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TAREA — Derivadas por Números Duales

1. $f(x) = \frac{(x-2)(x+2)}{(x-3)(x-4)}$

Comenzamos escribiendo cada número como dual:

$$f(x) = \frac{(x-2)(x+2)}{(x-3)(x-4)} = \frac{[(x,1)-(2,0)]\times[(x,1)+(2,0)]}{[(x,1)-(3,0)]\times[(x,1)-(4,0)]}$$

Para $x = 6$, se tiene:

$$f(6) = \frac{[(6,1)-(2,0)]\times[(6,1)+(2,0)]}{[(6,1)-(3,0)]\times[(6,1)-(4,0)]} = \frac{(4,1)\times(8,1)}{(3,1)\times(2,1)} =$$

$$= \frac{(32,12)}{(6,5)} = \left(\frac{32}{6}, \frac{12(6)-32(5)}{36}\right) = \left(\frac{32}{6}, \frac{12(6)-32(5)}{36}\right) = \left(\frac{16}{3}, -\frac{22}{9}\right)$$

Por tanto:

$$f'(6) = -\frac{22}{9} \approx -2.444$$

2. $f(x) = x^2 sinx$

Escribimos los números como duales:

$$f(x) = (x,1)\times(x,1)\times sin(x,1)$$

Para $x = 0.5$, se tiene:

$$f(0.5) = (0.5,1)\times(0.5,1)\times sin(0.5,1) =$$

$$= (0.25, 0.5)\times sin(0.5,1)$$

$$= (0.25, 0.5)\times(sin(0.5), cos(0.5))$$

$$= (0.25[sin(0.5)], 0.5[sin(0.5)] + 0.5[cos(0.5)])$$

$$= \left(\frac{sin(1/2)}{4}, \frac{sin(1/2)}{2} + \frac{cos(1/2)}{4}\right)$$

$$= \left(\frac{sin(1/2)}{4}, \frac{2sin(1/2)+cos(1/2)}{4}\right)$$

Por lo tanto:

$$f'(0.5) = \frac{2sin(1/2)+cos(1/2)}{4} \approx 0.459$$

3. $f(x) = (sinx)^{tanx}$

Escribimos en forma de números duales:

$$f(x) = [sin((x,1))]^{tan((x,1))}$$

Para $x = 0.5$, se tiene:

$$f(0.5) = [sin((0.5,1))]^{tan((0.5,1))}$$

$$= e^{Ln[sin((0.5,1))]^{tan((0.5,1))}}$$

$$= e^{[tan(0.5),sec^2(0.5)]\times Ln|sin(0.5),cos(0.5)|}$$

$$= e^{[tan(0.5),sec^2(0.5)]\times \left(Ln|sin(0.5)|,\frac{cos(0.5)}{sin(0.5)}\right)}$$

$$= e^{\left(tan(0.5)Ln|sin(0.5)|,sec^2(0.5)Ln|sin(0.5)|+tan(0.5)\frac{cos(0.5)}{sin(0.5)}\right)}$$

$$= e^{\left(tan(0.5)Ln|sin(0.5)|,sec^2(0.5)Ln|sin(0.5)|+1\right)}$$

$$\left(e^{(tan(0.5)Ln|sin(0.5)|)}, [sec^2(0.5)Ln|sin(0.5)|+1]e^{(tan(0.5)Ln|sin(0.5)|)}\right)$$

$$\left((sin(0.5))^{tan(0.5)}, [sec^2(0.5)Ln|sin(0.5)|+1]\,(sin(0.5))^{tan(0.5)}\right)$$

Por lo que:

$$f'(0.5) = [sec^2(0.5)Ln|sin(0.5)|+1]\,(sin(0.5))^{tan(0.5)} \approx 0.0304$$

4. $f(x) = \frac{1-cosx}{x^2}$

Escribimos en forma de números duales:

$$f(x) = \frac{(1,0)-cos(x,1)}{(x,1)^2} =$$

$$= \frac{(1,0)-(cosx,-sinx)}{(x^2,2x)}$$

$$= \frac{(1-cosx,sinx)}{(x^2,2x)}$$

$$= \left(\frac{1-cos}{x^2}, \frac{x^2sinx-2x(1-cosx)}{x^4}\right)$$

$$f(x) = \left(\frac{1-cosx}{x^2}, \frac{xsinx-2(1-cosx)}{x^3}\right)$$

Para $x = 0.004$, tenemos:

$$f(0.004) = \left(\frac{1 - cos(0.004)}{(0.004)^2}, \frac{(0.004)sin(0.004) - 2(1 - cos(0.004))}{(0.004)^3} \right)$$

Por lo tanto:

$$f'(0.004) = \frac{(0.004)sin(0.004) - 2(1 - cos(0.004))}{(0.004)^3} \approx -0.0003333$$

Submitted by Alan Axell Ramírez Pérez on 4 de marzo de 2024.