Communication and Error Correction via Polarisation of Single Photons and Time Ordering

Master’s Thesis in Engineering Physics (Quantum Technology) at KTH

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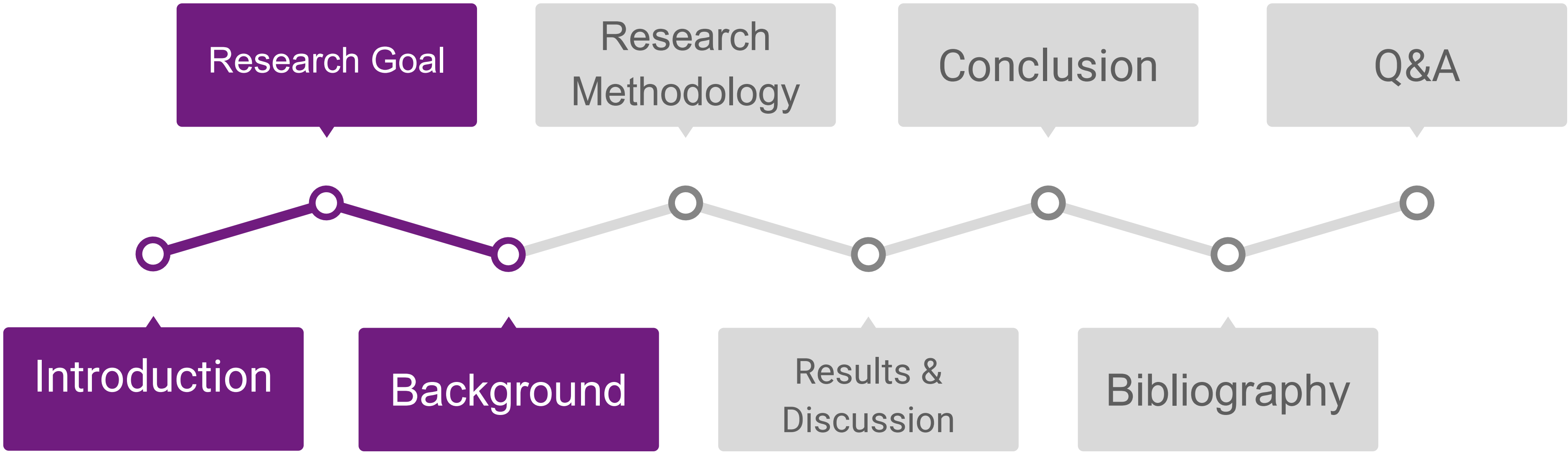
Introduction
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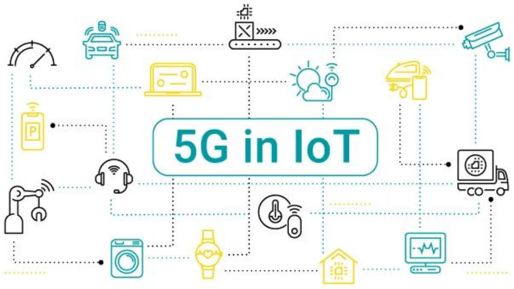
Q&A

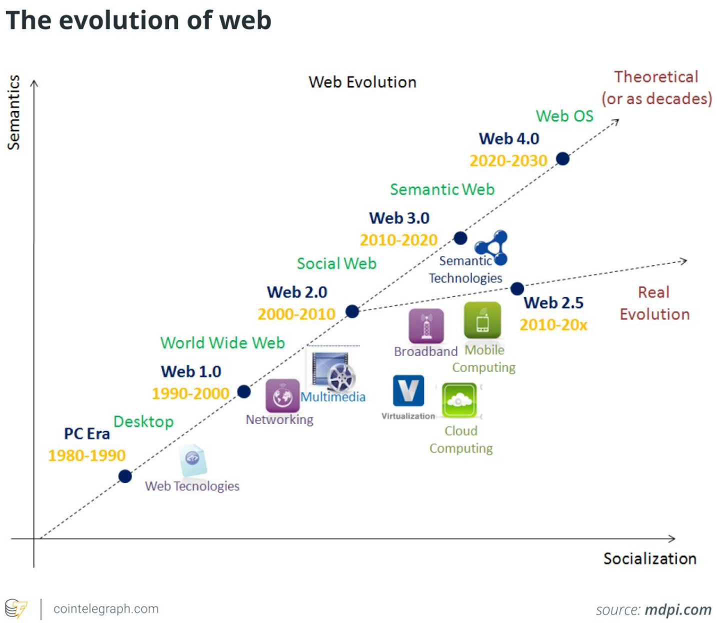
Introduction - Brief Overview of the Topic

Communication

=> telecommunication

1. Higher speed
2. Lower latency
3. Greater capacity





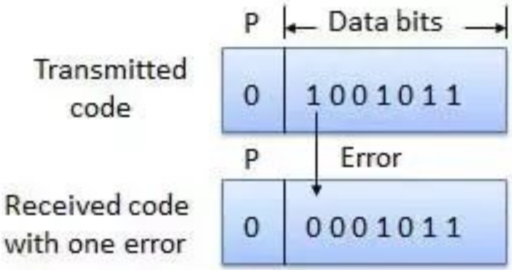
Brief Overview of the Topic

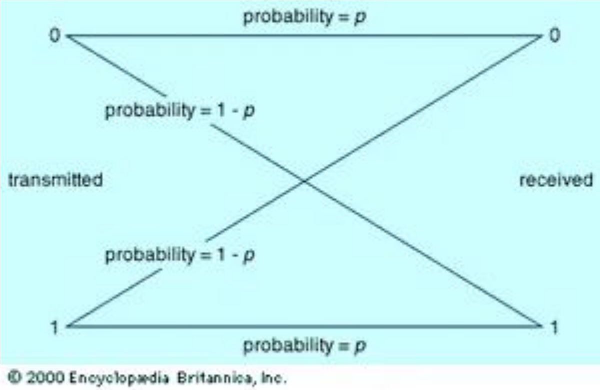
Communication

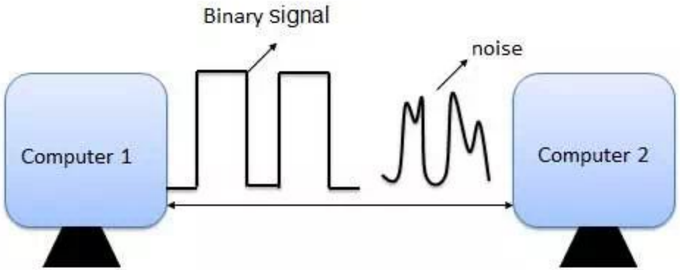
Application

Brief Overview of the Topic

Error Correction



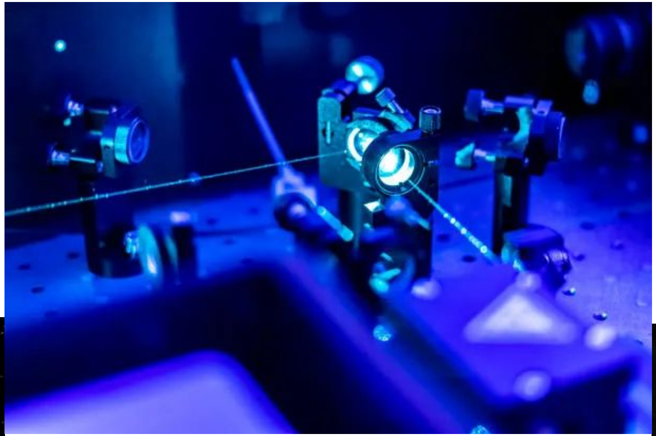


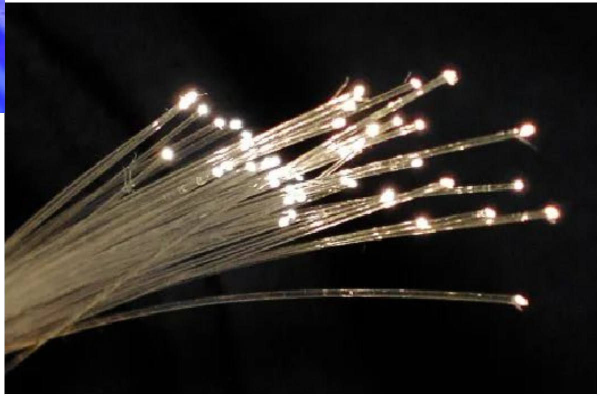


Single Photon Telecommunication

Polarization

Time Ordering





Research Question and Objectives

What is the maximum amount of information that a single photon can carry through polarization and time ordering?

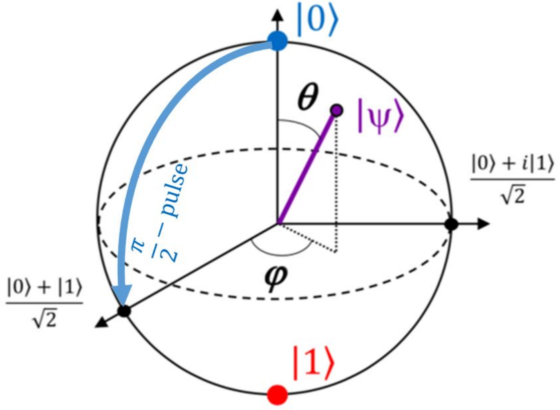
Information Capacity of a Single Photon => Error Correction Techniques

Literature:

Tristan BH Tentrup, Thomas Hummel, Tom AW Wolterink, Ravitej Uppu, Allard P Mosk, and Pepijn WH Pinkse. Transmitting more than 10 bit with a single photon. Optics express, 25(3):2826–2833, 2017.

Capacity: > 10 bits (10.5 bits) per photon
Size: one of the pixels of a grid of (alphabet) detectors in 9,072 spatial modes
Advantage: short distance communication

Motivation: improve reliability over long-distance communication



$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi}sin\frac{\theta}{2}|1\rangle$$

Research Question and Objectives

Literature:

A single photon can be used to convey more than 10 bits (10.5 bits) of information per detected photon between the sender and the receiver by associating each photon with one of the pixels of a grid of (alphabet) detectors with a size of 9,072 spatial modes

Tristan BH Tentrup, Thomas Hummel, Tom AW Wolterink, Ravitej Uppu, Allard P Mosk, and Pepijn WH Pinkse. Transmitting more than 10 bit with a single photon. Optics express, 25(3):2826–2833, 2017.

