Trellis-Coded Multiple-Pulse-Position Modulation for Wireless Infrared Communications

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***Abstract*—We present new trellis codes based on multiple-pulse-position modulation (MPPM) for wireless infrared communication. We assume that the receiver uses maximum-likelihood sequence detection to mitigate the effects of channel dispersion, which we model using a first-order lowpass filter. Compared to trellis codes based on PPM, the new codes are less sensitive to multipath dispersion and offer better power efficiency when the desired bit rate is large, compared with the channel bandwidth. For example, when the bit rate equals the bandwidth, trellis-coded $\binom{17}{2}$-MPPM requires 1.4 dB less optical power than trellis-coded 16-PPM having the same constraint length.**

***Index Terms*—Maximum-likelihood sequence detector (MLSD), minimum distance, symmetry, trellis-coded multiple-pulse-position modulation (TC-MPPM).**

I. Introduction

The rapid growth of the laptop and handheld computer industries has elevated the importance of indoor wireless communications and wireless local-area networks. Nondirected infrared radiation offers several advantages over radio as a medium for indoor wireless networks, including an abundance of unregulated bandwidth, immunity to multipath fading, and absence of interference between rooms. The channel model for diffuse infrared communications has unique properties affecting the choice of a modulation scheme. The intense background light that is typical of most indoor environments induces a shot noise at the receiver that is accurately modeled as white Gaussian noise. Furthermore, the temporal dispersion due to multipath propagation results in intersymbol interference (ISI). Because multipath propagation destroys spatial coherence, the effects of multipath propagation can be characterized by a baseband linear filter. Thus, an equivalent baseband channel model for wireless infrared communications using intensity modulation and direct detection is [1]

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau + n(t) \tag{1}$$

where $x(t)$ represents the instantaneous optical power of the transmitter, $y(t)$ represents the instantaneous current of the receiving photodetector, $h(t)$ represents the multipath impulse response, and $n(t)$ is white Gaussian noise with two-sided power spectral density $N_0/2$. In this paper, we use an exponential model for the channel impulse response

$$h(t) = We^{-Wt}u(t) \tag{2}$$

where W is a 3-dB bandwidth and $u(t)$ is the unit-step function. Note that the channel has unity direct current (dc) gain, and the delay spread of this channel is $1/(2W)$. This channel model is simple and agrees well with experimental channel measurements [2].

When the model (1) is used for conventional radio channels, the input $x(t)$ represents amplitude, and so a power constraint P_0 on the transmitter takes the form

$$\langle x^2(t)\rangle \le P_0, \text{ where } \langle\cdot\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(\cdot)dt. \tag{3}$$

However, because the channel input $x(t)$ represents optical power in our application, it must instead satisfy the following constraints:

$$x(t) \ge 0 \text{ and } \langle x(t)\rangle \le P \tag{4}$$

where P is the average optical power constraint of the transmitter. These constraints significantly impact modulation design.

Multiple-pulse-position modulation (MPPM) is a variation of pulse-position modulation (PPM) offering improved bandwidth efficiency and improved power efficiency [3]. Like PPM, however, the power efficiency of MPPM degrades rapidly in the face of multipath dispersion, even with maximum-likelihood (ML) sequence detection [4]. Trellis-coded modulation (TCM) is an efficient way of improving the bit-error rate (BER) performance without sacrificing bandwidth efficiency [5]. The optimum receiver for TCM on a multipath channel consists of a ML sequence detector (MLSD) based on a superstate trellis that combines the states of the code with the states of the ISI [6]. Georghiades [7] applied Ungerboeck trellis coding to the photon-counting optical channel. Lee *et al.* [8] developed power-efficient trellis codes based on PPM by accounting for ISI in the set-partitioning procedure.

In Section II, we describe the system model for uncoded MPPM over a multipath channel. In Section III, we examine the power efficiency and bandwidth efficiency of uncoded MPPM on a multipath channel. In Section IV, we describe the results of a computer search for good trellis codes based on MPPM. Finally, we evaluate the power efficiency of trellis-coded MPPM (TC-MPPM) on a multipath channel when the receiver uses superstate ML sequence detection.

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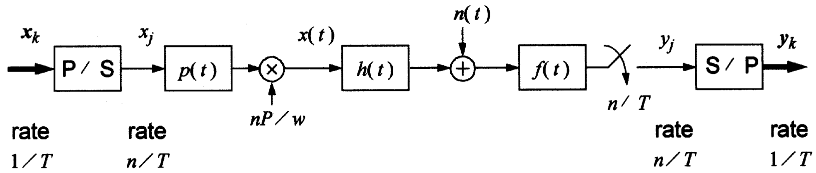

Fig. 1. Block diagram of MPPM system.

II. System Model

Let $C_{n,w}$ denote the binary block code consisting of the set of all binary vectors of length n and Hamming weight w. We refer to this code as the

$$\binom{n}{w}$$

MPPM codes, because the number of such codewords is

$$\binom{n}{w} = \frac{n!}{w!(n-w)!}. \tag{5}$$

The MPPM scheme results from a cascade of this simple block code with traditional on–off keying (OOK) modulation. In other words, each symbol period of duration T is divided into n time slots of duration T/n, and during each symbol period, a pulse of light is transmitted in precisely w slots. For the special case of $w = 1$, MPPM reduces to conventional PPM.

