Alan Axell Ramírez Pérez
Licenciatura en Física, DCI
Universidad de Guanajuato

March 4, 2024

TAREA — Derivadas Caso Discreto Derecho

1. Segunda Derivada por Diferencias Finitas, Caso Discreto Derecho

Comenzamos con la serie de Taylor de $f(x)$ alrededor de $\hat{x}$, que se define como:

$$f(x) = f(\hat{x}) + f'(\hat{x})(x-\hat{x}) + \frac{f''(\hat{x})}{2!}(x-\hat{x})^2 + \frac{f'''(\hat{x})}{3!}(x-\hat{x})^3 + \frac{f^{(4)}(\hat{x})}{4!}(x-\hat{x})^4 + \dots$$

Luego evaluamos la función en $\hat{x}+h$, como sigue:

$$f(\hat{x}+h) = f(\hat{x}) + f'(\hat{x})h + \frac{f''(\hat{x})}{2!}h^2 + \frac{f'''(\hat{x})}{3!}h^3 + \frac{f^{(4)}(\hat{x})}{4!}h^4 + \dots$$

Además:

$$-2f(\hat{x}+h) = -2f(\hat{x}) - 2f'(\hat{x})h - f''(\hat{x})h^2 - \frac{2f'''(\hat{x})}{3!}h^3 - \frac{2f^{(4)}(\hat{x})}{4!}h^4 + \dots$$

y

$$f(\hat{x}+2h) = f(\hat{x}) + 2f'(\hat{x})h + 2f''(\hat{x})h^2 + \frac{8f'''(\hat{x})}{3!}h^3 + \frac{16f^{(4)}(\hat{x})}{4!}h^4 + \dots$$

Por lo tanto:

$$f(\hat{x}+2h) - 2f(\hat{x}+h) = -f(\hat{x}) + h^2 f''(\hat{x}) + h^3 f'''(\hat{x}) + \frac{14h^4 f^{(4)}(\hat{x})}{4!} + \dots$$

Despejando la segunda derivada:

$$f''(\hat{x}) = \frac{f(\hat{x}+2h) - 2f(\hat{x}+h) + f(\hat{x})}{h^2} + h(-f'''(\hat{x}) - \frac{8}{4!}f^{(4)}(\hat{x})h + \dots)$$

si:

$$O = -f'''(\hat{x}) - \frac{8}{4!}f^{(4)}(\hat{x})h + \dots$$

Esto queda:

$$f''(\hat{x}) = \frac{f(\hat{x}+2h) - 2f(\hat{x}+h) + f(\hat{x})}{h^2} + h(O)$$

Llegando así a la expresión.

2. Cuarta Derivada por Diferencias Finitas, Caso Discreto Derecho

Comenzamos con la serie de Taylor de $f(x)$ alrededor de $\hat{x}$, que se define como:

$$f(x) = f(\hat{x}) + f'(\hat{x})(x-\hat{x}) + \frac{f''(\hat{x})}{2!}(x-\hat{x})^2 + \frac{f'''(\hat{x})}{3!}(x-\hat{x})^3 + \frac{f^{(4)}(\hat{x})}{4!}(x-\hat{x})^4 + \dots$$

Además:

$$f(\hat{x}+h) = f(\hat{x}) + f'(\hat{x})h + \frac{f''(\hat{x})}{2!}h^2 + \frac{f'''(\hat{x})}{3!}h^3 + \frac{f^{(4)}(\hat{x})}{4!}h^4 + \dots$$

$$f(\hat{x}+2h) = f(\hat{x}) + 2f'(\hat{x})h + 2f''(\hat{x})h^2 + \frac{8f'''(\hat{x})}{3!}h^3 + \frac{16f^{(4)}(\hat{x})}{4!}h^4 + \dots$$

$$f(\hat{x}+3h) = f(\hat{x}) + 3f'(\hat{x})h + \frac{9f''(\hat{x})h^2}{2!} + \frac{27f'''(\hat{x})}{3!}h^3 + \frac{81f^{(4)}(\hat{x})}{4!}h^4 + \dots$$

$$f(\hat{x}+4h) = f(\hat{x}) + 4f'(\hat{x})h + \frac{16f''(\hat{x})h^2}{2!} + \frac{64f'''(\hat{x})}{3!}h^3 + \frac{256f^{(4)}(\hat{x})}{4!}h^4 + \dots$$

Por tanto:

$$f(\hat{x}+4h) - 4f(\hat{x}+4h) + 6f(\hat{x}+4h) - 4f(\hat{x}+4h) = -f(\hat{x}) + h^4 f^{(4)}(\hat{x}) + h^5(O)$$

Despejando la cuarta derivada:

$$f^{(4)}(\hat{x}) = \frac{f(\hat{x}+4h) - 4f(\hat{x}+4h) + 6f(\hat{x}+4h) - 4f(\hat{x}+4h) + f(\hat{x}) + h^5(O)}{h^4}$$

$$f^{(4)}(\hat{x}) = \frac{f(\hat{x}+4h) - 4f(\hat{x}+4h) + 6f(\hat{x}+4h) - 4f(\hat{x}+4h) + f(\hat{x})}{h^4} + \frac{h^5(O)}{h^4}$$

$$f^{(4)}(\hat{x}) = \frac{f(\hat{x}+4h) - 4f(\hat{x}+4h) + 6f(\hat{x}+4h) - 4f(\hat{x}+4h) + f(\hat{x})}{h^4} + h(O)$$

Quedando así demostrado.

Submitted by Alan Axell Ramírez Pérez on 4 de marzo de 2024.