Motivation:

- Facilitate energy conservation when available power is limited
- Investigate the information capacity of a single photon through polarization and time ordering
- Develop error correction technique to improve reliability of photon-based communication over long distance

Significance of the Study

PPM

Lack of error correction

Demonstrate a new protocol - Beyond PPM (BPPM)

Using a vacuum or unexcited state in the encoding to send polarize single photon in the form of a polarized electromagnetic pulse

Message transmission > 1 bit per photon

Code to correct lost pulses and new unwanted pulses (due to reflections)

Significance of the Study

PPM < BPPM

- Lack of error correction

- Require more time for transmission
- it is energy-efficient
- Not sending wide spectrum
- Scheme resembles the permutation code by Blake
- Reliable Transmission
- Significant implication to space application

Cons and Pros of BPPM

Require more time for transmission than traditional block-coded communication protocol

it is energy-efficient

Not sending wide spectrum
- No potential health problems to the environment
- No interference with other telecommunication

Scheme resembles the permutation code
- Information transmitted more reliably
- Special type of error correction for an asymmetric channel
- <1 photon / information bit being sent

Significant implication to space application

Background

BPPM

- *Using a vacuum or unexcited state in the encoding to send polarize single photon in the form of a polarized electromagnetic pulse*
- *Message transmission > 1 bit per photon*
- *Code to correct lost pulses and new unwanted pulses (due to reflections)*

BPPM

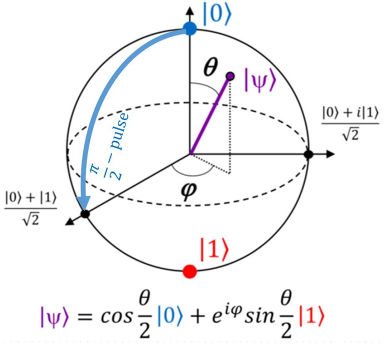
$$a(n) = (1, 2, 4, 7, 12, 20, 29, 38, 52, 73, 94, 127, 151, 181, 211, 257, 315, 373, 412, ...) \tag{2.5}$$

$$M(n) = (1, 3, 7, 14, 26, 46, 75, 113, 165, 238, 332, 459, 610, 791, 1002, 1259, 1574, 1947, 2359, ...)$$
$$= \sum_{i=1}^{n} a(i) \tag{2.6}$$

$$(1, 2, 4)$$

Three blocks of length 1,2,4 in a super block

Ways of encoding with Polarization



$$|\phi\rangle = cos\frac{\theta}{2}\,|H\rangle + e^{i\phi}sin\frac{\theta}{2}\,|V\rangle \tag{2.1}$$

$$\begin{aligned} l_i < l_{i+1} &\implies H_i \\ l_i > l_{i+1} &\implies V_i \\ l_1 > l_n &\implies H_n \\ l_1 < l_n &\implies V_n \end{aligned} \tag{2.10}$$

$$(1, 2, 4) \to V0HV000 \tag{2.11}$$

Error Correction Capabilities

Loss of Photon in (1, 2, 4, 7)

=> (3 ,4 ,7) **(1+2)**

=> (1 ,6, 7) **(2+4)**

=> (1, 2, 11) **(4+7)**

A block with the length is not allowed in the list, will be present and signal an error

$$(1, 2, 4, 7) \rightarrow HH0H000V000000 \qquad (2.12)$$

If the received superblock is (1,2,11) and know the polarization

(1,2,11) -> HH0H0000000000

(1,2,7,4) -> HH0V000000V000

The block of length 11 must be composed of (4,7) but not (7,4) by viewing the polarization

Addition of Photon in (1,2,4,7,12)

=> (1, 2, 4 , 1, 6, 12) **(1+6)**

=> (1, 2, 4, 2, 5, 12) **(2+5)**

=> (1, 2, 4, 3, 4, 12) **(3+4)**

=> (1, 2, 4, 4, 3, 12) **(4+3)**

=> (1, 2, 4, 2, 5, 2, 12) **(5+2)**

=> (1, 2, 4 ,6, 1, 12) **(6+1)**

Disallowed blocks indicate the position of error.

Correctable Multiple Errors

For multiple non-adjacent photons were lost

(1,2,4,7) -> (3,11) | HH0H000V000000 -> H00H0000000000

Likely: 2 loss errors (possibly 3 loss + 1 add)

Initial photon: H => (2,1) ruled out | second photon: H => (7,4) ruled out

Probability Distribution in loss error

$$(1,2,4) = HH0V000 \tag{2.14}$$

$$P(l,0,3,7,P_l,0) = (1-P_l)^3 + 3(1-P_l)^2 P_l + 3(1-P_l)P_l^2 + P_l^3 \tag{2.15}$$

P (exactly 2 loss errors)

P(0 loss error)

P(at most 1 loss error)

(1,2), (1,4), (2,4) for 1 loss error

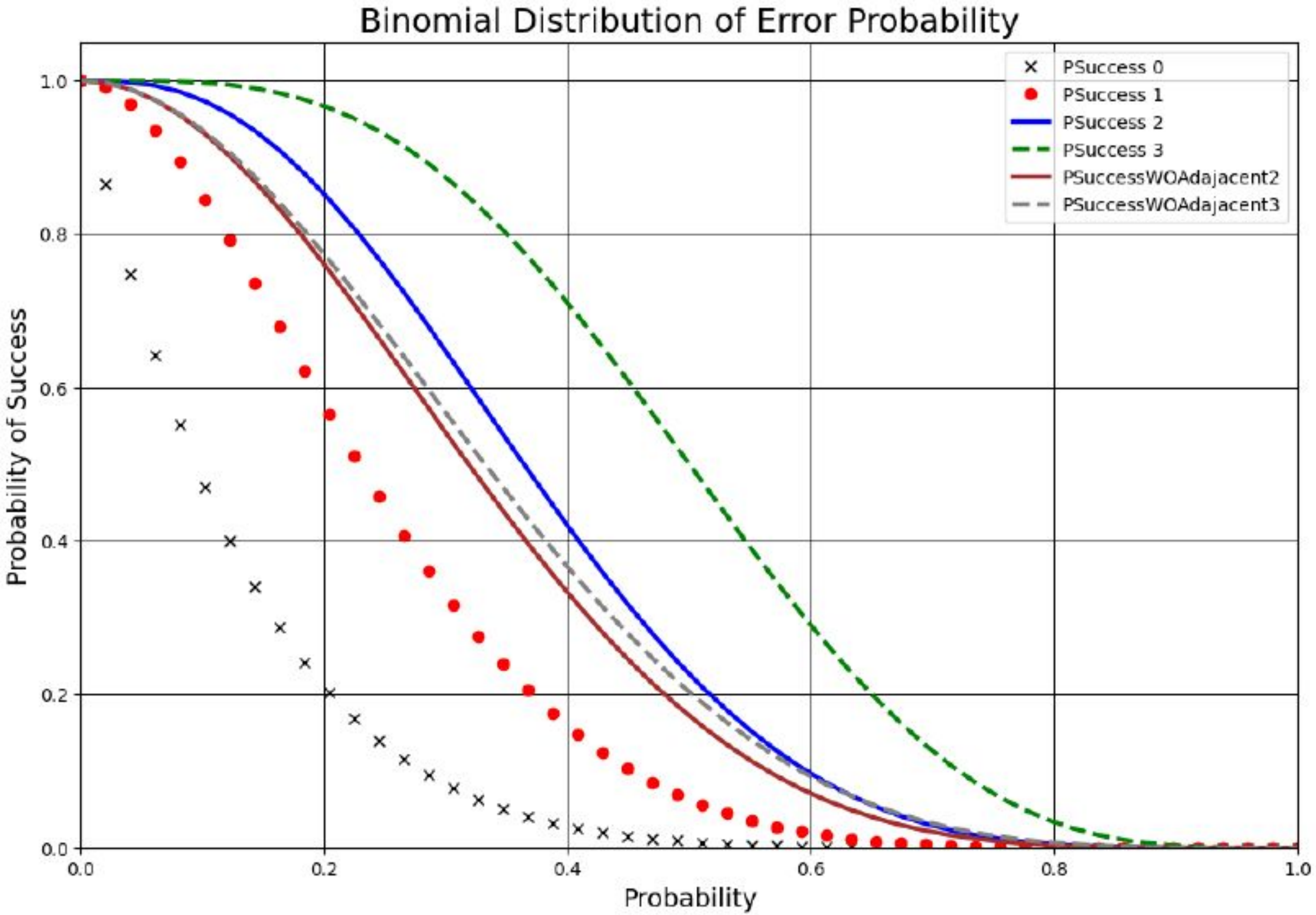
(1), (2), (4) for 2 loss errors

2 errors cannot occur in the same block since only one photon exists in each subblock

Most Detectable but Non-correctable Errors

The code cannot correct for 2 or more adjacent loss errors

$$
\begin{aligned}
&(1,2,4) = HH0V000 \\
&(1 \text{ loss}) \rightarrow 0H0V000 \\
&(1 \text{ loss}) \rightarrow 000V000 \\
&(1 \text{ add}) \rightarrow V00V000 \\
&(1 \text{ add}) \rightarrow V0HV000 = (2,1,4), \text{ which is a valid codeword}
\end{aligned}
\tag{2.19}
$$



Binomial Distribution of Error Probability
PSuccess 0
PSuccess 1
PSuccess 2
PSuccess 3
PSuccessWOAdajacent2
PSuccessWOAdajacent3
Probability of Success
Probability
0.0
0.2
0.4
0.6
0.8
1.0

Transmission in Beyond Pulse Position Modulation (BPPM)

The length of a codeword corresponds to the number of ways we can organize the blocks in a superblock (permutation):

$$K = n! \tag{4.1}$$

where K is the length of the codeword and n is the number of photons in a superblock.