We now describe the model for MPPM over a channel with multipath dispersion, with the aid of Fig. 1. Let $\mathbf{x}_k \in C_{n,w}$ denote the MPPM codeword transmitted during symbol period k. An uncoded MPPM transmitter chooses the codewords $\{\mathbf{x}_k\}$ independently and uniformly from $C_{n,w}$, whereas a TC-MPPM transmitter chooses the codewords $\{\mathbf{x}_k\}$ with the aid of a convolutional encoder, as described in Section IV-B. In either case, the sequence $\{\mathbf{x}_k\}$ is serialized to produce the binary chip sequence $\{x_j\}$ with rate n/T, where $\mathbf{x}_k = [x_{kn}, x_{kn+1}, \ldots, x_{kn+n-1}]^T$. The binary chip sequence drives a transmitter filter with a rectangular pulse shape $p(t)$ of duration T/n and unity height. To satisfy the power constraint of (4), the filter output is multiplied by (nP/w) before transmission.

The wireless infrared channel model of (1) is shown in Fig. 1. The receiver uses a unit-energy filter $f(t)$ and samples the output at the chip rate n/T, producing y_j. The receiver groups the samples y_j into blocks of length n, producing a sequence of observation vectors $\{\mathbf{y}_k\}$, where $\mathbf{y}_k = [y_{kn}, y_{kn+1}, \ldots, y_{kn+n-1}]^T$. The equivalent discrete-time channel between transmitted and received chips is

$$y_j = \sum_{i=-\infty}^{\infty} h_i x_{j-i} + n_j \tag{6}$$

where h_j is the equivalent chip-rate impulse response

$$h_j = \frac{nP}{w}(p(t) * h(t) * f(t))\,|_{t=jT/n}\,. \tag{7}$$

We assume that $f(t)$ is the unit-energy whitened matched filter, so that the noise samples $\{n_j\}$ will be independent zero-mean Gaussian random variables with variance $N_0/2$. The equivalent vector channel between transmitted codewords $\mathbf{x}_k$ and observation vectors $\mathbf{y}_k$ is given by [9]

$$\mathbf{y}_k = \sum_{l=0}^{\infty} \mathbf{H}_l \mathbf{x}_{k-l} + \mathbf{n}_k \tag{8}$$

where the channel impulse response is a Toeplitz sequence $\mathbf{H}_k$, with $[\mathbf{H}_k]_{ij} = h_{kn+i-j}$, and the noise component is $\mathbf{n}_k = [n_{kn}, n_{kn+1}, \ldots, n_{kn+n-1}]^T$. For the special case of an ideal channel with infinite bandwidth ($W = \infty$), the transfer function $\mathbf{H}(z) = \Sigma_k \mathbf{H}_k z^{-k}$ reduces to a distortionless diagonal matrix of the form $\mathbf{H}(z) = A\mathbf{I}$, where from (7) it is not hard to show that the channel gain A is given by

$$A = \frac{\sqrt{nT}P}{w}. \tag{9}$$

Alternatively, using the relationship $R_b T = \log_2 \binom{n}{w}$ for the case of uncoded MPPM, we can write the constant gain as

$$A = \sqrt{\frac{n \log_2 \binom{n}{w}}{w^2 R_b}}P. \tag{10}$$

In this special case, the received vector is simply $\mathbf{y}_k = A\mathbf{x}_k + \mathbf{n}_k$.

III. Performance of Uncoded MPPM

A. On an Ideal Channel

For an ideal channel with infinite bandwidth $(W = \infty)$, the received vector is $\mathbf{y}_k = A\mathbf{x}_k + \mathbf{n}_k$, and the ML detector decides on the codeword $\mathbf{c} \in \mathbf{C}_{n,w}$ that minimizes $\|\mathbf{y}_k - A\mathbf{c}\|^2$. The

$$\binom{n}{w}$$

MPPM set $C_{n,w}$ has perfect symmetry; all of the codewords have the same energy, the same set of distances to the other codewords, and the same conditional error probability. We can thus assume that a particular codeword $\mathbf{c}'$ was transmitted. For $i \in \{1, \ldots, w\}$, there are precisely

$$M_i = \binom{w}{i}\binom{n-w}{i}$$

codewords with Hamming distance $2i$ from $\mathbf{c}'$ [7]. Observe that the Euclidean distance $d_E = \|\mathbf{c} - \mathbf{c}'\|$ from $\mathbf{c}'$ to another codeword $\mathbf{c}$ is related to the Hamming distance $d_H(\mathbf{c}, \mathbf{c}')$ by $d_E = \sqrt{d_H}$. Thus, the union bound on symbol-error probability (SEP) for uncoded MPPM with ML detection is [4], [7]

$$\Pr[\text{error}] \le \sum_{i=1}^{w} M_i Q\left(\sqrt{\frac{A^2 i}{N_0}}\right). \tag{11}$$

Let P_{MPPM} denote the average optical power required by MPPM to achieve a given bit rate and a given error probability.

