



MOTO

Whether you love or hate math class, this formula sheet will help you survive math in one piece.

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Algebra – Calculus III

Formulas & Functions

<u>Arithmetic Operations</u>	<u>Exponents/Radicals</u>			<u>Logs</u>	<u>Circle</u>
$a(b \pm c) = ab \pm ac$	$x^a x^b = x^{a+b}$			$\log_b 1 = 0$	$(x-h)^2 + (y-k)^2 = r^2$
$\frac{a}{b} \pm \frac{x}{y} = \frac{ay \pm bx}{by}$	$\frac{x^a}{x^b} = x^{a-b}$			$\log_b b = 1$	<u>Ellipse</u>
$\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$	$(x^a)^b = x^{ab}$			$\log_b(xy) = \log_b x + \log_b y$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
$\frac{a}{b} * \frac{x}{y} = \frac{ax}{by}$	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$			$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
$\frac{a/b}{x/y} = \frac{ay}{bx}$	$(xy)^a = x^a y^a$			$\log_b x^n = n \log_b x$	$c^2 = a^2 - b^2$ (foci)
<u>Linear</u>	$x^{\frac{a}{b}} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$			$\log_b x = y \leftrightarrow b^y = x$	<u>Hyperbola</u>
$Ax + By = C$	$\sqrt[a]{xy} = \sqrt[a]{x} * \sqrt[a]{y}$			$b^{\log_b x} = x$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$\sqrt[a]{\frac{x}{y}} = \frac{\sqrt[a]{x}}{\sqrt[a]{y}}$			$\log_b x = \frac{\log_a x}{\log_a b}$	$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$
$y = mx + b$	<u>Functions</u>			<u>Natural Logs</u>	$y = \pm \left(\frac{b}{a}\right)(x-h) + k$ (asm)
$y - y_1 = m(x - x_1)$	$y = af(b(x-h)) + k$			$\ln 1 = 0$	$c^2 = a^2 + b^2$ (foci)
<u>Quadratic</u>	a : vertical stretch/shrink			$\ln e = 1$	<u>Parabola</u>
$ax^2 + bx + c = 0$	b : horizontal stretch/shrink			$\ln(xy) = \ln x + \ln y$	$y = a(x-h)^2 + k$
$a(x-h)^2 + k = 0$	h : horizontal shift			$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$	$x = a(y-k)^2 + h$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	h : vertical shift			$\ln x^n = n \ln x$	$a = \frac{1}{4c}$ (focus)
<u>Special Factor Rules</u>	<u>Domain/Range</u>			$\ln x = y \leftrightarrow e^y = x$	<u>Rational</u>
$x^2 - a^2 = (x-a)(x+a)$	<u>Interval</u>			$e^{\ln x} = x$	$y = \frac{1}{f(x)}$
$x^2 + a^2 = (x-ai)(x+ai)$	$x \in [a, b]$	$x \in (a, b)$	$x \in (-\infty, \infty)$	$\log_b x = \frac{\ln x}{\ln b}$	$f(x) \neq 0$
$x^3 \pm a^3 = (x \pm a)(x^2 \mp ab + b^2)$	$y \in [c, d]$	$y \in (c, d)$	$y \in (-\infty, \infty)$	<u>Temp Conversions</u>	<u>Pythagorean THM</u>
<u>Binomial Theorem</u>	<u>Set Builder</u>			$K = 273 + ^\circ\text{C}$	$a^2 + b^2 = c^2$
$(x+a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$	$a \leq x \leq b$	$a < x < b$	\mathbb{R}	$^\circ\text{C} = 5(^{\circ}\text{F} - 32)/9$	
	$c \leq y \leq d$	$c < y < d$	\mathbb{R}	$^\circ\text{F} = 9^{\circ}\text{C}/5 + 32$	

