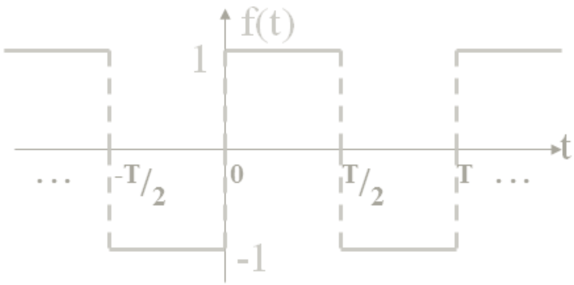
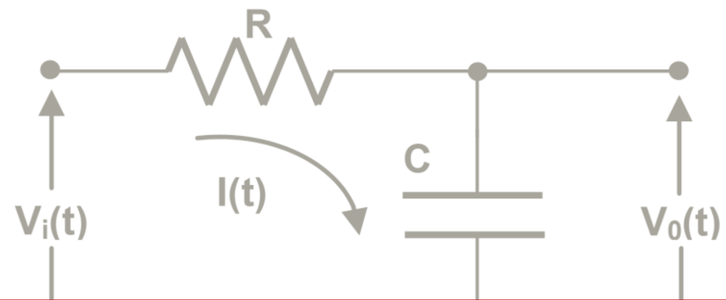
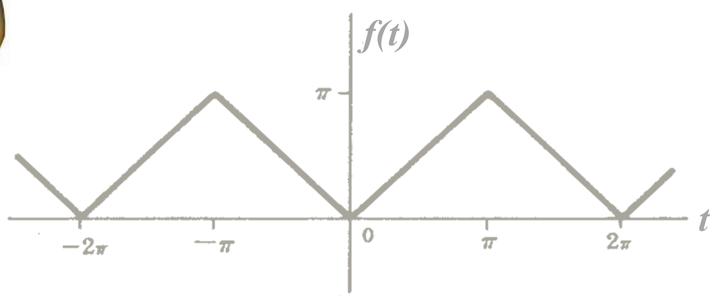
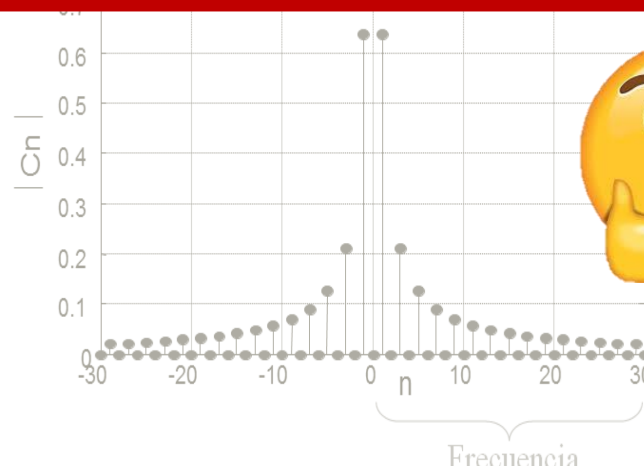




Actividad Práctica

●Convolución 1P●





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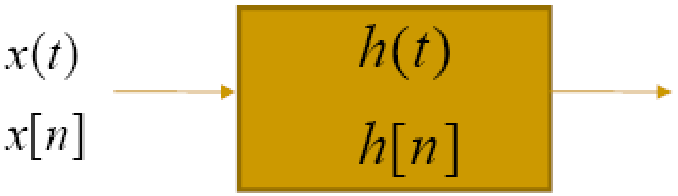
Sumatoria/Integral de Convolución



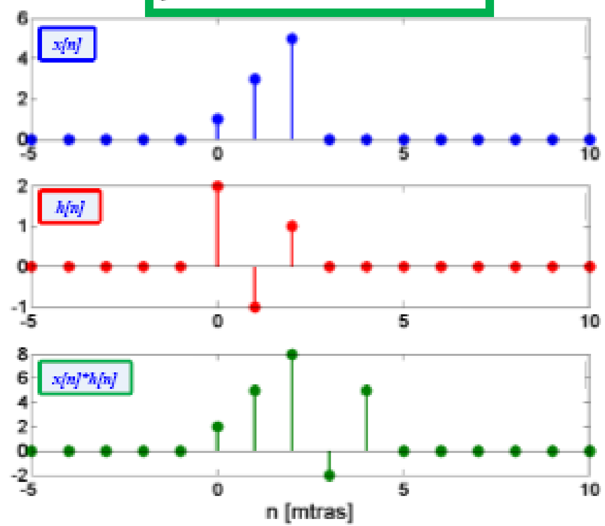
$$y(t) = x(t) * h(t)$$
$$y[n] = x[n] * h(n)$$

$$\boxed{y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]}$$

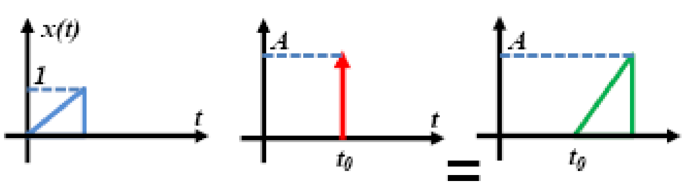
$$\boxed{y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$$

