Selection at Two Loci

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Model with random mating, no selection

$$
\begin{aligned}
x_1 &= \text{frequency of } AB\text{-gametes} \\
p_A &= \text{frequency of } A\text{-gametes} \\
p_B &= \text{frequency of } B\text{-gametes} \\
c &= \text{probability of recombination}
\end{aligned}
$$

Change in frequency of AB-gametes during one generation:

$$x_1' = x_1 - cD$$

All four gametes, still no selection

Gamete	*Recurrence*		
AB	x_1'	$=$	$x_1 - cD$
Ab	x_2'	$=$	$x_2 + cD$
aB	x_3'	$=$	$x_3 + cD$
ab	x_4'	$=$	$x_4 - cD$

Selection affecting gametes

Gamete			*Recurrence*
AB	x_1'	$=$	$w_1(x_1 - cD)/\bar{w}$
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where $\bar{w} = \sum x_i w_i$ is mean fitness.

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What if selection acts on adults?

The effect of recombination

What gametes are produced by the following genotypes?

Genotype	Heterozygous loci	Gametes produced			
		AB	Ab	aB	ab
AB/AB	0				

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Only double heterozygotes make recombinant gametes.

If these genotypes have low fitness, few recombinants appear.

Selection affecting diploid adults

Gamete			*Recurrence*
AB	x_1'	$=$	$\bar{w}_1(x_1 - cw_hD)/\bar{w}$
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- Fitnesses become $\bar{w}_i$: weighted mean over genotypes in which gamete i appears.

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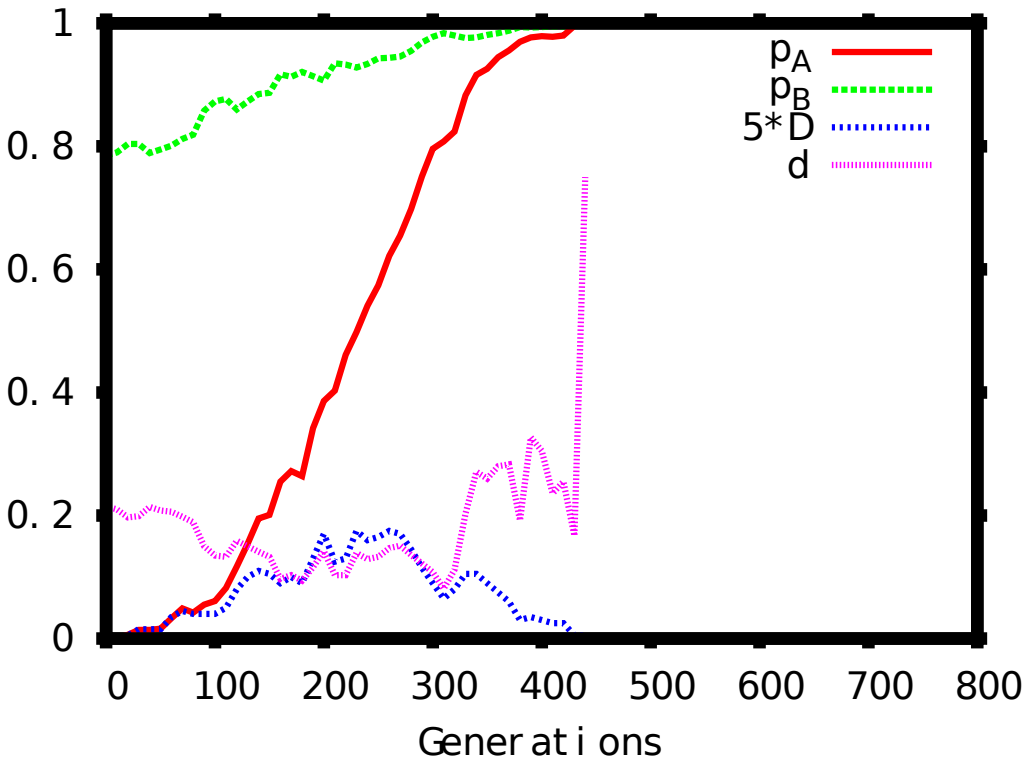
- Fitnesses become $\bar{w}_i$: weighted mean over genotypes in which gamete i appears.
- Recombination limited by the fitness (w_h) of double heterozygotes: only these contribute recombinant gametes.
- Useful as a recipe for calculation.

A sweeps; B hitch-hikes

Parameters: $s = 0.02$, $c = 0.001$, $N = 5000$



Selective sweep of allele A.

Allele B hitch-hikes to fixation.

