

# PERCOLATION AND ITS VARIATIONS

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An exploration of the current state of percolation models

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## ABSTRACT

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The current state of percolation theory is hard to assess. Research often uses different methods, language and notation, is hard to reproduce, and sometimes even provides conflicting results. We aim to give an overview of the current state of some of the most common and simple percolation models, alongside with open source code that allow the reader to easily reproduce the results presented, as well as present an analysis of a novel model called Stateful Mandelbrot Percolation. We hope to facilitate further research, as well as provide a starting point for anyone interested in the topic.



*We have seen that computer programming is an art,  
because it applies accumulated knowledge to the world,  
because it requires skill and ingenuity, and especially  
because it produces objects of beauty.*

— knuth:1974 [knuth:1974]

## ACKNOWLEDGEMENTS

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## ACRONYMS

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DRY Don't Repeat Yourself



## Part I

### CURRENT STATE



## CLASSIC PERCOLATION

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### 1.1 1D CASE: TOY MODEL

#### 1.1.1 *Percolation threshold $p_c$*

#### 1.1.2 *Critical exponent $\sigma$*

#### 1.1.3 *Critical exponent $\gamma$*

#### 1.1.4 *Critical exponent $\nu$*

### 1.2 2D CASE IN SQUARE LATTICE

#### 1.2.1 *Critical exponents*

#### 1.2.2 *Correlation length*

#### 1.2.3 *Phase transitions*

#### 1.2.4 *Real space renormalisation*

### 1.3 BETHE LATTICE





## MANDELBROT PERCOLATION

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### 2.1 INTRODUCTION

### 2.2 DEFINITIONS

### 2.3 RESULTS



## Part II

### NOVEL MODEL



## CORRELATED MANDELBROT PERCOLATION

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### 3.1 INTRODUCTION

### 3.2 DEFINITIONS

### 3.3 RESULTS



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