The information content per symbol of the superblock is:

$$log_2(n!) \quad \text{bits/symbol} \tag{4.2}$$

The information content per photon of the superblock is:

$$\frac{log_2(n!)}{n} \quad \text{bits/photon} \tag{4.3}$$

and the information content per time bin of the superblock is:

$$\frac{log_2(n!)}{n} \times \frac{n}{M} = \frac{log_2(n!)}{M} \quad \text{bits/time bin} \tag{4.4}$$

For 4 photons in a superblock, we have

$$log_2(4!) = 4.6 \quad \text{bits/symbol}$$

In each photon we have:

$$\frac{log_2(4!)}{4} = 1.15 \quad \text{bits/photon}$$

and for each time bin, it has

$$\frac{log_2(4!)}{4} \times \frac{4}{1+2+4+7} = 0.33 \quad \text{bits/time bin}$$

Transmission in Pulse Position Modulation (PPM)

$$n = 1, \text{Time Bin} = M \tag{4.6}$$

The information content per symbol of the superblock:

$$\log_2 \mathrm{K} = log_2\mathrm{M} \quad \text{bits/symbol} \tag{4.7}$$

The information content per photon of the superblock:

$$\frac{log_2\mathrm{M}}{1} = log_2 M \quad \text{bits/photon} \tag{4.8}$$

Since we have 1 photon, the value of information bits per symbol is the same as that of information bits per photon.
The information content per time bin of the superblock:

$$\frac{log_2\mathrm{M}}{1} * \frac{1}{\mathrm{M}} = \frac{log_2\mathrm{M}}{\mathrm{M}} \quad \text{bits/time bin} \tag{4.9}$$

For 14 time bin in a superblock, we have

$$log_2 14 = 3.8 \quad \text{bits/symbol}$$

For each photon, it contains

$$\frac{log_2 14}{1} = 3.8 \quad \text{bits/photon}$$

For each time bin, it has

$$\frac{log_2 14}{14} = 0.27 \quad \text{bits/time bin}$$

Transmission using On-Off Keying (OOK)

The length of time bin in OOK is double its number of photons.

$$M = 2n \quad \text{time bins} \tag{4.10}$$

, where n is the number of photon in the protocol.
The length of a codeword corresponds 2 to the power of the time bin to organize the blocks in a superblock:

$$K = 2^{2n} = 2^M \tag{4.11}$$

, where K is the length of the codeword, n is the number of photons, and M is the length of the time bin in a superblock.
The information content per symbol of the superblock is:

$$log_2 K = M \tag{4.12}$$

The information content per photon of the superblock is:

$$\frac{log_2 K}{n} = \frac{M}{n} \quad \text{bits/photon} \tag{4.13}$$

The information content per time bin of the superblock is:

$$\frac{log_2 K}{n} * \frac{n}{M} = \frac{M}{M} = 1 \quad \text{bits/time bin} \tag{4.14}$$

In order to fairly compare OOK with BPPM, we choose 14-time bins, which correspond to 7 photons in BPPM.

$$M = 2 * 7 = 14 \quad \text{time bins}$$

OOK uses 7 photons on average in 14-time bins.

$$K = 2^{2*7} = 2^{14} = 16,384$$

For sending and receiving 7 photons in OOK protocol, we have 16,384 codewords for digital communication.
For 14 time bin in a superblock, we have

$$log_2 16,384 = 14 \quad \text{bits/symbol}$$

For each photon, it contains

$$\frac{14}{7} = 2 \quad \text{bits/photon}$$

For each time bin, it has

$$\frac{14}{14} = 1 \quad \text{bits/time bin}$$

Transmission using a general protocol

$$K = \binom{M}{n} = \frac{M!}{(M-n)!n!} \tag{4.15}$$

where K is the number of codewords, n is the number of photons, and M is the number of time bins in a superblock.
The information content per symbol of the superblock is:

$$log_2 K = log_2 \binom{M}{n} \quad \text{bits/symbol} \tag{4.16}$$

In order to fairly compare General with BPPM, we choose 4 photons and 14-time bins.
The Information content per photon of the superblock is:

$$\frac{log_2 \binom{M}{n}}{n} \quad \text{bits/photon} \tag{4.17}$$

The information content per time bin of the superblock is:

$$\frac{log_2 \binom{M}{n}}{n} * \frac{n}{M} = \frac{log_2 \binom{M}{n}}{M} \quad \text{bits/time bin} \tag{4.18}$$

$$K = \binom{14}{4} = \frac{14!}{(14-4)!4!} = 1,001$$

For sending and receiving 4 photons in the general protocol, we have 1,001 codewords for digital communication.

For 14 time bin in a superblock, we have

$$log_2 \binom{14}{4} = 9.97 \quad \text{bits/symbol}$$

Each photon contains

$$\frac{log_2 \binom{14}{4}}{4} = 2.49 \quad \text{bits/photon}$$

For each time bin, it has

$$\frac{log_2 \binom{14}{4}}{14} = 0.71 \quad \text{bits/time bin}$$

Research Methodology

Research Design-Comparative Metrics

On Information Bits (IB)

1. IB per Photon Vs. No. of Photons
2. IB per Photon Vs. No. of Time Bin
3. IB per Time Bin Vs. No. of Time Bin
4. IB per Time Bin Vs. No. of Photon

On Mutual Information (MI) for constant

1. Power (P)
2. Energy per IB (EpB)
3. Transmission Rate (R)

On MI

1. MI per Photon Vs. No. of Time Bin
2. MI per Time Bin Vs. No. of Time BIn

Description of the Simulation & Hypothesis

Python 3.8.5 | Visual Studio Code IDE 1.76.2

Simulation software:

NumPy 1.23.5 for numerical computation

Matplotlib 3.7.1 for visualization

Assumptions:

1. Idal source
2. Perfect detector
3. Lossless transmission medium
4. Single-mode propagation
5. Distance travel and error probability
6. Non-interacting photons
7. Low photon flux

Parameters:

1. Error probability
2. Number of photons
3. Number of time bin

Protocols in Effectiveness Comparison

1. BPPM
2. PPM
3. General Protocol
4. OOK

Results & Discussion

Transmission in Beyond Pulse Position Modulation (BPPM)

$[1,2,4,7] \to A, [1,2,7,4] \to B, [1,4,2,7] \to C,$

$[1,4,7,2] \to D, [1,7,2,4] \to E, [1,7,4,2] \to F$

$[2,1,4,7] \to G, [2,1,7,4] \to H, [2,4,1,7] \to I$

$[2,4,7,1] \to J, [2,7,1,4] \to K, [2,7,4,1] \to L$

$[4,1,2,7] \to M, [4,1,7,2] \to N, [4,2,1,7] \to O$

$[4,2,7,1] \to P, [4,7,1,2] \to Q, [4,7,2,1] \to R$

$[7,1,2,4] \to S, [7,1,4,2] \to T, [7,2,1,4] \to U$

$[7,2,4,1] \to V, [7,4,1,2] \to W, [7,4,2,1] \to X$

For 4 photons in a superblock, we have

$$log_2(4!) = 4.6 \quad \text{bits/symbol}$$

In each photon we have:

$$\frac{log_2(4!)}{4} = 1.15 \quad \text{bits/photon}$$

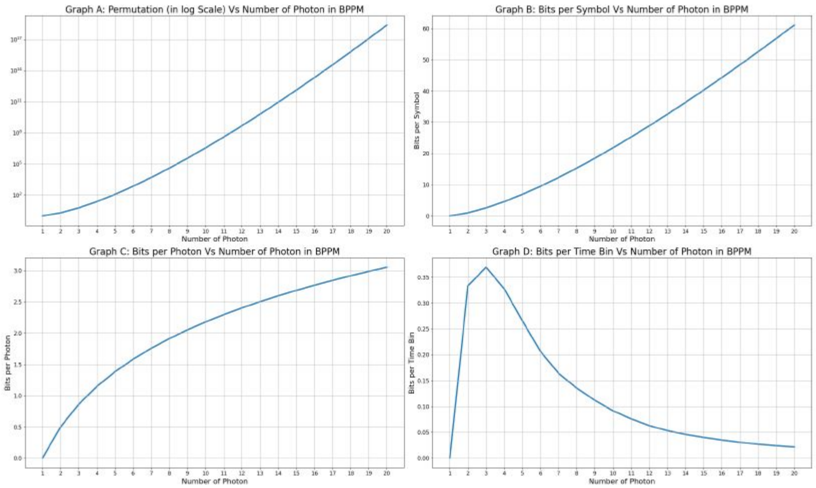
and for each time bin, it has

$$\frac{log_2(4!)}{4} \times \frac{4}{1+2+4+7} = 0.33 \quad \text{bits/time bin}$$