Quad.	Angle°	RA°	Angle (rad)	RA	$\cos\theta$	$\sin\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
X	0	0	0	0	1	0	0	1	und	und
I	30	30	$\pi/6$	$\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/3$	$2/\sqrt{3}$	2	$\sqrt{3}$
I	45	45	$\pi/4$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\sqrt{2}$	$\sqrt{2}$	1
I	60	60	$\pi/3$	$\pi/3$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}$	2	$2/\sqrt{3}$	$\sqrt{3}/3$
X	90	90	$\pi/2$	$\pi/2$	0	1	und	und	1	0
II	120	60	$2\pi/3$	$\pi/3$	$-1/2$	$\sqrt{3}/2$	$-\sqrt{3}$	-2	$2/\sqrt{3}$	$-\sqrt{3}/3$
II	135	45	$3\pi/4$	$\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
II	150	30	$5\pi/6$	$\pi/6$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}/3$	$-2/\sqrt{3}$	2	$-\sqrt{3}$
X	180	0	π	0	-1	0	0	-1	und	und
III	210	30	$7\pi/6$	$\pi/6$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$	$-2/\sqrt{3}$	-2	$\sqrt{3}$
III	225	45	$5\pi/4$	$\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
III	240	60	$4\pi/3$	$\pi/3$	$-1/2$	$-\sqrt{3}/2$	$\sqrt{3}/3$	-2	$-2/\sqrt{3}$	$\sqrt{3}/3$
X	270	90	$3\pi/2$	$\pi/2$	0	-1	und	und	-1	0
IV	300	60	$5\pi/3$	$\pi/3$	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}/3$	2	$-2/\sqrt{3}$	$-\sqrt{3}/3$
IV	315	45	$7\pi/4$	$\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
IV	330	30	$11\pi/6$	$\pi/6$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$	$2/\sqrt{3}$	-2	$-\sqrt{3}$
X	360	0	2π	0	1	0	0	1	und	und

Trig Identities		Geometric				Polar basics	
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$A_{\square} = s^2$	$A_{rec} = lw$	$A_{\Delta} = \frac{bh}{2}$	$A_{\circ} = \pi r^2$	$x = r\cos\theta$	$y = r\sin\theta$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$P_{\square} = 4s$	$P_{rec} = 2l + 2w$	$P_{\Delta} = a + b + c$	$C = 2\pi r$	$r = \sqrt{x^2 + y^2}$	$\theta = \arctan(\frac{y}{x})$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$	$V_{\square} = s^3$	$V_{prism} = Bh$	$V_{pyr} = \frac{lw h}{3}$	$V_{\circ} = \frac{\pi r^2 h}{3}$		

<u>Identities/Trig Formulas</u>				
Distance/Midpoint	Radian / Degree	Velocity	Angular Speed	Arc length / Area of Sector
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$1^\circ = \frac{180}{\pi} rad$	$v = \frac{s}{t}$	$v = r\omega$	$s = r\theta$
$m = \left(\frac{(x_2 + x_1)}{2}, \frac{(y_2 + y_1)}{2}\right)$	$1 rad = \frac{\pi}{180}^\circ$	$v = \frac{r\theta}{t}$	$\omega = \frac{\theta}{t}$	$\hat{A} = \frac{1}{2}r^2\theta$
Pythagorean Identities	Even/Odd Identities	Power Reducers	Sum/Difference Identities	
$\sin^2 x + \cos^2 x = 1$	$\sin(-x) = -\sin x$	$\sin^2 x = \frac{1 - \cos 2x}{2}$	$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	
$\tan^2 x + 1 = \sec^2 x$	$\cos(-x) = \cos x$	$\cos^2 x = \frac{1 + \cos 2x}{2}$	$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	
$1 + \cot^2 x = \csc^2 x$	$\tan(-x) = -\tan x$	$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$	$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	
Double Angle Identities	Half Angles	Co-function Identities		Reciprocal Identities
$\sin 2x = 2\sin x \cos x$	$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$	$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$		$\sin x = \frac{1}{\csc x}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$	$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$		$\cos x = \frac{1}{\sec x}$
$\cos 2x = 1 - 2\sin^2 x$	$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$	$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$		$\tan x = \frac{1}{\cot x}$
$\cos 2x = 2\cos^2 x - 1$	$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1 + \cos x}$	$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$		$\sec x = \frac{1}{\cos x}$
$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	$\tan\left(\frac{x}{2}\right) = \frac{(1 - \cos x)}{\sin x}$	$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$		$\csc x = \frac{1}{\sin x}$
$\tan 2x = \frac{\sin 2x}{\cos 2x}$	$\tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$		$\cot x = \frac{1}{\tan x}$
<u>Laws of Sines/Cosines</u>				
$\frac{\sin A}{a} = \frac{\sin B}{b}$	$A = \frac{ab}{2} \sin C$	$c^2 = a^2 + b^2 - 2ab * \cos C$	Semi perimeter/Area	
$\frac{\sin B}{b} = \frac{\sin C}{c}$	$A = \frac{ac}{2} \sin B$	$b^2 = a^2 + c^2 - 2ac * \cos B$	$A = \sqrt{s(s - a)(s - b)(s - c)}$	
$\frac{\sin A}{a} = \frac{\sin C}{c}$	$A = \frac{bc}{2} \sin A$	$a^2 = b^2 + c^2 - 2bc * \cos A$	$s = \frac{a+b+c}{2}$	

