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Joint Probabilities. (25 points)

For the following questions, let Influenza = I, smokes = S, sore throat = ST, fever = F, bronchitis = B, coughing = C, wheezing = W

1. Use the product formula of Bayes nets and the conditional probability parameters specified by AIspace to compute the probability that: all nodes are True.

P(I, S, ST, F, B, C, W) = P(I) × P(S) × P(ST|I) × P(F|I) × P(B|I,S) × P(C|B) × P(W|B) =

= 0.05 × 0.2 × 0.3 × 0.9 × 0.99 × 0.8 × 0.6 = **0.00128304**

2. Use the product formula of Bayes nets and the conditional probability parameters specified by AIspace to compute the probability that: all nodes are True except for Sore Throat, and that Sore Throat is False.

P(I, S, ST, F, B, C, W) = P(I) × P(S) × P(ST|I) × P(F|I) × P(B|I,S) × P(C|B) × P(W|B) =

= 0.05 × 0.2 × 0.7 × 0.9 × 0.99 × 0.8 × 0.6 = **0.00299376**

3. Show how can you use these two joint probabilities to compute the probability that: all nodes other than Sore Throat are True. (Where the value of Sore Throat is unspecified.)

Add the two probabilities using marginalization, P(sore throat) = P(all nodes are true) + P(all nodes true except sore throat and that sore throat is false)

= **0.00128304 + 0.00299376 = 0.00427680**

4. Verify the product rule:

P(all nodes are True) = P(Sore Throat = True | all other nodes are True) x P(all other nodes are True).

By using the conditional probability definition P(A|B) = $\frac{P(A\ and\ B)}{P(B)}$, the above equation becomes

P(all nodes are True) = $\frac{P(Sore\ Throat=True\ and\ \text{all other nodes are True})}{\cancel{P(all\ other\ nodes\ are\ true)}}$ x ~~P(all other nodes are True)~~

P(all other nodes are True) is canceled out, leaving P(Sore Throat=True and all other nodes are True) which is the same as P(all nodes are True).

5. Compute the probability that Sore Throat is False and that Fever is False. (Hint: If you use the right formula, you only need 4 conditional probabilities.)

P(Sore Throat = False, Fever = False) = P(influenza = true) x P(sore throat = false | influenza = true) x P(fever = false | influenza = true) + P(influenza = false) x P(sore throat = false | influenza = false) x P(sore fever = false | influenza = false)

= (0.05 x 0.7 x 0.1) + (0.95 x 0.999 x 0.95) = **0.905100**

Probability to be Computed	Your Result
P(all nodes True)	**0.00128304**
P(Sore Throat = False, all other nodes True)	**0.00299376**
P(all nodes other than Sore Throat True)	**0.00427680**
P(all nodes are True) = P(Sore Throat = True | all other nodes are True) x P(all other nodes are True)	**0.00128304**
P(Sore Throat = False, Fever = False)	**0.905100**

Bayes' Theorem

1) 0.974820
2) P(influenza = F | wheezing = F) = P(wheezing = F | influenza = F) P(influenza = F) / P(wheezing = F)

   = (0.915090 x 0.95) / 0.891790
   = 0.974820

Independence. 20 points

1) "If no evidence is observed, Influenza and Smoking are probabilistically independent." To prove this, all possible values for influenza and smokes were tested, however, nothing changed, therefore smokes and influenza are probabilistically independent when no evidence is observed. If evidence is observed, then they would no longer be probabilistically independent.

For influenza = Unknown	P(smokes = T) = 0.2	P(smokes = F) = 0.8
For influenza = T	P(smokes = T) = 0.2	P(smokes = F) = 0.8
For influenza = F	P(smokes = T) = 0.2	P(smokes = F) = 0.8