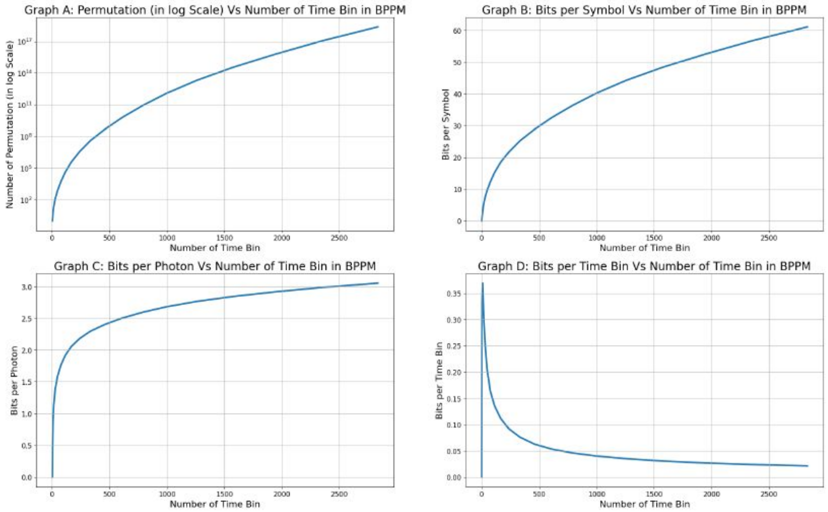
Number of Photon	Time Bin	Permutation	Bits/Symbol	Bits/Photon	Bits/Time Bin
1	1	1	0.000000	0.000000	0.000000
2	3	2	1.000000	0.500000	0.333333
3	7	6	2.584963	0.861654	0.369280
4	14	24	4.584963	1.146241	0.327497
5	26	120	6.906891	1.381378	0.265650
6	46	720	9.491853	1.581976	0.206345
7	75	5,400	12.299208	1.757030	0.163989
8	113	40,320	15.299208	1.912401	0.135391
9	165	362,880	18.469133	2.052126	0.111934
10	238	3,628,800	21.791061	2.179106	0.091559
11	332	3.991680e+07	25.250493	2.295499	0.076056
12	459	4.790016e+08	28.835455	2.402955	0.062822
13	610	6.227021e+09	32.535895	2.502761	0.053338
14	791	8.717829e+10	36.343250	2.595946	0.045946
15	1002	1.307674e+12	40.250140	2.683343	0.040170
16	1259	2.092279e+13	44.250140	2.765634	0.035147
17	1574	3.556874e+14	48.337603	2.843388	0.030710
18	1947	6.402374e+15	52.507528	2.917085	0.026968
19	2359	1.216451e+17	56.755456	2.987129	0.024059
20	2834	2.432902e+18	61.077384	3.053869	0.021552

Transmission in Beyond Pulse Position Modulation (BPPM)

4 Metrics vs Number of Photon



4 Metrics vs Number of Time Bin



Transmission in Pulse Position Modulation (PPM)

$$[10000000000000] \to A, [01000000000000] \to B$$

$$[00100000000000] \to C, [00010000000000] \to D$$

$$[00001000000000] \to E, [00000100000000] \to F$$

$$[00000010000000] \to G, [00000001000000] \to H$$

$$[00000000100000] \to I, [00000000010000] \to J$$

$$[00000000001000] \to K, [00000000000100] \to L$$

$$[00000000000010] \to M, [00000000000001] \to N$$

For 14 time bin in a superblock, we have

$$log_2 14 = 3.8 \quad \text{bits/symbol}$$

For each photon, it contains

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For each time bin, it has

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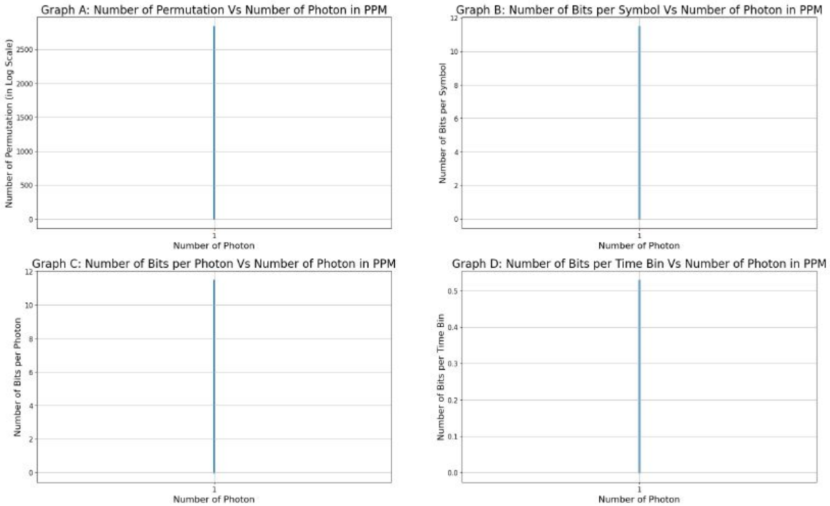
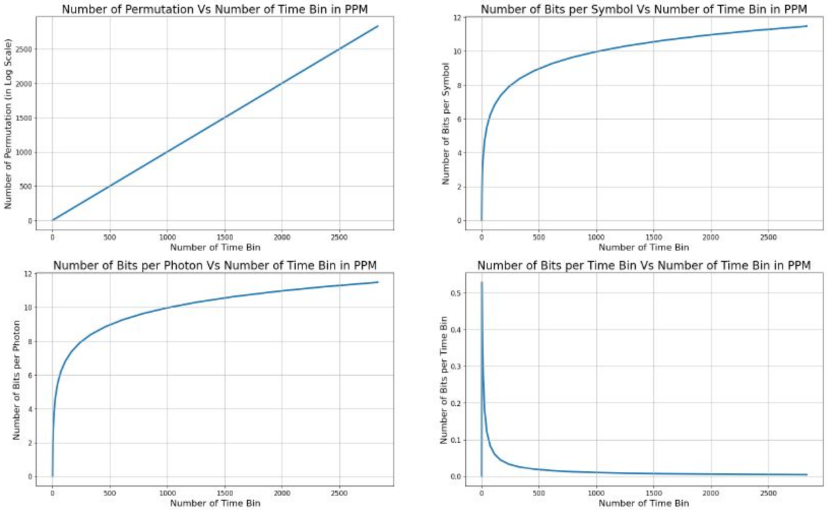
Number of Photon	Time Bin	Permutation	Bits/Symbol	Bits/Photon	Bits/Time Bin
1	1	1	0.000000	0.000000	0.000000
1	3	3	1.584963	1.584963	0.528321
1	7	7	2.807355	2.807355	0.401051
1	14	14	3.807355	3.807355	0.271954
1	26	26	4.700440	4.700440	0.180786
1	46	46	5.523562	5.523562	0.120077
1	75	75	6.228819	6.228819	0.083051
1	113	113	6.820179	6.820179	0.060356
1	165	165	7.366322	7.366322	0.044644
1	238	238	7.894818	7.894818	0.033172
1	332	332	8.375039	8.375039	0.025226
1	459	459	8.842350	8.842350	0.019264
1	610	610	9.252665	9.252665	0.015168
1	791	791	9.627534	9.627534	0.012171
1	1002	1002	9.968667	9.968667	0.009949
1	1259	1259	10.298063	10.298063	0.008180
1	1574	1574	10.620220	10.620220	0.006747
1	1947	1947	10.927037	10.927037	0.005612
1	2359	2359	11.203960	11.203960	0.004749
1	2834	2834	11.468624	11.468624	0.004047

Transmission in Pulse Position Modulation (PPM)

4 Metrics vs Number of Photon



4 Metrics vs Number of Time Bin



Transmission using On-Off Keying (OOK)

In order to fairly compare OOK with BPPM, we choose 14-time bins, which correspond to 7 photons in BPPM.

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OOK uses 7 photons on average in 14-time bins.

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For 14 time bin in a superblock, we have

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For each photon, it contains

$$\frac{14}{7} = 2 \quad \text{bits/photon}$$

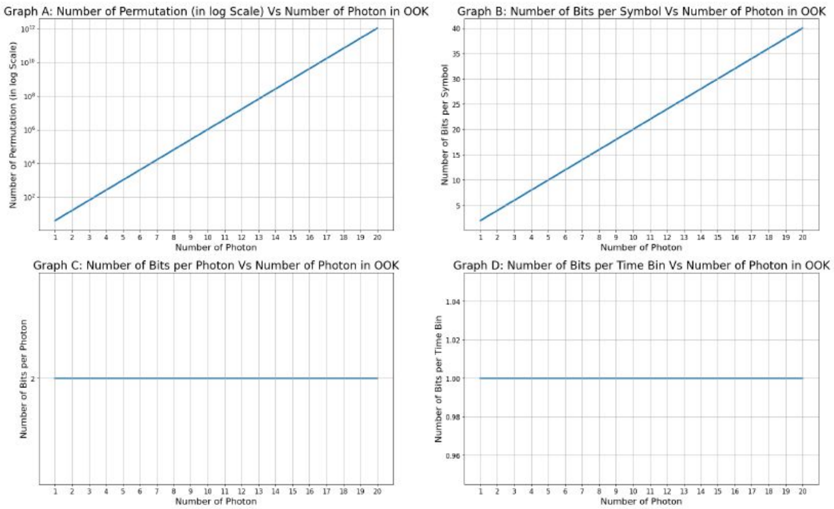
For each time bin, it has

$$\frac{14}{14} = 1 \quad \text{bits/time bin}$$

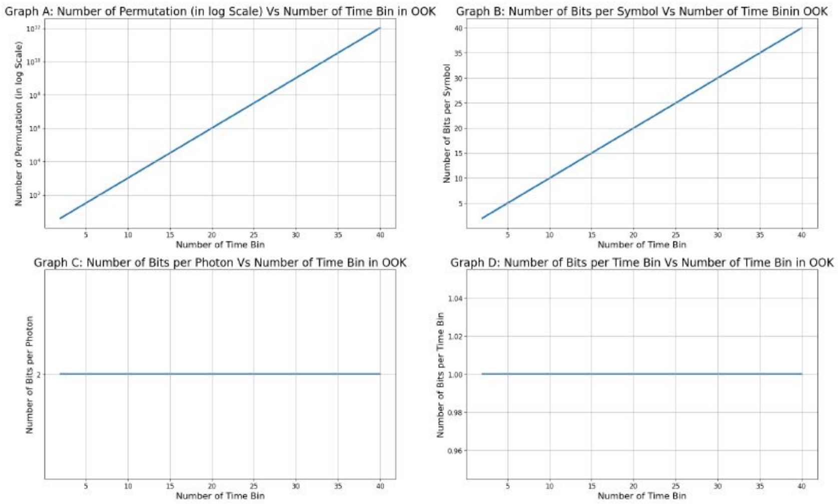
Number of Photon	Time Bin	Permutation	Bits/Symbol	Bits/Photon	Bits/Time Bin
1	2	4	2	2	1
2	4	16	4	2	1
3	6	64	6	2	1
4	8	256	8	2	1
5	10	1,024	10	2	1
6	12	4,096	12	2	1
7	14	16,384	14	2	1
8	16	65,536	16	2	1
9	18	262,144	18	2	1
10	20	1,048,576	20	2	1
11	22	4,194,304	22	2	1
12	24	1.677216e+07	24	2	1
13	26	6.7108864e+07	26	2	1
14	28	2.684355e+08	28	2	1
15	30	1.073742e+09	30	2	1
16	32	4.294967e+09	32	2	1
17	34	1.717987e+10	34	2	1
18	36	6.871948e+10	36	2	1
19	38	2.748779e+11	38	2	1
20	40	1.099512e+12	40	2	1

Transmission using On-Off Keying (OOK)

4 Metrics vs Number of Photon



4 Metrics vs Number of Time Bin



Transmission using a general protocol

$$K = \binom{14}{4} = \frac{14!}{(14-4)!4!} = 1,001$$

For sending and receiving 4 photons in the general protocol, we have 1,001 cod words for digital communication.