The calculation of this useful performance metric requires that (11) be solved for P, a formidable task. We can simplify the calculation by assuming that the error probability is dominated by the minimum distance term and neglecting the multiplicity M_i in (11), yielding the following approximation for the error probability:

$$\Pr[\text{error}] \approx Q\left(\sqrt{\frac{n\log_2\binom{n}{w}}{N_0 R_b}}\frac{P}{w}\right). \tag{12}$$

Solving this for P_{MPPM} yields the following as the power requirement for uncoded MPPM:

$$\frac{P_{\text{MPPM}}}{P_{\text{OOK}}} \approx \sqrt{\frac{2w^2}{n\log_2\binom{n}{w}}} \tag{13}$$

where $P_{\text{OOK}} = \sqrt{0.5N_0R_b}Q^{-1}(\Pr[\text{error}])$ is the average optical power required by OOK to achieve a bit rate of R_b and a BER equal to $\Pr[\text{error}]$. When $w = 1$, (13) reverts to the power requirement for conventional n-ary PPM. The bandwidth of MPPM is roughly approximated as the inverse of chip duration

$$\frac{B_{\text{MPPM}}}{R_b} \approx \frac{n}{\log_2\binom{n}{w}}. \tag{14}$$

This approximation was shown to be accurate for low duty-cycle MPPM in [4].

B. On a Multipath Channel

When MLSD is used at the receiver, the probability of a symbol (block) error of uncoded MPPM on a multipath channel is well approximated at high signal-to-noise ratio (SNR) by [11]

$$\Pr[\text{error}] \approx Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) \tag{15}$$

where $d_{\min}$ is the minimum Euclidean distance between received sequences

$$d_{\min}^2 = \min_{\{\mathbf{e}_k\}}\sum_k\left\|\sum_m \mathbf{H}_m\mathbf{e}_{k-m}\right\|^2. \tag{16}$$

The above minimization is performed over all nonzero error sequences $\{\mathbf{e}_k\}$ starting at time zero, using an error alphabet of $\{\mathbf{u}-\mathbf{v} : \mathbf{u},\mathbf{v} \in C_{n,w}\}$. We calculated the optical power required to achieve a 10^{-6} BER over this ISI channel. To reduce computational complexity, we truncate the vector channel to four terms, so that $\mathbf{y}_k = \sum_{l=0}^{3}\mathbf{H}_l\mathbf{x}_{k-l} + \mathbf{n}_k$. This truncation has no appreciable effect when n is large or when R_b/W is small, although it may not be accurate for small n and large R_b/W. To confirm this claim, we calculated the ratio of the fractional energy of $h(t)$ contained outside the truncation interval to the total energy of $h(t)$

$$\varepsilon = \frac{\int_{4T}^{\infty}h^2(t)dt}{\int_0^{\infty}h^2(t)dt}. \tag{17}$$

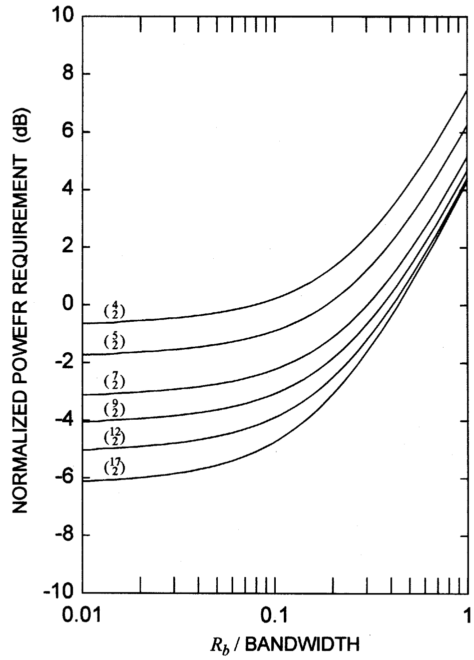

Fig. 2. Normalized power requirement versus normalized bit rate on an ISI channel with MLSD for MPPM.

We tabulate this ratio for various R_b/W in [12]. Except for OOK and 2-PPM at $R_b/W = 1$ and 2, less than 0.001% of the energy is discarded by the truncation.

The results are summarized in Fig. 2, where the normalized power requirement is plotted versus the bit-rate-to-bandwidth ratio R_b/W. The power requirements are normalized by $P_{\text{OOK}} = \sqrt{0.5N_0R_b}Q^{-1}(10^{-6})$, the power required by OOK in the ideal case ($W = \infty$) to achieve a 10^{-6} BER. We see that the power requirement grows as the target bit rate approaches the channel bandwidth. This increase in signal power can be interpreted as an ISI penalty. The ISI penalty is particularly severe when the bit rate approaches the channel bandwidth. Specifically, when the bit rate equals the bandwidth, the ISI power penalties for

$$\binom{4}{2}, \binom{5}{2}, \binom{7}{2}, \binom{9}{2}, \binom{12}{2}, \text{ and } \binom{17}{2}$$

MPPM with ML detection are 8, 8, 8, 8.5, 9, and 10 dB, respectively.

