# BeyondPPM_updated

February 15, 2023

## 1 Installation and Import of packages, mount the drive

pip install dataframe_image, the package to display the table from dataframe

[1]:
```
pip install dataframe-image
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-
wheels/public/simple/
Collecting dataframe-image
  Downloading dataframe_image-0.1.5-py3-none-any.whl (6.6 MB)
                           6.6/6.6 MB
27.4 MB/s eta 0:00:00
Requirement already satisfied: pandas>=0.24 in
/usr/local/lib/python3.8/dist-packages (from dataframe-image) (1.3.5)
Requirement already satisfied: matplotlib>=3.1 in /usr/local/lib/python3.8/dist-
packages (from dataframe-image) (3.2.2)
Requirement already satisfied: packaging in /usr/local/lib/python3.8/dist-
packages (from dataframe-image) (23.0)
Requirement already satisfied: nbconvert>=5 in /usr/local/lib/python3.8/dist-
packages (from dataframe-image) (5.6.1)
Requirement already satisfied: mistune in /usr/local/lib/python3.8/dist-packages
(from dataframe-image) (0.8.4)
Requirement already satisfied: requests in /usr/local/lib/python3.8/dist-
packages (from dataframe-image) (2.25.1)
Requirement already satisfied: aiohttp in /usr/local/lib/python3.8/dist-packages
(from dataframe-image) (3.8.3)
Requirement already satisfied: beautifulsoup4 in /usr/local/lib/python3.8/dist-
packages (from dataframe-image) (4.6.3)
Requirement already satisfied: kiwisolver>=1.0.1 in
/usr/local/lib/python3.8/dist-packages (from matplotlib>=3.1->dataframe-image)
(1.4.4)
Requirement already satisfied: python-dateutil>=2.1 in
/usr/local/lib/python3.8/dist-packages (from matplotlib>=3.1->dataframe-image)
(2.8.2)
Requirement already satisfied: pyparsing!=2.0.4,!=2.1.2,!=2.1.6,>=2.0.1 in
/usr/local/lib/python3.8/dist-packages (from matplotlib>=3.1->dataframe-image)
(3.0.9)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.8/dist-
```

```
packages (from matplotlib>=3.1->dataframe-image) (0.11.0)
Requirement already satisfied: numpy>=1.11 in /usr/local/lib/python3.8/dist-
packages (from matplotlib>=3.1->dataframe-image) (1.21.6)
Requirement already satisfied: pygments in /usr/local/lib/python3.8/dist-
packages (from nbconvert>=5->dataframe-image) (2.6.1)
Requirement already satisfied: jinja2>=2.4 in /usr/local/lib/python3.8/dist-
packages (from nbconvert>=5->dataframe-image) (2.11.3)
Requirement already satisfied: defusedxml in /usr/local/lib/python3.8/dist-
packages (from nbconvert>=5->dataframe-image) (0.7.1)
Requirement already satisfied: traitlets>=4.2 in /usr/local/lib/python3.8/dist-
packages (from nbconvert>=5->dataframe-image) (5.7.1)
Requirement already satisfied: entrypoints>=0.2.2 in
/usr/local/lib/python3.8/dist-packages (from nbconvert>=5->dataframe-image)
(0.4)
Requirement already satisfied: jupyter-core in /usr/local/lib/python3.8/dist-
packages (from nbconvert>=5->dataframe-image) (5.2.0)
Requirement already satisfied: nbformat>=4.4 in /usr/local/lib/python3.8/dist-
packages (from nbconvert>=5->dataframe-image) (5.7.3)
Requirement already satisfied: testpath in /usr/local/lib/python3.8/dist-
packages (from nbconvert>=5->dataframe-image) (0.6.0)
Requirement already satisfied: pandocfilters>=1.4.1 in
/usr/local/lib/python3.8/dist-packages (from nbconvert>=5->dataframe-image)
(1.5.0)
Requirement already satisfied: bleach in /usr/local/lib/python3.8/dist-packages
(from nbconvert>=5->dataframe-image) (6.0.0)
Requirement already satisfied: pytz>=2017.3 in /usr/local/lib/python3.8/dist-
packages (from pandas>=0.24->dataframe-image) (2022.7.1)
Requirement already satisfied: multidict<7.0,>=4.5 in
/usr/local/lib/python3.8/dist-packages (from aiohttp->dataframe-image) (6.0.4)
Requirement already satisfied: async-timeout<5.0,>=4.0.0a3 in
/usr/local/lib/python3.8/dist-packages (from aiohttp->dataframe-image) (4.0.2)
Requirement already satisfied: frozenlist>=1.1.1 in
/usr/local/lib/python3.8/dist-packages (from aiohttp->dataframe-image) (1.3.3)
Requirement already satisfied: charset-normalizer<3.0,>=2.0 in
/usr/local/lib/python3.8/dist-packages (from aiohttp->dataframe-image) (2.1.1)
Requirement already satisfied: aiosignal>=1.1.2 in
/usr/local/lib/python3.8/dist-packages (from aiohttp->dataframe-image) (1.3.1)
Requirement already satisfied: yarl<2.0,>=1.0 in /usr/local/lib/python3.8/dist-
packages (from aiohttp->dataframe-image) (1.8.2)
Requirement already satisfied: attrs>=17.3.0 in /usr/local/lib/python3.8/dist-
packages (from aiohttp->dataframe-image) (22.2.0)
Requirement already satisfied: certifi>=2017.4.17 in
/usr/local/lib/python3.8/dist-packages (from requests->dataframe-image)
(2022.12.7)
Requirement already satisfied: idna<3,>=2.5 in /usr/local/lib/python3.8/dist-
packages (from requests->dataframe-image) (2.10)
Requirement already satisfied: urllib3<1.27,>=1.21.1 in
/usr/local/lib/python3.8/dist-packages (from requests->dataframe-image) (1.24.3)
```

```
Requirement already satisfied: chardet<5,>=3.0.2 in
/usr/local/lib/python3.8/dist-packages (from requests->dataframe-image) (4.0.0)
Requirement already satisfied: MarkupSafe>=0.23 in
/usr/local/lib/python3.8/dist-packages (from
jinja2>=2.4->nbconvert>=5->dataframe-image) (2.0.1)
Requirement already satisfied: jsonschema>=2.6 in /usr/local/lib/python3.8/dist-
packages (from nbformat>=4.4->nbconvert>=5->dataframe-image) (4.3.3)
Requirement already satisfied: fastjsonschema in /usr/local/lib/python3.8/dist-
packages (from nbformat>=4.4->nbconvert>=5->dataframe-image) (2.16.2)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.8/dist-
packages (from python-dateutil>=2.1->matplotlib>=3.1->dataframe-image) (1.15.0)
Requirement already satisfied: webencodings in /usr/local/lib/python3.8/dist-
packages (from bleach->nbconvert>=5->dataframe-image) (0.5.1)
Requirement already satisfied: platformdirs>=2.5 in
/usr/local/lib/python3.8/dist-packages (from jupyter-
core->nbconvert>=5->dataframe-image) (3.0.0)
Requirement already satisfied: pyrsistent!=0.17.0,!=0.17.1,!=0.17.2,>=0.14.0 in
/usr/local/lib/python3.8/dist-packages (from
jsonschema>=2.6->nbformat>=4.4->nbconvert>=5->dataframe-image) (0.19.3)
Requirement already satisfied: importlib-resources>=1.4.0 in
/usr/local/lib/python3.8/dist-packages (from
jsonschema>=2.6->nbformat>=4.4->nbconvert>=5->dataframe-image) (5.10.2)
Requirement already satisfied: zipp>=3.1.0 in /usr/local/lib/python3.8/dist-
packages (from importlib-
resources>=1.4.0->jsonschema>=2.6->nbformat>=4.4->nbconvert>=5->dataframe-image)
(3.12.1)
Installing collected packages: dataframe-image
Successfully installed dataframe-image-0.1.5
```