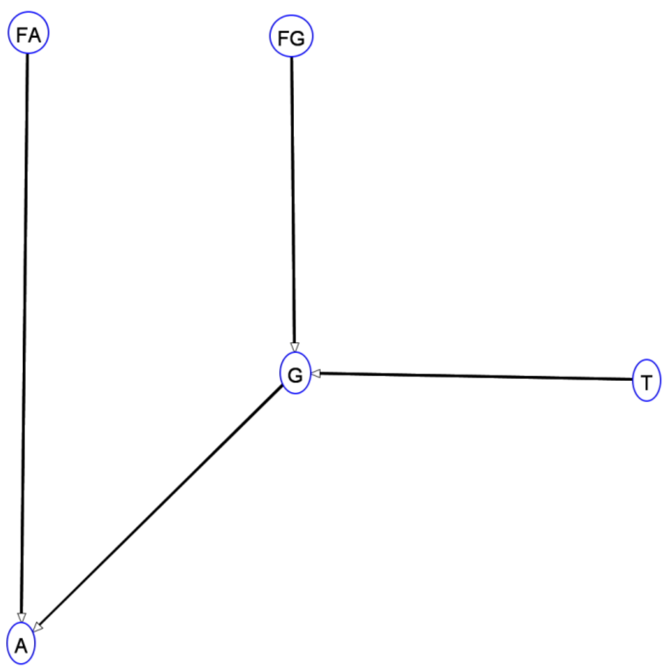
For smokes = Unknown	P(influenza = T) = 0.05	P(influenza = F) = 0.95
For smokes = T	P(influenza = T) = 0.05	P(influenza = F) = 0.95
For smokes = F	P(influenza = T) = 0.05	P(influenza = F) = 0.95

2) Markov's condition states that "node is conditionally independent of all other nodes in the network, given its parents, children, and children's parents".

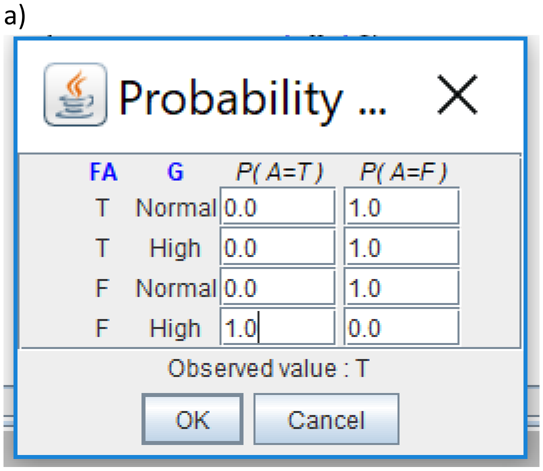
Influenza and Smoking are parent nodes themselves, therefore the parents part of the Markov condition does not apply since we are not given its parents. As for the children nodes, Influenza has children sore throat, fever, and bronchitis, and Smoking has child bronchitis; however, since there is no evidence observed, or in other words, is given, for their children, the children nodes do not affect the parent nodes Influenza and Smoking. All together, Influenza and Smoking are probabilistically independent

Chapter 14. Knowledge Representation With Bayesian Networks. 34 points total.

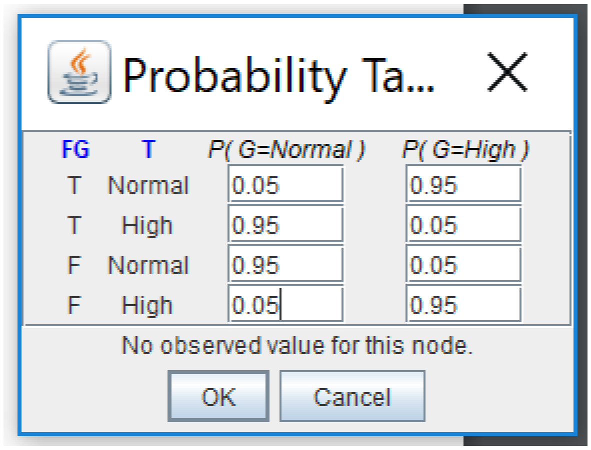
1)



2) a)

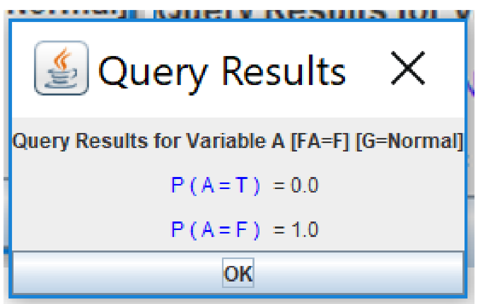


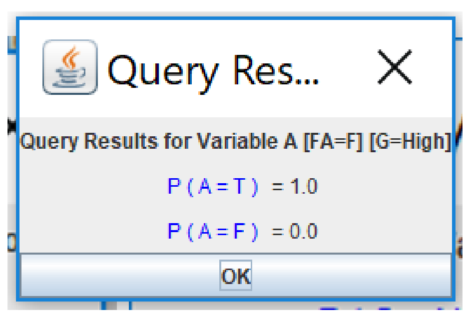
b)

