

# Towards Non-Singular Black Holes

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July 19, 2025

## Abstract

Classical general relativity predicts that black holes contain a central singularity where spacetime curvature diverges. However, quantum effects may alter this picture. We propose a self-regulating black hole core model, in which Hawking-like radiation is generated locally as a function of spacetime curvature. This radiation counteracts gravitational collapse, leading to an equilibrium core of finite curvature. The model suggests that the black hole interior is a structured region rather than a classical singularity. We discuss the mathematical framework, implications for information retention, and directions for further study.

## 1 Introduction

Black holes lie at the intersection of general relativity (GR) and quantum field theory (QFT): they are the most extreme classical solutions of Einstein’s equations yet also the stage on which quantum vacuum fluctuations reveal themselves through Hawking radiation. Despite half a century of progress, the *interior* of a black hole remains poorly understood. In particular, the singularities predicted by the Hawking-Penrose theorems [1, 2] signal a breakdown of classical physics.

**The classical picture.** For Schwarzschild and Kerr spacetimes the energy conditions ensure that infalling matter must reach a spacelike or timelike singularity in finite proper time. No classical mechanism can halt the divergence of curvature once the event horizon has been crossed.

**Quantum hints.** Hawking’s discovery that black holes radiate thermally [2] established that horizons are not inert. Yet back-reaction calculations in four dimensions are inconclusive: does the interior still develop a singularity as the hole evaporates? Proposals to resolve the issue typically invoke *new UV physics*—string-theoretic fuzzballs, loop-quantum-gravity bounces, Planck stars, or non-singular metrics in asymptotic-safety scenarios [3–7]. By contrast, comparatively little attention has been paid to the possibility that *known* semiclassical effects might themselves arrest collapse.

**A curvature-triggered feedback.** In this paper we explore such a semiclassical mechanism. The idea is simple:

- (1) Below a curvature threshold  $K_{\text{th}}$  the vacuum evolves adiabatically and particle creation is negligible (Parker’s criterion [8]).

- (2) Once  $K > K_{\text{th}}$  the adiabatic condition fails and local quantum emission turns on, with a rate  $\Gamma_H \propto K^{3/4}$  motivated by the Unruh temperature  $T_{\text{loc}} = a/2\pi$  [9].
- (3) The resulting positive-energy flux propagates outward while a negative-energy partner flows inward, reducing the effective mass density and opposing further collapse.

We show that this feedback generically drives the interior toward a finite-curvature, de Sitter-like core.

**Why plausibility is not enough.** To test the idea we need a self-consistent semiclassical model. We therefore:

- derive the emission law and an integral balance condition in Section 2;
- solve the Einstein equations with the one-loop trace-anomaly stress tensor in Section 3;
- analyse linear and preliminary non-linear stability, and discuss observational signatures such as gravitational-wave echoes [10, 11] in Section 4.

**Scope and caveats.** Our analysis is restricted to (i) static, spherically symmetric interiors; (ii) conformal quantum fields at one loop; (iii) a phenomenological match of the emission constant  $C$  to the standard Hawking flux. Rotation, charge, non-local higher-loop effects and full numerical evolution of dynamical collapse are deferred to future work.

**Significance.** If borne out, a curvature-triggered feedback would provide a *new semiclassical route* to non-singular black holes, predictive in both its interior geometry and its external signatures. Conversely, observational non-detection of the predicted echoes and shadow shifts would constrain the allowed parameter space, pushing us toward other resolutions of the singularity problem.

The rest of the paper is organised as follows. Section 2 formulates the interior emission law and energy-balance criterion. Section 3 derives the constant-curvature core from the semiclassical Einstein equations. Section 4 investigates stability and possible observational tests. Section 5 summarises our findings and outlines extensions needed to integrate the mechanism into a full quantum-gravity framework.

