2D Example for **SUTRA**

Island2D

(Clifford I. Voss, USGS)

This 2D problem is presented in the SUTRA documentation (Voss and Provost, 2002). It is intended for comparison with the 3D solution to the same problem, also described in the SUTRA documentation. It involves a transient saturated-unsaturated variable-density flow and transient solute transport simulation.

A circular island in the sea undergoes a prolonged drought, during which the water table declines to sea level, and all ground water beneath the island becomes saline. Then, renewed freshwater recharge finally restores the island's freshwater lens. This example concerns simulation of the post-drought recharge and restoration of the freshwater lens.

Following the drought, the water table is at sea level and both the saturated aquifer below sea level and the unsaturated zone above sea level contain only seawater. Fresh rainwater recharge to the surface of the island begins and continues at a constant rate, raising the water table on the island, flushing out seawater, and eventually establishing a stable freshwater lens and a diffuse saltwater-freshwater interface. The aquifer on the island is unconfined with both unsaturated and saturated zones and the material properties are generally homogeneous but permeability is anisotropic.

The problem may be simulated twice, using both a 2D radial mesh and a fully 3D mesh (see Island3D problem). The 3D steady-state solution may be compared with the 2D solution to verify that these give equivalent results. The solution is radially symmetric.

This description follows the discussion in the **SUTRA** documentation where figures showing the mesh and results may also be found.

The main parameter values are:

α = 1.0 x 10^{-8} [m·s^2/kg]

(The α and β values imply that S_{op} = 9.0447 x 10^{-9} [m·s^2/kg].)

β = 4.47 x 10^{-10} [m·s^2/kg]

k_H = 5.0 x 10^{-12} [m^2]

ε = 0.1

k_V = 5.0 x 10^{-13} [m^2]

μ = 1.0 x 10^{-3} [kg/m·s]

$|\underline{g}|$ = 9.81 [m/s^2]

σ_w = 1.0 x 10^{-9} [m^2/s]

ρ_0 = 1000. [kg/m^3]

$$C_{sea} = 0.0357 \left[\frac{\text{(kg dissolved solids)}}{\text{(kg seawater)}}\right]$$

ρ_{sea} = 1024.99 [kg/m^3]

$$\frac{\partial\rho}{\partial C} = 700. \left[\frac{\text{(kg seawater)/m}^3}{\text{(kg dissolved solids)}}\right]$$

$$\alpha_L = \begin{cases} 10. & \text{for horizontal flow} \\ 2.5 & \text{for vertial flow} \end{cases} \quad \text{[m]}$$

D_m = 1.0 x 10^{-9} [m^2/s]

α_T = 0.1 [m]

Q_{IN} = 2.3766x10^{-5} [kg/(m^2 of horizontally projected land surface area)·s] on land (equivalent to 75. [cm/yr] of recharge)

$$C_{IN} = 0. \left[\frac{\text{(kg dissolved solids)}}{\text{(kg seawater)}}\right]$$

$$\left.\begin{aligned} S_w &= \begin{cases} 1 & \text{for } p \geq 0 \\ 0.3 + 0.7\left[1 + \left(5 \text{ x } 10^{-5}\ p\right)^2\right]^{-0.5} & \text{for } p < 0 \end{cases} \\ k_r &= \left(\frac{S_w - 0.3}{0.7}\right)^{0.5} \left(1 - \left\{1 - \left(\frac{S_w - 0.3}{0.7}\right)^2\right\}^{0.5}\right)^2 \end{aligned}\right\} \begin{array}{l} \text{Unsaturated properties functions} \\ \text{of Van Genuchten (1980)} \\ \text{with } S_{wres} = 0.3,\ a = 5 \text{ x } 10^{-5}\ [\text{m} \cdot \text{s}^2/\text{kg}], \\ \text{and } n = 2. \\ \text{(See eqns. 2.8 and 2.21 in SUTRA documentation.)} \end{array}$$

Boundary Conditions:

In the 2D cylindrical model, no flow crosses the inner boundary (the axis of radial symmetry, r = 0) and the bottom boundary (z = -100 m). Specified pressure is set at hydrostatic seawater pressure along the vertical outer boundary (r = 800 m). Along the top boundary, nodes above sea level (r ≤ 500 [m]) receive freshwater recharge (equivalent to 75.0 cm/yr) totaling 18.6658 kg/s of recharge for the entire circular island. The amount of recharge at each node is determined by the surface area of its cell on the top surface of the cylindrical 2D model; the concentric ring-shaped cells have areas that increase as $2\pi r$. In the region where the island surface slopes down towards the coast, 400 m ≤ r ≤ 500 m, the surface area used for calculating recharge is the horizontal projection of this sloping area (area reduced by cosine of the dip angle). At nodes below sea level, the pressure is specified to be hydrostatic seawater pressure. Any fluid that enters at points of specified pressure has the concentration of seawater. The value for the specified pressure boundary condition factor, GNUP, in 2D is 1.0x10^{+5}.

Note: When 2D and 3D simulations are set up using the graphical preprocessor, **SutraGUI** (Winston and Voss, 2003), small discrepancies may be expected between these models in some parameters, such as recharge to the top surface. However, despite the obvious differences in spatial discretization, according to the fluid budgets output by SUTRA, the total recharge to the top surface of the entire island in 2D and 3D representations matches to five significant figures (18.665 kg/s).