<u>Standard Derivatives</u>				
Basic Rules	Trig	Inverse Trig	Exponents	Logs
$\frac{d}{dx} k = 0$	$\frac{d}{dx} \sin x = \cos x$	$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx} e^x = e^x$	$\frac{d}{dx} \ln x = \frac{1}{x}$
$\frac{d}{dx} (ku) = ku'$	$\frac{d}{dx} \cos x = -\sin x$	$\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx} b^x = b^x \ln b$	$\frac{d}{dx} \log_b x = \frac{1}{x \ln b}$
$\frac{d}{dx} (u \pm v) = u' \pm v'$	$\frac{d}{dx} \tan x = \sec^2 x$	$\frac{d}{dx} \arctan x = \frac{1}{x^2+1}$	Definition	LH's Rules
$\frac{d}{dx} (uv) = uv' + vu'$	$\frac{d}{dx} \cot x = -\csc^2 x$	$\frac{d}{dx} \operatorname{arccot} x = \frac{-1}{x^2+1}$	$\frac{d}{dx} f = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$	$\lim_{x \rightarrow \infty} \frac{u}{v} = \lim_{x \rightarrow \infty} \frac{u'}{v'}$
$\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{vu'-uv'}{v^2}$	$\frac{d}{dx} \sec x = \sec x \tan x$	$\frac{d}{dx} \operatorname{arcsec} x = \frac{1}{x \sqrt{x^2-1}}$	$\frac{d}{dx} f = \lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$	$\lim_{x \rightarrow 0} \frac{u}{v} = \lim_{x \rightarrow 0} \frac{u'}{v'}$
$\frac{d}{dx} x^n = nx^{n-1}$	$\frac{d}{dx} \csc x = -\csc x \cot x$	$\frac{d}{dx} \operatorname{arccsc} x = \frac{-1}{x \sqrt{x^2-1}}$		
<u>Chain Rules/Misc.</u>				
Basic Rules	Trig	Inverse Trig	Exponents	Logs
$\frac{d}{dx} u(v(x)) = u'(v) * v'(x)$	$\frac{d}{dx} \sin u = u' \cos u$	$\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}}$	$\frac{d}{dx} e^u = u' e^u$	$\frac{d}{dx} \ln u = \frac{u'}{u}$
$\frac{d}{dx} u^n = u' * nu^{n-1}$	$\frac{d}{dx} \cos u = -u' \sin u$	$\frac{d}{dx} \arccos u = \frac{-u'}{\sqrt{1-u^2}}$	$\frac{d}{dx} b^u = u' b^u \ln b$	$\frac{d}{dx} \log_b u = \frac{u'}{u \ln b}$
Linear Approx./Differential	$\frac{d}{dx} \tan u = u' \sec^2 u$	$\frac{d}{dx} \arctan u = \frac{u'}{u^2+1}$	Polar	
$L(x) = f'(a)(x-a) + f(a)$	$\frac{d}{dx} \cot u = -u' \csc^2 u$	$\frac{d}{dx} \operatorname{arccot} u = \frac{-u'}{u^2+1}$	$\frac{dy}{dx} = \frac{\left[\frac{dr}{d\theta} \sin \theta + r \cos \theta\right]}{\left[\frac{dr}{d\theta} \cos \theta - r \sin \theta\right]}$	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$
$\Delta y = f(x+h) - f(x)$	$\frac{d}{dx} \sec u = u' \sec u \tan u$	$\frac{d}{dx} \operatorname{arcsec} u = \frac{u'}{u \sqrt{u^2-1}}$		
$dy = f'(x) dx$	$\frac{d}{dx} \csc u = -u' \csc u \cot u$	$\frac{d}{dx} \operatorname{arccsc} u = \frac{-u'}{u \sqrt{u^2-1}}$		
<u>Special Chain Rules</u>				
Function-Power			Implicit/Logarithmic Differentiation	
$\frac{d}{dx} \sin^n u = nu' \sin^{n-1} u \cos u$	$\frac{d}{dx} \sec^n u = nu' \sec^n u \tan u$	$\frac{d}{dx} e^{u^n} = nu' u^{n-1} e^{u^n}$	$[f(x) + f(y) = a] \rightarrow f'(x) + \frac{dy}{dx} f'(y) = 0$	
$\frac{d}{dx} \cos^n u = -nu' \cos^{n-1} u \sin u$	$\frac{d}{dx} \csc^n u = -nu' \csc^n u \cot u$	$\frac{d}{dx} b^{u^n} = nu' u^{n-1} b^{u^n} \ln b$	$\frac{d}{dx} x^x = x^x (\ln(x) + 1)$	
$\frac{d}{dx} \tan^n u = nu' \tan^{n-1} u \sec^2 u$	$\frac{d}{dx} \cot^n u = -nu' \cot^{n-1} u \csc^2 u$	$\frac{d}{dx} \ln^n u = \frac{nu' \ln^{n-1} u}{u}$	$\frac{d}{dx} \left[\frac{(x \pm a)^n}{(x \pm b)^m}\right] = \left[\frac{(x \pm a)^n}{(x \pm b)^m}\right] \left[\frac{n}{(x \pm a)} - \frac{m}{(x \pm b)}\right]$	