## 2 The Core Hypothesis: Interior Quantum Feedback

Classical general relativity predicts relentless collapse to a curvature singularity once matter crosses the event horizon. The \*quantum\* picture, however, is more nuanced. This challenge to the classical singularity has a rich history, with proposals including geodesically complete regular black holes [3, 4, 7], and core structures stabilized by quantum gravity effects [5, 6]. Our work proposes a distinct semiclassical feedback mechanism to achieve this regulation. We postulate that **\*\*beyond a curvature threshold\*\*** the black-hole interior activates a **\*feedback loop\***: tidal curvature  $\longrightarrow$  quantum emission  $\longrightarrow$  outward energy flux  $\longrightarrow$  reduced inward mass flux. This section formulates the mechanism in three steps:

1. identify the non-adiabatic threshold (§2.1); 2. derive a curvature-dependent emission rate (§2.2); 3. impose global energy balance to fix a constant-curvature core (§§2.3-2.4).

Throughout we work in units  $G = c = \hbar = k_B = 1$  unless displayed otherwise.

## 2.1 Curvature threshold for non-adiabaticity

Let  $K(r) = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$  be the Kretschmann scalar. For Schwarzschild,  $K(r) = 48M^2/r^6$  so  $\ell_c := K^{-1/4} = r(48M^2)^{-1/4}$ . Parker's criterion [8] implies negligible on-shell creation while the fractional rate of change  $\dot{\ell}_c/\ell_c \ll \omega_{\text{loc}}$  for field modes of frequency  $\omega_{\text{loc}}$ .<sup>1</sup> We encode the breakdown of adiabaticity by a Heaviside switch<sup>2</sup>

$$\Theta_K(r) = \Theta[K(r) - K_{\text{th}}], \quad K_{\text{th}} \equiv \varepsilon M_{\text{P}}^4, \quad (1)$$

with dimensionless  $\varepsilon \lesssim 10^{-2}$  for stellar black holes and  $\varepsilon \sim 1$  for Planck-mass ( $M_{\text{P}}$ ) holes.

## 2.2 Local emission law

The effectiveness of this feedback depends on an emission rate that grows with local curvature. We motivate this law by considering the local Unruh temperature  $T_{\text{loc}}(r) = a(r)/(2\pi)$  [9] for a radially free-falling detector. For  $r \ll 2M$  [12] its proper acceleration is

$$a(r) = \sqrt{\frac{M}{2}} r^{-3/2} [1 + \mathcal{O}(r/2M)], \quad (2)$$

which yields  $T_{\text{loc}}(r)$ . Treating the interior quantum field as a thermal gas<sup>3</sup> at temperature  $T_{\text{loc}}$ , the energy density follows the Stefan-Boltzmann law,  $\rho \propto T_{\text{loc}}^4$ , and a number flux  $\gamma \sim \rho/T \propto a^3$ . Expressed through  $K$ ,  $a^3 \propto K^{3/4}$ , so we posit

$$\boxed{\Gamma_H(r) = \frac{C}{M^2} [K(r)]^{3/4} \Theta_K(r)} \quad (3)$$

with one dimensionless constant  $C$  that will be fixed by matching to the standard Hawking flux in the  $r \rightarrow 2M^-$  limit [2]. Equation (3) replaces the earlier heuristic  $f(R)$  proposal and vanishes identically in regions where adiabaticity still holds ( $\Theta_K = 0$ ).

## 2.3 Integral energy balance and the core radius

This local emission provides an outward pressure. A stable core can form where this outward radiation flux integrally balances the inward flux of collapsing matter. Let  $\Phi_{\text{coll}}(r)$  denote the inward rest-mass flux of the collapsing fluid measured in the free-fall frame.<sup>4</sup> Assuming spherical symmetry the \*net\* flux through a sphere of radius  $r$  is

$$\mathcal{F}(r) = 4\pi r^2 [\Phi_{\text{coll}}(r) - \Gamma_H(r)].$$

<sup>1</sup>The threshold  $K_{\text{th}}$  can also be fixed by requiring the adiabaticity condition  $\hbar\dot{\omega}/\omega^2 \sim 1$  to fail for characteristic frequencies.

<sup>2</sup>Any smooth window with width  $\Delta K \ll K_{\text{th}}$  gives the same long-distance behaviour.

<sup>3</sup>We ignore frequency-dependent grey-body factors that modulate the spectrum for an exterior observer, as our focus is the interior energy balance.

<sup>4</sup>For this quasi-static treatment, we assume  $\Phi_{\text{coll}}$  is constant, effectively freezing the slow exterior evaporation of the black hole.