Sumatoria de Convolución

$$\boxed{y[n] = x[n] * h[n]}$$



Convolución con una función impulso



$$\boxed{x(t) * A\delta(t-t_0) = Ax(t-t_0)}$$

$$\boxed{x[n] * A\delta[n-n_0] = Ax[n-n_0]}$$

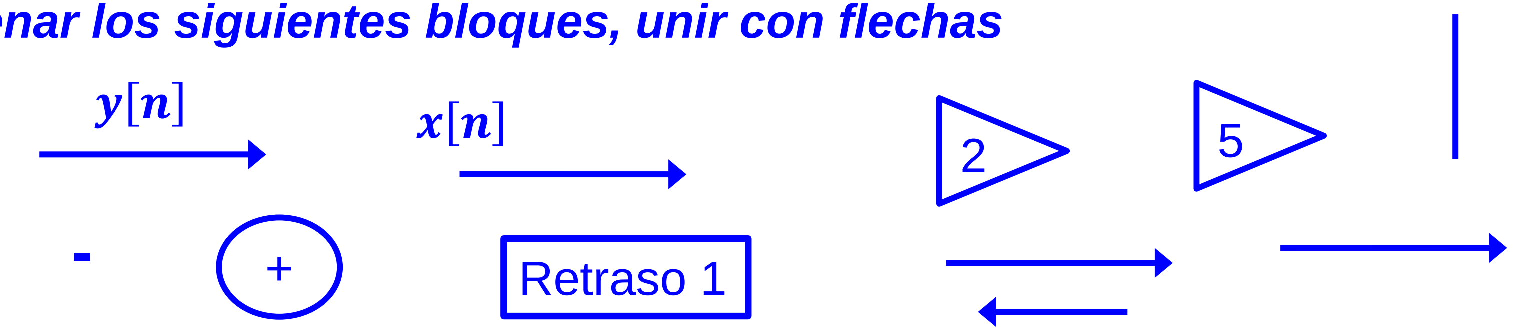
Propiedades

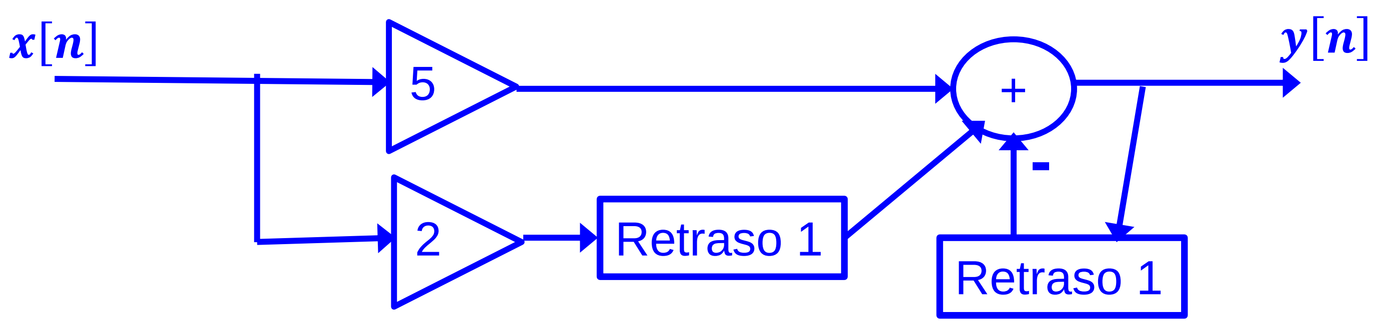
$$(x * h_1) * h_2 = x * (h_1 * h_2)$$
$$x * h = h * x$$
$$x * (h_1 + h_2) = x * h_1 + x * h_2$$

Repaso Diagrama en bloques de $\boldsymbol{y[n] = 5.x[n] + 2.x[n-1] - y[n-1]}$

$\delta[n-1]$ retraso 1

Ordenar los siguientes bloques, unir con flechas



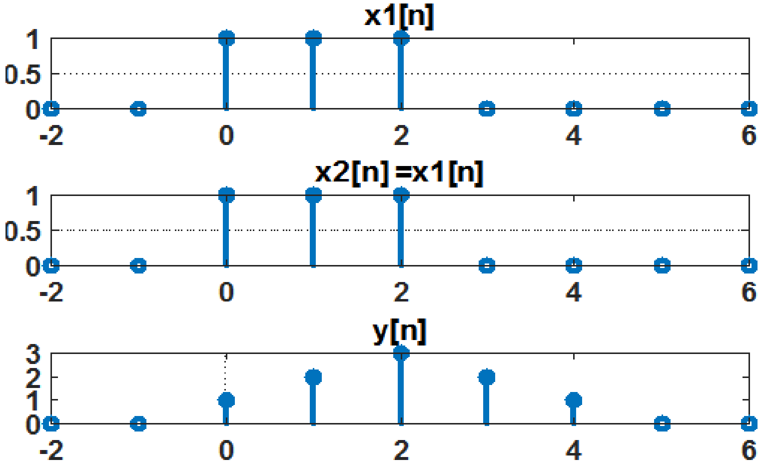


Ejercicio - Convolución Discreta

Sean las siguientes señales discretas, calcular la convolución y graficar:

1) $x_1[n] = u[n] - u[n-3]$

Calcular: $y[n] = x_1[n] * x_1[n]$



Resolución con Matlab / Octave

```
% Convolución de Señales Discretas
n= -2 : 6 ;   %Vector discreto. Se utiliza dt=1
%Señales a convolucionar
x1=escalon(n)-escalon(n-3) ;
x2= x1 ; % dt=1
%Algoritmo de convolución: Utilizamos conv(v1,v2) provista por MatLab
y=conv(x1,x2) ;
% Convolución inicia en: suma de inicios, finaliza en: suma de finales
nc = (-2-2) : (6+6) ; % Nuevo vector temporal más largo
subplot(311); stem(n,x1); grid on; title('x1[n]')
subplot(312); stem(n,x2); grid on; title('x2[n] =x1[n]')
subplot(313); stem(nc,y); grid on; title('y[n]'); xlim([-2 6])
```

$$y[n] = x[n] * g[n] = \sum_{k=-\infty}^{\infty} x[k].\, g[n-k]$$

cont. Resolución Analítica :

$x_1[n] = u[n] - u[n-3]$

Calcular: $y[n] = x_1[n] * x_1[n]$

$x_1[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$y[n] = x_1[n] * x_1[n] = x_1[n] \ * (\delta[n] + \delta[n-1] + \delta[n-2])$

$y[n] = x_1[n] + x_1[n-1] + x_1[n-2]$ Sumamos las 3 señales

Otra forma:

$y[n] = (\delta[n] + \delta[n-1] + \delta[n-2]) * (\delta[n] + \delta[n-1] + \delta[n-2])$

$y[n] = \delta[n] + \delta[n-1] + \delta[n-2] \ + \ \delta[n-1] + \delta[n-2] + \delta[n-3] \ + \delta[n-2] +$
$+ \delta[n-3] + \delta[n-4]$

$\boxed{y[n] = \delta[n] + 2.\delta[n-1] + 3.\delta[n-2] + 2.\delta[n-3] + \delta[n-4]}$

cont. Resolución Analítica : $x_1[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ $\qquad y[n] = x_1[n] * x_1[n]$

Por definición (otra forma)