D high when p_A has high heterozygosity.

d high throughout

Linked allele may fail to increase

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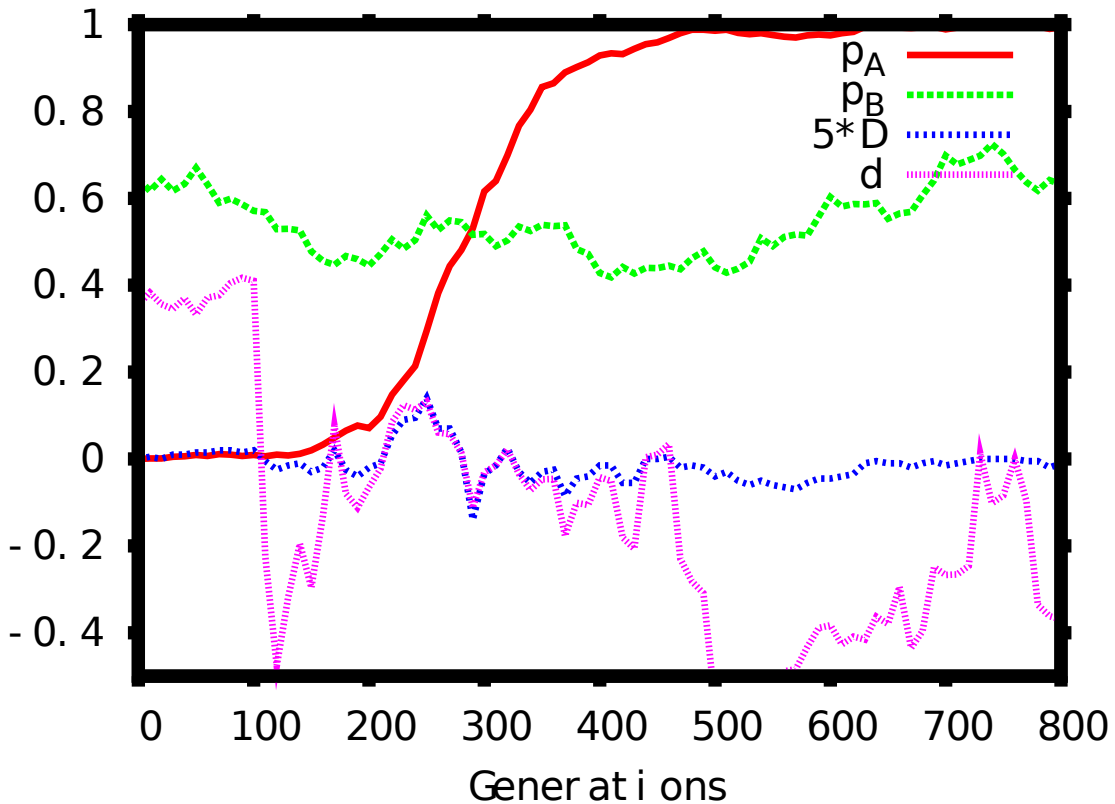
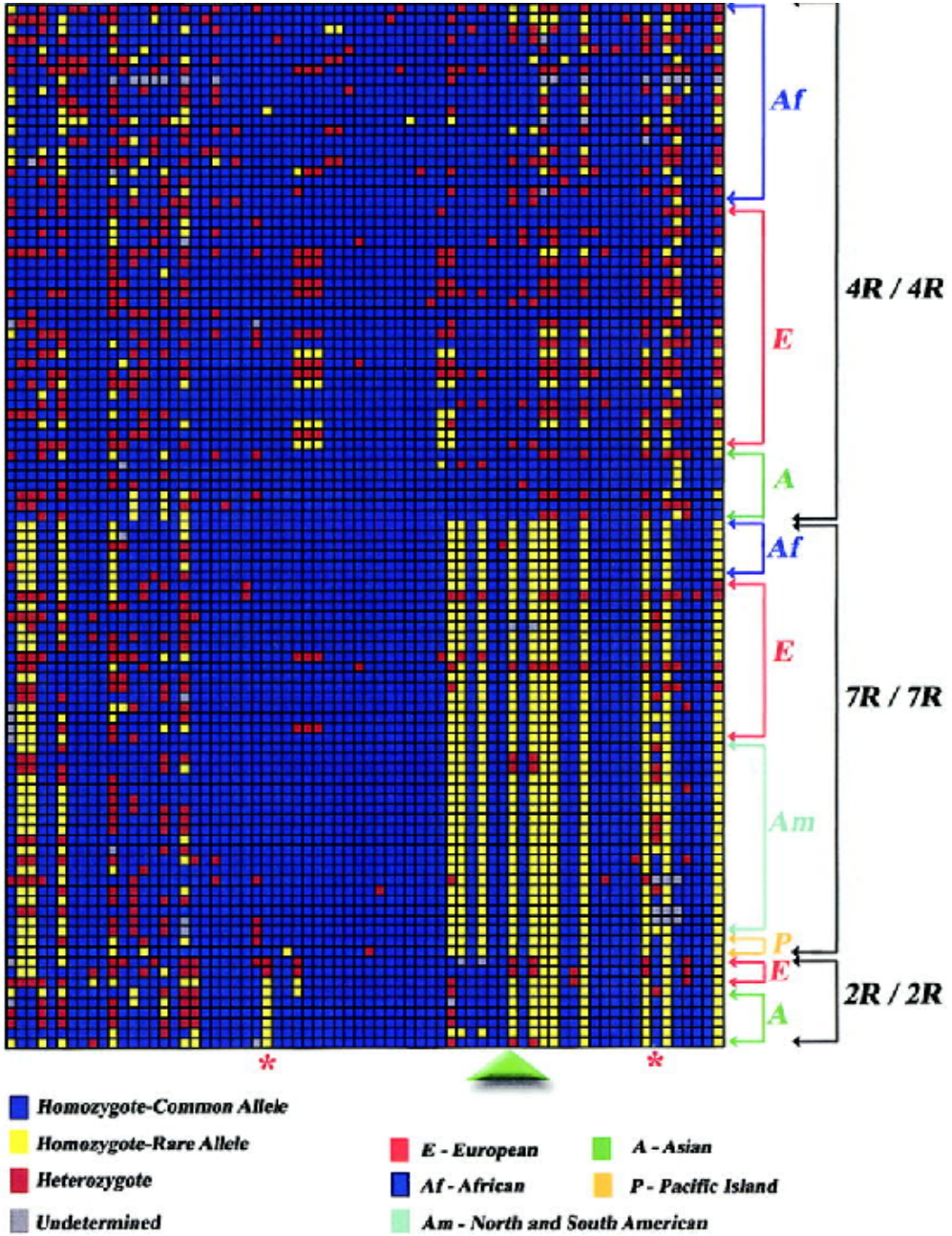