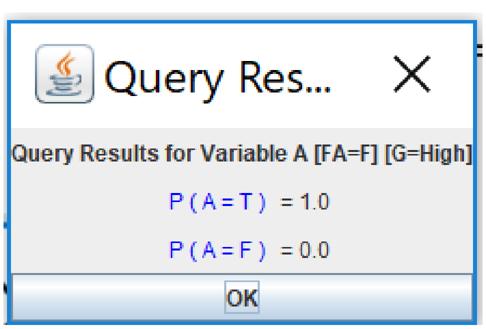


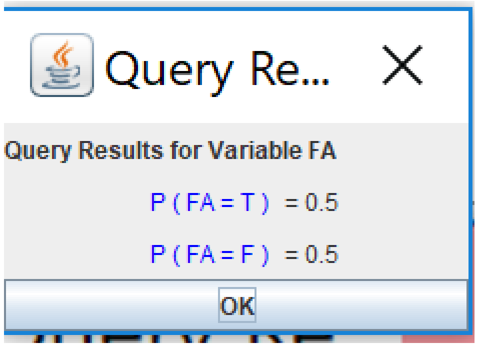
c) P(A = T) = P(A = T| G = normal, FA = F) x P(G = normal | T = high, FG = F)
+ P(A = T| G = high, FA = F) x P(G = high | T = high, FG = F)
= 0 x 0.05 + 1 x 0.95 = 0.95

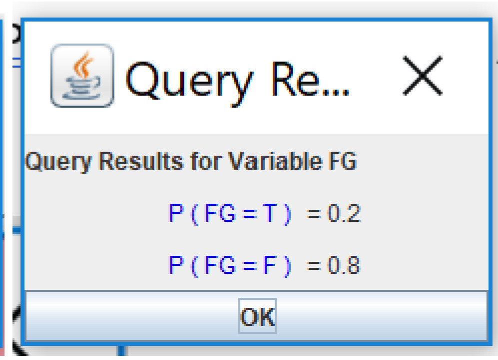
P(T) = P(T = normal) x P(G = normal | T = normal, FG = F) + P(T = high) x P(G = normal | T = high, FG = F)
= (0.8 x 0.05) + (0.2 x 0.95) = 0.23

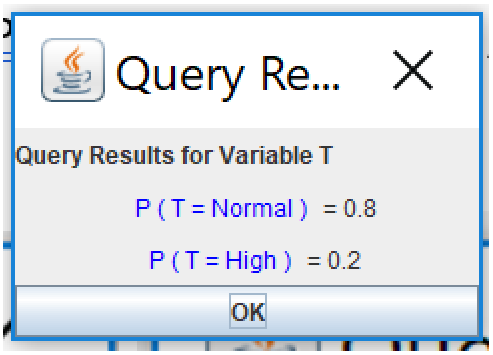






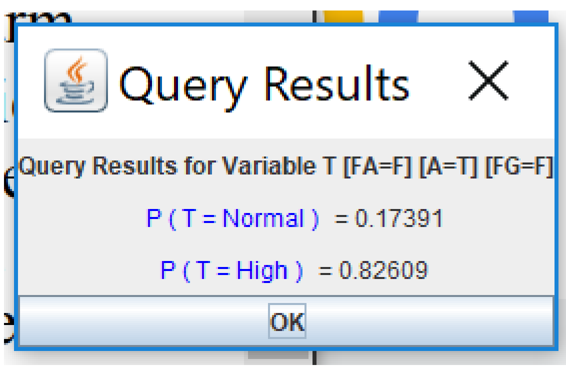






P(T = High | FA = F, FG = F, A = T) = P(T = High, FA = F, FG = F, A = T) / P(FA = F, FG = F, T)
= (0.2 x 0.5 x 0.8 x 0.95)/(0.5 x 0.8 x 0.23) = 0.076/0.092 = 0.826090

d) **0.826090**



Chapter 20. Bayesian Network Parameter Learning. 30 points total

1)