[2]:
```
# Import the packages
import math
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
from pandas.plotting import table
import dataframe_image as dfi
```

[3]:
```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

## 2 A scheme Beyond PPM

We temporarily call the scheme as Jonas. \ We have 4 photons 14 timeslots 1 symbol created by 14 timeslots

A1. The number of slots n with Jonas method. n=Sum(First k integers in the series) \

A2. The number of possible symbols, m, will be m=k \

A3. The number of bits (the information content per symbol) will be b=log2(m) \

### 2.1 Sequence to design the length of time slot / time bin

We can generate the desired number of sequence we want from the below code.

A010672 A B_2 sequence: a(n) = least value such that sequence increases and pairwise sums of distinct elements are all distinct. \ Author: Dan Hoey

1. https://oeis.org/A010672
2. https://oeis.org/A011185
3. https://oeis.org/A010672

The below function generates a list of 100 sequences.

[4]:
```
from itertools import islice

def agen(): # generator of terms

    aset, sset, k = set(), set(), 0

    while True:

        k += 1

        while any(k+an in sset for an in aset): k += 1

        yield k; sset.update(k+an for an in aset); aset.add(k)

a = list(islice(agen(), 100))
b = list(map(lambda v: v-1, a))
print(b)
```

```
[0, 1, 2, 4, 7, 12, 20, 29, 38, 52, 73, 94, 127, 151, 181, 211, 257, 315, 373,
412, 475, 530, 545, 607, 716, 797, 861, 964, 1059, 1160, 1306, 1385, 1434, 1555,
1721, 1833, 1933, 2057, 2260, 2496, 2698, 2873, 3060, 3196, 3331, 3628, 3711,
3867, 4139, 4446, 4639, 5021, 5064, 5322, 5613, 6003, 6273, 6493, 6641, 6979,
7275, 7587, 8017, 8373, 9071, 9167, 9760, 10105, 10489, 11109, 11374, 11516,
12101, 12330, 12867, 13426, 13923, 14535, 14911, 15469, 15904, 16136, 16900,
17041, 17822, 19421, 19933, 20288, 20996, 21491, 22065, 22612, 22659, 23724,
24399, 24969, 25360, 26071, 26680, 27601]
```

[5]:
```
len(b)
```

[5]:
```
100
```

### 2.2 The Number of Time Bin (Time Slot)

Let us consider the following sequence: [0, 1, 2, 4, 7, 12, 20, 29, 38, 52, 73, 94, 127] as an example

Since we should have at least 1 photon in a time slot, we drop out the 1st term and start with the value 1.

The length of the nth time bin is the sum of the first nth sequence.

[6]:
```
l = [1, 2, 4, 7, 12, 20, 29, 38, 52, 73, 94, 127, 151, 181, 211] # or l =␣
 ↪range(1, 21)
series = [sum(l[:i]) for i in range(1,len(l)+1)]
print (series)
```

```
[1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002]
```

If we have 4 photons to encode into the superblock. We represent them into a sequence in a list as [1,2,4,7], which is encoded as follow: \ H H0 H000 V000000, which comprie of 14 time slots. We can find all the permutation, i.e. the number of ways to order them, which is 4! = 24. It means that we have 24 possible way to represent a symbol in this time bin.

[7]:
```
# A Python program to print all
# permutations using library function
from itertools import permutations


# Get all permutations of [1, 2, 3]
perm = permutations([1, 2, 4, 7])

lists = []
# Print the obtained permutations
for i in (perm):
    #print (i)
    lists.append(i)

lists
```

[7]:
```
[(1, 2, 4, 7),
 (1, 2, 7, 4),
 (1, 4, 2, 7),
 (1, 4, 7, 2),
 (1, 7, 2, 4),
 (1, 7, 4, 2),
 (2, 1, 4, 7),
 (2, 1, 7, 4),
 (2, 4, 1, 7),
 (2, 4, 7, 1),
 (2, 7, 1, 4),
 (2, 7, 4, 1),
 (4, 1, 2, 7),
```

```
 (4, 1, 7, 2),
 (4, 2, 1, 7),
 (4, 2, 7, 1),
 (4, 7, 1, 2),
 (4, 7, 2, 1),
 (7, 1, 2, 4),
 (7, 1, 4, 2),
 (7, 2, 1, 4),
 (7, 2, 4, 1),
 (7, 4, 1, 2),
 (7, 4, 2, 1)]
```

# 3 Number of ways to order the photons

$$n!$$

[8]:
```python
def ways(n):
    return math.factorial(n)
```

# 4 Number of Bits per Symbol

It is the information content per symbol

$$log_2 n!$$

[9]:
```python
def bps(W):
    return math.log2(W)
```

# 5 Number of Bits per Photon

math.log2(Jonasways) / n

$$log_2 \frac{n!}{n}$$

[10]:
```python
def bpph(b,n):
    return b/n
```

# 6 Number of Bits per Timeslot

$$log_2 \frac{n!}{n} \times \frac{n}{T}$$

[11]:
```python
def bpt(B, n, T):
    return B*n/T
```

[12]:
```python
bpph(bps(24), 4)*4/14
```

[12]: 0.3274973214800826

# 7 Putting all the functions together

[13]:
```python
series, series[4]
```

[13]: ([1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002], 26)

[14]:
```python
Jonasnumber = []
JonasPermutation = []
JonasBPS = []
JonasBPP = []
JonasBPT = []


series = [1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002]

df_Jonas = pd.DataFrame(columns=['|Number of Photon|',
                                 'Time Bins|',
                                 'Permutation|',
                                 'Bits/Symbol|',
                                 'Bits/Photon|',
                                 'Bits/Time Bin|']
                   )

for n in range(1,11):
    Jonasways = math.factorial(n)          # A2. The number of possible symbols,␣
 ↪m, will be m=k \\
    JonasTimeBin = series[n-1]             # A1. The number of slots n with␣
 ↪Jonas method. n=Sum(First k integers in the series) \\
    Jonasbps = math.log2(Jonasways)        # A3. The number of bits (the␣
 ↪information content per symbol) will be b=log2(m)
    Jonasbpp = math.log2(Jonasways) / n
    Jonasbpt = Jonasbpp * n/series[n-1]

    JonasTimeBin_SigFig = "{:.2e}".format(JonasTimeBin)
    Jonasways_SigFig = "{:.2e}".format(Jonasways)
    Jonasbps_round = round(Jonasbps, 2)
    Jonasbpp_round = round(Jonasbpp, 2)
    Jonasbpt_round = round(Jonasbpt, 2)
```

```
    df_Jonas.loc[len(df_Jonas)] = [n, JonasTimeBin_SigFig, Jonasways_SigFig,␣
↪Jonasbps_round, Jonasbpp_round, Jonasbpt_round]
```

[15]:
```
df_Jonas # 4th one data is correct
```

[15]:
```
  |Number of Photon| Time Bins| Permutation|  Bits/Symbol|  Bits/Photon|  \
0                  1    1.00e+00      1.00e+00         0.00         0.00
1                  2    3.00e+00      2.00e+00         1.00         0.50
2                  3    7.00e+00      6.00e+00         2.58         0.86
3                  4    1.40e+01      2.40e+01         4.58         1.15
4                  5    2.60e+01      1.20e+02         6.91         1.38
5                  6    4.60e+01      7.20e+02         9.49         1.58
6                  7    7.50e+01      5.04e+03        12.30         1.76
7                  8    1.13e+02      4.03e+04        15.30         1.91
8                  9    1.65e+02      3.63e+05        18.47         2.05
9                 10    2.38e+02      3.63e+06        21.79         2.18

   Bits/Time Bin|
0           0.00
1           0.33
2           0.37
3           0.33
4           0.27
5           0.21
6           0.16
7           0.14
8           0.11
9           0.09
```

[16]:
```
df_Jonas[df_Jonas.columns[0]]
```

[16]:
```
0     1
1     2
2     3
3     4
4     5
5     6
6     7
7     8
8     9
9    10
Name: |Number of Photon|, dtype: object
```

[17]:
```
df_Jonas[df_Jonas.columns[1]]
```

[17]:
```
0    1.00e+00
1    3.00e+00
```

