

PERCOLATION AND ITS VARIATIONS

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An exploration of the current state of percolation models

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ABSTRACT

The current state of percolation theory is hard to assess. Research often uses different methods, language and notation, is hard to reproduce, and sometimes even provides conflicting results. We aim to give an overview of the current state of some of the most common and simple percolation models, alongside with open source code that allow the reader to easily reproduce the results presented, as well as present an analysis of a novel model called Stateful Mandelbrot Percolation. We hope to facilitate further research, as well as provide a starting point for anyone interested in the topic.

*We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity, and especially
because it produces objects of beauty.*

— **knuth:1974** [**knuth:1974**]

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CONTENTS

I	CURRENT STATE	1
1	CLASSIC PERCOLATION	3
1.1	1D Case: toy model	3
1.2	2D case in square lattice	3
1.3	2D case in more general lattices	3
2	CLASSIC PERCOLATION	5
2.1	Introduction	5
2.2	Definitions	5
2.3	Results	5
II	NOVEL MODEL	7
3	CORRELATED MANDELBROT PERCOLATION	9
3.1	Introduction	9
3.2	Definitions	9
3.3	Results	9

LIST OF FIGURES

LIST OF TABLES

LISTINGS

ACRONYMS

DRY Don't Repeat Yourself

Part I

CURRENT STATE

CLASSIC PERCOLATION

1.1 1D CASE: TOY MODEL

1.2 2D CASE IN SQUARE LATTICE

1.3 2D CASE IN MORE GENERAL LATTICES

CLASSIC PERCOLATION

2.1 INTRODUCTION

2.2 DEFINITIONS

2.3 RESULTS

Part II

NOVEL MODEL

CORRELATED MANDELBROT PERCOLATION

3.1 INTRODUCTION

3.2 DEFINITIONS

3.3 RESULTS

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