For 14 time bin in a superblock, we have

$$log_2\binom{14}{4} = 9.97 \quad \text{bits/symbol}$$

Each photon contains

$$\frac{log_2\binom{14}{4}}{4} = 2.49 \quad \text{bits/photon}$$

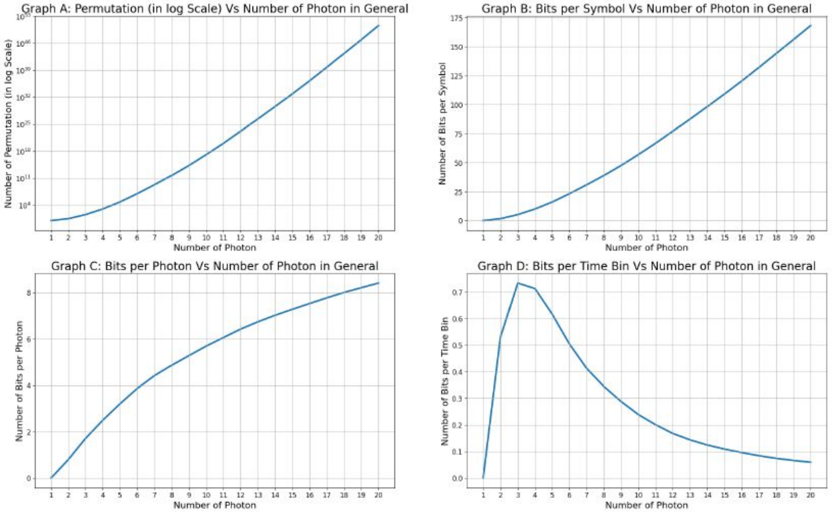
For each time bin, it has

$$\frac{log_2\binom{14}{4}}{14} = 0.71 \quad \text{bits/time bin}$$

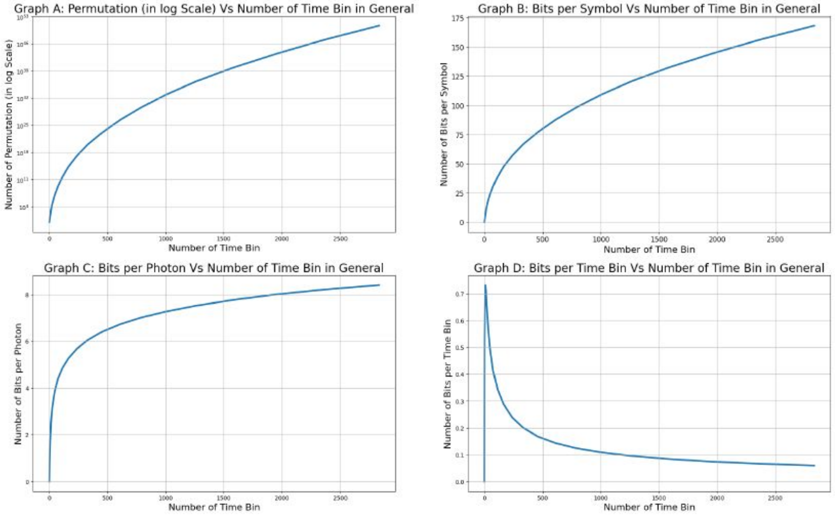
Number of Photon	Time Bin	Permutation	Bits/Symbol	Bits/Photon	Bits/Time Bin
1	1	1	0	0	0
2	3	3	1.584963	0.792481	0.528321
3	7	35	5.129283	1.709761	0.732755
4	14	1,001	9.967226	2.491807	0.711945
5	26	65,780	16.005361	3.201072	0.615591
6	46	9,366,819	23.159128	3.859855	0.503459
7	75	1.984830e+09	30.886368	4.412338	0.411818
8	113	5.117190e+11	38.896561	4.862070	0.344217
9	165	2.000631e+14	47.507449	5.278605	0.287924
10	238	1.326714e+17	56.880635	5.688063	0.238994
11	332	1.144580e+20	66.633380	6.057580	0.200703
12	459	1.579209e+23	77.063548	6.421962	0.167894
13	610	2.286081e+26	87.563007	6.735616	0.143546
14	791	3.835418e+29	98.275298	7.019664	0.124242
15	1,002	7.092269e+32	109.127946	7.275196	0.108910
16	1,259	1.730720e+36	120.380784	7.523799	0.095616
17	1,574	5.759108e+39	132.081041	7.769473	0.083914
18	1,947	2.333541e+43	144.065429	8.003635	0.073994
19	2,359	9.229376e+46	156.014926	8.211312	0.066136
20	2,834	4.292288e+50	168.198152	8.409908	0.059350

Transmission using a general protocol

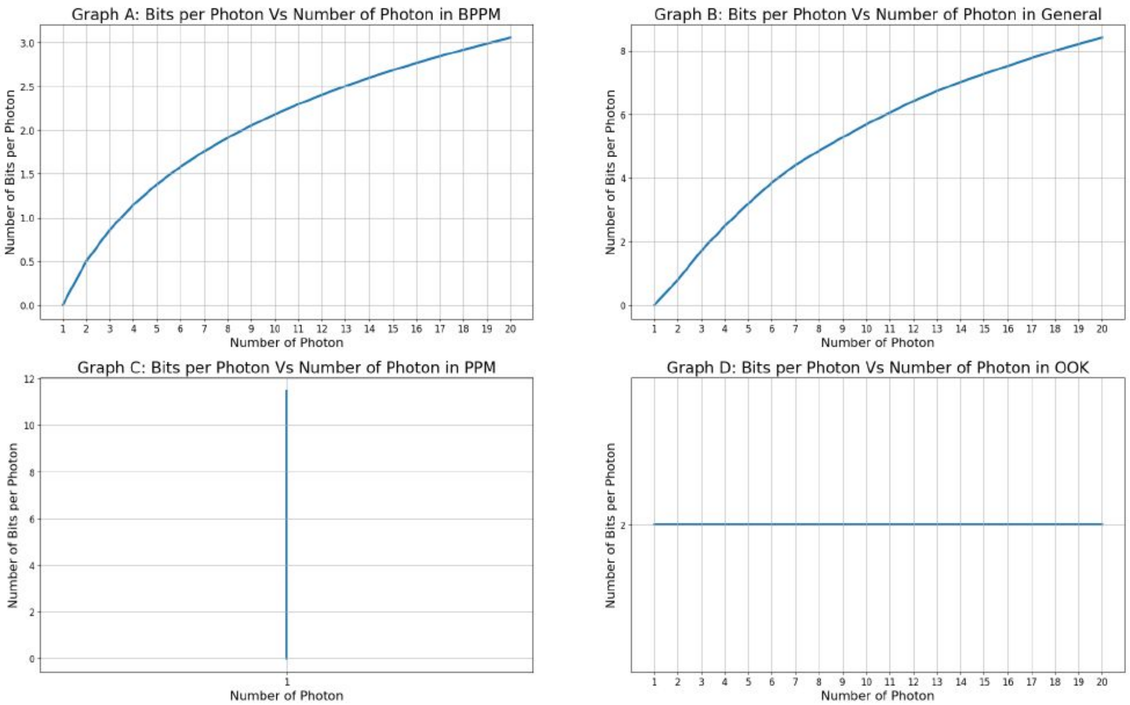
4 Metrics vs Number of Photon



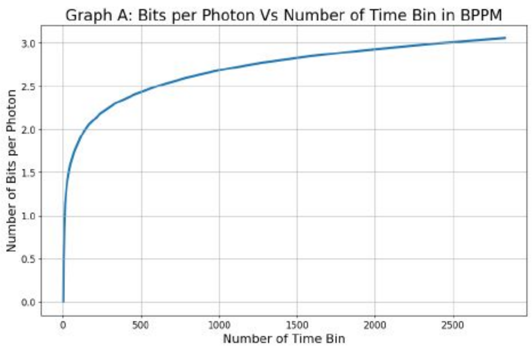
4 Metrics vs Number of Time Bin

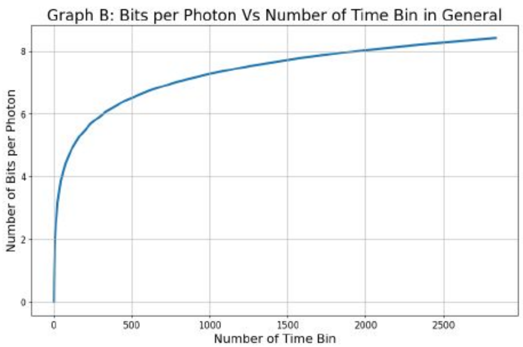


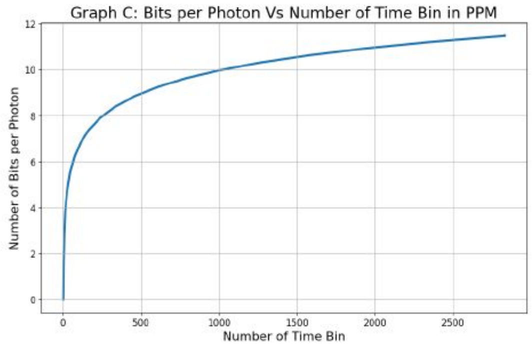
Information Bits per Photon Versus Number of Photon