```
2     7.00e+00
3     1.40e+01
4     2.60e+01
5     4.60e+01
6     7.50e+01
7     1.13e+02
8     1.65e+02
9     2.38e+02
Name: Time Bins|, dtype: object
```

[18]:
```
df_Jonas[df_Jonas.columns[2]]
```

[18]:
```
0     1.00e+00
1     2.00e+00
2     6.00e+00
3     2.40e+01
4     1.20e+02
5     7.20e+02
6     5.04e+03
7     4.03e+04
8     3.63e+05
9     3.63e+06
Name: Permutation|, dtype: object
```

[19]:
```
df_Jonas[df_Jonas.columns[3]]
```

[19]:
```
0      0.00
1      1.00
2      2.58
3      4.58
4      6.91
5      9.49
6     12.30
7     15.30
8     18.47
9     21.79
Name: Bits/Symbol|, dtype: float64
```

[20]:
```
df_Jonas[df_Jonas.columns[4]]
```

[20]:
```
0     0.00
1     0.50
2     0.86
3     1.15
4     1.38
5     1.58
6     1.76
```

```
7    1.91
8    2.05
9    2.18
Name: Bits/Photon|, dtype: float64
```

[21]:
```
df_Jonas[df_Jonas.columns[5]]
```

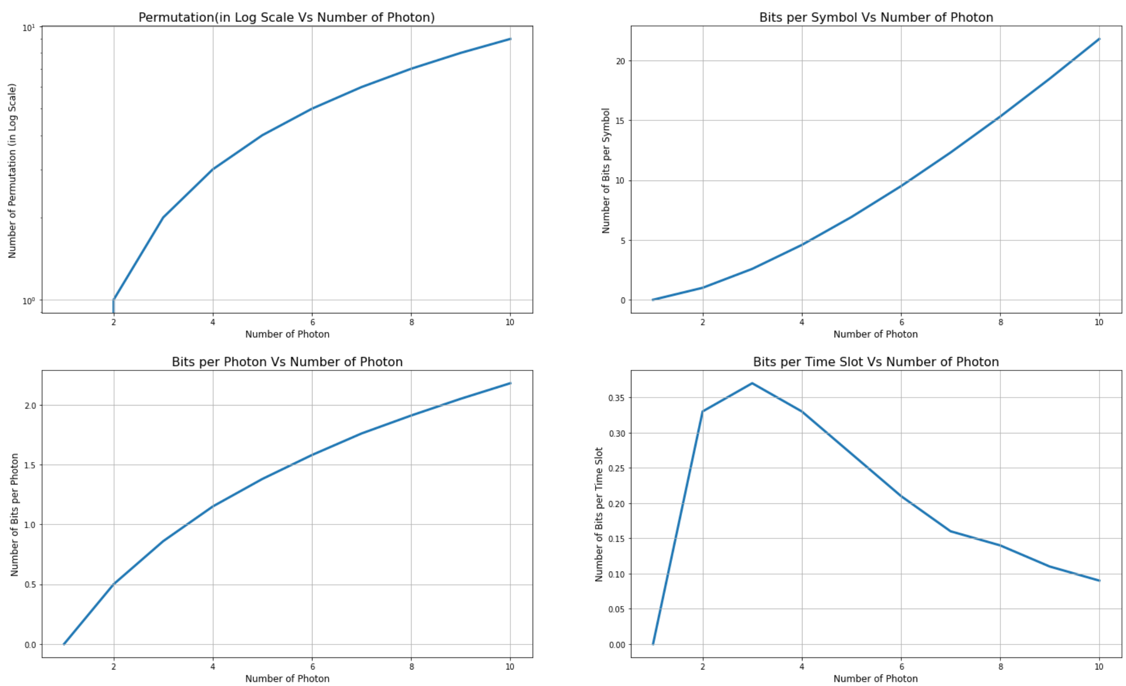
[21]:
```
0    0.00
1    0.33
2    0.37
3    0.33
4    0.27
5    0.21
6    0.16
7    0.14
8    0.11
9    0.09
Name: Bits/Time Bin|, dtype: float64
```

[22]:
```
#df_Jonas = df_Jonas.style.background_gradient() #adding a gradient based on␣
↪values in cell
dfi.export(
    df_Jonas,
    "Jonas_Table.png",
    table_conversion="matplotlib"
)
```

## 8 Plot Graph

[23]:
```
figure, axis = plt.subplots(2,2,figsize=(25,15))

axis[0, 0].plot(df_Jonas[df_Jonas.columns[0]],df_Jonas[df_Jonas.
↪columns[2]],linewidth=3,zorder=1, label = "bits")
axis[0, 0].set_title('Permutation(in Log Scale Vs Number of Photon)', fontsize␣
↪= 16)
axis[0, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 0].set_ylabel('Number of Permutation (in Log Scale)', fontsize = 12)
axis[0, 0].set_yscale('log')
#axis[0, 0].set_xlim([0, 21])
axis[0, 0].grid(True)

axis[0, 1].plot(df_Jonas[df_Jonas.columns[0]],df_Jonas[df_Jonas.
↪columns[3]],linewidth=3,zorder=1, label = "bits")
axis[0, 1].set_title('Bits per Symbol Vs Number of Photon', fontsize = 16)
axis[0, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 1].set_ylabel('Number of Bits per Symbol', fontsize = 12)
```

```
axis[0, 1].grid(True)

axis[1, 0].plot(df_Jonas[df_Jonas.columns[0]],df_Jonas[df_Jonas.columns[4]],linewidth=3,zorder=1, label = "bits")

axis[1, 0].set_title('Bits per Photon Vs Number of Photon', fontsize = 16)
axis[1, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 0].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[1, 0].grid(True)

axis[1, 1].plot(df_Jonas[df_Jonas.columns[0]],df_Jonas[df_Jonas.columns[5]],linewidth=3, zorder=1, label = "bits")
axis[1, 1].set_title('Bits per Time Slot Vs Number of Photon', fontsize = 16)
axis[1, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 1].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[1, 1].grid(True)

figure.set_facecolor("white")
plt.savefig('Jonas_Plot.png', dpi=450, bbox_inches='tight')
plt.show()
```



# 9 General

The number of symbols m in general method will be the number ow ways k photons can be placed in n bins, i.e.

$$m = \binom{n}{k}$$

If we have 4 photons, 14 timeslots, then there will be 1,0001 ways to order them by binomial where order does not matter.

The number of bits (the information content per symbol) will be

$$b = log_2 m$$

However, if we consider the repeition are not allowed, then we use Permutations = r! x Combinations

B1. Compare with "general" method using same number of slots, n, and same number of photons, k \

B2. The number of symbols m in general method will be the number ow ways k photons can be placed in n bins, i.e. m=(n over k) \

B3. The number of bits (the information content per symbol) will be b=log2(m)

### 9.0.1 Combination

[24]:
```
def combination(n,r):
  return math.factorial(n) / (math.factorial(n-r)*math.factorial(r))

combination(14,4)
```

[24]: 1001.0

[25]:
```
Generalnumber_c = []
GeneralPermutation_c = []
GeneralBPS_c = []
GeneralBPP_c = []
GeneralBPT_c = []

series = [1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002]

df_General_c = pd.DataFrame(columns=['|Number of Photon|',
                            'Time Bins|',
                            'Permutation|',
                            'Bits/Symbol|',
                            'Bits/Photon|',
                            'Bits/Time Bin|']
                  )
```