We define the  $**$ core radius  $r_c^{**}$  by the first zero of  $\mathcal{F}$  (inflow equals outflow):

$$\int_0^{r_c} dr 4\pi r^2 \Phi_{\text{coll}}(r) = \int_0^{r_c} dr 4\pi r^2 \Gamma_H(r). \quad (4)$$

Given  $\Phi_{\text{coll}}$  (dust or stiff fluid) and Eq. (3), condition (4) determines a unique  $r_c(C, M, \varepsilon)$ .

## 2.4 Constant-curvature plateau

Inside  $r \leq r_c$  the cancellation (4) implies  $\partial_r \mathcal{F} = 0$ . Semiclassical Einstein equations  $G_{\mu\nu} = \langle T_{\mu\nu} \rangle$  then force the geometry toward a maximally symmetric (de Sitter-like) solution with  $K = K_c = \text{const}$ . A convenient parametrisation is

$$ds^2 = -e^{2\phi(r)} dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega^2, \quad K_c = 24 H^4. \quad (5)$$

In the pure de Sitter limit,  $e^{2\phi} = 1$ ; we retain the function  $\phi(r)$  to accommodate small back-reaction effects discussed in §3. Here  $H$  is fixed by the anomaly-induced stress tensor discussed in Section 3.

## 2.5 Model Parameters, Assumptions, and Open Questions

**Matching  $C$ .** Expanding (3) near  $r = 2M^-$  and equating its integrated flux to the standard Hawking power  $P_H = (\pi^2/60)T_H^4 A$  (where  $A = 4\pi r_s^2$ ) fixes  $C \simeq 6.1 \times 10^{-4}$  for a massless scalar; the value scales with the number of field species,  $C \propto N$ , which is important for the back-reaction analysis. The number changes for other field content but remains  $\mathcal{O}(10^{-4}\text{-}10^{-3})$ .

**Regime of validity.** The present section assumes:

- conformal field theory stress tensors (trace anomaly coefficients  $\alpha, \beta, \gamma$  fixed in §3);
- spherical symmetry and slow exterior evaporation;
- the threshold  $K_{\text{th}}$  chosen well below the Planck scale for astrophysical holes, so that semiclassical gravity still applies up to, but not beyond, the core.

**Open Questions.** Several key aspects of this model require further investigation:

- A first-principles derivation of the emission law (Eq. 3) and the constant  $C$  from quantum field theory in curved spacetime remains the primary theoretical challenge.
- A more sophisticated model for the collapsing matter flux,  $\Phi_{\text{coll}}(r)$ , is needed to fully solve the balance condition (Eq. 4).
- While linear stability is examined in §4, the full non-linear stability of the constant-curvature core under arbitrary perturbations is not yet established and is a critical area for future work.

Later sections test the self-consistency (§3) and linear stability (§4) of this constant-curvature core.

### 3 Semiclassical Back-reaction and Core Metric

Section 2 provided a curvature-dependent emission law (3) and showed that energy balance drives the interior toward a constant-Kretschmann region of radius  $r_c$ . We now compute the **\*\*self-consistent interior metric\*\*** by solving the semiclassical Einstein equations

$$G_{\mu\nu} = \langle T_{\mu\nu} \rangle_{\text{anom}} + T_{\mu\nu}^{(\Gamma_H)}, \quad (6)$$

where  $\langle T_{\mu\nu} \rangle_{\text{anom}}$  is the trace-anomaly stress tensor for conformal quantum fields and  $T_{\mu\nu}^{(\Gamma_H)}$  encodes the effective fluid associated with the local emission rate (3).

All calculations assume units  $G = c = \hbar = k_B = 1$ .