$y[n] = x_1[n] * x_1[n]$

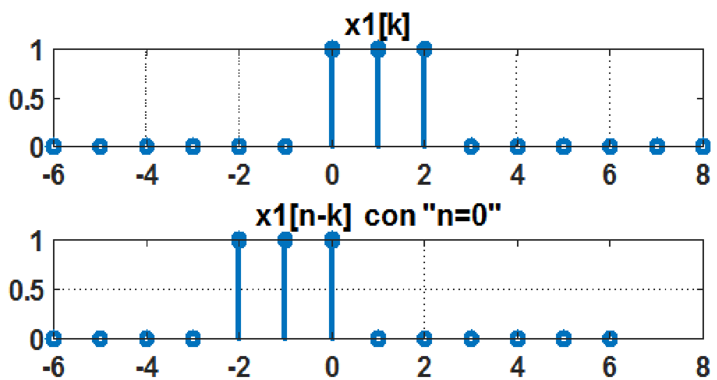
$$\boldsymbol{y[n] = \sum_{k=-\infty}^{\infty} x_1[k].\, x_1[n-k]}$$

En convolución: «elegimos la señal menos compleja y la Invertimos»

$\boldsymbol{n = 0}$

$y[0] = \sum_{k=-\infty}^{\infty} x_1[k].\, x_1[0-k] =$

$= 1x1 = 1$

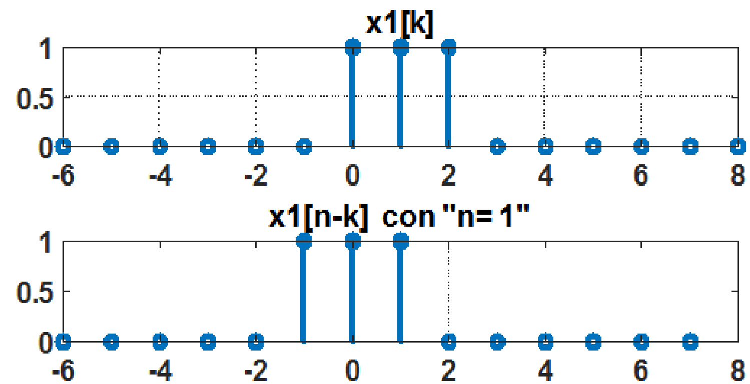


Desplazamos a la derecha señal x1

$\boldsymbol{n = 1}$

$y[1] = \sum_{k=-\infty}^{\infty} x_1[k].\, x_1[1-k] =$

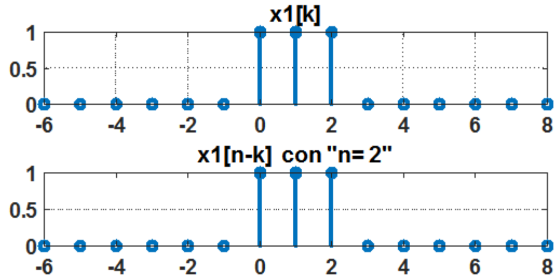
$= 1x1 + 1x1 = 2$



cont.

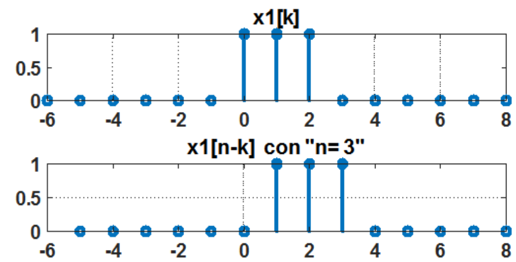
$\underline{\boldsymbol{n=2}}$

$y[2] = \sum_{k=-\infty}^{\infty} x_1[k].x_1[2-k] = 1x1 + 1x1 + 1x1 = 3$



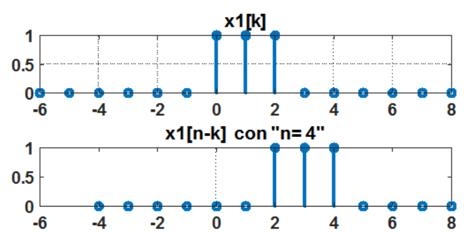
$\underline{\boldsymbol{n=3}}$

$y[3] = \sum_{k=-\infty}^{\infty} x_1[k].x_1[3-k] = 1x1 + 1x1 = 2$



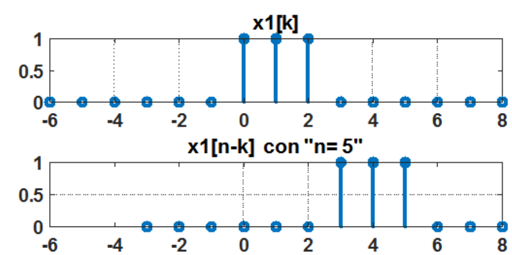
$\underline{\boldsymbol{n=4}}$

$y[4] = \sum_{k=-\infty}^{\infty} x_1[k].x_1[4-k] = 1x1 = 1$



$\underline{\boldsymbol{n=5}}$

$y[5] = \sum_{k=-\infty}^{\infty} x_1[k].x_1[5-k] = 0$



Finalmente:

$$\boxed{\boldsymbol{y[n] = \delta[n] + 2.\delta[n-1] + 3.\delta[n-2] + 2.\delta[n-3] + \delta[n-4]}}$$

$y[n] = x[n] * g[n] = \sum_{k=-\infty}^{\infty} x[k].g[n-k]$ Tiempo Discreto

Convolución de tiempo continuo

$\boxed{y(t) = x(t) * g(t) = \int_{-\infty}^{\infty} x(\tau).g(t-\tau).d\tau}$ Tiempo Continuo

$$y(t) = x(t) * g(t) = g(t) * x(t)$$

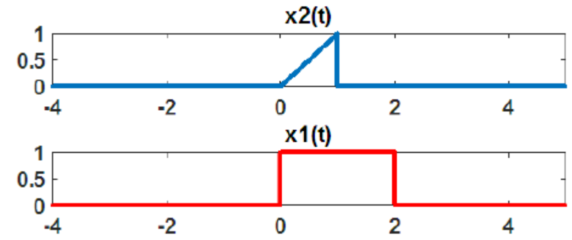
$$y(t) = \int_{-\infty}^{\infty} g(\tau).x(t-\tau).d\tau$$

Ejercicio Conv. Continua) Sean las siguientes señales contínuas, calcular la operatoria de convolución que se detallan a continuación:

$x_1(t) = u(t) - u(t-2)$;