<u>Basic Integrals</u>	<u>Integration By Parts</u>	<u>Trig Sub Rules</u>
$\int k \, dx = kx + c$	$\int u \, dv = uv - \int v \, du$ (Remember <i>LIPTE</i>)	$\sqrt{a^2 + x^2} \mid x = a \tan \theta$
$\int k f \, dx = k \int f \, dx$	Special IBP rules	$\sqrt{a^2 - x^2} \mid x = a \sin \theta$
$\int u \pm v \, dx = \int u \, dx \pm \int v \, dx$	$\int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$	$\sqrt{x^2 - a^2} \mid x = a \sec \theta$
$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$	$\int x^n \sin(ax) \, dx = -\frac{x^n \cos(ax)}{a} + \frac{n}{a} \int x^{n-1} \cos(ax) \, dx$	<u>Definite Integrals</u>
$\int e^x \, dx = e^x + c$	$\int x^n \cos(ax) \, dx = \frac{x^n \sin(ax)}{a} - \frac{n}{a} \int x^{n-1} \sin(ax) \, dx$	$\int_a^b f(x) \, dx = F(b) - F(a)$
$\int b^x \, dx = \frac{b^x}{\ln b} + c$	$\int \arcsin x \, dx = x \arcsin x + \sqrt{1 - x^2} + c$	$\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$
$\int \frac{1}{x} \, dx = \ln x + c$	$\int \arccos x \, dx = x \arccos x - \sqrt{1 - x^2} + c$	<u>IBP Priority (LIPTE)</u>
$\int \sin x \, dx = -\cos x + c$	$\int \arctan x \, dx = x \arctan x - \frac{\ln(1 + x^2)}{2} + c$	Log
$\int \cos x \, dx = \sin x + c$	$\int e^{ax} \sin(bx) \, dx = \frac{e^{ax}(a \sin(bx) - b \cos(bx))}{a^2 + b^2} + c$	Inverse trig
$\int \sec x \tan x \, dx = \sec x + c$	$\int e^{ax} \cos(bx) \, dx = \frac{e^{ax}(a \cos(bx) + b \sin(bx))}{a^2 + b^2} + c$	Polynomial
$\int \csc x \cot x \, dx = -\csc x + c$	$\int x^n \ln ax \, dx = \frac{(x^{n+1} (n+1) \ln(ax) - 1)}{(n+1)^2} + c$	Trig
$\int \sec^2 x \, dx = \tan x + c$	$\int \ln ax \, dx = x(\ln(ax) - 1) + c$	Exponent
$\int \csc^2 x \, dx = -\cot x + c$	$\int \sec^3 x \, dx = \frac{\sec x \tan x + \ln \sec x + \tan x }{2} + c$	<u>FTC</u>
$\int \sec x \, dx = \ln \sec x + \tan x + c$	$\int \csc^3 x \, dx = \frac{\csc x \cot x - \ln \csc x + \cot x }{2} + c$	$\frac{d}{dx} \int_a^x f(t) \, dt = f(x)$
$\int \csc x \, dx = -\ln \csc x + \cot x + c$	$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$	
$\int \tan x \, dx = -\ln \cos x + c$	$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$	
$\int \cot x \, dx = \ln \sin x + c$	<u>Partial Fraction Decomposition</u>	
$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin\left(\frac{x}{a}\right) + c$	$\int \frac{P(x)}{(ax+b)(cx+z)} \, dx = \int \frac{A}{(ax+b)} \, dx + \int \frac{B}{(cx+z)} \, dx$	
$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c$	$\int \frac{P(x)}{(ax+b)(cx+z)^n} \, dx = \int \frac{A}{(ax+b)} \, dx + \int \frac{B}{(cx+z)} \, dx + \int \frac{C}{(cx+z)^2} \, dx + \dots + \int \frac{U}{(cx+z)^n} \, dx$	
$\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right) + c$	$\int \frac{P(x)}{(ax+b)(ux^2+vx+w)} \, dx = \int \frac{A}{(ax+b)} \, dx + \int \frac{Ux+V}{(ux^2+vx+w)} \, dx$	