IV. TC-MPPM

A. Signal Set Decimation and MPPM Constellations

If $\binom{n}{w}$ is a power of two, an integer number of information bits can be used to select each codeword, greatly simplifying implementation. For this reason, practical PPM systems with $w = 1$ typically use $n = 2, 4, 8, 16$, etc. Unfortunately, when $w > 1$

$$\binom{n}{w}$$

TABLE I
BANDWIDTH AND POWER PENALTIES FOR DECIMATED MPPM WITH $w = 2$

n	BW Penalty(%)	Power Penalty(dB)	n	BW Penalty(%)	Power Penalty(dB)
4	29	0.56	18	4	0.08
5	11	0.22	19	6	0.13
6	30	0.57	20	8	0.17
7	10	0.20	21	10	0.21
8	20	0.40	22	12	0.25
9	3	0.07	23	14	0.29
10	10	0.20	24	1	0.03
11	16	0.32	25	3	0.06
12	1	0.02	26	4	0.09
13	5	0.10	27	6	0.12
14	8	0.18	28	7	0.15
15	12	0.24	29	8	0.17
16	15	0.31	30	10	0.20
17	1	0.03			

is rarely a power of two. To simplify the implementation of an MPPM transmitter, then, we propose to decimate the natural MPPM code by selecting 2^N of the

$$\binom{n}{w}$$

codewords, where

$$N = \left\lfloor \log_2 \binom{n}{w} \right\rfloor$$

so that 2^N is the largest power of two not exceeding

$$\binom{n}{w}.$$

Let $\tilde{C}_{n,w} \subseteq C_{n,w}$ denote the 2^N selected codewords. The question of which 2^N codewords to select will be addressed below. From (14), we see that, roughly speaking, this decimation increases the bandwidth requirement by the ratio

$$\frac{\log_2 \binom{n}{w}}{\left\lfloor \log_2 \binom{n}{w} \right\rfloor}. \tag{18}$$

Furthermore, from (13), we see that the decimation increases the power requirement by roughly the square root of the ratio (18). These penalties can be significant for certain values of n and w. For example, Table I summarizes the bandwidth and power penalties for decimating the

$$\binom{n}{2}$$

MPPM code for $n = 4, 5, \ldots, 31$. The table shows that the penalties are particularly small for

$$\binom{9}{2} = 36, \binom{12}{2} = 66, \binom{17}{2} = 136,$$
$$\binom{24}{2} = 276, \text{ and } \binom{25}{2} = 300$$

because all are close to a power of two.

We now describe a useful geometric description of MPPM codewords for the special case of $w = 2$ [13]. Let i_1 and i_2

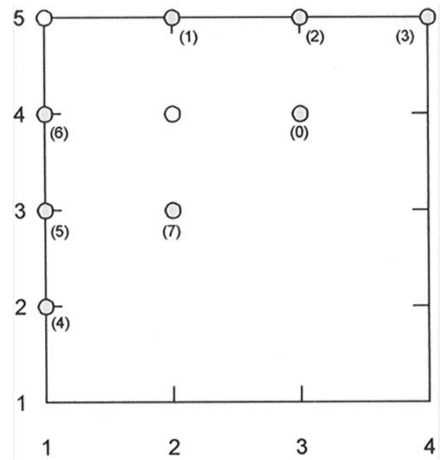

Fig. 3. Constellations for $\binom{5}{2}$-MPPM; the shaded circles represent the chosen L codewords, and unshaded circles represent the unused codewords.

denote the indexes of the two ones within a particular MPPM codeword, where $i_1, i_2 \in \{1, \ldots, n\}$ with $i_1 < i_2$. Then each MPPM codeword can then be mapped to the unique point (i_1, i_2) in two-dimensional space. For example, the codeword $[11000]^T$ is mapped into $(1, 2)$, the codeword $[00101]^T$ is mapped to $(3, 5)$, and so on. A similar mapping was used in [14] for pulse-width modulation, where i_1 and i_2 represented the starting and ending indexes of each pulse, respectively. In Fig. 3, we illustrate this mapping for the

$$\binom{5}{2}$$

MPPM codes.

Note that the Hamming distance between two codewords in the same row or column is two, and the Hamming distance between two codewords having different rows and columns is four.

B. Model for TC-MPPM System

The model for a TC-MPPM system is shown in Fig. 4. Information bits with rate R_b enter the trellis encoder, which consists of a linear $(N-1)/N$ convolutional encoder followed by a signal mapper. The convolutional encoder outputs a sequence of N-bit blocks $\{\mathbf{c}_k\}$, whereas the signal mapper converts each block of N coded bits into one of the 2^N codewords $\tilde{C}_{n,w}$. The output of the trellis encoder is a sequence of MPPM codewords $\{\mathbf{x}_k\}$ with rate $1/T = R_b/(N-1)$. These MPPM codewords are then transmitted across the multipath channel, as described in Section II. In Fig. 4, we use the equivalent vector channel model of Section II. Based on the receiver sequence $\{\mathbf{y}_k\}$, the receiver makes decisions using superstate ML sequence detection on the combined trellis formed by the convolutional encoder and channel ISI.

Let $d_{\min}$ denote the minimum Euclidean distance between coded sequences

$$d_{\min}^2 = \min \sum_k w_H(\mathbf{x}_k - \tilde{\mathbf{x}}_k) \tag{19}$$

where the minimization is performed over all distinct trellis-coded sequences $\{\mathbf{x}_k\}$ and $\{\tilde{\mathbf{x}}_k\}$, and where the Hamming weight $w_H(\mathbf{x})$ of a vector $\mathbf{x}$ is the number of nonzero

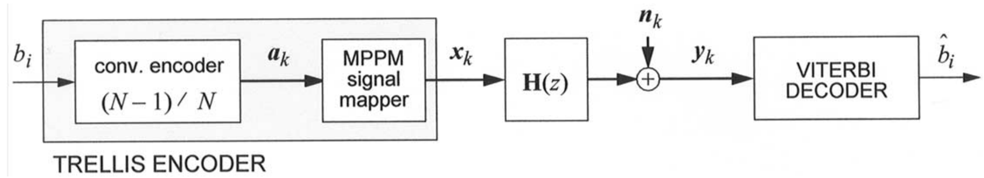

Fig. 4. System model for TC-MPPM.