$x_2(t) = \rho(t) - \rho(t-1) - u(t-1)$

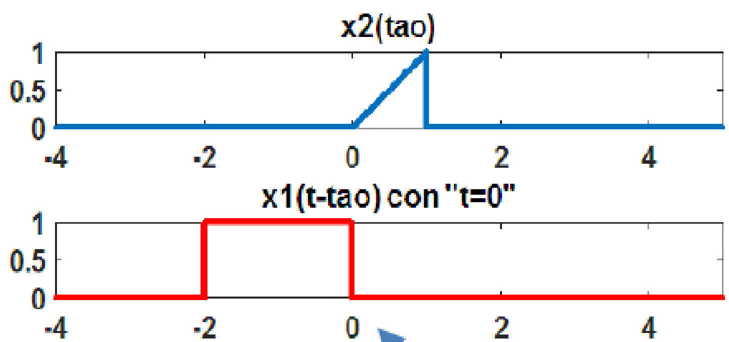
$y(t) = x_1(t) * x_2(t)$



Resolución

En convolución Invertimos la señal más simple

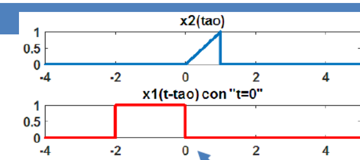
Graficamos: $x_2(\tau)$ y $x_1(-\tau)$



$$\boxed{x_1(t-\tau)|_{t=0} = x_1(-\tau)}$$

$\boxed{t}$ Este punto en el orígen le asignamos t

Continuación

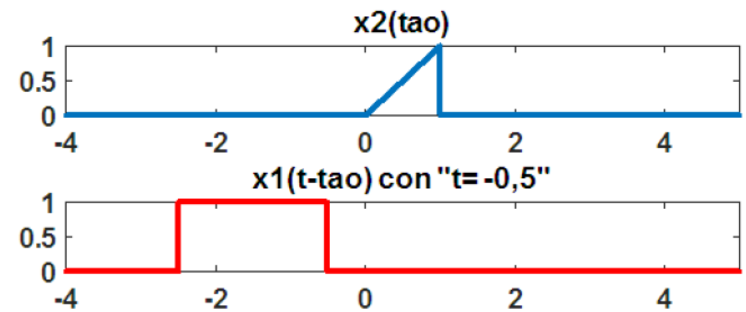


$\boxed{x_1(t-\tau)|_{t=0} = x_1(-\tau)}$

$\boxed{t}$ Este punto en el orígen le asignamos t

$\underline{t < 0 \qquad y(t) = 0}$

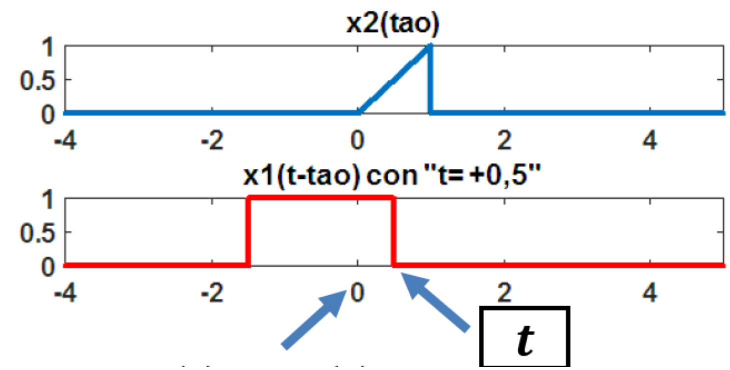
$$y(t) = \int_{-\infty}^{\infty} x_2(\tau).x_1(t-\tau).d\tau = 0$$



$\underline{0 < t < 1}$ Siendo: $\underline{x_1(t) = u(t) - u(t-2)}$

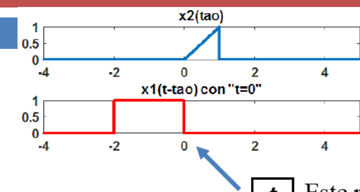
$x_2(t) = \rho(t) - \rho(t-1) - u(t-1)$

$y(t) = \int_{-\infty}^{\infty} x_2(\tau).x_1(t-\tau).d\tau =$



$$y(t) = \int_0^t \tau.\underline{1}.d\tau = \left[\frac{\tau^2}{2}\right]_0^t = \frac{t^2}{2}$$

Continuación



$x_1(t-\tau)|_{t=0} = x_1(-\tau)$

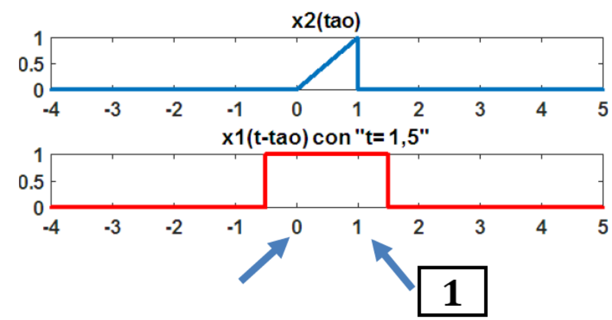
$\boxed{t}$ Este punto en el orígen le asignamos t

$\underline{1 < t < 2}$

$$y(t) = x_2(t) * x_1(t)$$

$$y(t) = \int_{-\infty}^{\infty} x_2(\tau).x_1(t-\tau).d\tau = \int_{0}^{1} \tau.1.d\tau$$

$$y(t) = \left[\frac{\tau^2}{2}\right]_0^1 = \frac{1}{2}$$

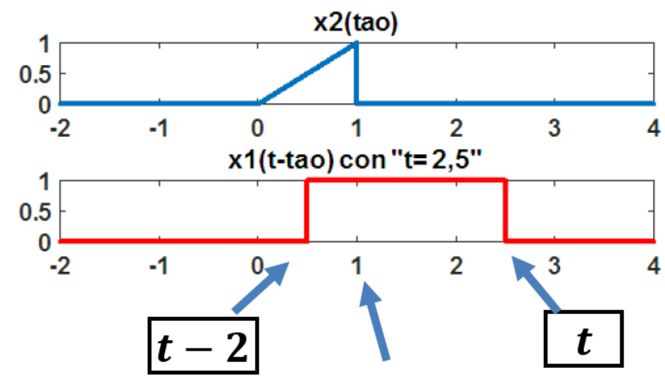


$\underline{2 < t < 3}$

$$y(t) = \int_{-\infty}^{\infty} x_2(\tau).x_1(t-\tau).d\tau = \int_{t-2}^{1} \tau.1.d\tau = \left[\frac{\tau^2}{2}\right]_{t-2}^{1}$$