Arc Length		Area	Volumes	Probability		
Rectangular			Discs	PDF		
$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$	$A_x = \int_a^b f(x) - g(x) dx$	$V_x = \pi \int_a^b f(x)^2 dx$	$\int_{-\infty}^{\infty} f(x)dx = 1$			
	$A_y = \int_c^d f(y) - g(y) dy$	$V_y = \pi \int_c^d f(y)^2 dy$	Median PDF			
Parametric		Washers	$\int_m^{\infty} f(x)dx = \frac{1}{2}$			
$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$	$A = \int_a^b y dx$	$V_x = \pi \int_a^b f(x)^2 - g(x)^2 dx$	Mean PDF			
	$A = \int_c^d x dy$	$V_y = \pi \int_c^d f(y)^2 - g(y)^2 dy$	$\mu = \int_{-\infty}^{\infty} fx(x)dx$			
Polar		Shells	Average Value			
$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$	$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$	$V_x = 2\pi \int_c^d y[f(y) - g(y)]dy$	$S = \frac{1}{b-a} \int_a^b f(x)dx$			
		$V_y = 2\pi \int_a^b x[f(x) - g(x)]dx$				
Improper Integrals		Tabular (PT)		Tabular (PE)		
$\int_a^{\infty} f(x)dx = \lim_{b \rightarrow \infty} \int_a^b f(x)dx$	u	dv	\pm	u	dv	\pm
$\int_{-\infty}^b f(x)dx = \lim_{a \rightarrow -\infty} \int_a^b f(x)dx$	x ²	cosx	+	x ²	e ^x	+
$\int_{-\infty}^{\infty} f(x)dx = \lim_{a \rightarrow -\infty} \int_a^b f(x)dx + \lim_{b \rightarrow \infty} \int_a^b f(x)dx$	2x	sinx	-	2x	e ^x	-
Manual Integration	2	cosx	+	2	e ^x	+
('n' is the number of rectangles to split by)	0	sinx	-	0	e ^x	-
	0	cosx	+	0	e ^x	+
$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=0}^n f(x_i)\Delta x$	$\Delta x = \frac{b-a}{n}$	$\underline{x^2 sinx - 2xcosx + 2sinx + c}$		$\underline{e^x(x^2 - 2x + 2) + c}$		
Midpoint Rule: $\int_a^b f(x)dx = \Delta x [f_0 + f_1 + f_2 + f_3 + \cdots + f_{n-3} + f_{n-2} + f_{n-1} + f_n]$ where $x = \frac{x_{i-1} + x_i}{2}$						
Trapezoid Rule: $\int_a^b f(x)dx = \frac{\Delta x}{2} [f_0 + 2(f_1 + f_2 + f_3 + \cdots + f_{n-3} + f_{n-2} + f_{n-1}) + f_n]$						
Simpson Rule: $\int_a^b f(x)dx = \frac{\Delta x}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \cdots + 2f_{n-4} + 4f_{n-3} + 2f_{n-2} + 4f_{n-1} + f_n]$						

