + x^2} \mid x = a\tan\theta$
$\int kf\,dx = k\int f\,dx$	**Special IBP rules**	$\sqrt{a^2 - x^2} \mid x = a\sin\theta$
$\int u \pm v\,dx = \int u\,dx \pm \int v\,dx$	$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax} dx$	$\sqrt{x^2 - a^2} \mid x = a\sec\theta$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$	$\int x^n \sin(ax)\,dx = -\frac{x^n\cos(ax)}{a} + \frac{n}{a}\int x^{n-1}\cos(ax)\,dx$	**Definite Integrals**
$\int e^x dx = e^x + c$	$\int x^n \cos(ax)\,dx = \frac{x^n\sin(ax)}{a} - \frac{n}{a}\int x^{n-1}\sin(ax)\,dx$	$\int_a^b f(x)\,dx = F(b) - F(a)$
$\int b^x dx = \frac{b^x}{\ln b} + c$	$\int \arcsin x\,dx = x\,arcsinx + \sqrt{1 - x^2} + c$	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \arccos x\,dx = x\,arccosx - \sqrt{1 - x^2} + c$	**IBP Priority (LIPTE)**
$\int \sin x\,dx = -\cos x + c$	$\int \arctan x\,dx = x\,arctanx - \frac{\ln(1+x^2)}{2} + c$	**L**og
$\int \cos x\,dx = \sin x + c$	$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}(a\sin(bx) - b\cos(bx))}{a^2+b^2} + c$	**I**nverse trig
$\int \sec x\tan x\,dx = \sec x + c$	$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}(a\cos(bx) + b\sin(bx))}{a^2+b^2} + c$	**P**olynomial
$\int \csc x\cot x\,dx = -\csc x + c$	$\int x^n \ln ax\,dx = \frac{(x^{n+1}\,(n+1)\ln(ax)-1)}{(n+1)^2} + c$	**T**rig
$\int \sec^2 x\,dx = \tan x + c$	$\int \ln ax\,dx = x(\ln(ax) - 1) + c$	**E**xponent
$\int \csc^2 x\,dx = -\cot x + c$	$\int \sec^3 x\,dx = \frac{secx\,tanx + \ln\lvert secx + tanx\rvert}{2} + c$	**FTC**
$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + c$	$\int \csc^3 x\,dx = \frac{cscx\,cotx - \ln\lvert cscx + cotx\rvert}{2} + c$	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
$\int \csc x\,dx = -\ln\lvert \csc x + \cot x\rvert + c$	$\int \sin^n x\,dx = -\frac{1}{n}sin^{n-1}x\,cosx + \frac{n-1}{n}\int cos^{n-2}x\,dx$	
$\int \tan x\,dx = -\ln\lvert\cos x\rvert + c$	$\int \cos^n x\,dx = \frac{1}{n}cos^{n-1}x\,sinx + \frac{n-1}{n}\int cos^{n-2}x\,dx$	
$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$	**Partial Fraction Decomposition**	
$\int \frac{1}{\sqrt{a^2-x^2}}dx = arcsin\left(\frac{x}{a}\right) + c$	$\int \frac{P(x)}{(ax+b)(cx+z)}dx = \int \frac{A}{(ax+b)}dx + \int \frac{B}{(cx+z)}dx$	
$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}arctan\left(\frac{x}{a}\right) + c$	$\int \frac{P(x)}{(ax+b)(cx+z)^n}dx = \int \frac{A}{(ax+b)}dx + \int \frac{B}{(cx+z)}dx + \int \frac{C}{(cx+z)^2}dx + \cdots + \int \frac{U}{(cx+z)^n}dx$	
$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}arcsec\left(\frac{x}{a}\right) + c$	$\int \frac{P(x)}{(ax+b)(ux^2+vx+w)}dx = \int \frac{A}{(ax+b)}dx + \int \frac{Ux+V}{(ux^2+vx+w)}dx$	

Arc Length

Rectangular

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Parametric

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Polar

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Area

Rectangular:

$$A_x = \int_a^b f(x) - g(x)\, dx$$

$$A_y = \int_c^d f(y) - g(y)\, dy$$

Parametric:

$$A = \int_a^b y\, dx$$

$$A = \int_c^d x\, dy$$

Polar:

$$A = \frac{1}{2}\int_\alpha^\beta r^2\, d\theta$$

Volumes

Discs

$$V_x = \pi \int_a^b f(x)^2\, dx$$

$$V_y = \pi \int_c^d f(y)^2\, dy$$

Washers

$$V_x = \pi \int_a^b f(x)^2 - g(x)^2 dx$$

$$V_y = \pi \int_c^d f(y)^2 - g(y)^2 dy$$

Shells

$$V_x = 2\pi \int_c^d y[f(y) - g(y)]dy$$

$$V_y = 2\pi \int_a^b x[f(x) - g(x)]dx$$

Probability

PDF

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Median PDF

$$\int_m^{\infty} f(x)dx = \frac{1}{2}$$

Mean PDF

$$\mu = \int_{-\infty}^{\infty} fx(x)dx$$

Average Value

$$S = \frac{1}{b-a}\int_a^b f(x)dx$$

Improper Integrals

$$\int_a^{\infty} f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$$

$$\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx + \lim_{b\to\infty} \int_a^b f(x)dx$$