Allele A sweeps to fixation.

Little change in linked allele. Why?

LD lost early on this run, so D near 0.

Loss of LD shows as big drop in d.



- LD at D4 dopamine receptor
- Rows are diploid genotypes
- Blue: common homozygote
- Yellow: rare homozygote
- Red: heterozygote
- Note LD w/i 7R genotypes

DNA sequences from region of human lactase gene

```
     cgcttcaggcattcctatctaaacagaccaacgtaAgggtacaatgcctaacccagacgtttcaactct
20   ............................................................................
21   ............................................................................
22   ............................................................................
23   ............................................................................
24   ............................................................................
25   ............................................................................
26   ............................................................................
27   ..................t.........................................................
28   ..................t.........................................................
29   ..................................................c.........................
37   ..................................G..a.gt.....t.........gac.c.tgtct.
38   ...ccgga....gat..at..gg..c.....tc.gGaaa.g..ccttt...tg......c...t.t...
39   ...ccgga....gat..at..gg..c.....tc.gGaaa.g..ccttt...tg......c...t.t...
40   ..tcc...agtag.t.cat..g.....t..ttccgG..a.gt.....t.........gac.c.tgtct.
41   ..tcc...agtag.t.cat..g.....t.gttccgG..a.gt.....t.........gac.c.tgtct.
42   ..tcc...agtag.t.cat..g.....t.gttccgG..a.gt.....t.........gac.c.tgtct.
43   ..tcc...agtag.t.cat..g.....t.g.tc.gG..a.gt.....t.........gac.c.tgtct.
44   ..tcc...agtag.t.cat..g.....t..ttc.gG..acgt.....t.........gac.c.tgtct.
45   ..tcc...agtag.t.cat..g.....t.gttc.gG..a.gt.....t.........gac.c.tgtct.
46   ...ccgga....gat..at..gg..c.....tc.gGaaa.g..ccttt...tg......cg.gt.t..c
47   ..tcc...agtag.t.cat..g.....t.gttccgG..a.gt.....t.........gac.c.tgtct.
48   ..tcc...agtag.t.cat..g.....t.gttccgG..a.gt.....t.........gac.c.tgtct.
49   ..tcc...agtag.t.cat..g.....t.gttccgG..a.gt.....t.........gac.c.tgtct.
50   tatccgga....g.tc.atcgg.tc.g.tg.tc.gG..a.g.g....tg....ggt...cg.gt.t..c
51   ta.ccgga....g.t..atcgg.tc.g.tg.tc.gG..a.g.g....tg....ggt...cg.gt.t..c
52   ta.ccgga....g.t..atc.g.tc.g.tg.tc.gG..a.g.g....tg....ggt...cg.gt.t..c
53   ta.ccgga....g.t..atcgg.tc.g.tg.tc.gG..a.g.g....tg....ggt...cg.gt.t..c
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Summary

- Two-locus gametic selection is very simple.
- When selection acts on diploids, the recombination rate is weighted by the fitness of double heterozygotes.
- Hitch-hiking: selection at one locus may change allele frequencies at linked loci.
- If enough recombination happens early in the process, linked loci do not hitch-hike.