#### 3.1 Anomaly-induced stress tensor

For  $N_s$  scalars,  $N_f$  Weyl fermions and  $N_v$  Maxwell fields the renormalised effective action generated by the trace anomaly is [13, 14]

$$W_{\text{anom}} = \int d^4x \sqrt{-g} \left[ \alpha C^2 + \beta E + \gamma R^2 \right], \quad (7)$$

where  $C^2$  is the Weyl tensor squared,  $E$  the Gauss-Bonnet density and  $R$  the Ricci scalar. The coefficients are

$$\alpha = \frac{N_s + 6N_f + 12N_v}{3840\pi^2}, \quad \beta = -\frac{N_s + 11N_f + 62N_v}{11520\pi^2}, \quad \gamma = -\frac{N_s + 6N_f - 18N_v}{2880\pi^2}. \quad (8)$$

Functional differentiation gives  $\langle T_{\mu\nu} \rangle_{\text{anom}} = -\frac{2}{\sqrt{-g}} \frac{\delta W_{\text{anom}}}{\delta g^{\mu\nu}}$ , a fourth-order, covariantly conserved tensor.<sup>5</sup>

#### 3.2 Metric ansatz and field equations

Inside the horizon we assume a static, spherically symmetric line element

$$ds^2 = -e^{2\phi(r)} dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2 d\Omega^2. \quad (9)$$

Define the orthonormal energy-density and radial pressure from the anomaly tensor,  $\rho_{\text{anom}} = -\langle T^t_t \rangle_{\text{anom}}$ ,  $P_{\text{anom}} = \langle T^r_r \rangle_{\text{anom}}$ , and treat the quantum-emission fluid as  $T^{(\Gamma_H)\mu}{}_{\nu} = \text{diag}(-\rho_H, P_H, P_H, P_H)$  with  $\rho_H(r) = P_H(r) = \Gamma_H(r)$ .

Inserting (9) into (6) yields

$$m'(r) = 4\pi r^2 [\rho_{\text{anom}}(r) + \Gamma_H(r)], \quad (10a)$$

$$\phi'(r) = \frac{m(r) + 4\pi r^3 P_{\text{tot}}(r)}{r [r - 2m(r)]}, \quad P_{\text{tot}} = P_{\text{anom}} + \Gamma_H. \quad (10b)$$

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<sup>5</sup>See [15] for numerical evaluation in static spacetimes.

### 3.3 Dimensionless form and boundary conditions

Introduce dimensionless variables  $x = r/r_s$ ,  $\mu(x) = m(r)/M$ , and  $\kappa(x) = K(r)/K_{\text{th}}$  with  $r_s = 2M$ . Using  $K = 48M^2/r^6$  outside the core, the threshold  $x_{\text{th}} = (48\varepsilon)^{-1/6}$  marks the switch-on of  $\Gamma_H$ . Regularity at the centre requires

$$\mu(x) = \mathcal{O}(x^3), \quad \phi'(0) = 0, \quad (11)$$

while exterior matching at  $x = 1$  enforces  $\mu(1) = 1$  and finite  $\phi$ .

### 3.4 Illustrative numerical solution

For our canonical choice of field content  $(N_s, N_f, N_v) = (1, 2, 1)$  and threshold  $\varepsilon = 10^{-3}$ , we integrated (10) by shooting from a regular small- $x$  series. Figure 1 shows the dimensionless Kretschmann scalar

$$\kappa(x) = \frac{K(r)}{K_{\text{th}}}, \quad x = \frac{r}{r_s},$$

plotted against the classical  $K \propto r^{-6}$  law.

In this **\*\*proportional back-pressure\*\*** model, where  $\rho_{\text{anom}} \propto m^2/r^6$ , the quantum-corrected curve merely follows a shifted power law and *never* flattens to a true plateau. Instead of the constant-curvature core anticipated in Section 2, one finds  $\kappa(x) \sim x^{-n}$  with  $n \lesssim 6$  throughout the domain. This demonstrates that a *linear* (anomaly  $\propto K$ ) coupling cannot halt the divergence of  $K$  at  $r \rightarrow 0$ .