```
for n in range(1,11):
    Generalways_c = combination(series[n-1],n)     # B2. The number of symbols m␣
 ↪in general method will be the number ow ways k photons can be placed in n␣
 ↪bins, i.e. m=(n over k) \\
    GeneralTimeBin = series[n-1]                    # B1. Compare with "general"␣
 ↪method using same number of slots, n, and same number of photons, k \\
    Generalbps_c = math.log2(Generalways_c)         # B3. The number of bits (the␣
 ↪information content per symbol) will be b=log2(m)
    Generalbpp_c = math.log2(Generalways_c) / n
    Generalbpt_c = Generalbpp_c * n/series[n-1]

    GeneralTimeBin_SigFig_c = "{:.2e}".format(GeneralTimeBin)
    Generalways_SigFig_c = "{:.2e}".format(Generalways_c)
    Generalbps_round_c = round(Generalbps_c, 2)
    Generalbpp_round_c = round(Generalbpp_c, 2)
    Generalbpt_round_c = round(Generalbpt_c, 2)

    df_General_c.loc[len(df_General_c)] = [n, GeneralTimeBin_SigFig_c,␣
 ↪Generalways_SigFig_c, Generalbps_round_c , Generalbpp_round_c,␣
 ↪Generalbpt_round_c]



    #round(answer, 2)
```

[26]:
```
# The first is zero because, the first item in the time bin is 1, which give␣
 ↪the log based 2 to be 0.
math.log2(combination(1,1))
```

[26]: 0.0

[27]:
```
df_General_c # the 4 data is correct
```

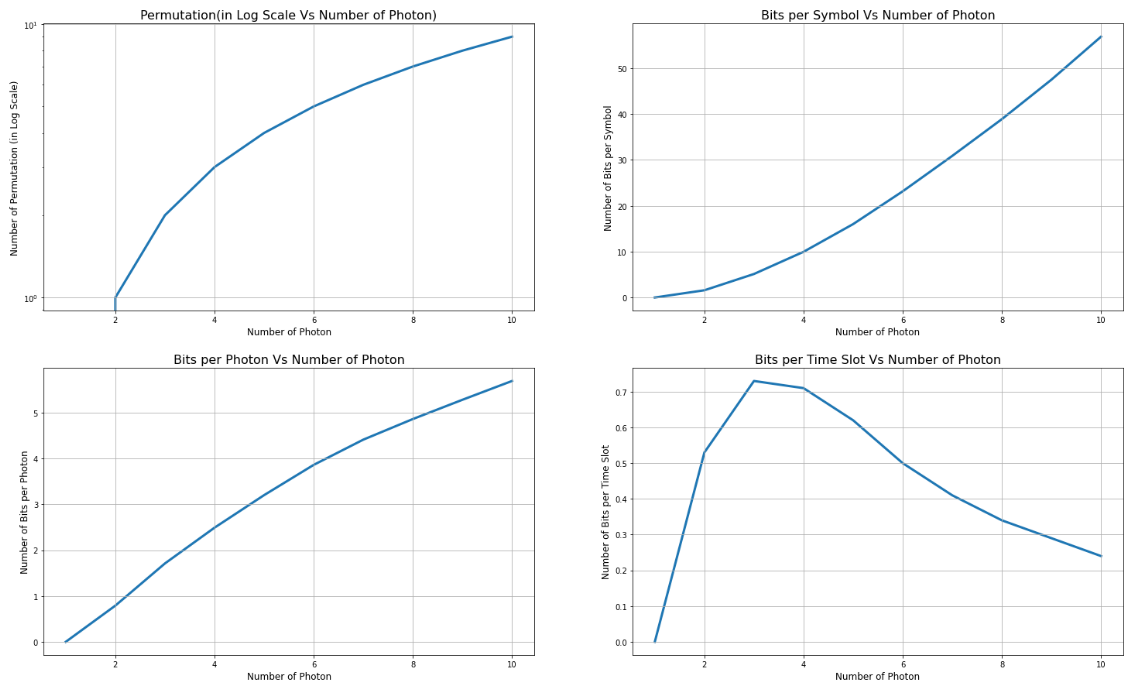
[27]:
```
  |Number of Photon| Time Bins| Permutation|  Bits/Symbol|  Bits/Photon|  \
0                 1   1.00e+00     1.00e+00          0.00          0.00
1                 2   3.00e+00     3.00e+00          1.58          0.79
2                 3   7.00e+00     3.50e+01          5.13          1.71
3                 4   1.40e+01     1.00e+03          9.97          2.49
4                 5   2.60e+01     6.58e+04         16.01          3.20
5                 6   4.60e+01     9.37e+06         23.16          3.86
6                 7   7.50e+01     1.98e+09         30.89          4.41
7                 8   1.13e+02     5.12e+11         38.90          4.86
8                 9   1.65e+02     2.00e+14         47.51          5.28
9                10   2.38e+02     1.33e+17         56.88          5.69

   Bits/Time Bin|
0           0.00
```

```
1             0.53
2             0.73
3             0.71
4             0.62
5             0.50
6             0.41
7             0.34
8             0.29
9             0.24
```

[28]:
```
dfi.export(
    df_General_c,
    "General_c_Table.png",
    table_conversion="matplotlib"
)
```

[29]:
```
figure, axis = plt.subplots(2,2,figsize=(25,15))

axis[0, 0].plot(df_General_c[df_General_c.columns[0]],df_General_c[df_General_c.
 ↪columns[2]],linewidth=3,zorder=1, label = "bits")
axis[0, 0].set_title('Permutation(in Log Scale Vs Number of Photon)', fontsize␣
 ↪= 16)
axis[0, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 0].set_ylabel('Number of Permutation (in Log Scale)', fontsize = 12)
axis[0, 0].set_yscale('log')
#axis[0, 0].set_yscale('log')
#axis[0, 0].set_xlim([0, 21])
axis[0, 0].grid(True)

axis[0, 1].plot(df_General_c[df_General_c.columns[0]],df_General_c[df_General_c.
 ↪columns[3]],linewidth=3,zorder=1, label = "bits")
axis[0, 1].set_title('Bits per Symbol Vs Number of Photon', fontsize = 16)
axis[0, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 1].set_ylabel('Number of Bits per Symbol', fontsize = 12)
axis[0, 1].grid(True)

axis[1, 0].plot(df_General_c[df_General_c.columns[0]],df_General_c[df_General_c.
 ↪columns[4]],linewidth=3,zorder=1, label = "bits")
axis[1, 0].set_title('Bits per Photon Vs Number of Photon', fontsize = 16)
axis[1, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 0].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[1, 0].grid(True)

axis[1, 1].plot(df_General_c[df_General_c.columns[0]],df_General_c[df_General_c.
 ↪columns[5]],linewidth=3, zorder=1, label = "bits")
axis[1, 1].set_title('Bits per Time Slot Vs Number of Photon', fontsize = 16)
axis[1, 1].set_xlabel('Number of Photon', fontsize = 12)
```

```
axis[1, 1].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[1, 1].grid(True)

figure.set_facecolor("white")
#plt.xticks(x)
plt.savefig('General_c_Plot.png', dpi=450, bbox_inches='tight')
plt.show()
```




### 9.0.2 Permutation

[30]:
```
# A Python program to print all permutations of given length
from itertools import permutations

# Get all permutations of length 2
#perm = permutations([1, 2, 3], 2)

# Print the obtained permutations
#for i in list(perm):
#     print (i)
```

[31]:
```
def permutation(n,r):
  return math.factorial(n)/math.factorial(n-r)
```

[32]:
```
Generalnumber_p = []
GeneralPermutation_p = []
```

```
GeneralBPS_p = []
GeneralBPP_p = []
GeneralBPT_p = []


series = [1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002]

df_General_p = pd.DataFrame(columns=['|Number of Photon|',
                           'Time Bins|',
                           'Permutation|',
                           'Bits/Symbol|',
                           'Bits/Photon|',
                           'Bits/Time Bin|']
                  )

for n in range(1,11):
    Generalways_p = permutation(series[n-1],n)
    GeneralTimeBin = series[n-1]
    Generalbps_p = math.log2(Generalways_p)
    Generalbpp_p = math.log2(Generalways_p) / n
    Generalbpt_p = Generalbpp_p * n/series[n-1]