Manual Integration

('n' is the number of rectangles to split by)

$$\int_a^b f(x)dx = \Delta x \sum_{i=0}^{n} f(x_i) \qquad \Delta x = \frac{b-a}{n}$$

Tabular (PT)

u	dv	±
x^2	cosx	+
2x	sinx	-
2	cosx	+
0	sinx	-
0	cosx	+

$x^2 sinx - 2xcosx + 2sinx + c$

Tabular (PE)

u	dv	±
x^2	e^x	+
2x	e^x	-
2	e^x	+
0	e^x	-
0	e^x	+

$e^x(x^2 - 2x + 2) + c$

Midpoint Rule: $\int_a^b f(x)dx = \Delta x\,[f_0 + f_1 + f_2 + f_3 + \cdots + f_{n-3} + f_{n-2} + f_{n-1} + f_n]$ where $x = \frac{x_{i-1}+x_i}{2}$

Trapezoid Rule: $\int_a^b f(x)dx = \frac{\Delta x}{2}[f_0 + 2(f_1 + f_2 + f_3 + \cdots + f_{n-3} + f_{n-2} + f_{n-1}) + f_n]$

Simpson Rule: $\int_a^b f(x)dx = \frac{\Delta x}{3}[f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \cdots + 2f_{n-4} + 4f_{n-3} + 2f_{n-2} + 4f_{n-1} + f_n]$

Diff. EQ

General Forms

$\frac{dy}{dx} = f(x)$ | $\frac{dy}{dx} = f(y)$ | $\frac{dy}{dx} = f(x,y)$

Newton's Law of Cooling

$$\frac{dT}{dt} = k(T - T_a)$$

Growth/Decay

$\frac{dP}{dt} = kP$ | $\frac{dP}{dt} = kP(M - P)$ | $\frac{dP}{dt} = k(M - P)$

Mixing

$$\frac{dx}{dt} = r_i c_i - \frac{r_o x}{V}$$

Euler Method

$$y_{n-1} = y_n + f(x_n, y_n) * h$$

Series

Arithmetic Series

$$S_n = \frac{n\,(a + a_n)}{2}$$

Alternating Series

$$\sum_{n=0}^{\infty} (-1)^n \; a_n$$

Power Series

$$\sum_{n=0}^{\infty} x^n$$

$$S_n = \frac{1}{1 - x}$$

Test	Series	Converges	Diverges	Comment
nth -term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	Cannot be used to show convergence
Geometric series	$\sum_{n=0}^{\infty} ar^{n-1}$	$\lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	Sum is $S = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (a_n - a_{n+1})$	$\lim_{n\to\infty} a_n = L$		Sum is $S = a_1 - L$
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder $R_n = S - S_n$: $\lvert R_n \rvert \leq a_{n+1}$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	Remainder: $0 \leq R_n \leq \int_n^{\infty} f(x)dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{a_n} < 1$	$\lim_{n\to\infty} \sqrt[n]{a_n} > 1$	Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$.
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$.
Direct Comparison ($a_n, c_n, d_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq c_n$ and $\sum_{n=1}^{\infty} c_n$ converges	$0 \leq d_n \leq a_n$ and $\sum_{n=1}^{\infty} d_n$ diverges	
Limit Comparison ($a_n, c_n, d_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{c_n} = L > 0$ and $\sum_{n=1}^{\infty} c_n$ converges	$\lim_{n\to\infty} \frac{a_n}{d_n} = L > 0$ and $\sum_{n=1}^{\infty} d_n$ diverges	

Taylor Series	Expanded
$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$	$f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \frac{f^4(a)(x-a)^4}{4!} + \ldots + \frac{f^{(n)}(a)(x-a)^n}{n!}$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \frac{x^9}{9!} + \frac{x^{10}}{10!} + \ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$
$sinx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$	$\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \ldots$
$cosx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \ldots$