Initial Conditions:

Seawater concentration and natural steady-state pressures are initially set everywhere in the aquifer. The natural initial pressure values are obtained through an extra initial simulation that calculates steady pressures for the conditions of seawater concentration throughout, zero recharge at the surface of the island, and specified hydrostatic pressures along the sea bottom and the outer boundary.

Mesh:

The 2D mesh has 40 elements in the radial direction and 25 elements vertically, giving NN=1,066 and NE=1,000. See Figure 6.21 in the **SUTRA** documentation. Elements are 20 m wide and 5 m high, except within 5 m of the top surface, where they are 1 m high. Vertical discretization in the unsaturated zone is relatively coarse because, in this problem, the intent is to approximately locate the water table and the details of the saturation distribution are of less interest. Mesh thickness at node i is given by $B_i=2\pi r_i$, thereby providing a radial coordinate system. The outer boundary approximates a circle of radius 800 m and is sufficiently distant from the island that it does not significantly influence the results.

Simulation:

A natural steady-state pressure distribution is obtained in a pre-simulation (described above under *Initial Conditions*).

For the transient runs, for both pressure and concentration, the time step size is Δt = 6311520. s (0.2 yr). Because only the long-time (steady-state) behavior of the system is of interest, a single iteration for resolving nonlinearities is used per time step. The system essentially achieves a new steady state after 100 time steps (20 yr). For 2D, the direct solver is used for both simulations.

The transient run takes about 3 seconds on a PC running Windows 2000 with a 2GHz Xeon processor; it uses about 2 Mbytes of RAM.

***SUTRA** files:*

For initial run to obtain starting pressures

input :
Island2D_initial-p.inp, *Island2D_initial-p.ics*, and SUTRA.FIL

output:
Island2D_initial-p.smy, *Island2D_initial-p.lst*, *Island2D_initial-p.rst*, *Island2D_initial-p.nod*, and *Island2D_initial-p.ele*.

For transient run

input :
Island2D.inp, *Island2D.ics*, and SUTRA.FIL

output:
Island2D.smy, *Island2D.lst*, *Island2D.rst*, *Island2D.nod*, *Island2D.ele*, and *Island2D.obs*.

Preprocessing:

The ArgusONE setup files, *Island2D_initial-p.mmb* and *Island2D.mmb*, are included for use with **SutraGUI**. In **SutraGUI**, this problem uses a fishnet mesh with eight blocks. The top of the model represents the top of the island on land and the sea bottom below the sea. The top is assigned an elevation, given as a function of radius from the center of the island. The bottom of the upper block is 5. m below the top and the bottom of the model is at 100 m depth below sea level. Recharge is specified on the land portion of the island on the top surface of the model with zero concentration of inflow. It is adjusted with radius as described above. On the sea bottom, the pressure is specified as hydrostatic seawater. The outermost vertical side of the model is assigned a specified pressure of hydrostatic seawater. An observation well is placed on the island penetrating the entire vertical extent of the model for the transient simulation.

In order to begin the transient simulation with a natural pressure distribution, an initial steady-state simulation is done (using the ArgusONE setup file *Island2D_initial-p.mmb*) to determine the pressures in the model during the drought (when there is no recharge and all ground water is seawater). The pressures resulting from this simulation are used as the initial pressure condition for the transient simulation (that uses using the ArgusONE setup file *Island2D.mmb*). This is specified below "Select restart file:" on the Initial Conditions Controls page of the SUTRA Project Information dialog.

> *Note:* The user must change the path of the restart file so that it is correct on the user's computer).

Results:

Results are reported 20 years after recharge begins, by which time the system has nearly reached a new steady state. (Note that more-exact steady-state solutions may be obtained by running longer simulations, e.g. 40 years or more.)

Postprocessing:

The match of specified and calculated pressures may be checked using **CheckMatchBC**.

Hydrographs of pressure, saturation and concentration at the observation nodes may be viewed using **GW_Chart**.

Spatial distributions of results may be viewed with both **SutraPlot** and **ModelViewer**.

> For **SutraPlot**, two setup files are provided; *Island2D_mesh.spl*, shows the mesh and, *Island2D_conc.spl*, shows contours of percent seawater (between 10% and 90%) in a vertical section at the end of the simulation.
>
> For **ModelViewer**, a setup file, *Island2D.mv*, is provided for viewing the results with isosurfaces (contours) for concentration (10% to 90% seawater) and the finite element mesh. The first image shows the initial conditions. To see the end of the simulation, select Toolbox|Animation|Set to time: (select 100)|Set. **ModelViewer** can only visualize results from FishNet meshes in 2D, not from irregular meshes.

Suggestions:

A longer transient simulation may bring the 2D and 3D results into even closer agreement. Viewing the hydrographs of concentration and saturation using **GW_Chart**, it is apparent that some adjustment towards steady state conditions will still occur after the 20-year simulation described above. The user is encouraged to try this. Spatial variations of hydrologic properties such as permeability may also be implemented, for example using SutraGUI to place low and high permeability lenses within the aquifer and the results examined.