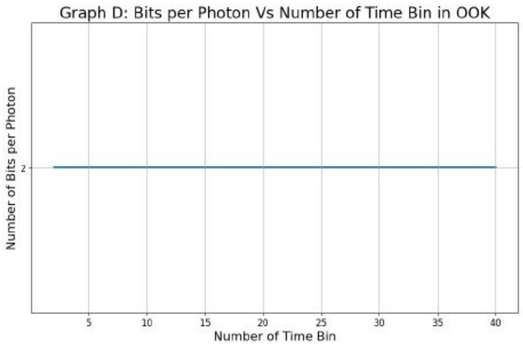


Information Bits per Photon Versus Number of Time Bin

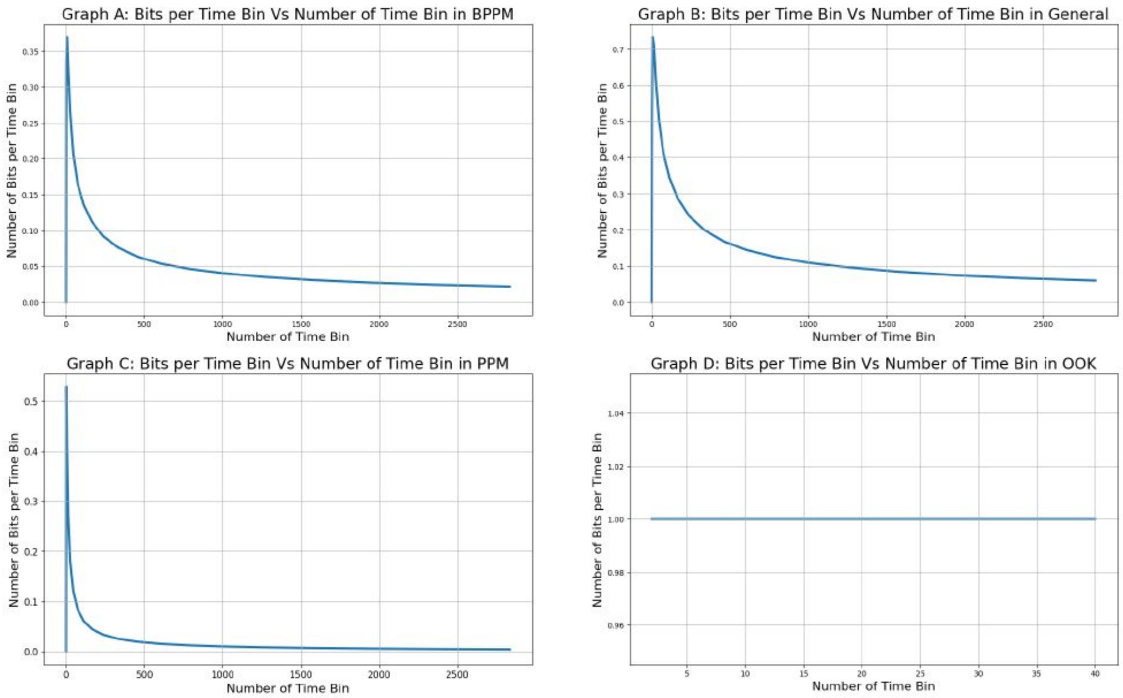




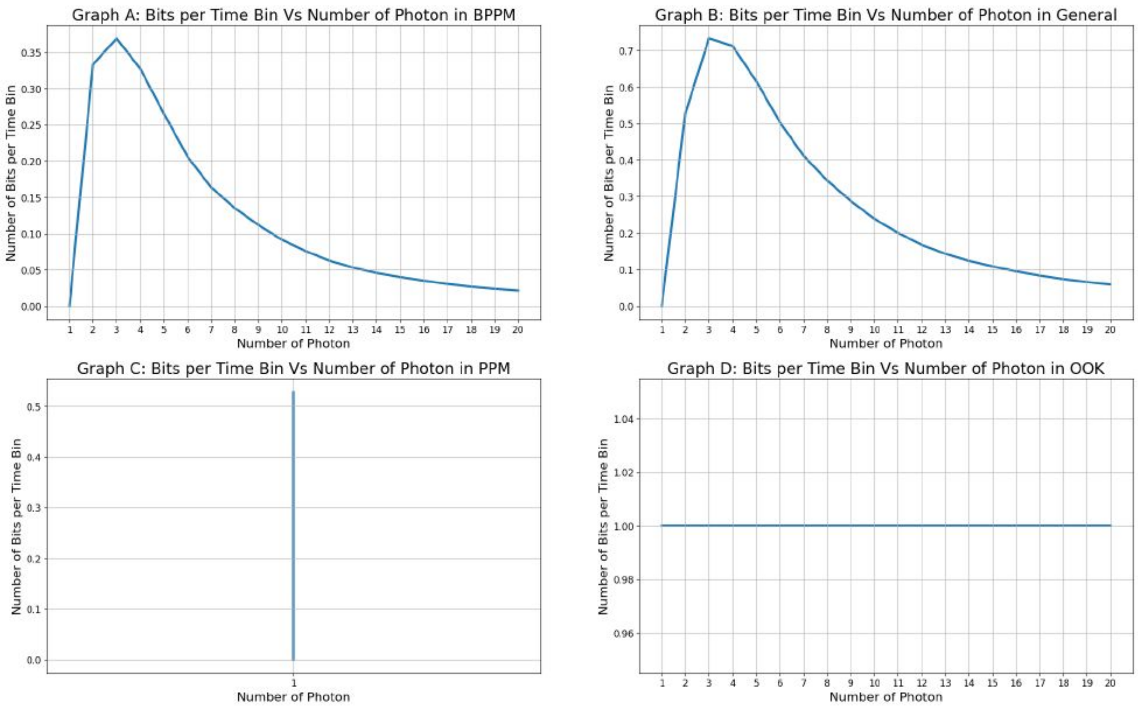




Information Bits per Time Bin Versus Number of Time Bin



Information Bits per Time Bin Versus Number of Photons



Mutual Information for Constant Power

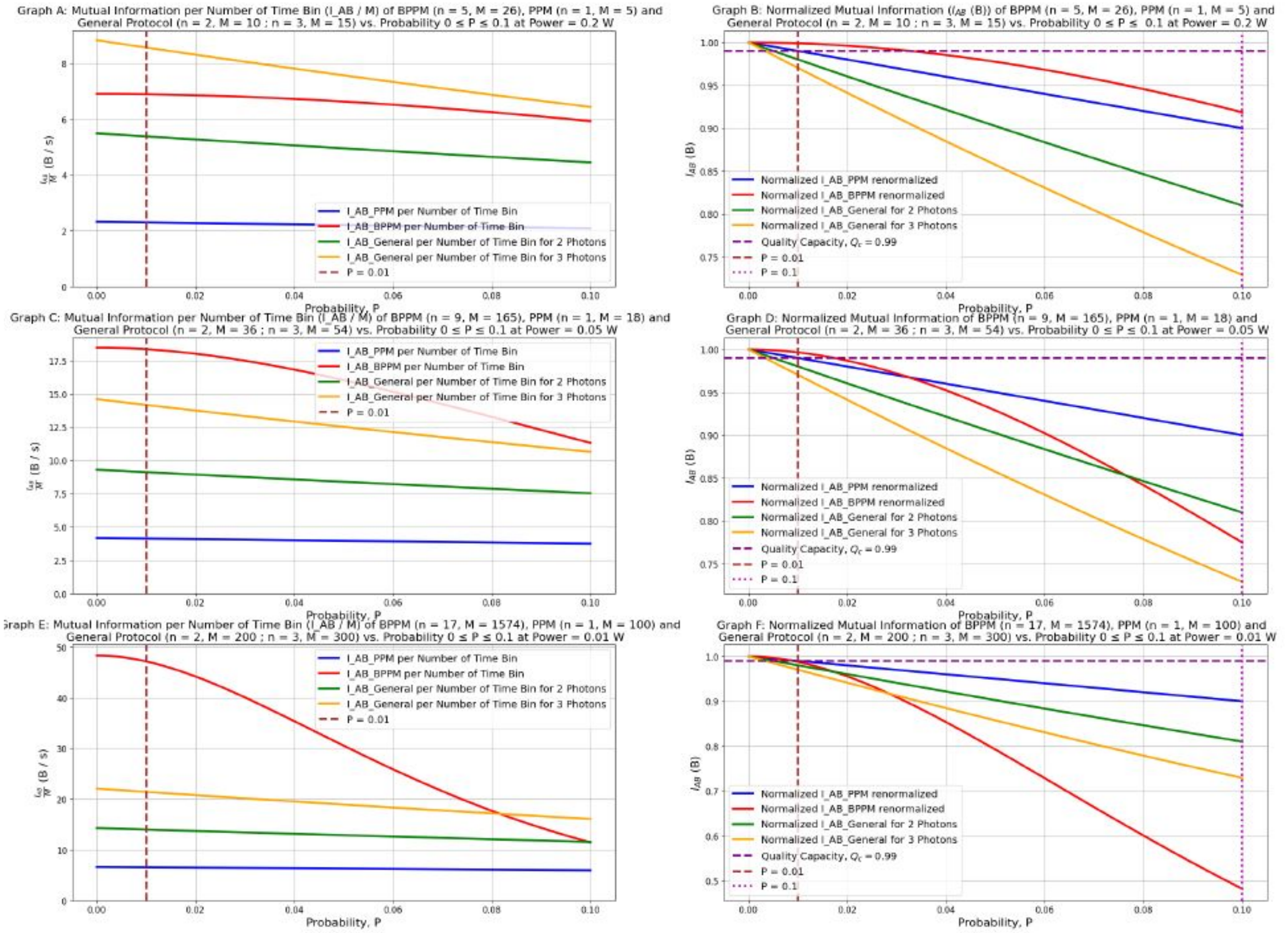
$$P = \frac{E}{T} = \frac{n}{M} \tag{4.19}$$