    GeneralTimeBin_SigFig_p = "{:.2e}".format(GeneralTimeBin)
    Generalways_SigFig_p = "{:.2e}".format(Generalways_p)
    Generalbps_round_p = round(Generalbps_p, 2)
    Generalbpp_round_p = round(Generalbpp_p, 2)
    Generalbpt_round_p = round(Generalbpt_p, 2)

    df_General_p.loc[len(df_General_p)] = [n, GeneralTimeBin_SigFig_p,␣
↪Generalways_SigFig_p, Generalbps_round_p , Generalbpp_round_p,␣
↪Generalbpt_round_p]



    #round(answer, 2)
```

[33]:
```
df_General_p # the 4 data is not correct
```

[33]:

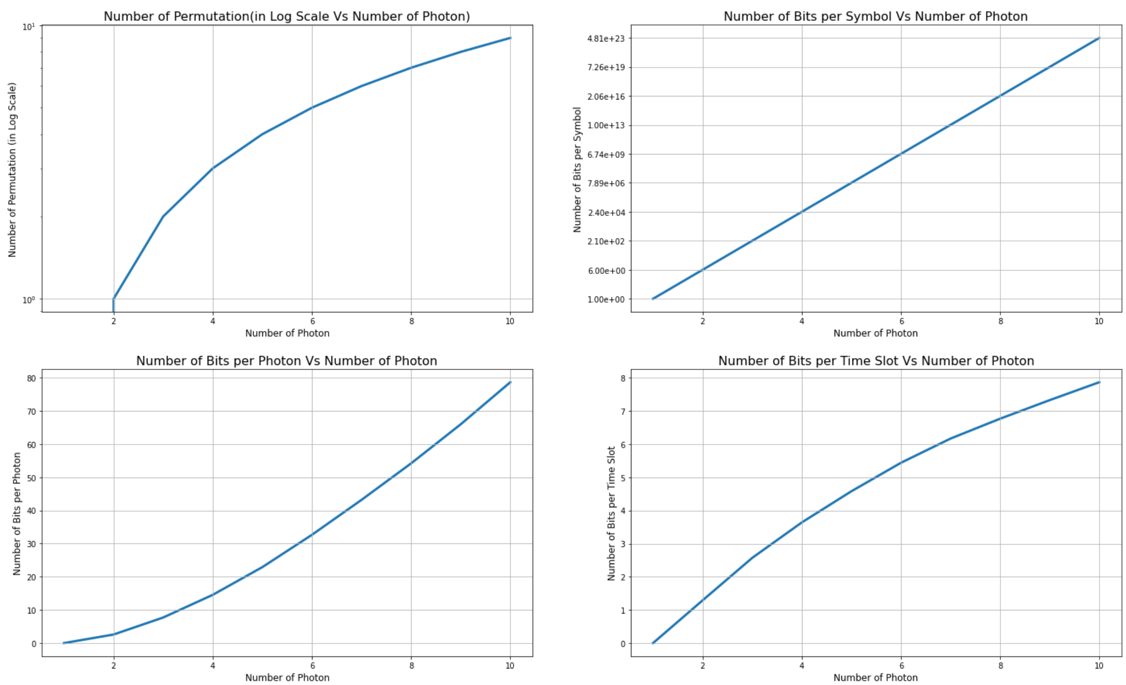
| | \|Number of Photon\| | Time Bins\| | Permutation\| | Bits/Symbol\| | Bits/Photon\| | \ |
|---|---|---|---|---|---|---|
| 0 | 1 | 1.00e+00 | 1.00e+00 | 0.00 | 0.00 | |
| 1 | 2 | 3.00e+00 | 6.00e+00 | 2.58 | 1.29 | |
| 2 | 3 | 7.00e+00 | 2.10e+02 | 7.71 | 2.57 | |
| 3 | 4 | 1.40e+01 | 2.40e+04 | 14.55 | 3.64 | |
| 4 | 5 | 2.60e+01 | 7.89e+06 | 22.91 | 4.58 | |
| 5 | 6 | 4.60e+01 | 6.74e+09 | 32.65 | 5.44 | |
| 6 | 7 | 7.50e+01 | 1.00e+13 | 43.19 | 6.17 | |
| 7 | 8 | 1.13e+02 | 2.06e+16 | 54.20 | 6.77 | |
| 8 | 9 | 1.65e+02 | 7.26e+19 | 65.98 | 7.33 | |

```
9                 10   2.38e+02     4.81e+23        78.67          7.87

   Bits/Time Bin|
0            0.00
1            0.86
2            1.10
3            1.04
4            0.88
5            0.71
6            0.58
7            0.48
8            0.40
9            0.33
```

[34]:
```python
dfi.export(
    df_General_p,
    "General_p_Table.png",
    table_conversion="matplotlib"
)
```

[35]:
```python
figure, axis = plt.subplots(2,2,figsize=(25,15))

axis[0, 0].plot(df_General_p[df_General_p.columns[0]],df_General_p[df_General_p.
 ↪columns[1]],linewidth=3,zorder=1, label = "bits")
axis[0, 0].set_title('Number of Permutation(in Log Scale Vs Number of Photon)',␣
 ↪fontsize = 16)
axis[0, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 0].set_ylabel('Number of Permutation (in Log Scale)', fontsize = 12)
axis[0, 0].set_yscale('log')
#axis[0, 0].set_yscale('log')
#axis[0, 0].set_xlim([0, 21])
axis[0, 0].grid(True)

axis[0, 1].plot(df_General_p[df_General_p.columns[0]],df_General_p[df_General_p.
 ↪columns[2]],linewidth=3,zorder=1, label = "bits")
axis[0, 1].set_title('Number of Bits per Symbol Vs Number of Photon', fontsize␣
 ↪= 16)
axis[0, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 1].set_ylabel('Number of Bits per Symbol', fontsize = 12)
axis[0, 1].grid(True)

axis[1, 0].plot(df_General_p[df_General_p.columns[0]],df_General_p[df_General_p.
 ↪columns[3]],linewidth=3,zorder=1, label = "bits")
axis[1, 0].set_title('Number of Bits per Photon Vs Number of Photon', fontsize␣
 ↪= 16)
axis[1, 0].set_xlabel('Number of Photon', fontsize = 12)
```

```
axis[1, 0].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[1, 0].grid(True)

axis[1, 1].plot(df_General_p[df_General_p.columns[0]],df_General_p[df_General_p.columns[4]],linewidth=3, zorder=1, label = "bits")
axis[1, 1].set_title('Number of Bits per Time Slot Vs Number of Photon', fontsize = 16)
axis[1, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 1].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[1, 1].grid(True)

figure.set_facecolor("white")
plt.savefig('General_p_Plot.png', dpi=450, bbox_inches='tight')
plt.show()
```




# 10 PPM

We have 1 photon 14 timeslots 14 ways to order them

C1. Compare with PPM method using same number of slots, n. It will only use k=1 photons per symbol \

C2. The number of symbols m in PPM will be the number ow ways 1 photon can be placed in n bins, i.e. m=n \

C3. The number of bits (the information content per symbol) will be b=log2(m) \

[36]:
```python
PPMnumber = []
PPMPermutation = []
PPMBPS = []
PPMBPP = []
PPMBPT = []

series = [1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002]

df_PPM = pd.DataFrame(columns=['|Number of Photon|',
                               'Time Bins|',
                               'Permutation|',
                               'Bits/Symbol|',
                               'Bits/Photon|',
                               'Bits/Time Bin|']
                     )

for n in range(1,11):
    PPMWays = series[n-1]
    #PPMWays = n                       # C2 The number of symbols m in PPM will␣
 ↪be the number ow ways 1 photon can be placed in n bins, i.e. m=n \\
    PPMTimeBin = series[n-1]           # C1 Compare with PPM method using same␣
 ↪number of slots, n. It will only use k=1 photons per symbol \\
    PPMbps = math.log2(PPMTimeBin)        # C3 The number of bits (the␣
 ↪information content per symbol) will be b=log2(m)
    PPMbpp = math.log2(PPMTimeBin) / 1    # C1 Compare with PPM method using␣
 ↪same number of slots, n. It will only use k=1 photons per symbol \\
    PPMbpt = PPMbpp * 1/series[n-1]