Diff. EQ			<table><tr><th>Test</th><th>Series</th><th>Converges</th><th>Diverges</th><th>Comment</th></tr><tr><td>nth -term</td><td>$\sum_{n=1}^{\infty} a_n$</td><td></td><td>$\lim_{n \rightarrow \infty} a_n \neq 0$</td><td>Cannot be used to show convergence</td></tr><tr><td>Geometric series</td><td>$\sum_{n=0}^{\infty} ar^{n-1}$</td><td>$r < 1$</td><td>$r \geq 1$</td><td>Sum is $S = \frac{a}{1-r}$</td></tr><tr><td>Telescoping Series</td><td>$\sum_{n=1}^{\infty} (a_n - a_{n+1})$</td><td>$\lim_{n \rightarrow \infty} a_n = L$</td><td></td><td>Sum is $S = a_1 - L$</td></tr><tr><td>p-series</td><td>$\sum_{n=1}^{\infty} \frac{1}{n^p}$</td><td>$p > 1$</td><td>$p \leq 1$</td><td></td></tr><tr><td>Alternating Series</td><td>$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$</td><td>$0 < a_{n+1} \leq a_n$ and $\lim_{n \rightarrow \infty} a_n = 0$</td><td></td><td>Remainder $R_n = S - S_n$: $R_n \leq a_{n+1}$</td></tr><tr><td>Integral (f is continuous, positive, and decreasing)</td><td>$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$</td><td>$\int_1^{\infty} f(x)dx$ converges</td><td>$\int_1^{\infty} f(x)dx$ diverges</td><td>Remainder: $0 \leq R_n \leq \int_n^{\infty} f(x)dx$</td></tr><tr><td>Root</td><td>$\sum_{n=1}^{\infty} a_n$</td><td>$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} < 1$</td><td>$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} > 1$</td><td>Test is inconclusive if $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 1$.</td></tr><tr><td>Ratio</td><td>$\sum_{n=1}^{\infty} a_n$</td><td>$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1$</td><td>$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1$</td><td>Test is inconclusive if $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$.</td></tr><tr><td>Direct Comparison ($a_n, c_n, d_n > 0$)</td><td>$\sum_{n=1}^{\infty} a_n$</td><td>$0 \leq a_n \leq c_n$ and $\sum_{n=1}^{\infty} c_n$ converges</td><td>$0 \leq d_n \leq a_n$ and $\sum_{n=1}^{\infty} d_n$ diverges</td><td></td></tr><tr><td>Limit Comparison ($a_n, c_n, d_n > 0$)</td><td>$\sum_{n=1}^{\infty} a_n$</td><td>$\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = L > 0$ and $\sum_{n=1}^{\infty} c_n$ converges</td><td>$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = L > 0$ and $\sum_{n=1}^{\infty} d_n$ diverges</td><td></td></tr></table>				Test	Series	Converges	Diverges	Comment	nth -term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	Cannot be used to show convergence	Geometric series	$\sum_{n=0}^{\infty} ar^{n-1}$	$ r < 1$	$ r \geq 1$	Sum is $S = \frac{a}{1-r}$	Telescoping Series	$\sum_{n=1}^{\infty} (a_n - a_{n+1})$	$\lim_{n \rightarrow \infty} a_n = L$		Sum is $S = a_1 - L$	p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$		Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n \rightarrow \infty} a_n = 0$		Remainder $R_n = S - S_n$: $ R_n \leq a_{n+1}$	Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	Remainder: $0 \leq R_n \leq \int_n^{\infty} f(x)dx$	Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} < 1$	$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} > 1$	Test is inconclusive if $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 1$.	Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1$	Test is inconclusive if $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$.	Direct Comparison ($a_n, c_n, d_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq c_n$ and $\sum_{n=1}^{\infty} c_n$ converges	$0 \leq d_n \leq a_n$ and $\sum_{n=1}^{\infty} d_n$ diverges		Limit Comparison ($a_n, c_n, d_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \frac{a_n}{c_n} = L > 0$ and $\sum_{n=1}^{\infty} c_n$ converges	$\lim_{n \rightarrow \infty} \frac{a_n}{d_n} = L > 0$ and $\sum_{n=1}^{\infty} d_n$ diverges	
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General Forms																																																													
$\frac{dy}{dx} = f(x)$	$\frac{dy}{dx} = f(y)$	$\frac{dy}{dx} = f(x, y)$																																																											
Newton's Law of Cooling																																																													
$\frac{dT}{dt} = k(T - T_a)$																																																													
Growth/Decay																																																													
$\frac{dP}{dt} = kP$	$\frac{dP}{dt} = kP(M - P)$	$\frac{dP}{dt} = k(M - P)$																																																											
Mixing																																																													
$\frac{dx}{dt} = r_i c_i - \frac{r_o x}{V}$																																																													
Euler Method																																																													
$y_{n-1} = y_n + f(x_n, y_n) * h$																																																													
Series																																																													
Arithmetic Series	Alternating Series																																																												
$S_n = \frac{n(a + a_n)}{2}$	$\sum_{n=0}^{\infty} (-1)^n a_n$																																																												
Power Series																																																													
$\sum_{n=0}^{\infty} x^n$																																																													
$S_n = \frac{1}{1-x}$																																																													
Taylor Series			Expanded																																																										
$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$			$f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \frac{f^4(a)(x-a)^4}{4!} + \dots + \frac{f^{(n)}(a)(x-a)^n}{n!}$																																																										
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$			$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \frac{x^9}{9!} + \frac{x^{10}}{10!} + \dots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$																																																										
$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$			$\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \dots$																																																										
$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$			$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \dots$																																																										