Photon Number_BPPM	Time Bin_BPPM	Power Ratio of BPPM	Time Bin_PPM	Power Ratio_PPM	Time Bin_General_2	Power Ratio_General_2	Time Bin_General_3	Power Ratio_General_3
1	1	1	1	1	2	1	3	1
2	3	0.666666667	2	0.5	4	0.5	6	0.5
3	7	0.428571429	3	0.333333333	6	0.333333333	9	0.333333333
4	14	0.285714286	4	0.25	8	0.25	12	0.25
5	26	0.192307692	5	0.2	10	0.2	15	0.2
6	46	0.130434783	6	0.166666667	12	0.166666667	18	0.166666667
7	75	0.093333333	7	0.142857143	14	0.142857143	21	0.142857143
8	113	0.07079646	8	0.125	16	0.125	24	0.125
9	165	0.054545455	9	0.111111111	18	0.111111111	27	0.111111111
10	238	0.042016807	10	0.1	20	0.1	30	0.1
11	332	0.03313253	11	0.090909091	22	0.090909091	33	0.090909091
12	459	0.026143791	12	0.083333333	24	0.083333333	36	0.083333333
13	610	0.021311475	13	0.076923077	26	0.076923077	39	0.076923077
14	791	0.017699115	14	0.071428571	28	0.071428571	42	0.071428571
15	1002	0.01497006	15	0.066666667	30	0.066666667	45	0.066666667
16	1259	0.012708499	16	0.0625	32	0.0625	48	0.0625
17	1574	0.010800508	17	0.058823529	34	0.058823529	51	0.058823529
18	1947	0.009244992	18	0.055555556	36	0.055555556	54	0.055555556
19	2359	0.00805426	19	0.052631579	38	0.052631579	57	0.052631579
20	2834	0.007057163	20	0.05	40	0.05	60	0.05
21	3364	0.006242568	21	0.047619048	42	0.047619048	63	0.047619048
22	3909	0.005628038	22	0.045454545	44	0.045454545	66	0.045454545
23	4516	0.005093003	23	0.043478261	46	0.043478261	69	0.043478261

Photon Number_BPPM	Time Bin_BPPM	Power Ratio of BPPM	Time Bin_PPM	Power Ratio_PPM	Time Bin_General_2	Power Ratio_General_2	Time Bin_General_3	Power Ratio_General_3
73	259324	0.000281501	73	0.01369863	146	0.01369863	219	0.01369863
74	272191	0.000271868	74	0.013513514	148	0.013513514	222	0.013513514
75	285617	0.000262589	75	0.013333333	150	0.013333333	225	0.013333333
76	299540	0.000253722	76	0.013157895	152	0.013157895	228	0.013157895
77	314075	0.000245164	77	0.012987013	154	0.012987013	231	0.012987013
78	328986	0.000237092	78	0.012820513	156	0.012820513	234	0.012820513
79	344455	0.000229348	79	0.012658228	158	0.012658228	237	0.012658228
80	360359	0.000222001	80	0.0125	160	0.0125	240	0.0125
81	376495	0.000215142	81	0.012345679	162	0.012345679	243	0.012345679
82	393395	0.000208442	82	0.012195122	164	0.012195122	246	0.012195122
83	410436	0.000202224	83	0.012048193	166	0.012048193	249	0.012048193
84	428258	0.000196143	84	0.011904762	168	0.011904762	252	0.011904762
85	447679	0.000189868	85	0.011764706	170	0.011764706	255	0.011764706
86	467612	0.000183913	86	0.011627907	172	0.011627907	258	0.011627907
87	487900	0.000178315	87	0.011494253	174	0.011494253	261	0.011494253
88	508896	0.000172923	88	0.011363636	176	0.011363636	264	0.011363636
89	530387	0.000167802	89	0.011235955	178	0.011235955	267	0.011235955
90	552452	0.00016291	90	0.011111111	180	0.011111111	270	0.011111111
91	575064	0.000158243	91	0.010989011	182	0.010989011	273	0.010989011
92	597723	0.000153917	92	0.010869565	184	0.010869565	276	0.010869565
93	621447	0.000149651	93	0.010752688	186	0.010752688	279	0.010752688
94	645846	0.000145546	94	0.010638298	188	0.010638298	282	0.010638298
95	670815	0.000141619	95	0.010526316	190	0.010526316	285	0.010526316
96	696175	0.000137896	96	0.010416667	192	0.010416667	288	0.010416667
97	722246	0.000134303	97	0.010309278	194	0.010309278	291	0.010309278
98	748926	0.000130854	98	0.010204082	196	0.010204082	294	0.010204082
99	776527	0.000127491	99	0.01010101	198	0.01010101	297	0.01010101
100	804545	0.000124294	100	0.01	200	0.01	300	0.01

Protocol	P = 0.2 W	P = 0.05 W	P = 0.01 W
BPPM	n = 5, M = 26	n = 9, M = 165	n = 17, M = 1,574
PPM for 1 photon	M = 5	M = 18	M = 100
General for 2 photons	M = 10	M = 36	M = 200
General for 3 photons	M = 15	M = 54	M = 300

MI for Constant Power



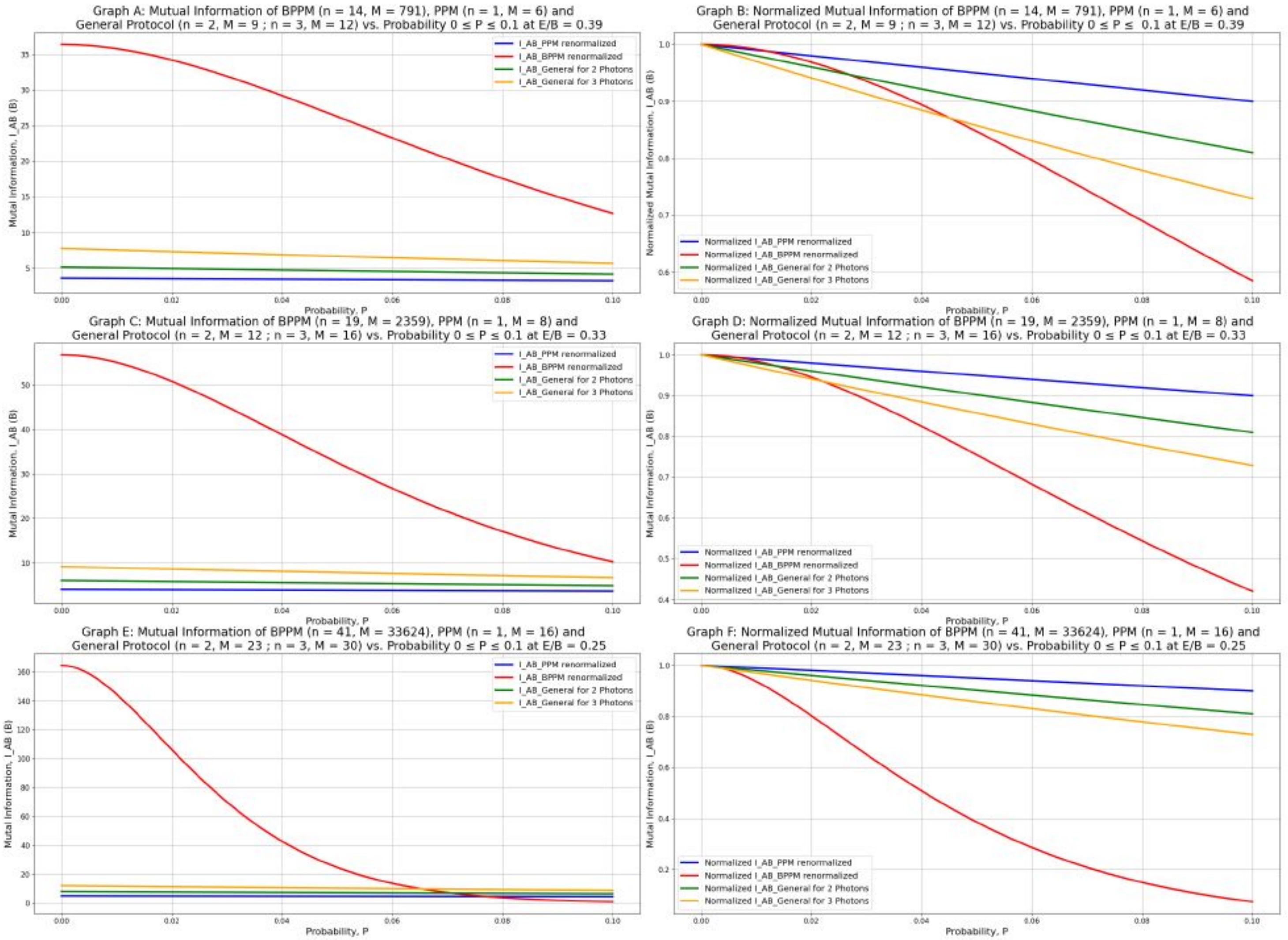
Mutual Information for Constant Energy per Information Bit

$$\mathbf{EpB} = \frac{E}{B} = \frac{n}{log_2 K}, \tag{4.20}$$

Time Bin_PPM and G	Energy per Information_G for n = 2	Energy per Information_G for n = 3	Energy per Information_PPM	Energy per Information_BPPM	Photon Number_BPPM	Time Bin_BPPM
3	1.261859507	inf	0.630929754	1.160558422	3	7
4	0.773705614	1.5	0.5	0.872417168	4	14
5	0.602059991	0.903089987	0.430676558	0.723914753	5	26
6	0.51191605	0.694134639	0.386852807	0.632121035	6	46
7	0.455340497	0.584877066	0.356207187	0.569142337	7	75
8	0.416029195	0.516586301	0.333333333	0.522902884	8	113
9	0.386852807	0.469313365	0.315464877	0.487299539	9	165
10	0.364175801	0.434348852	0.301029996	0.458903766	10	238
11	0.345939381	0.407258862	0.289064826	0.435635063	11	332
12	0.330885108	0.385536733	0.278942946	0.416154345	12	459
13	0.318197616	0.367652856	0.270238154	0.399558703	13	610
14	0.307323773	0.35261782	0.262649535	0.385215963	14	791
15	0.297874124	0.339761518	0.255958025	0.372669507	15	1002
16	0.289565901	0.328612882	0.25	0.361580773	16	1259
17	0.282188428	0.318830413	0.244650542	0.351693068	17	1574
18	0.275581248	0.310160057	0.239812467	0.342807985	18	1947
19	0.269619812	0.302408598	0.235408913	0.334769578	19	2359
20	0.264205832	0.295426266	0.231378213	0.327453449	20	2834
21	0.25926061	0.289095031	0.227670249	0.320759062	21	3364
22	0.254720316	0.283320507	0.224243824	0.314604215	22	3909
23	0.250532583	0.278026238	0.221064729	0.308920987	23	4516
24	0.246654001	0.273149573	0.218104292	0.303652724	24	5232
25	0.243048252	0.268638638	0.215338279	0.298751766	25	6029
26	0.239684693	0.264450083	0.212746054	0.294177703	26	6890