Vectors

Cross Product	Dot Product	Triple Product	Projections
$\vec{u} \times \vec{v} = (\lvert\vec{u}\rvert\lvert\vec{v}\rvert sin\theta)\vec{n}$	$\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$	$A = \vec{u} \cdot (\vec{v} \times \vec{w})$	$comp_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert}$
$\lvert\vec{u} \times \vec{v}\rvert = \lvert\vec{u}\rvert\lvert\vec{v}\rvert sin\theta$	$\vec{u} \cdot \vec{v} = \lvert\vec{u}\rvert\lvert\vec{v}\rvert cos\theta$	$V = (\vec{u} \times \vec{v}) \cdot \vec{w}$	
$\vec{u} \times \vec{v} = \langle det\begin{bmatrix} u_2 & u_3 \\ v_2 & v_3 \end{bmatrix}, -det\begin{bmatrix} u_1 & u_3 \\ v_1 & v_3 \end{bmatrix}, det\begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}\rangle$	**Magnitude**		$proj_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert} * \frac{\vec{a}}{\lvert\vec{a}\rvert}$
	$\lvert\vec{u}\rvert = \sqrt{u_1{}^2 + u_2{}^2 + u_3{}^2}$		

Tangent Vectors/Curvature

$\vec{r}'(t) = \langle\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\rangle$	$\vec{T}(t) = \frac{\vec{r}'(t)}{\lvert\vec{r}'(t)\rvert}$	$\vec{N}(t) = \frac{\vec{T}'(t)}{\lvert\vec{T}'(t)\rvert}$	$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t) = \frac{r'(t) \times r''(t)}{\lvert r'(t) \times r''(t)\rvert}$	**Arc Length**
$k = \frac{\vec{T}'(t)}{\lvert\vec{r}'(t)\rvert}$	$k = \frac{\lvert\vec{r}'(t)\times\vec{r}''(t)\rvert}{\lvert\vec{r}'(t)\rvert^3}$	$k = \frac{\lvert f''(x)\rvert}{(1+[f'(x)]^2)^{3/2}}$	$k = \frac{\lvert d\vec{T}\rvert}{\lvert d\vec{s}\rvert}$	$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$

Coordinate Systems

Cylindrical $f(r, \theta, z)$ Spherical $f(\rho, \theta, \emptyset)$

$x = rcos\theta$	$y = rsin\theta$	$z = z$	$\theta = arctan\left(\frac{y}{x}\right)$	$r = \sqrt{x^2 + y^2}$
$x = \rho sin\emptyset cos\theta$	$y = \rho sin\emptyset sin\theta$	$z = \rho cos\emptyset$	$\rho = \sqrt{x^2 + y^2 + z^2}$	$\emptyset = arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

Partial Derivatives

General		Gradient Vector	LaGrange	Normal Line	Implicit	Directional Derivative
$f_x = \frac{\partial f}{\partial x}$	$f_{xy} = \frac{\partial f}{\partial x \partial y}$	$\nabla f = \langle\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\rangle$	$\nabla f = \lambda \nabla g$	$\frac{(x-a)}{f_a} = \frac{(y-b)}{f_b} = \frac{(z-c)}{f_c}$	$\frac{dy}{dx} = -\frac{f_x}{f_y}$	$D_u = \nabla f(x_0, y_0, z_0) \cdot \frac{\langle A,B,C\rangle}{\sqrt{A^2+B^2+C^2}}$

Tangent Planes	Laplace/Wave	Linearization/Differential
$\nabla f \cdot \langle(x-a), (y-b), (z-c)\rangle = 0$	$f_{xx} + f_{tt} = 0$	$L(x, y, z) = f_a(x-a) + f_b(y-b) + f_c(z-c) + f(a, b, c)$
$z - c = z_a(x-a) + z_b(y-b)$	$f_{tt} = a^2 f_{xx}$	$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

Chain rule

$$\frac{d}{dt}u(x, y, z) = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

Second Derivative Tests

$D = f_{xx} + f_{yy} - (f_{xy})^2$ D>0, local extrema, D<0, saddle point, D=0, inconclusive

$f_{xx}\ or\ f_{yy} > 0: local\ min,\ f_{xx}\ or\ f_{yy} < 0: local\ max,\ f_{xx}\ or\ f_{yy} = 0: test\ D$

Double Integrals

General		Probability Average Value	Surface Area/Polar Conversion
$\int_a^b \int_c^d f(x,y)\, dydx = \int_c^d \int_a^b f(x,y)\, dxdy$		$\int_c^d \int_a^b f(x,y)\, dxdy = 1$ (CDF)	$A = \iint \sqrt{1 + {f_x}^2 + {f_y}^2}dA$
$\int_a^b \int_c^d f(x)\, f(y)\, dydx = \int_a^b f(x)dx \int_c^d f(y)dy$		$Avg = \frac{1}{Area(D)} \iint f(x,y)dA$	$\int_a^b \int_c^d f(x,y)\, dydx = \int_m^n \int_\alpha^\beta rf(r,\theta)\, drd\theta$