    PPMTimeBin_SigFig = "{:.2e}".format(PPMTimeBin)
    #PPMways_SigFig = "{:.2e}".format(PPMWays)
    PPMWays_SigFig = PPMWays
    PPMbps_round = round(PPMbps, 2)
    PPMbpp_round = round(PPMbpp, 2)
    PPMbpt_round = round(PPMbpt, 2)

    df_PPM.loc[len(df_PPM)] = [1, PPMTimeBin_SigFig, PPMWays_SigFig,␣
 ↪PPMbps_round , PPMbpp_round, PPMbpt_round]


    #round(answer, 2)
```

[37]:
```python
df_PPM.style # the 4th data is not correct
```

[37]:
```
<pandas.io.formats.style.Styler at 0x7eff1b2e2640>
```

[38]:
```python
math.log2(15)
```

[38]: 3.9068905956085187

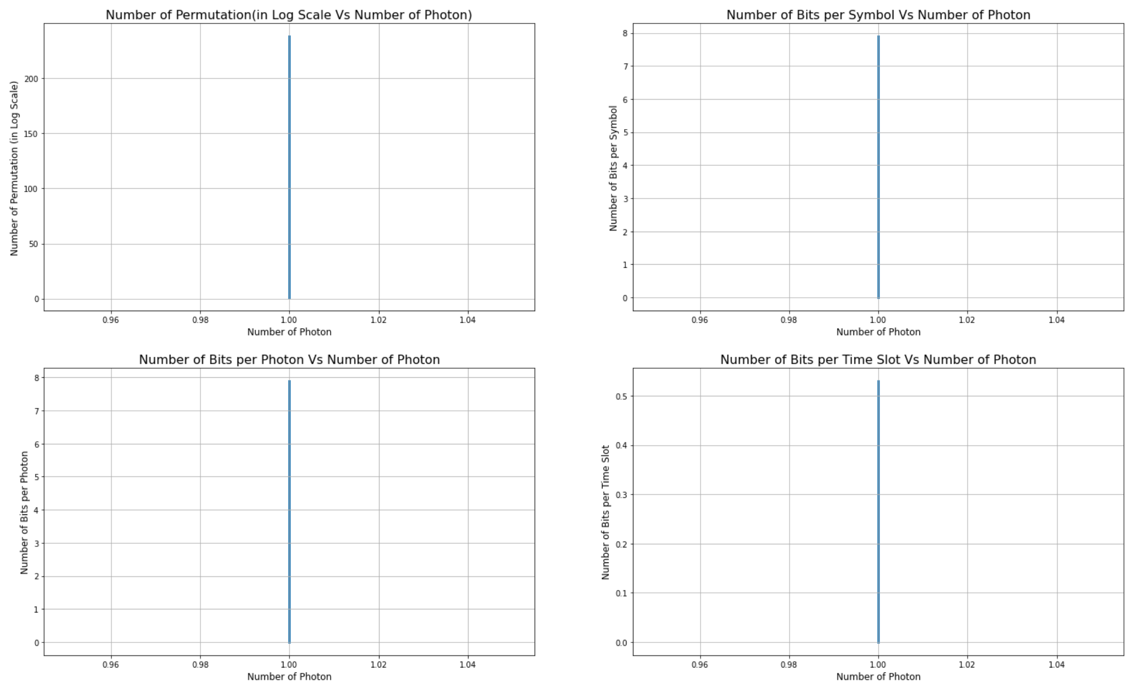
[39]:
```
df_PPM.columns[0]
```

[39]: '|Number of Photon|'

[40]:
```
dfi.export(
    df_PPM,
    "PPM_Table.png",
    table_conversion="matplotlib"
)
```

[41]:
```
figure, axis = plt.subplots(2,2,figsize=(25,15))

axis[0, 0].plot(df_PPM[df_PPM.columns[0]],df_PPM[df_PPM.
 ↪columns[2]],linewidth=3,zorder=1, label = "bits")
axis[0, 0].set_title('Number of Permutation(in Log Scale Vs Number of Photon)',␣
 ↪fontsize = 16)
axis[0, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 0].set_ylabel('Number of Permutation (in Log Scale)', fontsize = 12)
#axis[0, 0].set_yscale('log')
#axis[0, 0].set_xlim([0, 21])
axis[0, 0].grid(True)

axis[0, 1].plot(df_PPM[df_PPM.columns[0]],df_PPM[df_PPM.
 ↪columns[3]],linewidth=3,zorder=1, label = "bits")
axis[0, 1].set_title('Number of Bits per Symbol Vs Number of Photon', fontsize␣
 ↪= 16)
axis[0, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 1].set_ylabel('Number of Bits per Symbol', fontsize = 12)
axis[0, 1].grid(True)

axis[1, 0].plot(df_PPM[df_PPM.columns[0]],df_PPM[df_PPM.
 ↪columns[4]],linewidth=3,zorder=1, label = "bits")
axis[1, 0].set_title('Number of Bits per Photon Vs Number of Photon', fontsize␣
 ↪= 16)
axis[1, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 0].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[1, 0].grid(True)

axis[1, 1].plot(df_PPM[df_PPM.columns[0]],df_PPM[df_PPM.
 ↪columns[5]],linewidth=3, zorder=1, label = "bits")
axis[1, 1].set_title('Number of Bits per Time Slot Vs Number of Photon',␣
 ↪fontsize = 16)
axis[1, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 1].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
```

```
axis[1, 1].grid(True)

figure.set_facecolor("white")
plt.savefig('PPM_Plot.png', dpi=450, bbox_inches='tight')
plt.show()
```




# 11 OOK

D1. Compare with OOK method using same number of slots, n. It will in average use k=n/2 photons per symbol \

D2. The number of symbols m in PPM will be m=2^n \

D3. The number of bits (the information content per symbol) will be b=log2(m)=n \

[42]:
```
OOKnumber = []
OOKPermutation = []
OOKBPS = []
OOKBPP = []
OOKBPT = []

series = [1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002]

df_OOK = pd.DataFrame(columns=['|Number of Photon|',
                               'Time Bins|',
```

```
                              'Permutation|',
                              'Bits/Symbol|',
                              'Bits/Photon|',
                              'Bits/Time Bin|']
                    )

for n in range(1,11):
    #OOKways = (2)**series[n-1]
    OOKTimeBin = series[n-1]                              # D1. Compare with OOK␣
↪method using same number of slots, n. It will in average use k=n/2 photons␣
↪per symbol
    OOKWays = (2)**series[n-1]                            # D2. The number of␣
↪symbols m in PPM will be m=2^n, n is the number of time bin
    OOKbps = math.log2(OOKWays)                           # D3. The number of bits␣
↪(the information content per symbol) will be b=log2(m)=n, n is the number of␣
↪time bin
    #OOKbpp = (math.log2(OOKWays) * series[n-1] / n
    OOKbpp = 2
    OOKbpt = OOKbpp * n / series[n-1]