Vectors						
Cross Product => <i>vector</i>		Dot Product => <i>scalar</i>	Triple Product	Projections		
$\vec{u} \times \vec{v} = (\vec{u} \vec{v} \sin\theta)\vec{n}$		$\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$	$A = \vec{u} \cdot (\vec{v} \times \vec{w})$	$comp_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} }$		
$ \vec{u} \times \vec{v} = \vec{u} \vec{v} \sin\theta$		$\vec{u} \cdot \vec{v} = \vec{u} \vec{v} \cos\theta$	$V = (\vec{u} \times \vec{v}) \cdot \vec{w}$			
$\vec{u} \times \vec{v} = \langle \det \begin{bmatrix} u_2 & u_3 \\ v_2 & v_3 \end{bmatrix}, -\det \begin{bmatrix} u_1 & u_3 \\ v_1 & v_3 \end{bmatrix}, \det \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix} \rangle$			Magnitude		$proj_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} } * \frac{\vec{a}}{ \vec{a} }$	
			$ \vec{u} = \sqrt{u_1^2 + u_2^2 + u_3^2}$			
Tangent Vectors/Curvature						
$\vec{r}'(t) = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$	$\vec{T}(t) = \frac{\vec{r}'(t)}{ \vec{r}'(t) }$	$\vec{N}(t) = \frac{\vec{T}'(t)}{ \vec{T}'(t) }$	$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t) = \frac{\vec{r}'(t) \times \vec{r}''(t)}{ \vec{r}'(t) \times \vec{r}''(t) }$	Arc Length		
$k = \frac{ \vec{T}'(t) }{ \vec{r}'(t) }$	$k = \frac{ \vec{r}'(t) \times \vec{r}''(t) }{ \vec{r}'(t) ^3}$	$k = \frac{ f''(x) }{(1+[f'(x)]^2)^{3/2}}$	$k = \frac{ d\vec{T} }{ d\vec{s} }$	$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$		
Coordinate Systems						
Cylindrical $f(r, \theta, z)$ Spherical $f(\rho, \theta, \phi)$						
$x = r\cos\theta$	$y = r\sin\theta$	$z = z$	$\theta = \arctan\left(\frac{y}{x}\right)$	$r = \sqrt{x^2 + y^2}$		
$x = \rho\sin\phi\cos\theta$	$y = \rho\sin\phi\sin\theta$	$z = \rho\cos\phi$	$\rho = \sqrt{x^2 + y^2 + z^2}$	$\phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$		
Partial Derivatives						
General		Gradient Vector	LaGrange	Normal Line	Implicit	Directional Derivative
$f_x = \frac{\partial f}{\partial x} \quad f_{xy} = \frac{\partial f}{\partial x \partial y}$		$\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$	$\nabla f = \lambda \nabla g$	$\frac{(x-a)}{f_a} = \frac{(y-b)}{f_b} = \frac{(z-c)}{f_c}$	$\frac{dy}{dx} = -\frac{f_x}{f_y}$	$D_u = \nabla f(x_0, y_0, z_0) \cdot \frac{\langle A, B, C \rangle}{\sqrt{A^2 + B^2 + C^2}}$
Tangent Planes			Laplace/Wave	Linearization/Differential		
$\nabla f \cdot \langle (x-a), (y-b), (z-c) \rangle = 0$			$f_{xx} + f_{tt} = 0$	$L(x, y, z) = f_a(x-a) + f_b(y-b) + f_c(z-c) + f(a, b, c)$		
$z-c = z_a(x-a) + z_b(y-b)$			$f_{tt} = a^2 f_{xx}$	$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$		
Chain rule						
$\frac{d}{dt} u(x, y, z) = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \frac{dz}{dt}$						
Second Derivative Tests						
$D = f_{xx} + f_{yy} - (f_{xy})^2$ D>0, local extrema, D<0, saddle point, D=0, inconclusive						
f_{xx} or $f_{yy} > 0$: local min, f_{xx} or $f_{yy} < 0$: local max, f_{xx} or $f_{yy} = 0$: test D						