Probability Table for Hungry

Hun	P(Hun)
T	0.67
F	0.33

Probability Table for Patron

Pat	P(Pat)
Some	0.33
Full	0.58
None	0.08

Conditional Probability Table for Will Wait

Hun	Pat	WW	P (WW|Pat, Hun)
T	Some	T	1/3 = 0.33
T	Some	F	2/3 = 0.66
F	Some	T	1/1 = 1.00
F	Some	F	0/1 = 0.00 (undefined)
F	Full	T	0/2 = 0.00 (undefined)
F	Full	F	2/2 = 1.00
T	Full	T	2/5 = 0.40
T	Full	F	3/5 = 0.60
F	None	T	0/1 = 0.00 (undefined)
F	None	F	1/1 = 1.00
T	None	T	0/0 = 0.00 (undefined)
T	None	F	0/0 = 0.00 (undefined)

Dataset + joint probabilities

P(WW, Pat, Hun) = P(WW|Pat, Hun) x P(Pat) x P(Hun)

P(WW = T, Pat = some, Hun = T) = P(WW = T |Pat = some, Hun = T) x P(Pat = some) x P(Hun = T) = 0.33 x 0.33 x 0.67 = 0.072963

P(WW = F, Pat = some, Hun = T) = P(WW = F |Pat = some, Hun = T) x P(Pat = some) x P(Hun = T) = 0.66 x 0.33 x 0.67 = 0.145826

P(WW = T, Pat = some, Hun = F) = P(WW = T |Pat = some, Hun = F) x P(Pat = some) x P(Hun = F) = 1.00 x 0.33 x 0.33 = 0.108900

P(WW = F, Pat = some, Hun = F) = P(WW = F |Pat = some, Hun = F) x P(Pat = some) x P(Hun = F) = 0.00 x 0.33 x 0.33 = 0.00

P(WW = T, Pat = full, Hun = F) = P(WW = T |Pat = full, Hun = F) x P(Pat = full) x P(Hun = F) = 0.00 x 0.58 x 0.33 = 0.00

P(WW = F, Pat = full, Hun = F) = P(WW = F |Pat = full, Hun = F) x P(Pat = full) x P(Hun = F) =
1.00 x 0.58 x 0.33 = 0.191400

P(WW = T, Pat = full, Hun = T) = P(WW = T |Pat = full, Hun = T) x P(Pat = full) x P(Hun = T) =
0.40 x 0.58 x 0.67 = 0.155440

P(WW = F, Pat = full, Hun = T) = P(WW = F |Pat = full, Hun = T) x P(Pat = full) x P(Hun = F) =
0.60 x 0.58 x 0.67 = 0.233160

P(WW = T, Pat = none, Hun = F) = P(WW = T |Pat = none, Hun = F) x P(Pat = none) x P(Hun = F) =
0.00 x 0.08 x 0.33 = 0.00

P(WW = F, Pat = none, Hun = F) = P(WW = F |Pat = none, Hun = F) x P(Pat = none) x P(Hun = F) =
1.00 x 0.08 x 0.33 = 0.0264000

P(WW = T, Pat = none, Hun = T) = P(WW = T |Pat = none, Hun = T) x P(Pat = none) x P(Hun = T) =
0.00 x 0.08 x 0.67 = 0.00

P(WW = F, Pat = none, Hun = T) = P(WW = F |Pat = none, Hun = T) x P(Pat = none) x P(Hun = T) =
0.00 x 0.08 x 0.67 = 0.00

Hun	Pat	WW	P (WW, Pat, Hun)
T	Some	F	0.145826
T	Full	F	0.233160
F	Some	T	0.108900
T	Full	T	0.155440
F	Full	F	0.191400
T	Some	T	0.072963
F	Full	F	0.191400
T	Some	F	0.145826
T	Full	F	0.233160
T	Full	F	0.233160
F	None	F	0.0264000
T	Full	T	0.155440

The likelihood of the dataset D is approximately **$5.0046763563 \times 10^{-11}$**