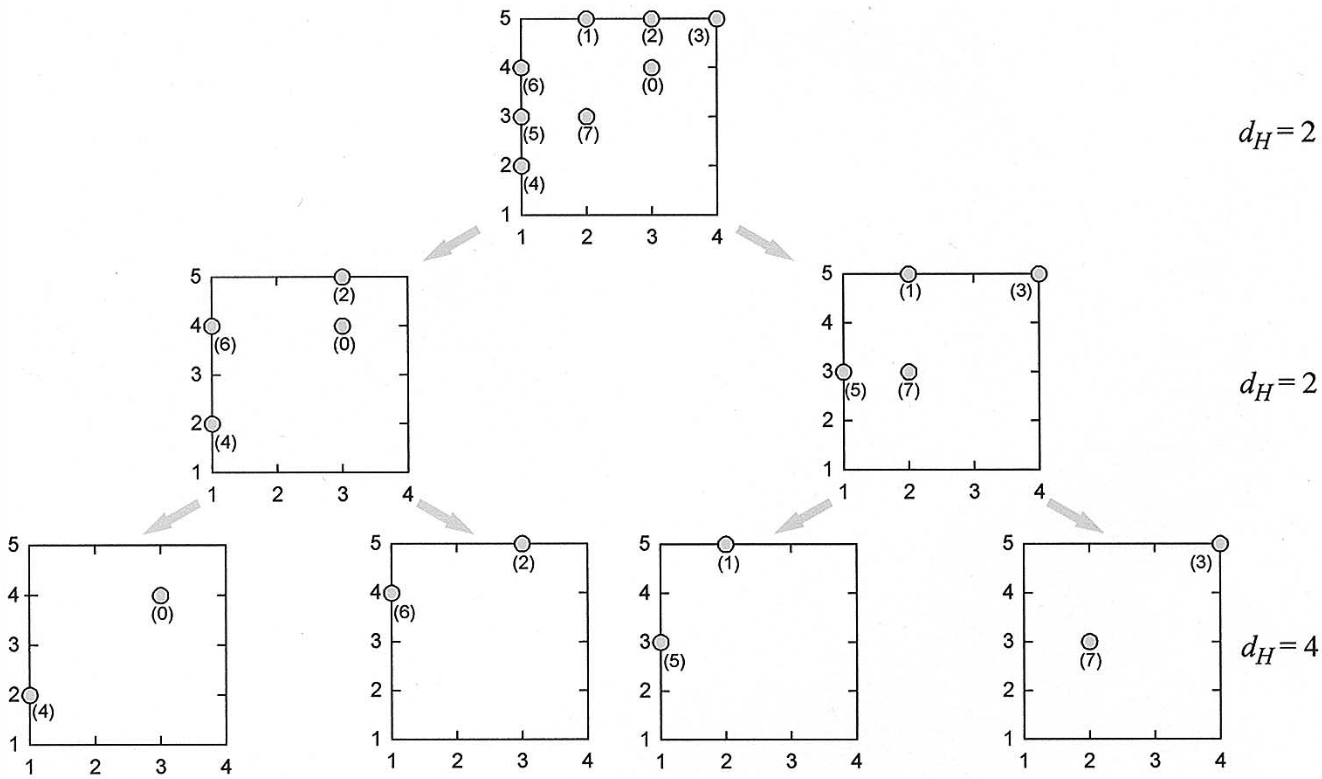

Fig. 5. Set partitioning for the decimated $\binom{5}{2}$-MPPM signal set.

components in $\mathbf{x}$. The probability of sequence error after ML sequence detection can be roughly approximated by

$$\Pr[\text{error}] \approx Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right). \tag{20}$$

For this reason, we will use $d_{\min}$ as a performance metric.

C. Symmetry Properties of the Decimated MPPM Signal Set

The calculation of $d_{\min}$ can be significantly simplified when the underlying signal set satisfies a symmetry property referred to as the Zehavi and Wolf (Z–W) condition [15], because this condition allows the all-zero path to serve as a reference path. In other words, it allows the sequence $\{\tilde{\mathbf{x}}_k\}$ to be fixed, so that the minimization in (19) need be performed over $\{\mathbf{x}_k\}$ only. The Z–W condition requires that when the signal set is partitioned into two subsets $S^{(1)}$ and $S^{(2)}$, the distance weight profiles of the two subsets be identical. The distance weight profile of a subset with respect to an error vector $\mathbf{e}$ is defined as [15]

$$F\left(S^{(i)}, \mathbf{e}, Z\right) = \sum_d N_d Z^{d_H(f(\mathbf{c}), f(\mathbf{c}\oplus\mathbf{e}))} \quad i = 1, 2 \tag{21}$$

where N_d is the number of codewords of having Hamming distance d_H between the codeword $f(\mathbf{c})$ and codeword $f(\mathbf{c}\oplus\mathbf{e})$, and the summation is taken over all the possible d_H.