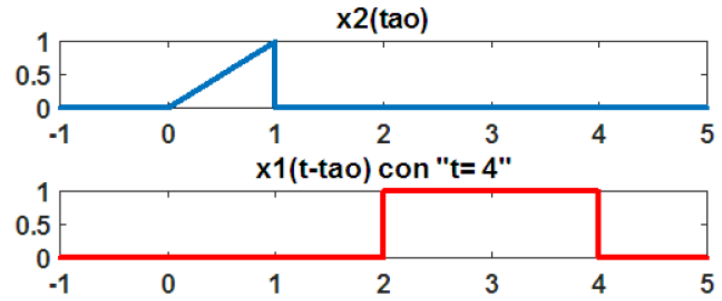
$$= \frac{1}{2} - \frac{(t-2)^2}{2} = \frac{1}{2} - \frac{t^2-4.t+4}{2} = \frac{1}{2} - \frac{t^2}{2} + 2.t - 2$$

$$y(t) = -\frac{t^2}{2} + 2.t - \frac{3}{2}$$

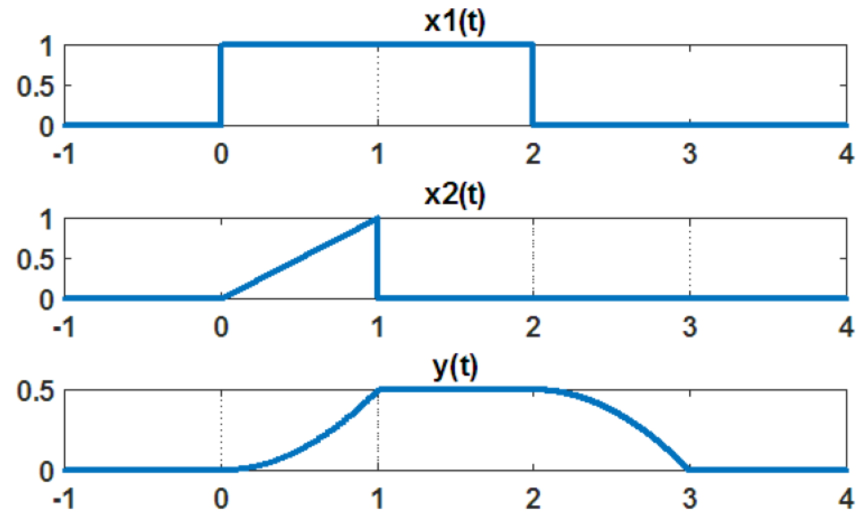


Continuación

$\underline{t > 3 \qquad y(t) = 0}$



$$y(t) = \begin{cases} 0 & t < 0 \\ \dfrac{t^2}{2} & 0 < t < 1 \\ 1/2 & 1 < t < 2 \\ -\dfrac{t^2}{2} + 2.t - \dfrac{3}{2} & 2 < t < 3 \\ 0 & t > 3 \end{cases}$$



Continuación

```
%Resolución Numérica con Matlab
% Usamos mismo tiempo en ambas señales, podría ser distinto
dt = 0.01 ; t= -1: dt: 5 ; % largo L
x1= escalon(t)-escalon(t-2);
x2= rampa(t) -rampa(t-1) - escalon(t-1);
y=conv(x1,x2)* dt; % largo resultante L+L-1
subplot(311), plot(t,x1, 'linewidth',3, 'color','r') ; grid on;
subplot(312), plot(t,x2, 'linewidth',3) ; grid on;
% Convolución inicia en: suma de inicios,
% finaliza en: suma de finales
tc = (-1-1) : dt : (5+5) ; % Nuevo vector temporal más largo
subplot(313), plot(tc,y, 'linewidth',11, 'color','y') ;
grid on; xlim([-1 5])
% Opcional gráfico analítico
hold on;
ya = t.^2 /2 .*(escalon(t)-escalon(t-1)) +...
    1/2 .*(escalon(t-1)-escalon(t-2)) +...
    (-t.^2 /2  +2*t -3/2) .*(escalon(t-2)-escalon(t-3))  ;
plot(t,ya)
```

Resumen del código

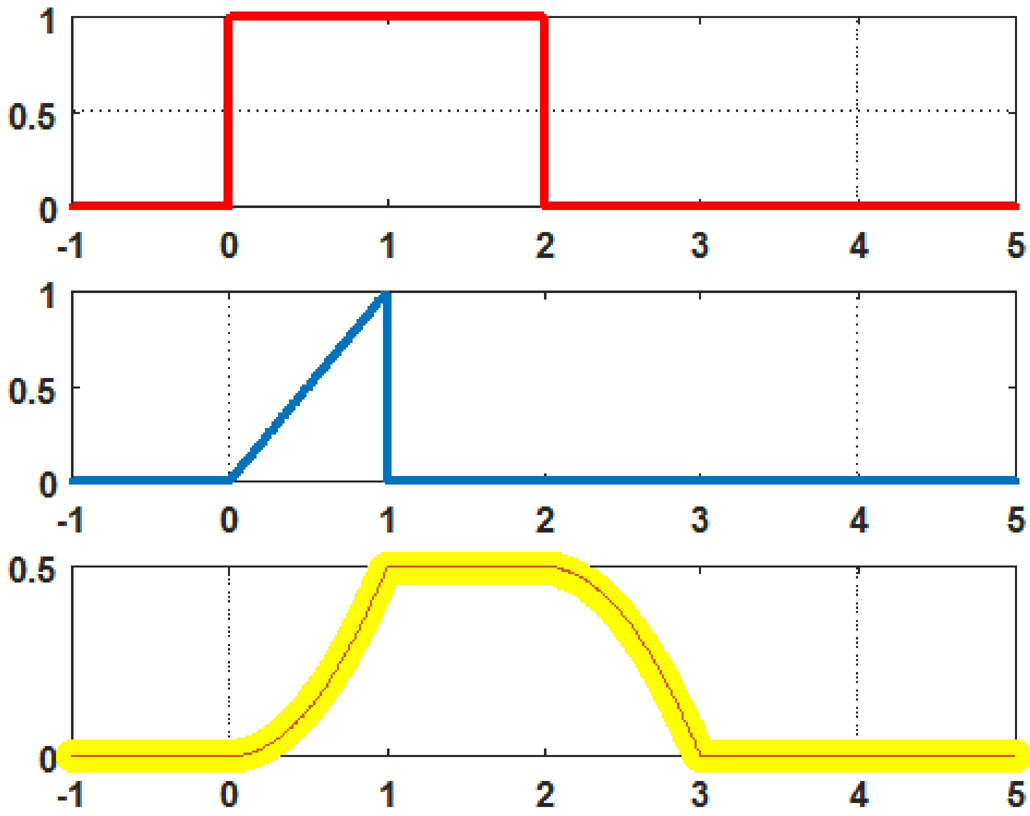
Definimos: dt, t, x1, x2

y=conv(x1,x2) dt;*

tc = (-1-1) : dt : (5+5) ;