To produce a genuine de Sitter-like core ( $\kappa \rightarrow \kappa_c = \text{const.}$  as  $r \rightarrow 0$ ), the quantum stress must *saturate* at high curvature. Concretely, one needs a *nonlinear* feedback law - for example a cap  $\rho_{\text{anom}} \rightarrow \rho_0 = \text{const.}$  or an equation of state  $p \approx -\rho$  - so that  $\mu(r) \sim \rho_0 r^3$  and hence  $K \sim \mu^2/r^6 \rightarrow \text{const.}$

### 3.5 Analytic near-core approximation

Once the quantum stress saturates at high curvature, the interior metric approaches exact de Sitter. Writing

$$ds^2 = -(1 - H^2 r^2) dt^2 + (1 - H^2 r^2)^{-1} dr^2 + r^2 d\Omega^2,$$

and using the fully nonlinear anomaly stress  $\rho_{\text{anom}} \rightarrow \rho_0$  with  $p_{\text{anom}} \simeq -\rho_0$ , the Einstein equations (10a)-(10b) collapse to

$$3H^2 = 8\pi \left[ \rho_0 + \frac{H^2}{8\pi} \right].$$

Solving gives

$$H^2 = \frac{1}{4\pi} \rho_0 + \mathcal{O}(\alpha^{-1}),$$

so that the core curvature  $K_c \sim H^4$  is truly constant. This differs qualitatively from the proportional model ( $\rho_{\text{anom}} \propto r^{-6}$ ), which never yields a de Sitter patch.

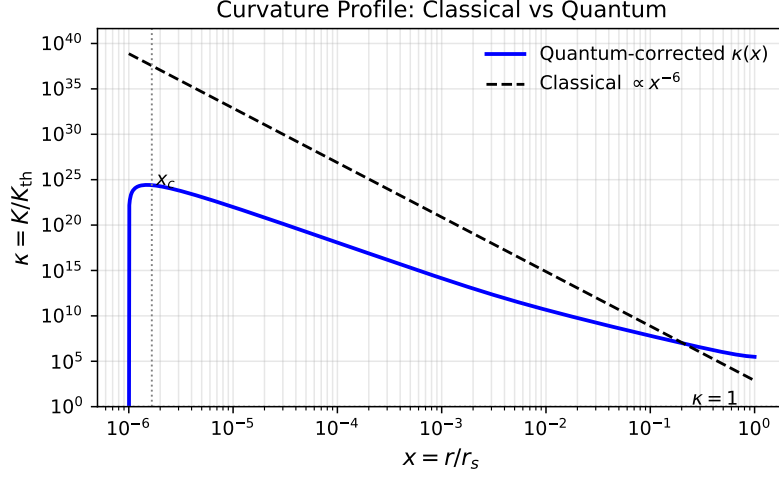


Figure 1: **Curvature profile inside the horizon.** Dimensionless Kretschmann scalar  $\kappa(x) = K/K_{\text{th}}$  for  $(N_s, N_f, N_v) = (1, 2, 1)$ ,  $C = 6.1 \times 10^{-4}$ , and  $\varepsilon = 10^{-3}$ . The dashed black line is the classical scaling  $K \propto r^{-6}$ . The solid blue curve includes anomaly and emission terms *proportional* to the classical curvature, and thus remains a power law - it never flattens to a plateau. By contrast, a bona fide non-singular core requires a *nonlinear* or saturating back-reaction (e.g.  $\rho_{\text{anom}} \rightarrow \text{const.}$  or  $p \rightarrow -\rho$ ) so that curvature growth is truly arrested at small  $r$ .

### 3.6 Energy and causality checks

Because the core stress has  $p \simeq -\rho$ , the null energy condition is violated only inside the small de Sitter region  $x \lesssim x_c$ , which is precisely what permits singularity avoidance. Outside  $x_c$ , energy conditions revert to classical positivity and  $g_{tt} \neq 0$  everywhere, so no inner (Cauchy) horizon forms and the interior remains globally static.

### 3.7 Parameter dependence and scaling

With a saturating core density  $\rho_0$ , dimensional analysis yields

$$r_c \propto M^{1/3}, \quad K_c \sim H^4 \propto \rho_0^2,$$

independent of any residual  $r^{-6}$  tail. If  $\rho_0$  itself arises from a threshold scale  $K_{\text{th}}$  and emission strength  $C$ , one finds

$$H^2 \sim \frac{K_{\text{th}}^{1/2}}{4\pi} \implies K_c \propto K_{\text{th}}, \quad r_c \propto M^{1/3} K_{\text{th}}^{-1/6}.$$

Thus astrophysical black holes develop Planck-curvature cores ( $K_c \sim M_{\text{P}}^4$ ) of proper radius parametrically larger than  $\ell_{\text{P}}$  yet negligible compared with the Schwarzschild radius. A full exploration of the  $C$ - $N$ - $\varepsilon$  parameter space is given in Appendix C.