Fig. 5 shows the set partitioning of the decimated

$$\binom{5}{2}$$

MPPM signal set. In Fig. 5, we show the selected 2^N codewords (shaded circle), and the number below the constellation represents the labeling of the codeword. The decimated

$$\binom{5}{2}$$

MPPM signal set is partitioned into two subsets $S^{(1)} = \{0, 2, 4, 6\}$ and $S^{(2)} = \{1, 3, 5, 7\}$. The distance weight profile of these subsets are listed in Table II, and they are identical. Therefore, this signal set satisfies the Z–W condition.

D. Trellis Code Search for $\binom{17}{2}$-MPPM

We now present trellis codes based on the

$$\binom{17}{2}$$

TABLE II
DISTANCE WEIGHT PROFILE FOR DECIMATED $\binom{5}{2}$-MPPM SIGNAL SET

Error vector $\mathbf{e}$	$S^{(1)} = \{0, 2, 4, 6\}$	$S^{(2)} = \{1, 3, 5, 7\}$
000	4	4
001	$2Z^2 + 2Z^4$	$2Z^2 + 2Z^4$
010	$4Z^2$	$4Z^2$
011	$4Z^2$	$4Z^2$
100	$4Z^4$	$4Z^4$
101	$4Z^2$	$4Z^2$
110	$2Z^2 + 2Z^4$	$2Z^2 + 2Z^4$
111	$2Z^2 + 2Z^4$	$2Z^2 + 2Z^4$

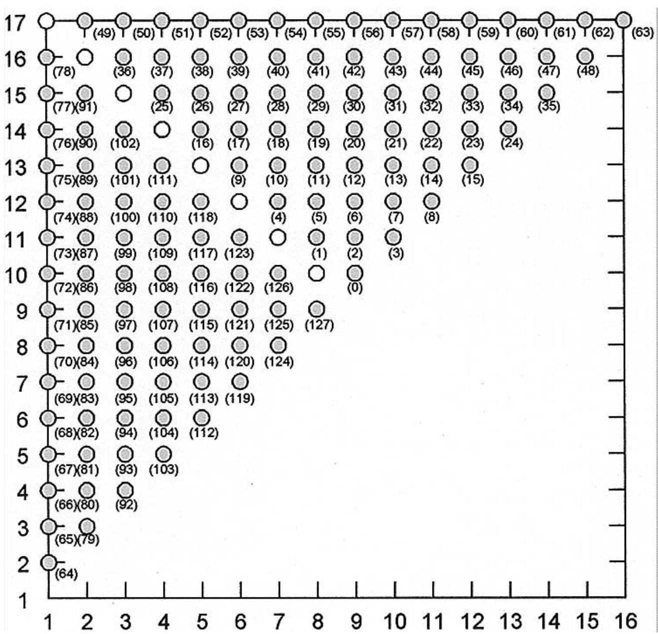

Fig. 6. Constellations for $\binom{17}{2}$_MPPM; the shaded circles represent the chosen L codewords, and unshaded circles represent the unused codewords.

MPPM signal set. Of the

$$\binom{17}{2} = 136$$

natural MPPM codewords, we choose the 128 shaded codewords of Fig. 6. This decimated signal set calls for a rate-6/7 convolutional code, which we choose to be systematic and recursive. This convolutional encoder operates on six bits, $\mathbf{b}_k = [b_k^1, b_k^2, b_k^3, b_k^4, b_k^5, b_k^6]^T$ and produces seven encoded bits, $\mathbf{c}_k = [c_k^0, c_k^1, c_k^2, c_k^3, c_k^4, c_k^5, c_k^6]^T$. The coded bits are mapped into one of the

$$\binom{17}{2}$$

MPPM codewords according to the mapping rule. This mapping rule is depicted in Fig. 6, where each constellation point is labeled by the decimal representation of the corresponding seven-bit block of coded bits.

A recursive and systematic configuration for the encoder is beneficial, because it reduces the number of coefficients to search, as compared with a feedforward configuration, and also because it is free from catastrophic condition [5]. Optimal codes may be selected by performing an exhaustive search. However, as the code complexity increases, an exhaustive search becomes impractical. In particular, for

$$\binom{17}{2}$$

MPPM with constraint length ν, the number of coefficients to search is $2^{7(\nu-1)}$, making it impractical to perform an exhaustive search for large constraint lengths. In such cases, limited searches are necessary, even if they may not always provide optimal codes. One simple limited-search algorithm is a random search, whereby a large number of generator polynomials are generated at random, and ones with the best performance are selected [16].

We performed a random search for the best generator polynomials $\boldsymbol{h}^0, \boldsymbol{h}^1, \ldots, \boldsymbol{h}^6$. We generated 200 polynomials, calculated the minimum Hamming distance using (19) where $\{\tilde{\mathbf{x}}_k\}$ is an all-zero path, and stored the polynomials if the minimum distance was larger than any previous. We repeated this search for all constraint lengths ν between 4 and 12. The coefficient vectors $\{\boldsymbol{h}^i\}$ were generated independently and uniformly distributed over $[0, 2^\nu - 2]$ for $i \neq 0$, and over $[2^\nu + 1, 2^{\nu+1} - 1]$ for $i = 0$. The random search results are shown in Table III, where the generator polynomials $\boldsymbol{h}^0, \ldots, \boldsymbol{h}^6$ are tabulated in octal form for constraint lengths $\nu = 4, \ldots, 12$, along with the corresponding squared-minimum-Euclidean distance achieved by the resulting trellis code.