Manual Integration	$\Delta x = \frac{b-a}{m}$	$\Delta y = \frac{d-c}{n}$	Midpoint Rule
$\int_c^d \int_a^b f(x,y)\, dxdy = \Delta x \Delta y \sum_i^m \sum_j^n f(x_i, y_j)$			$\int_c^d \int_a^b f(x,y)\, dxdy = \Delta x \Delta y \sum_i^m \sum_j^n f(x_i, y_j)$ $where\ x = \frac{x_{i-1}+x_i}{2}, y = \frac{y_{i-1}+y_i}{2}$

Mass		Inertia
$M = \iint p(x,y)dA$	$\bar{x} = \frac{M_y}{M} = \frac{1}{M}\iint x\, p(x,y)dA$	$I_x = \iint y^2 f(x,y)dA$
$C = \frac{1}{M}(M_y, M_x)$	$\bar{y} = \frac{M_x}{M} = \frac{1}{M}\iint y\, p(x,y)dA$	$I_y = \iint x^2 f(x,y)dA$
$C = (\bar{x}, \bar{y})$	$M_x = \iint y\, p(x,y)dA$	$I_0 = \iint (x^2 + y^2) f(x,y)dA$
$C = \frac{1}{M}(\iint x\, p(x,y)dA, \iint y\, p(x,y)dA)$	$M_y = \iint x\, p(x,y)dA$	$I_0 = I_x + I_y$

Triple Integrals

General

$\int_a^b \int_c^d \int_e^f f(x)\, f(y)\, f(z)dzdydx = \int_a^b f(x)dx \int_c^d f(y)dy \int_e^f f(z)dz$

$\int_e^f \int_c^d \int_a^b f(x,y,z)\, dxdydz = \Delta x \Delta y \Delta z \sum_i^m \sum_j^n \sum_k^o f(x_i, y_j, z_k)$

Volume			Average Value
$V(x,y,z) = \iiint 1\, dV$	$V(r,\Theta,z) = \iiint r\, dV$	$V(\rho,\Theta,\emptyset) = \iiint \rho^2 sin\emptyset\, dV$	$Avg = \frac{1}{Volume(D)} \iiint f(x,y,z)dV$

Probability

$\int_e^f \int_c^d \int_a^b f(x,y,z)\, dxdydz = 1$ (CDF)

Mass

$M = \iiint p(x,y,z)dV$	$\bar{x} = \frac{M_{yz}}{M} = \frac{1}{M}\iiint xp(x,y,z)dV$
$C = \frac{1}{M}(M_{yz}, M_{xz}, M_{xy})$	$\bar{y} = \frac{M_{xz}}{M} = \frac{1}{M}\iiint yp(x,y,z)dV$
$C = \frac{1}{M}(\iiint xp(x,y,z)dV, \iiint yp(x,y,z)dV, \iiint zp(x,y,z)dV)$	$\bar{z} = \frac{M_{xy}}{M} = \frac{1}{M}\iiint zp(x,y,z)dV$

Vector Calculus

Line Integrals $\vec{F} = \langle P, Q, R\rangle$ $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle$

$\int f(x,y,z)ds = \int_a^b f(x(t),y(t),z(t))\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}dt$	$\int \vec{F}\, dr = \int P\, dx + Q\, dy + R\, dz$	$\int \vec{F}\, dr = \int_a^b F(r) \cdot \frac{dr}{dt} dt$
$\int f(x,y,z)dx = \int_a^b f(x(t),y(t),z(t))\frac{dx}{dt}dt$	$\int f(x,y,z)dy = \int_a^b f(x(t),y(t),z(t))\frac{dy}{dt}dt$	$\int f(x,y,z)dz = \int_a^b f(x(t),y(t),z(t))\frac{dz}{dt}dt$

Green's THM	**Surface Integral**	**Divergence**
$\oint Pdx + Qdy = \iint \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA$	$\iint f(x,y,z)ds = \iint f(x,y,z(x,y))\sqrt{{z_x}^2 + {z_y}^2 + 1}\, dA$	$div(\vec{F}) = \nabla \cdot \vec{F} = \frac{\partial \mathrm{P}}{\partial \mathrm{x}} + \frac{\partial \mathrm{Q}}{\partial \mathrm{y}} + \frac{\partial \mathrm{R}}{\partial \mathrm{z}}$

Curl	**Flux**	**Stokes THM**	**Divergence THM**
$curl(\vec{F}) = \nabla \times \vec{F} = \langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\rangle$	$\iint F \cdot dS = \iint -P\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} + R\, dA$	$\int \vec{F}\, dr = \iint \mathrm{curl}\vec{F} \cdot dS$	$\iint F \cdot dS = \iiint \mathrm{div}\vec{F}\, dV$