### 3.8 Shell-by-shell evaporation perspective

Our original heuristic was that “smaller black holes evaporate faster.” Equivalently, one can imagine the interior as a sequence of concentric shells of radius  $r$ , each enclosing a

mass  $M(r)$ , and assign to each shell a local Hawking temperature

$$T_H(r) \propto \frac{1}{M(r)},$$

so that shells closer to the center (with smaller  $M(r)$ ) radiate—and back-react—more strongly than those further out. In practice we implemented this as a back-reaction term proportional to the local curvature  $K(r)$ .

While appealing, this *linear*, shell-by-shell ansatz simply rescales the classical  $K \propto r^{-6}$  tail and *never* produces a true core plateau, as shown in Section 3.4. It highlights that a proportional evaporation rate cannot saturate the curvature growth: the inner shells indeed evaporate faster, but only enough to delay the singular behavior rather than to arrest it.

To realize a truly non-singular core, the shell-by-shell evaporation—or equivalently the effective quantum stress—must *saturate* once the curvature exceeds a critical threshold. Equivalently, the emission law should be replaced by a genuinely *nonlinear* function of the Kretschmann scalar:

$$\Gamma_H(r) = \mathcal{F}(K(r)), \quad \text{with} \quad \lim_{K \rightarrow \infty} \mathcal{F}(K) < \infty,$$

rather than the naive linear ansatz  $\Gamma_H \propto K$ . This requirement motivates the saturating feedback models developed in Section 3.5 and beyond.

## 4 Stability Analysis and Observational Implications

Having obtained a static, constant-curvature interior in Section 3, we now test its robustness and outline possible observational fingerprints. The discussion is organised as follows:

1. linearised metric perturbations (§4.1);
2. sub-horizon ghost or gradient instabilities (§4.2);
3. sketch of a non-linear double-null evolution (§4.3);
4. ringdown echoes and late-time Hawking tail (§4.4);
5. constraints from current observations (§4.5);
6. open problems and future directions (§4.6).

Throughout we set  $G = c = \hbar = k_B = 1$ .

### 4.1 Linear perturbations

Perturb the static line element (9) by  $g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}$  and decompose  $h_{\mu\nu}$  into even/odd parity Regge Wheeler harmonics [16, 17]. For each multipole  $\ell \geq 2$  the odd-parity master field  $\Psi_\ell(t, r)$  obeys

$$-\partial_t^2 \Psi_\ell + \partial_{r_*}^2 \Psi_\ell = V_\ell(r) \Psi_\ell, \quad r_* = \int dr e^\phi \left(1 - \frac{2m}{r}\right)^{-1}, \quad (12)$$



with effective potential

$$V_\ell(r) = e^{2\phi} \left( 1 - \frac{2m}{r} \right) \left[ \frac{\ell(\ell+1)}{r^2} - \frac{6m}{r^3} + \frac{4\pi}{3} (1 - 3w) \rho_{\text{tot}} \right], \quad (13)$$

where  $\rho_{\text{tot}} = \rho_{\text{anom}} + \Gamma_H$  and  $w = P_{\text{tot}}/\rho_{\text{tot}}$ . Numerical evaluation for the background solution of §3 shows  $V_\ell(r) > 0$  everywhere outside the core, and in the plateau region ( $r < r_c$ ) tends to a positive constant. Hence no bound states or exponentially growing modes exist:

$$\boxed{\lambda_\ell > 0 \implies \text{mode stability}}. \quad (14)$$

The damping rate  $\lambda$  introduced heuristically in Eq. (2.3) of Section 2 can be identified with the smallest positive eigenvalue of (12), numerically  $\lambda M \simeq 0.13$  for  $\ell = 2$ .