Trellis-coded

$$\binom{n}{w}$$

MPPM has a bit rate of $R_b = (N-1)/T$, where

$$N = \left\lfloor \log_2 \binom{n}{w} \right\rfloor$$

and its bandwidth and power requirements are

$$\frac{B_{\text{TC-MPPM}}}{R_b} = \frac{n}{N-1} \tag{22}$$

$$\frac{P_{\text{TC-MPPM}}}{P_{\text{OOK}}} = \frac{2w}{d_{\min}\sqrt{n(N-1)}} \tag{23}$$

where $d_{\min}$ is the minimum Euclidean distance (19) between valid trellis-coded sequences.

Table III also tabulates a coding gain for each trellis code, defined to be the reduction in optical power required with the TC-MPPM, as compared with an uncoded modulation that has the same bandwidth efficiency.

For a given trellis-coded

$$\binom{n}{w}$$

MPPM scheme with a given bandwidth efficiency, let $\tilde{L}$ denote the integer such that $\tilde{L}$-PPM has the same bandwidth efficiency. For example, the bandwidth required by both 9-PPM and trellis-coded

$$\binom{17}{2}$$

TABLE III
GENERATOR COEFFICIENTS FOR TRELLIS-CODED MPPM IN OCTAL FORM

Constraint length ν	Generator Polynomial Coefficients							d^2_{min}	Coding Gain[dB]	$\tilde{d}^2_{avg}$ (28)	Simplex (29)
	$\mathbf{h}^0$	$\mathbf{h}^1$	$\mathbf{h}^2$	$\mathbf{h}^3$	$\mathbf{h}^4$	$\mathbf{h}^5$	$\mathbf{h}^6$				
4	23	10	06	14	16	00	04	4	1.4	4	4
5	65	14	34	22	12	20	16	4	1.4	4	4
6	103	054	072	016	014	024	066	4	1.4	7	8
7	357	144	014	024	040	140	102	6	2.3	7	8
8	443	102	040	064	276	022	164	6	2.3	7	8
9	1057	0516	0324	0546	0720	0604	0206	6	2.3	7	8
10	3341	1406	1330	0176	1266	1746	1156	6	2.3	7	8
11	7433	0736	1162	1316	2032	2272	3302	6	2.3	7	8
12	10017	02102	05242	03314	01374	07550	01044	8	2.9	11	12

MPPM is larger than the information bit rate by a factor of 2.8. The asymptotic coding gain of trellis-coded

$$\binom{n}{w}$$

MPPM over $\tilde{L}$-PPM is then

$$\text{Asymptotic Coding Gain} = 10 \cdot \log_{10}\left(\sqrt{\frac{n(N-1)d_c}{2w^2\tilde{L}\log_2\tilde{L}}}\right) \text{dB}. \tag{24}$$

Using the constraint lengths 4, 7, and 12, we achieve asymptotic coding gain of 1.4, 2.3, and 2.9 dB relative to uncoded 9-PPM, respectively.

E. Approximation for the Minimum Distance of TC-MPPM

In this section, to verify our trellis-code search results, we derive an approximation for the minimum distance of TC-MPPM for a given constraint length. The minimum distance of a trellis code is the smallest among the distances of pairs of sequences arising from an error event. Each trellis path associated with the error event of length l involves l MPPM codewords. We define the trellis path vector, $\mathbf{X} = [\mathbf{x}_0^T, \mathbf{x}_1^T, \ldots, \mathbf{x}_{l-1}^T]^T$ of dimension ln, where $\mathbf{x}_i$ is the

$$\binom{n}{w}$$

MPPM codeword corresponding to the ith branch in the path. Observe that the trellis vector $\mathbf{X}$ is a valid

$$\binom{N}{\Omega}$$

MPPM codeword with length $N = ln$, and weight $\Omega = lw$. As we indicated in Section III-A, any valid MPPM codeword has the same set of distance with respect to the other codewords, and the number of codewords for

$$\binom{N}{\Omega}$$

MPPM with mutual Hamming distance $2m$ is

$$N_m = \binom{\Omega}{m}\binom{N-\Omega}{m}.$$

We can calculate the average distance for

$$\binom{N}{\Omega}$$

MPPM as shown in (25)–(28) at the bottom of the page, where

$$L = \binom{N}{\Omega}$$

is the number of extended MPPM codewords. Since not all valid

$$\binom{N}{\Omega}$$

MPPM codewords are included in the set of $\{\mathbf{X}\}$, the $\tilde{d}_{\text{avg}}$ in (25) is only an *approximation* for $d_{\min}$.

We also can apply this approximation method to trellis-coded PPM by treating the trellis-coded PPM sequences of length l as extended MPPM codewords with length $N = lL$ and weight $\Omega = l$, and then apply (28).

$$\tilde{d}^2_{\text{avg}} = \frac{1}{L(L-1)} \sum_{i=0}^{L-1} \sum_{j=0, j\neq i}^{L-1} \|\mathbf{X}^i - \mathbf{X}^j\|^2 \tag{25}$$

$$= \frac{1}{L-1} \sum_{j=1}^{L-1} \|\mathbf{X}^0 - \mathbf{X}^j\|^2 \tag{26}$$

$$= \frac{1}{L-1} \sum_{m=1}^{\Omega} 2mN_m \tag{27}$$

$$= \frac{2\binom{\Omega}{1}\binom{N-\Omega}{1} + 4\binom{\Omega}{2}\binom{N-\Omega}{2} + \cdots + 2\Omega\binom{\Omega}{\Omega}\binom{N-\Omega}{\Omega}}{L-1} \tag{28}$$

The approximations based on average distance are listed in Table III. For comparison, the table also shows the simplex bound [17]

$$d_{\min}^2 \le \min_{t\ge 1} \frac{2^{\kappa t+1}}{2^{\kappa t}-1} w \left(1+\left\lfloor \frac{\nu}{\kappa}\right\rfloor\right) \tag{29}$$

where $\kappa = (\log_2 L - 1)$, and $\lfloor\cdot\rfloor$ takes the integer part of its argument. We can see that approximation method is tighter than the simplex bound.