    OOKTimeBin_SigFig = "{:.2e}".format(OOKTimeBin)
    OOKWays_SigFig = "{:.2e}".format(OOKWays)
    #OOKWays_SigFig = round(OOKWays, 2)
    OOKbps_round = round(OOKbps, 2)
    OOKbpp_round = round(OOKbpp, 2)
    OOKbpt_round = round(OOKbpt, 2)

    df_OOK.loc[len(df_OOK)] = [OOKTimeBin/2, OOKTimeBin_SigFig, OOKWays_SigFig,␣
↪OOKbps_round, OOKbpp_round, OOKbpt_round]
```

[43]:
```
#df_OOK.style
```

[44]:
```
df_OOK # not sure if it is correct
```

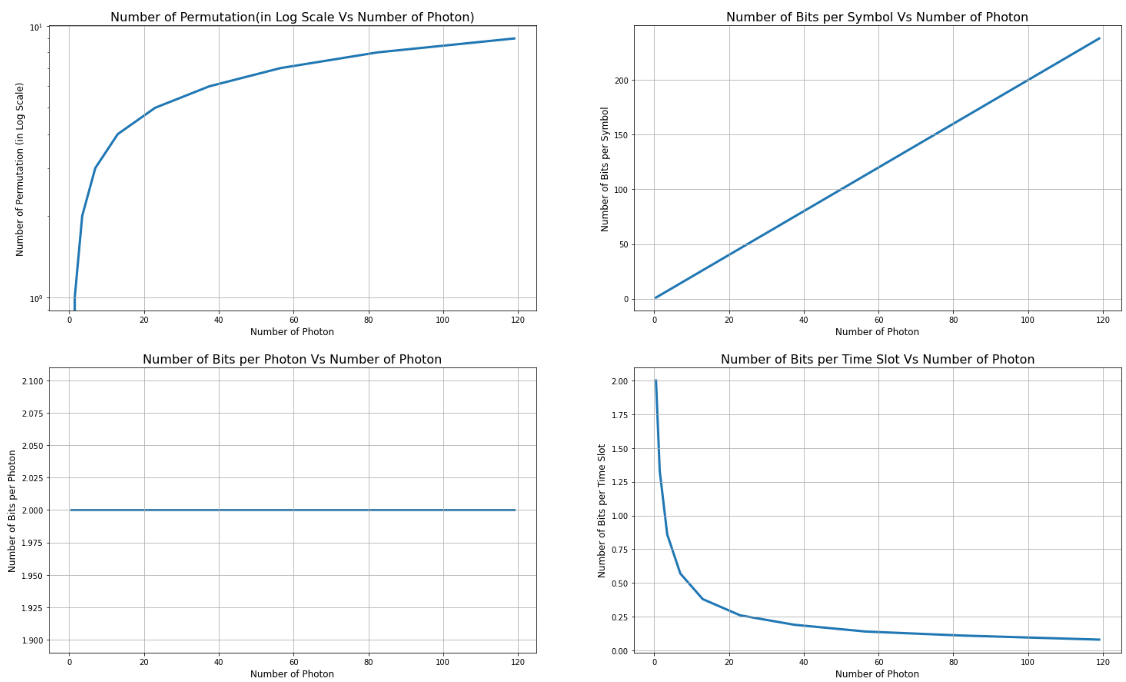
[44]:
```
  |Number of Photon| Time Bins| Permutation|  Bits/Symbol| Bits/Photon|  \
0                0.5   1.00e+00     2.00e+00           1.0            2
1                1.5   3.00e+00     8.00e+00           3.0            2
2                3.5   7.00e+00     1.28e+02           7.0            2
3                7.0   1.40e+01     1.64e+04          14.0            2
4               13.0   2.60e+01     6.71e+07          26.0            2
5               23.0   4.60e+01     7.04e+13          46.0            2
6               37.5   7.50e+01     3.78e+22          75.0            2
7               56.5   1.13e+02     1.04e+34         113.0            2
8               82.5   1.65e+02     4.68e+49         165.0            2
9              119.0   2.38e+02     4.42e+71         238.0            2

  Bits/Time Bin|
```

| | |
|---|---|
| 0 | 2.00 |
| 1 | 1.33 |
| 2 | 0.86 |
| 3 | 0.57 |
| 4 | 0.38 |
| 5 | 0.26 |
| 6 | 0.19 |
| 7 | 0.14 |
| 8 | 0.11 |
| 9 | 0.08 |

[45]:
```
dfi.export(
    df_OOK,
    "OOK_Table.png",
    table_conversion="matplotlib"
)
```

[46]:
```
figure, axis = plt.subplots(2,2,figsize=(25,15))

axis[0, 0].plot(df_OOK[df_OOK.columns[0]],df_OOK[df_OOK.
 ↪columns[2]],linewidth=3,zorder=1, label = "bits")

axis[0, 0].set_title('Number of Permutation(in Log Scale Vs Number of Photon)',␣
 ↪fontsize = 16)
axis[0, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 0].set_ylabel('Number of Permutation (in Log Scale)', fontsize = 12)
axis[0, 0].set_yscale('log')
#axis[0, 0].set_yscale('log')
#axis[0, 0].set_xlim([0, 21])
axis[0, 0].grid(True)

axis[0, 1].plot(df_OOK[df_OOK.columns[0]],df_OOK[df_OOK.
 ↪columns[3]],linewidth=3,zorder=1, label = "bits")
axis[0, 1].set_title('Number of Bits per Symbol Vs Number of Photon', fontsize␣
 ↪= 16)
axis[0, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 1].set_ylabel('Number of Bits per Symbol', fontsize = 12)
axis[0, 1].grid(True)

axis[1, 0].plot(df_OOK[df_OOK.columns[0]],df_OOK[df_OOK.
 ↪columns[4]],linewidth=3,zorder=1, label = "bits")
axis[1, 0].set_title('Number of Bits per Photon Vs Number of Photon', fontsize␣
 ↪= 16)
axis[1, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 0].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[1, 0].grid(True)
```

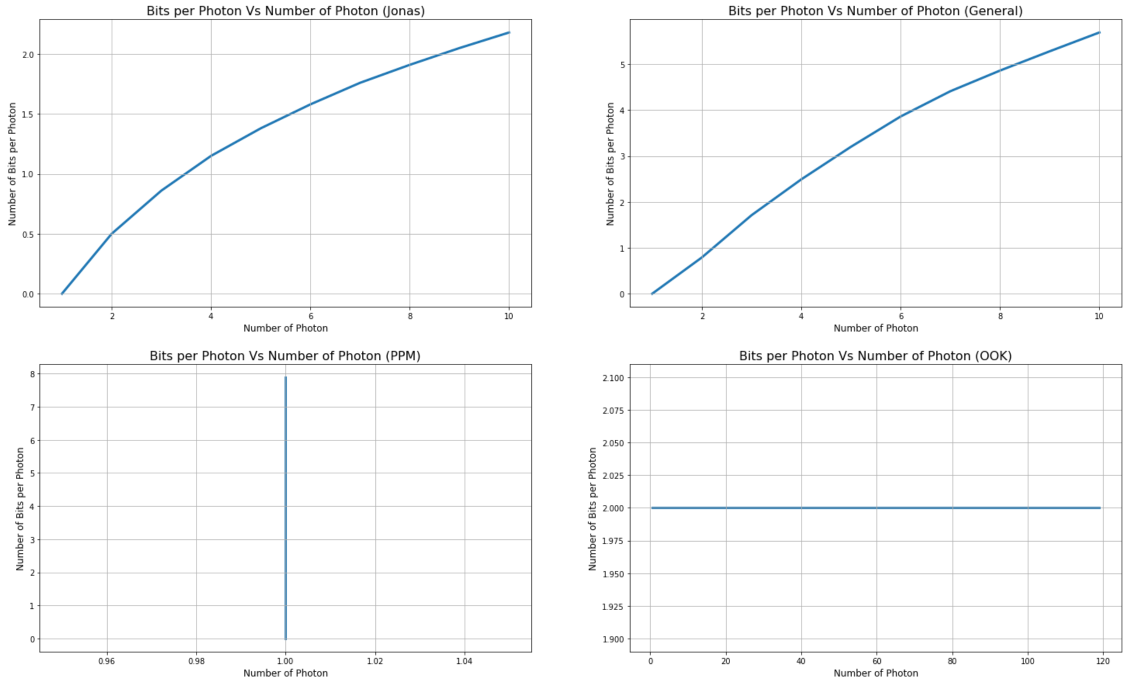
```
axis[1, 1].plot(df_OOK[df_OOK.columns[0]],df_OOK[df_OOK.
 ↪columns[5]],linewidth=3, zorder=1, label = "bits")
axis[1, 1].set_title('Number of Bits per Time Slot Vs Number of Photon',␣
 ↪fontsize = 16)
axis[1, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 1].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[1, 1].grid(True)