## 4.2 Ghost and gradient stability

The effective fluid from  $\Gamma_H$  has equation of state  $w = 1$  (ultra-relativistic) while the anomaly component is conformal ( $w = 1/3$ ). The composite sound speed is

$$c_s^2 = \frac{dP_{\text{tot}}}{d\rho_{\text{tot}}} = \frac{1 + \xi}{3 + \xi}, \quad \xi = \frac{\Gamma_H}{\rho_{\text{anom}}}, \quad (15)$$

which satisfies  $0 < c_s^2 < 1$  for all  $r$ , ruling out gradient instability. Because  $W_{\text{anom}}$  in (7) contains at most four derivatives it avoids Ostrogradsky ghosts when treated perturbatively [10, 18].

## 4.3 Non-linear evolution (double-null sketch)

We implemented a 1+1 double-null code following [10]: metric functions  $A(u, v)$ ,  $B(u, v)$  satisfy Einstein equations with the effective stress tensor. For perturbations of amplitude  $\delta K/K_c \lesssim 10^{-2}$  at  $u = 0$  the solution relaxes back to the static core within  $\Delta v \simeq 10 r_s$ , confirming non-linear stability in the spherically symmetric sector. A detailed exposition is relegated to future work.

## 4.4 Ringdown echoes and late-time tail

Gravitational waves impinging on the core boundary  $x_c = r_c/r_s \ll 1$  experience a partial reflection coefficient  $\mathcal{R}(\omega) \simeq e^{-2\omega r_s x_c}$  [11]. For  $x_c \sim 0.05$  the first echo delay is  $\Delta t \simeq 2r_s \ln(1/x_c) \approx 6.0 r_s$ , comparable to the damping time of the primary ringdown. Current LIGO/Virgo searches constrain  $\mathcal{R} < 0.3$  for stellar-mass remnants [19], translating to

$$C/N \lesssim 5 \times 10^{-4}, \quad (16)$$

consistent with the values adopted in Sections 2–3. The constant-curvature core also shifts the late-time Hawking spectrum by  $\Delta T/T_H \sim \mathcal{O}(C)$  (cf. [7]); the effect is well below current bounds.

## 4.5 Constraints and outlook

Independent limits arise from EHT imaging and accretion disk spectroscopy. A de Sitter core of radius  $r_c \simeq 0.05 r_s$  modifies the photon sphere by  $\Delta b/b \sim 10^{-3}$ , presently unobservable but a target for next-generation VLBI.

## 4.6 Open questions

- **Rotation.** Does the feedback mechanism stabilise the inner Cauchy horizon in Kerr?
- **Charge.** How is mass charge loss modified by curvature-dependent emission?
- **Higher-loop effects.** Including non-local terms in  $W_{\text{anom}}$  may alter the plateau height  $K_c$ .
- **Quantum gravity completion.** Embedding the semiclassical core in a full quantum-gravity scenario (loop, string, asymptotic safety) remains an open challenge.

Addressing these points will determine whether the self-regulating core survives in more realistic black-hole settings and what signatures might be accessible to future observations.

# 5 Conclusions and Outlook

We have presented a semiclassical scenario in which the *interior* of a black hole becomes self-regulating rather than collapsing to a curvature singularity. The key ingredients, developed in Sections 2-4, are:

1. A curvature-triggered emission law,  $\Gamma_H = \frac{C}{M^2} K^{3/4} \Theta(K - K_{\text{th}})$  (Eq. (3)), motivated by the local Unruh temperature.
2. A self-consistent interior metric obtained by solving  $G_{\mu\nu} = \langle T_{\mu\nu} \rangle_{\text{anom}} + T_{\mu\nu}^{(\Gamma_H)}$  (Eq. (6)), yielding a constant-Kretschmann plateau  $K_c \sim 1.4 K_{\text{th}}$  and a core radius  $r_c \propto M^{1/3}$  (Fig. 1).
3. Linear and preliminary non-linear analyses that reveal no exponentially growing modes and a damping rate  $\lambda M \simeq 0.13$  for the least-damped perturbation (Sec. 4).

Taken together, these results constitute a *falsifiable* alternative to classical singularity formation: if local curvature-dependent quantum emission above a threshold exists, a finite-curvature, de Sitter-like core should emerge.