F. TC-MPPM on Multipath Channel

In this section, we examine the performance of the proposed TC-MPPM scheme over a multipath channel. A trellis encoder is followed by an ISI channel whose impulse response is truncated, so that the effective vector channel has memory μ, as described in Section III-B. The kth transmitted codeword $\mathbf{x}_k$ is a function of the convolutional encoder state α_k and the information bits $\mathbf{b}_k$

$$\mathbf{x}_k = f(\mathbf{b}_k, \alpha_k) \tag{30}$$

and the state transition equation is

$$\alpha_{k+1} = g(\mathbf{b}_k, \alpha_k). \tag{31}$$

We consider a receiver that performs MLSD on the combined trellis formed by the convolutional encoder and the ISI channel. In other words, the trellis-coded signal in the presence of ISI is modeled using a single finite-state machine. For rate-$(\log_2 L - 1)/\log_2 L$ TC-MPPM, there are $(L/2)^\mu$ ISI states associated with each encoder state. The states for the combined finite-state machine are

$$\beta_k = (\alpha_{k-\mu}, \mathbf{b}_{k-\mu}, \mathbf{b}_{k-\mu+1}, \ldots, \mathbf{b}_{k-1}). \tag{32}$$

If the convolutional encoder has 2^m states, the combined trellis has $L^\mu 2^{m-\mu}$ states.

The performance of trellis-coded

$$\binom{17}{2}$$

MPPM with multipath is shown in Fig. 7. As a reference, the figure also shows the performance of trellis-coded 16-PPM with multipath, using the PPM encoder coefficients of [8]. We assume the same underlying channel as we considered for the uncoded case. As in the uncoded case, we calculate the optical power required to achieve a 10^{-6} BER over this ISI channel.

Trellis-coded 16-PPM shows better performance up to a bit-rate-to-bandwidth ratio of 0.15. Beyond that, trellis-coded

$$\binom{17}{2}$$

MPPM outperforms trellis-coded 16-PPM. At a bit-rate-to-bandwidth ratio of unity, the normalized power requirements for trellis-coded 16-PPM with constraint lengths $\nu = 4$ and 7 are 3.4 and 2.9 dB, respectively. But the power requirements for trellis-coded

$$\binom{17}{2}$$

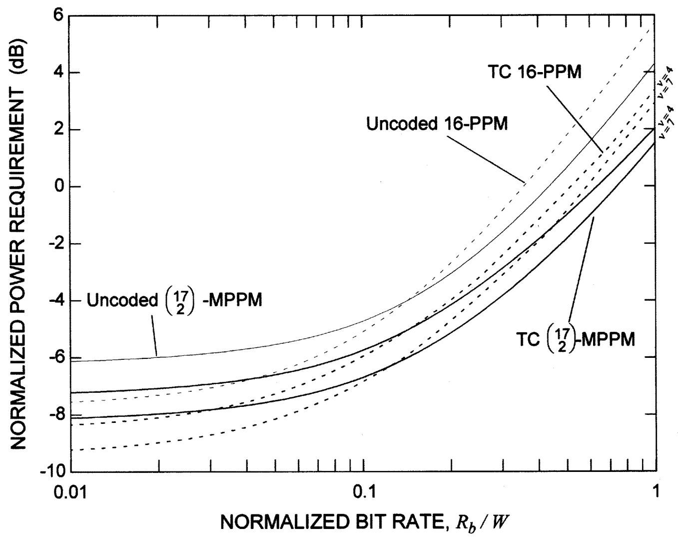

Fig. 7. Power requirement of trellis-coded $\binom{17}{2}$-MPPM and trellis-coded 16-PPM as a function of normalized bit rate.

MPPM with $\nu = 4$ and 7 are 2 and 1.5 dB, respectively. Therefore, trellis-coded

$$\binom{17}{2}$$

MPPM requires 1.4 dB less power than trellis-coded 16-PPM when both schemes use the same constraint length and when the target bit rate is equal to the channel bandwidth.

V. Conclusions

We have developed new trellis codes based on MPPM. Trellis codes with large minimum distance have been obtained through a random computer search. To verify our results, we derived an approximation for the minimum distance using the symmetry properties of MPPM, and compared our result with the well-known simplex bound. Code-search results show that trellis-coded

$$\binom{17}{2}$$

MPPM with constraint length seven provides a coding gain of 2.3 dB over uncoded 9-PPM. Furthermore, when the bit rate equals the bandwidth, trellis-coded

$$\binom{17}{2}$$

MPPM requires 1.4 dB less optical power than trellis-coded 16-PPM having the same constraint length.

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