figure.set_facecolor("white")
plt.savefig('OOK_Plot.png', dpi=450, bbox_inches='tight')
plt.show()
```




# 12 Bits/Photon over 4 Schemes

[47]:
```
figure, axis = plt.subplots(2,2,figsize=(25,15))

axis[0, 0].plot(df_Jonas[df_Jonas.columns[0]],df_Jonas[df_Jonas.
 ↪columns[4]],linewidth=3, zorder=1, label = "bits")
axis[0, 0].set_title('Bits per Photon Vs Number of Photon (Jonas)' , fontsize =␣
 ↪16)
axis[0, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 0].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[0, 0].grid(True)
```

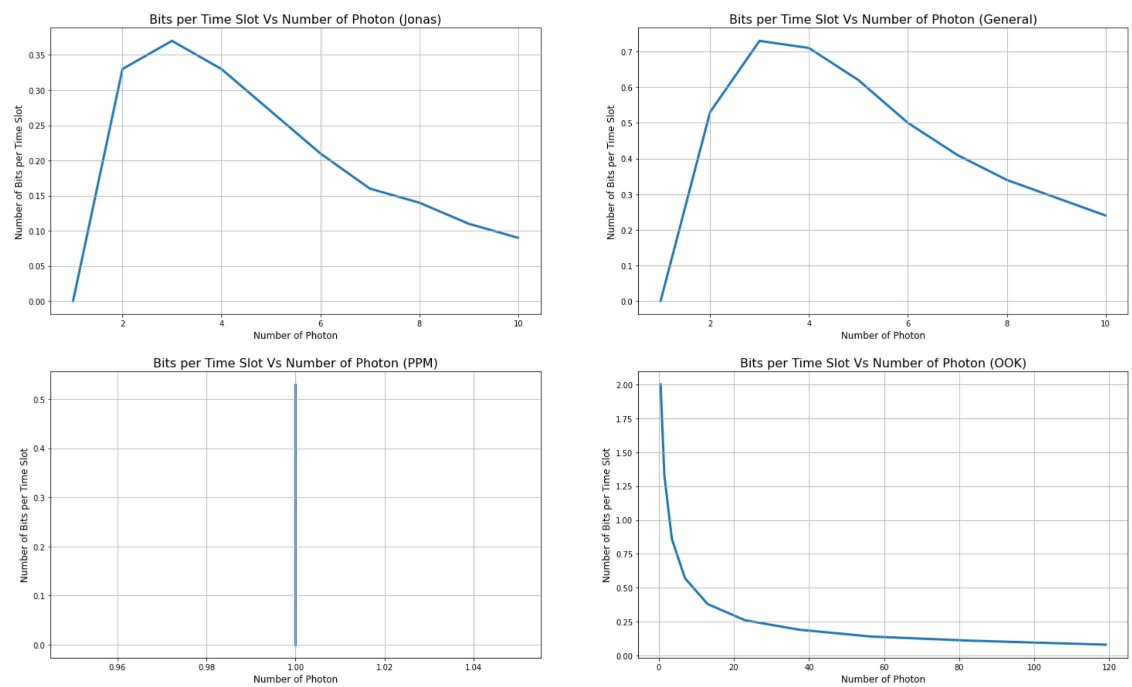
```
axis[0, 1].plot(df_General_c[df_General_c.columns[0]],df_General_c[df_General_c.
 ↪columns[4]],linewidth=3, zorder=1, label = "bits")
axis[0, 1].set_title('Bits per Photon Vs Number of Photon (General)' , fontsize␣
 ↪= 16)
axis[0, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 1].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[0, 1].grid(True)

axis[1, 0].plot(df_PPM[df_PPM.columns[0]],df_PPM[df_PPM.
 ↪columns[4]],linewidth=3, zorder=1, label = "bits")
axis[1, 0].set_title('Bits per Photon Vs Number of Photon (PPM)' , fontsize =␣
 ↪16)
axis[1, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 0].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[1, 0].grid(True)

axis[1, 1].plot(df_OOK[df_OOK.columns[0]],df_OOK[df_OOK.
 ↪columns[4]],linewidth=3, zorder=1, label = "bits")
axis[1, 1].set_title('Bits per Photon Vs Number of Photon (OOK)' , fontsize =␣
 ↪16)
axis[1, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 1].set_ylabel('Number of Bits per Photon', fontsize = 12)
axis[1, 1].grid(True)

figure.set_facecolor("white")
plt.savefig('4SchemesBPP.png', dpi=450, bbox_inches='tight')
plt.show()
```




# 13 Bits/ Time Bin over 4 Schemes

[48]:
```
figure, axis = plt.subplots(2,2,figsize=(25,15))

axis[0, 0].plot(df_Jonas[df_Jonas.columns[0]],df_Jonas[df_Jonas.
 ↪columns[5]],linewidth=3, zorder=1, label = "bits")
axis[0, 0].set_title('Bits per Time Slot Vs Number of Photon (Jonas)', fontsize␣
 ↪= 16)
axis[0, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 0].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[0, 0].grid(True)

axis[0, 1].plot(df_General_c[df_General_c.columns[0]],df_General_c[df_General_c.
 ↪columns[5]],linewidth=3, zorder=1, label = "bits")
axis[0, 1].set_title('Bits per Time Slot Vs Number of Photon (General)',␣
 ↪fontsize = 16)
axis[0, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[0, 1].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[0, 1].grid(True)

axis[1, 0].plot(df_PPM[df_PPM.columns[0]],df_PPM[df_PPM.
 ↪columns[5]],linewidth=3, zorder=1, label = "bits")
axis[1, 0].set_title('Bits per Time Slot Vs Number of Photon (PPM)', fontsize =␣
 ↪16)
```

```
axis[1, 0].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 0].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[1, 0].grid(True)

axis[1, 1].plot(df_OOK[df_OOK.columns[0]],df_OOK[df_OOK.
 ↪columns[5]],linewidth=3, zorder=1, label = "bits")
axis[1, 1].set_title('Bits per Time Slot Vs Number of Photon (OOK)', fontsize =␣
 ↪16)
axis[1, 1].set_xlabel('Number of Photon', fontsize = 12)
axis[1, 1].set_ylabel('Number of Bits per Time Slot', fontsize = 12)
axis[1, 1].grid(True)

figure.set_facecolor("white")
#plt.xticks(x)
plt.savefig('4schemesBPT.png', dpi=450, bbox_inches='tight')
plt.show()
```




[49]:
```
# https://www.geeksforgeeks.org/pandas-set_option-function-in-python/
# https://www.geeksforgeeks.org/pandas-set_option-function-in-python/
# https://stackoverflow.com/questions/35634238/
 ↪how-to-save-a-pandas-dataframe-table-as-a-png
# https://towardsdatascience.com/
 ↪6-pandas-display-options-you-should-memories-84adf8887bc3
# https://dash.plotly.com/datatable/width
```

```
# https://randomds.com/2021/12/23/
↪visualize-and-save-full-pandas-dataframes-as-images/
# https://randomds.com/2021/12/23/
↪visualize-and-save-full-pandas-dataframes-as-images/\
# https://www.adamsmith.haus/python/answers/
↪how-to-print-a-number-in-scientific-notation-in-python
# https://www.scaler.com/topics/python-scientific-notation/
# https://stackoverflow.com/questions/20457038/
↪how-to-round-to-2-decimals-with-python
# https://pythonfix.com/pkg/d/dataframe-image/
```

[49]:

```

```