Double Integrals			
General		Probability Average Value	Surface Area/Polar Conversion
$\int_a^b \int_c^d f(x, y) \, dydx = \int_c^d \int_a^b f(x, y) \, dxdy$		$\int_c^d \int_a^b f(x, y) \, dxdy = 1$ (CDF)	$A = \iint \sqrt{1 + f_x^2 + f_y^2} dA$
$\int_a^b \int_c^d f(x) f(y) \, dydx = \int_a^b f(x)dx \int_c^d f(y)dy$		$Avg = \frac{1}{Area(D)} \iint f(x, y)dA$	$\int_a^b \int_c^d f(x, y) \, dydx = \int_m^n \int_\alpha^\beta r f(r, \theta) \, drd\theta$
Manual Integration	$\Delta x = \frac{b-a}{m}$	$\Delta y = \frac{d-c}{n}$	Midpoint Rule
$\int_c^d \int_a^b f(x, y) \, dxdy = \sum_i^m \sum_j^n f(x_i, y_j) \Delta x \Delta y$		$\int_c^d \int_a^b f(x, y) \, dxdy = \sum_i^m \sum_j^n f(x_i, y_j) \Delta x \Delta y$ where $x = \frac{x_{i-1}+x_i}{2}, y = \frac{y_{j-1}+y_j}{2}$	
Mass		Inertia	
$M = \iint p(x, y)dA$	$\bar{x} = \frac{M_y}{M} = \frac{1}{M} \iint x p(x, y)dA$	$I_x = \iint y^2 f(x, y)dA$	
$C = \frac{1}{M} (M_y, M_x)$	$\bar{y} = \frac{M_x}{M} = \frac{1}{M} \iint y p(x, y)dA$	$I_y = \iint x^2 f(x, y)dA$	
$C = (\bar{x}, \bar{y})$	$M_x = \iint y p(x, y)dA$	$I_0 = \iint (x^2 + y^2) f(x, y)dA$	
$C = \frac{1}{M} (\iint x p(x, y)dA, \iint y p(x, y)dA)$	$M_y = \iint x p(x, y)dA$	$I_0 = I_x + I_y$	
Triple Integrals			
General			
$\int_e^f \int_c^d \int_a^b f(x, y, z) \, dxdydz = \lim_{m \rightarrow \infty} \sum_i^m \lim_{n \rightarrow \infty} \sum_j^n \lim_{o \rightarrow \infty} \sum_k^o f(x_i, y_j, z_k) \, \Delta x \Delta y \Delta z$			
$\int_a^b \int_c^d \int_e^f f(x) f(y) f(z) dzdydx = \int_a^b f(x)dx \int_c^d f(y)dy \int_e^f f(z)dz$			
Volume			Average Value
$V(x, y, z) = \iiint 1 \, dV$	$V(r, \theta, z) = \iiint r \, dV$	$V(\rho, \theta, \phi) = \iiint \rho^2 \sin\phi \, dV$	$Avg = \frac{1}{Volume(D)} \iiint f(x, y, z) dV$
Probability/Mass			
$\int_e^f \int_c^d \int_a^b f(x, y, z) \, dxdydz = 1$ (CDF)			
$M = \iiint p(x, y, z) dV$		$\bar{x} = \frac{M_{yz}}{M} = \frac{1}{M} \iiint xp(x, y, z) dV$	
$C = \frac{1}{M} (M_{yz}, M_{xz}, M_{xy})$		$\bar{y} = \frac{M_{xz}}{M} = \frac{1}{M} \iiint yp(x, y, z) dV$	
$C = \frac{1}{M} (\iiint xp(x, y, z) dV, \iiint yp(x, y, z) dV, \iiint zp(x, y, z) dV)$		$\bar{z} = \frac{M_{xy}}{M} = \frac{1}{M} \iiint zp(x, y, z) dV$	

<u>Vector Calculus</u>				
Line Integrals $\vec{F} = \langle P, Q, R \rangle \nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$				
$\int f(x, y, z) ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$		$\int \vec{F} dr = \int P dx + Q dy + R dz$		$\int \vec{F} dr = \int_a^b F(r) \cdot \frac{dr}{dt} dt$
$\int f(x, y, z) dx = \int_a^b f(x(t), y(t), z(t)) \frac{dx}{dt} dt$	$\int f(x, y, z) dy = \int_a^b f(x(t), y(t), z(t)) \frac{dy}{dt} dt$	$\int f(x, y, z) dz = \int_a^b f(x(t), y(t), z(t)) \frac{dz}{dt} dt$		
Green's THM	Surface Integral		Divergence	
$\oint P dx + Q dy = \iint \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA$	$\iint f(x, y, z) ds = \iint f(x, y, z(x, y)) \sqrt{z_x^2 + z_y^2 + 1} dA$		$div(\vec{F}) = \nabla \cdot \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$	
Curl	Flux		Stokes THM	Divergence THM
$curl(\vec{F}) = \nabla \times \vec{F} = \langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \rangle$	$\iint F \cdot dS = \iint -P \frac{\partial z}{\partial x} - Q \frac{\partial z}{\partial y} + R dA$		$\int \vec{F} dr = \iint curl \vec{F} \cdot dS$	$\iint F \cdot dS = \iiint div \vec{F} dV$