Graficamos

Y analitica



Ejercicio: *Dado* $x(t) = u(t+1) - u(t-1)$; $h(t) = \delta(t+5) - 2.\delta(t-5)$

$y(t) = x(t) * h(t)$

Hallar $y(t)$ analíticamente y numéricamente

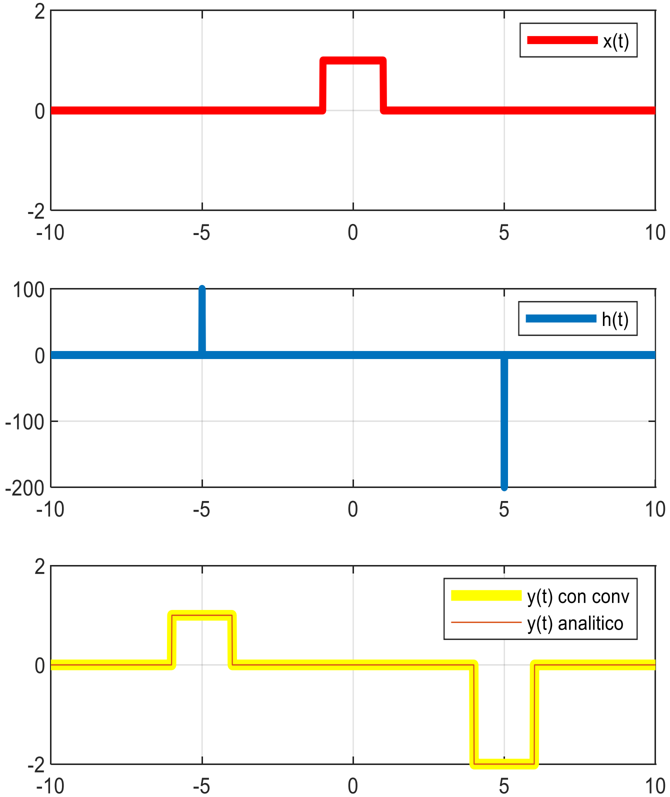
Hallar $y(5)$

$$y(t) = x(t) * h(t)$$

$$y(t) = x(t) * (\delta(t+5) - 2.\delta(t-5))$$

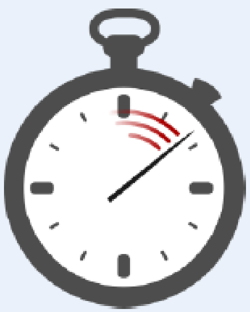
$$y(t) = x(t+5) - 2.x(t-5)$$

$$\boxed{y(5) = -2}$$



Consigna de la clase #A (30 minutos)

1. Utilizar MatLab para obtener la salida del siguiente sistema ***LIT***, cuya respuesta impulsional se detalla. ***Verificar*** ambos resultados ***analíticamente*** (las primeras muestras de ser necesario):



$$h[n]=\frac{1}{3}\left(\delta[n]+\delta[n-1]+\delta[n-2]\right)$$

$$\begin{cases} a)\ x[n]=\{\underline{1},-2,3,-4,5,-6\} \\ b)\ x[n]=\cos\left(\dfrac{\pi}{4}n\right)u[n] \end{cases}$$

2. Comparar en un único gráfico excitación vs. respuesta, el resultado del ***punto b)*** ***¿Qué efecto impone el sistema LIT sobre una señal de tipo sinusoidal?¿Se modifica su frecuencia?*** (comparar varios ciclos)

3. Efectuar el ***diagrama en bloques*** correspondiente al sistema representado por ***h[n]***

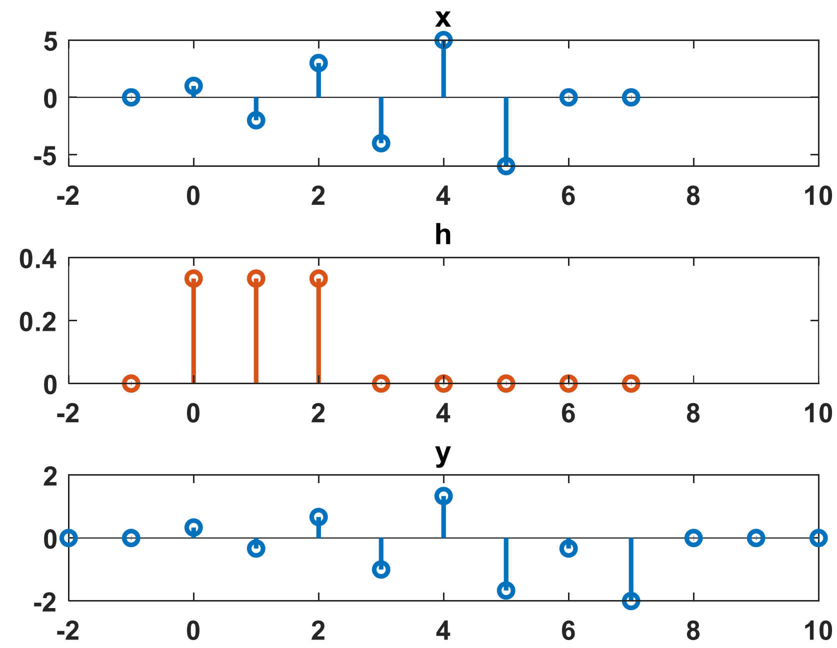
$$h[n] = \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2])$$

$$a)\ x[n] = \{\underline{1}, -2, 3, -4, 5, -6\}$$

```
close all; clear all;
%% Ej. 1.a) Convolución Discreta
% y[n] = x[n] * h[n]
n1= -1:7 ; % largo L1
x= [ 0,1,-2,3,-4,5,-6,0,0] ;% largo L1
%n2= n1 ;  % largo L2
%h= 1/3 * [ 0, 1,1,1,0,0,0,0,0]; % largo L2
h= 1/3 * ( delta(n1)+ delta(n1-1) + delta(n1-2))  ;
y=conv(x,h) ; % largo resultante L1+L2-1
% Grafico
figure  ;subplot(311), stem(n1,x) ;
subplot(312), stem(n1,h) ;
%Para graficar genero nuevo vector temporal nT de largo resultante
% de la convolución (L1+L2-1).
% Convolución comienza en: inicio n1 + inicio n2, finaliza en:  fin n1+ fin n2
nT = n1(1)+n1(1)  :  n1(end)+n1(end) ;
subplot(313), stem(nT,y) ;
```