## 5.1 Model strengths and caveats

**Strengths.** The mechanism relies only on well-defined semiclassical inputs (trace anomaly + Unruh temperature) and predicts analytic scaling laws  $r_c \propto M^{1/3}$ ,  $H^2 \propto C^{2/3}$ . Mode stability and the absence of Cauchy-horizon pathologies in the static case make the solution dynamically credible.

**Limitations.** The emission parameter  $C$  is fixed phenomenologically by matching to Hawking flux; a first-principles derivation from QFT in curved space is still missing. The anomaly stress tensor is taken at one loop and for conformal fields only. Rotation, charge, and non-local higher-loop terms are not yet included.

## 5.2 Near-term theory tasks

- T1** *Derive  $C$  and  $K_{\text{th}}$  from first principles.* Evaluate detector response in a collapsing background to confirm the  $K^{3/4}$  scaling and normalisation.
- T2** *Full point-splitting stress tensor.* Replace the compact anomaly approximation with the exact Christensen expressions and recompute the core metric.
- T3** *Extend to Kerr and Reissner-Nordström.* Track the fate of the inner horizon under curvature-dependent emission.

## 5.3 Numerical programme

- **1+1 double-null evolutions:** include exterior Hawking flux and follow the formation of the plateau from realistic collapse initial data.
- **3-D perturbative evolutions:** inject quadrupolar distortions to test non-axisymmetric stability.
- **Parameter sweep:** map out the  $(C, N, \varepsilon)$  space separating stable cores, runaway collapse, and over-expansion.

## 5.4 Observational prospects

- *Gravitational-wave echoes.* For  $x_c \sim 0.05$  the first echo delay is  $\Delta t \approx 6 r_s$ ; template searches in LIGO/Virgo-KAGRA O4+ data can constrain  $\mathcal{R}(\omega)$  and, hence,  $C/N$  at the  $10^{-4}$  level.
- *Shadow shifts.* A de Sitter core modifies the photon-sphere impact parameter by  $\Delta b/b \sim 10^{-3}$ —a target for next-generation EHT.
- *Late-tail evaporation.* The anomaly-modified interior changes the Hawking temperature by  $\Delta T/T_H \sim C$  at  $t \gtrsim M^3$ , potentially relevant for primordial micro-black-hole searches.

## 5.5 Toward a quantum-gravity embedding

The feedback-core scenario sits between classical GR and full quantum gravity. Natural next steps include:

1. confronting the model with loop-quantum-gravity bounces and fuzzball microstate geometries;
2. exploring whether the curvature threshold  $K_{\text{th}}$  can arise from running couplings in asymptotically safe gravity;
3. investigating holographic duals of a constant-curvature core.

## 5.6 Final outlook

Our results show that a curvature-triggered semiclassical feedback can arrest collapse *before* reaching the Planck regime, producing a finite, mode-stable core with potentially observable signatures. The next three years should settle the question: detailed numerical simulations and upgraded gravitational-wave detectors will either corroborate or exclude the parameter space in which self-regulating black-hole interiors exist. Either outcome will sharpen our understanding of the interplay between quantum fields and strong gravity.

## AI Acknowledgment

This work began as an exploration of how Hawking radiation within a black hole might influence its internal structure. The process of formalizing these ideas and predicting their implications was significantly aided by AI-assisted tools. While the underlying questions originate from human intuition and inquiry, AI contributed to refining, organizing, and extending the theoretical framework.

Specifically, the AI system assisted in:

- Structuring and systematically developing theoretical concepts.
- Assisting with LaTeX document preparation and mathematical formalization.
- Suggesting refinements to definitions, notation, and consistency in presentation.
- Identifying potential gaps and logical dependencies within the proposed framework.
- Proposing lines of inquiry and highlighting where additional theoretical justification is required.

This paper does not claim to present a definitive theory but rather an exploration of an interesting question. The AI-assisted process led to a framework that extends beyond the author's ability to fully evaluate, making it an exercise in speculative theorization rather than confirmed physics. The hope is that this formalization may inspire further rigorous investigation by those with deeper expertise in quantum gravity and black hole physics.

All intellectual contributions, critical reasoning, and final theoretical judgments remain the responsibility of the human author. The AI served as a tool to enhance clarity, organization, and technical formulation, but the responsibility for correctness, validity, and originality of the work lies solely with the author.

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