

Selection at Two Loci

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Model with random mating, no selection

x_1	=	frequency of AB -gametes
p_A	=	frequency of A -gametes
p_B	=	frequency of B -gametes
c	=	probability of recombination

Change in frequency of AB -gametes during one generation:

$$x'_1 = x_1 - cD$$

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All four gametes, still no selection

Gamete	Recurrence
AB	$x'_1 = x_1 - cD$
Ab	$x'_2 = x_2 + cD$
aB	$x'_3 = x_3 + cD$
ab	$x'_4 = x_4 - cD$

Selection affecting gametes

Gamete	Recurrence
AB	$x'_1 = w_1(x_1 - cD)/\bar{w}$
Ab	$x'_2 = w_2(x_2 + cD)/\bar{w}$
aB	$x'_3 = w_3(x_3 + cD)/\bar{w}$
ab	$x'_4 = w_4(x_4 - cD)/\bar{w}$

where $\bar{w} = \sum x_i w_i$ is mean fitness.

What if selection acts on adults?

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The effect of recombination

What gametes are produced by the following genotypes?

Genotype	Heterozygous loci	Gametes produced			
		AB	Ab	aB	ab
AB/AB	0	1			
AB/Ab	1	$1/2$	$1/2$		
AB/ab	2	$\frac{1-c}{2}$	$\frac{c}{2}$	$\frac{c}{2}$	$\frac{1-c}{2}$
Ab/ab	2	$\frac{c}{2}$	$\frac{1-c}{2}$	$\frac{1-c}{2}$	$\frac{c}{2}$

Only double heterozygotes make recombinant gametes.

If these genotypes have low fitness, few recombinants appear.

Selection affecting diploid adults

Gamete	Recurrence
AB	$x'_1 = \bar{w}_1(x_1 - cw_h D)/\bar{w}$
Ab	$x'_2 = \bar{w}_2(x_2 + cw_h D)/\bar{w}$
aB	$x'_3 = \bar{w}_3(x_3 + cw_h D)/\bar{w}$
ab	$x'_4 = \bar{w}_4(x_4 - cw_h D)/\bar{w}$

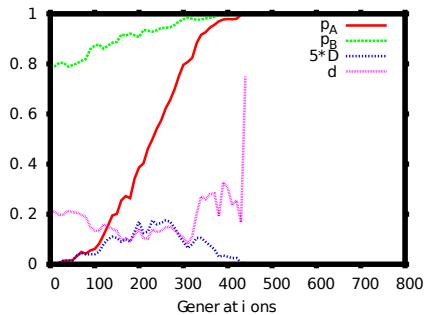
- ▶ Fitnesses become \bar{w}_i : weighted mean over genotypes in which gamete i appears.
- ▶ Recombination limited by the fitness (w_h) of double heterozygotes: only these contribute recombinant gametes.
- ▶ Useful as a recipe for calculation.

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A sweeps; B hitch-hikes

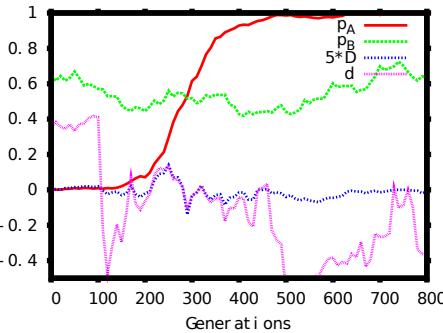
Parameters: $s = 0.02$, $c = 0.001$, $N = 5000$



- Selective sweep of allele A.
- Allele B hitch-hikes to fixation.
- D high when p_A has high heterozygosity.
- d high throughout

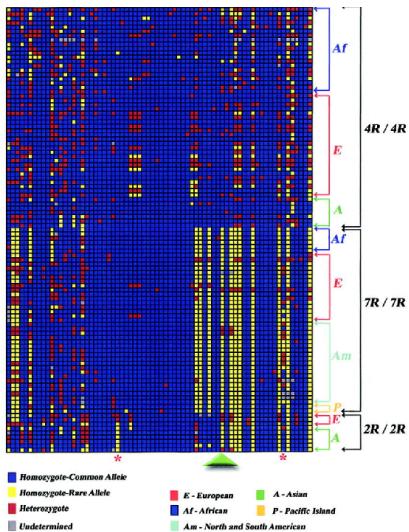
Linked allele may fail to increase

Parameters: $s = 0.02$, $c = 0.001$, $N = 5000$



- Allele A sweeps to fixation.
- Little change in linked allele. Why?
- LD lost early on this run, so D near 0.
- Loss of LD shows as big drop in d .

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- LD at D4 dopamine receptor
- Rows are diploid genotypes
- Blue: common homozygote
- Yellow: rare homozygote
- Red: heterozygote
- Note LD w/i 7R genotypes

DNA sequences from region of human lactase gene

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cgcttcaggcattctatctaaccaccaacgtaaaggtaatgcctaacccagacgtttcaactct
20 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
21 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
22 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
23 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
24 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
25 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
26 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
27 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
28 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
29 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
30 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
31 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
32 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
33 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
34 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
35 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
36 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
37 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
38 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
39 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
40 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
41 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
42 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
43 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
44 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
45 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
46 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
47 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
48 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
49 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
50 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
51 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
52 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t
53 .....c.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t.....t

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Summary

- Two-locus gametic selection is very simple.
- When selection acts on diploids, the recombination rate is weighted by the fitness of double heterozygotes.
- Hitch-hiking: selection at one locus may change allele frequencies at linked loci.
- If enough recombination happens early in the process, linked loci do not hitch-hike.

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