Continuación

```
limites = [-2  10] ;
xlim( limites )
subplot(311), xlim( limites )
subplot(312), xlim( limites )
% Conclusiones… COMPLETAR Textos, colores, etc
 'linewidth'
```



Del gráfico obtenemos:

$y[0] = 1/3$

$y[1] = -\frac{1}{3}$

$y[2] = \cdots .$

Completar

%% Ej 1.b)

Completar

Ayudas 1) Analítico $\boldsymbol{h}[n]=\frac{1}{3}\delta[n]+\frac{1}{3}\delta[n-1]+\frac{1}{3}\delta[n-2]$

$$x[n]=\{\underline{\mathbf{1},}-2,3,-4,5,-6\}$$

$$x[n]=\delta[n]-2.\delta[n-1]+3.\delta[n-2]-4.\delta[n-3]+5.\delta[n-4]-6.\delta[n-5]$$

La señal más complicada la dejamos fija y reemplazamos k:

$$x[k]=\{1,-2,3,-4,5,-6\}$$

Graficamos:

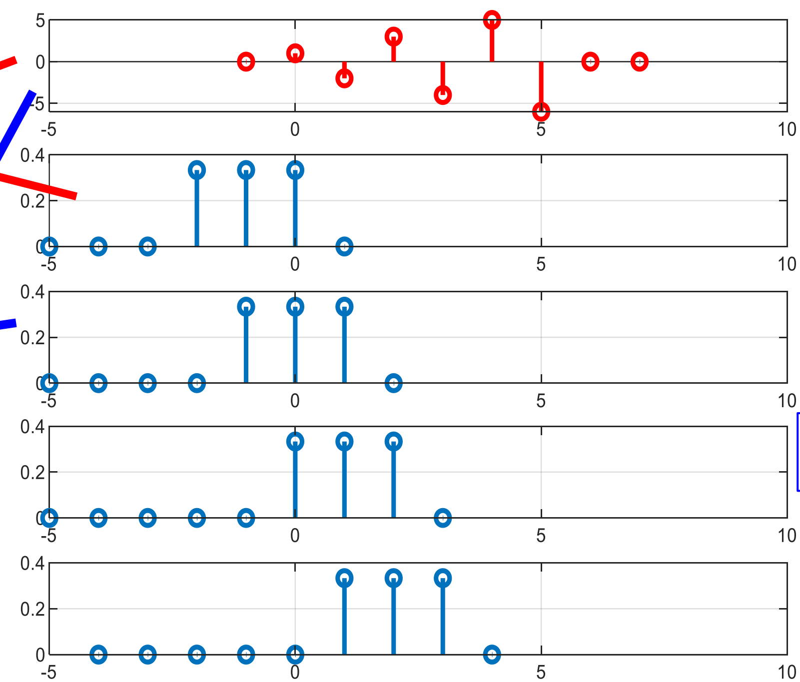
$$y[0]=1x1/3=1/3$$

$$y[1]=\frac{1x1}{3}+(-2)*\frac{1}{3}=-1/3$$

$$y[3]=\cdots$$

$$y[4]=\cdots$$

Ver página siguiente



$\boldsymbol{x[k]}$

k

$\boxed{\begin{matrix}\boldsymbol{h[n-k]\ con\ n=0}\\ \boldsymbol{h[-k]}\end{matrix}}$

$\boxed{\boldsymbol{h[1-k]\ ;\quad n=1}}$

$\boxed{\boldsymbol{h[2-k]\ ;\quad n=2}}$

$\boxed{\boldsymbol{h[3-k]\ ;\quad n=3}}$

k

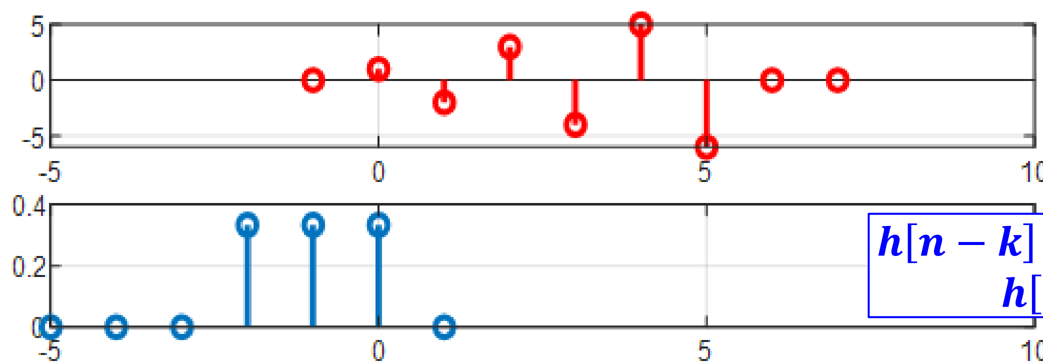
Ayudas 1) Analizamos por separado

La señal más complicada la dejamos fija y reemplazamos k: $x[k] = \{1, -2, 3, -4, 5, -6\}$

Para n=0

$$y[0] = 1x1/3 = 1/3$$

$x[k]$



k

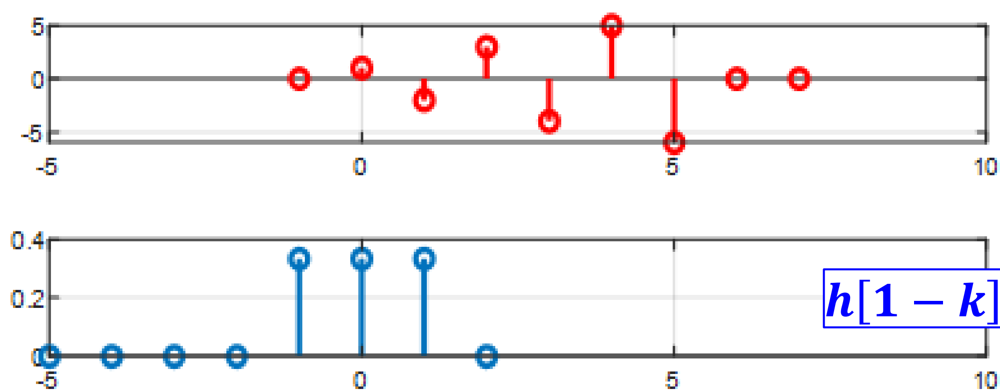
$h[n-k]\ con\ n = 0$

$h[-k]$

Para n=1

$$y[1] = \frac{1x1}{3} + (-2) * \frac{1}{3} = -1/3$$

$x[k]$



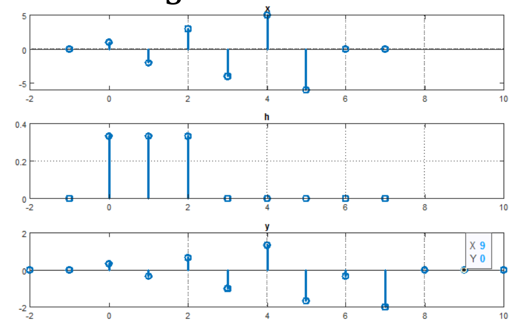
k

$h[1-k]\ ;\ \ n = 1$

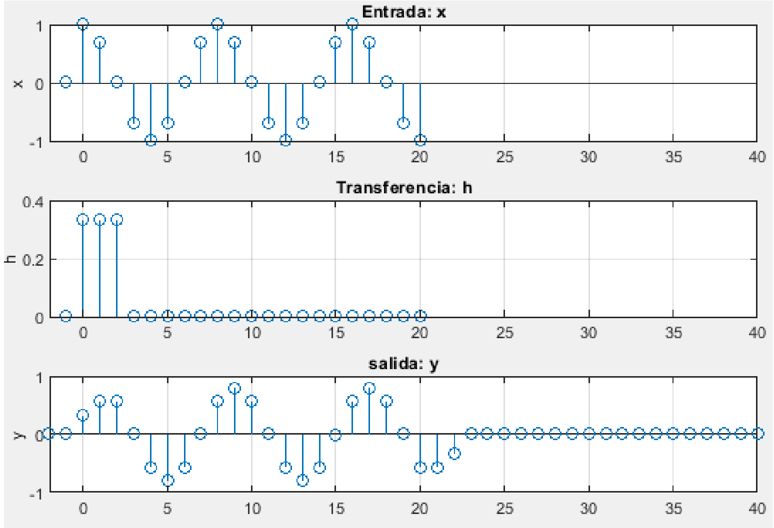
Para n=2 3 ...
completar

Resultado analítico y Matlab/Octave

a) $y[n] = \frac{1}{3}.\delta[n] - \frac{1}{3}.\delta[n-1] + \frac{2}{3}.\delta[n-2] - \delta[n-3] + \frac{4}{3}.\delta[n-4] - \frac{5}{3}.\delta[n-5] - \frac{1}{3}.\delta[n-6] - \delta[n-7]$



b)



3- Diagrama en bloques de

$$\boldsymbol{h}[n] = \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2])$$

$$y[n] = x[n] * h[n] = x[n] * \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2])$$

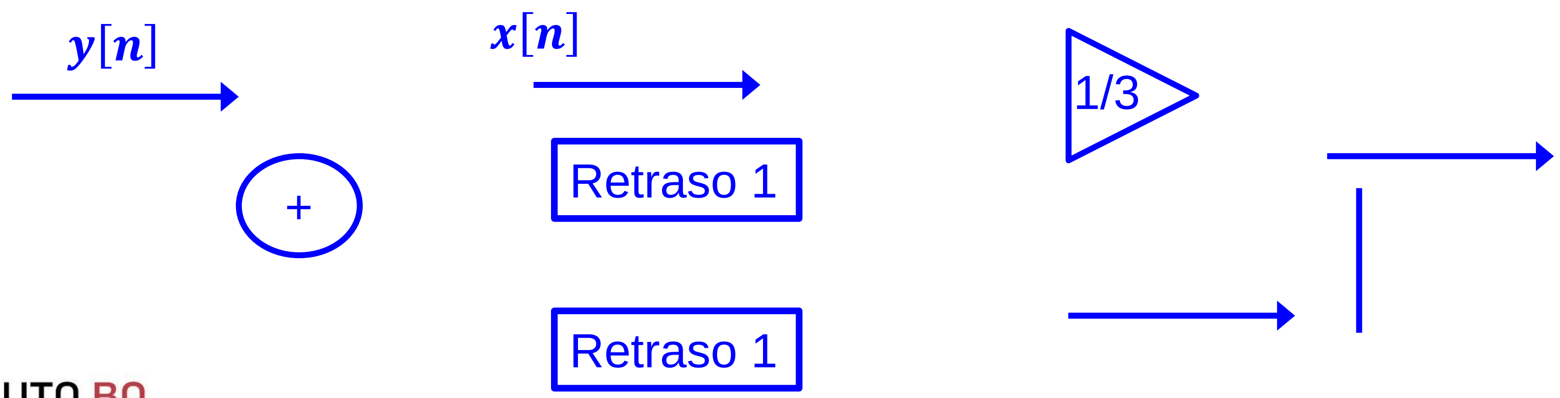
Aplicamos propiedad distributiva

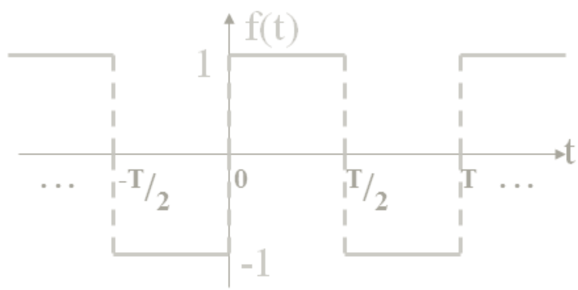
$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

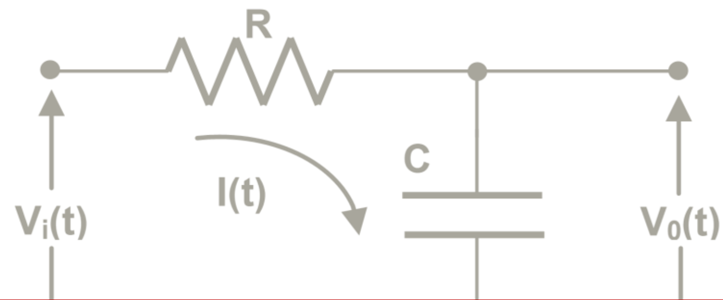
$\delta[n-1]$ retraso 1

$\delta[n-2]$ retraso 2

Ordenar los siguientes bloques, unir con flechas







Actividad Práctica

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