Time Bin_PPM and G	Energy per Information_G for n = 2	Energy per Information_G for n = 3	Energy per Information_PPM	Energy per Information_BPPM	Photon Number_BPPM	Time Bin_BPPM
22	0.254720316	0.283320507	0.224243824	0.314604215	22	3909
23	0.250532583	0.278026238	0.221064729	0.308920987	23	4516
24	0.246654001	0.273149573	0.218104292	0.303652724	24	5232
25	0.243048252	0.268638638	0.215338279	0.298751766	25	6029
26	0.239684693	0.264450083	0.212746054	0.294177703	26	6890
27	0.236537268	0.260547372	0.210309918	0.289896026	27	7854
28	0.233583669	0.256899471	0.208014598	0.285877073	28	8913
29	0.23080467	0.253479836	0.205846832	0.282095194	29	10073
30	0.228183605	0.250265615	0.203795047	0.278528083	30	11379
31	0.225705946	0.247237015	0.201849087	0.27515624	31	12764
32	0.223358963	0.2443768	0.2	0.27196254	32	14198
33	0.221131452	0.241669882	0.198239863	0.268931871	33	15753
34	0.219013505	0.239102992	0.196561632	0.266050846	34	17474
35	0.216996327	0.236664405	0.194959022	0.263307557	35	19307
36	0.215072079	0.234343723	0.193426404	0.260691372	36	21240
37	0.213233748	0.232131685	0.19195872	0.258192767	37	23297
38	0.21147504	0.230020013	0.190551412	0.255803182	38	25557
39	0.209790289	0.228001285	0.18920036	0.2535149	39	28053
40	0.208174377	0.22606882	0.187901825	0.251320943	40	30751
41	0.206622668	0.22421659	0.186652411	0.249214985	41	33624
42	0.205130952	0.222439141	0.185449023	0.247191276	42	36684
43	0.203695398	0.220731521	0.184288833	0.245244574	43	39880
44	0.202312505	0.219089225	0.183169251	0.243370093	44	43211
45	0.200979075	0.217508145	0.1820879	0.241563452	45	46839

Protocol	E/B = 0.39	E/B = 0.33	E/B = 0.25
BPPM	n = 14, M = 791	n = 19, M = 2359	n = 41, M = 33624
PPM for 1 photon	M = 6	M = 8	M = 16
General for 2 photons	M = 9	M = 12	M = 23
General for 3 photons	M = 12	M = 16	M = 30

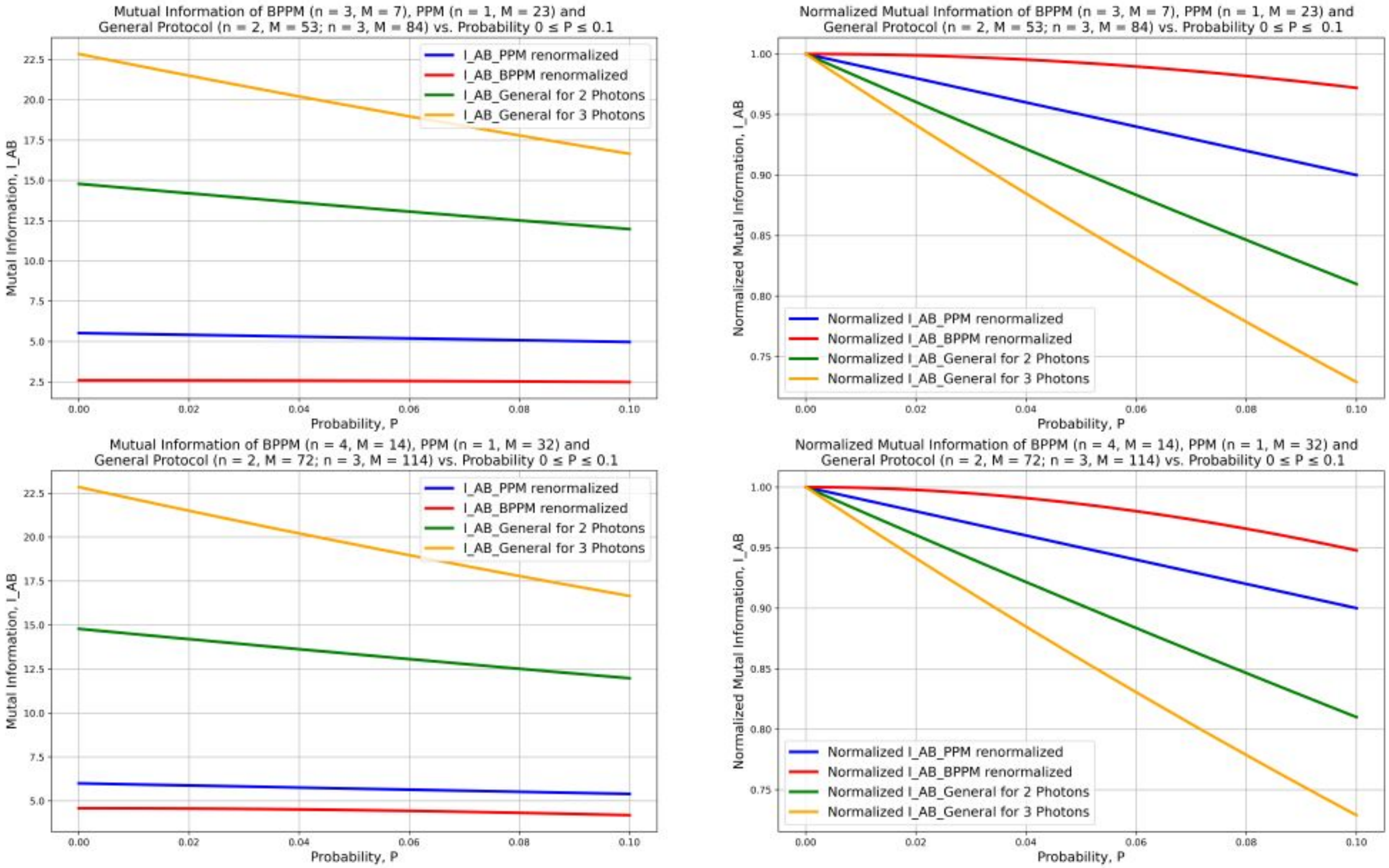

Graph A: Mutual Information of BPPM (n = 14, M = 791), PPM (n = 1, M = 6) and General Protocol (n = 2, M = 9 ; n = 3, M = 12) vs. Probability 0 ≤ P ≤ 0.1 at E/B = 0.39
I_AB_PPM renormalized
I_AB_BPPM renormalized
I_AB_General for 2 Photons
I_AB_General for 3 Photons
Mutal Information, I_AB (B)
Probability, P
Graph B: Normalized Mutual Information of BPPM (n = 14, M = 791), PPM (n = 1, M = 6) and General Protocol (n = 2, M = 9 ; n = 3, M = 12) vs. Probability 0 ≤ P ≤ 0.1 at E/B = 0.39
Normalized I_AB_PPM renormalized
Normalized I_AB_BPPM renormalized
Normalized I_AB_General for 2 Photons
Normalized I_AB_General for 3 Photons
Normalized Mutal Information, I_AB (B)
Probability, P
Graph C: Mutual Information of BPPM (n = 19, M = 2359), PPM (n = 1, M = 8) and General Protocol (n = 2, M = 12 ; n = 3, M = 16) vs. Probability 0 ≤ P ≤ 0.1 at E/B = 0.33
I_AB_PPM renormalized
I_AB_BPPM renormalized
I_AB_General for 2 Photons
I_AB_General for 3 Photons
Mutal Information, I_AB (B)
Probability, P
Graph D: Normalized Mutual Information of BPPM (n = 19, M = 2359), PPM (n = 1, M = 8) and General Protocol (n = 2, M = 12 ; n = 3, M = 16) vs. Probability 0 ≤ P ≤ 0.1 at E/B = 0.33
Normalized I_AB_PPM renormalized
Normalized I_AB_BPPM renormalized
Normalized I_AB_General for 2 Photons
Normalized I_AB_General for 3 Photons
Mutal Information, I_AB (B)
Probability, P
Graph E: Mutual Information of BPPM (n = 41, M = 33624), PPM (n = 1, M = 16) and General Protocol (n = 2, M = 23 ; n = 3, M = 30) vs. Probability 0 ≤ P ≤ 0.1 at E/B = 0.25
I_AB_PPM renormalized
I_AB_BPPM renormalized
I_AB_General for 2 Photons
I_AB_General for 3 Photons
Mutal Information, I_AB (B)
Probability, P
Graph F: Normalized Mutual Information of BPPM (n = 41, M = 33624), PPM (n = 1, M = 16) and General Protocol (n = 2, M = 23 ; n = 3, M = 30) vs. Probability 0 ≤ P ≤ 0.1 at E/B = 0.25
Normalized I_AB_PPM renormalized
Normalized I_AB_BPPM renormalized
Normalized I_AB_General for 2 Photons
Normalized I_AB_General for 3 Photons
Mutal Information, I_AB (B)
Probability, P

Mutual Information for Constant Transmission Rate

$$R = \frac{B}{T} = \frac{B}{M} = \frac{log_2 K}{M} \tag{4.24}$$

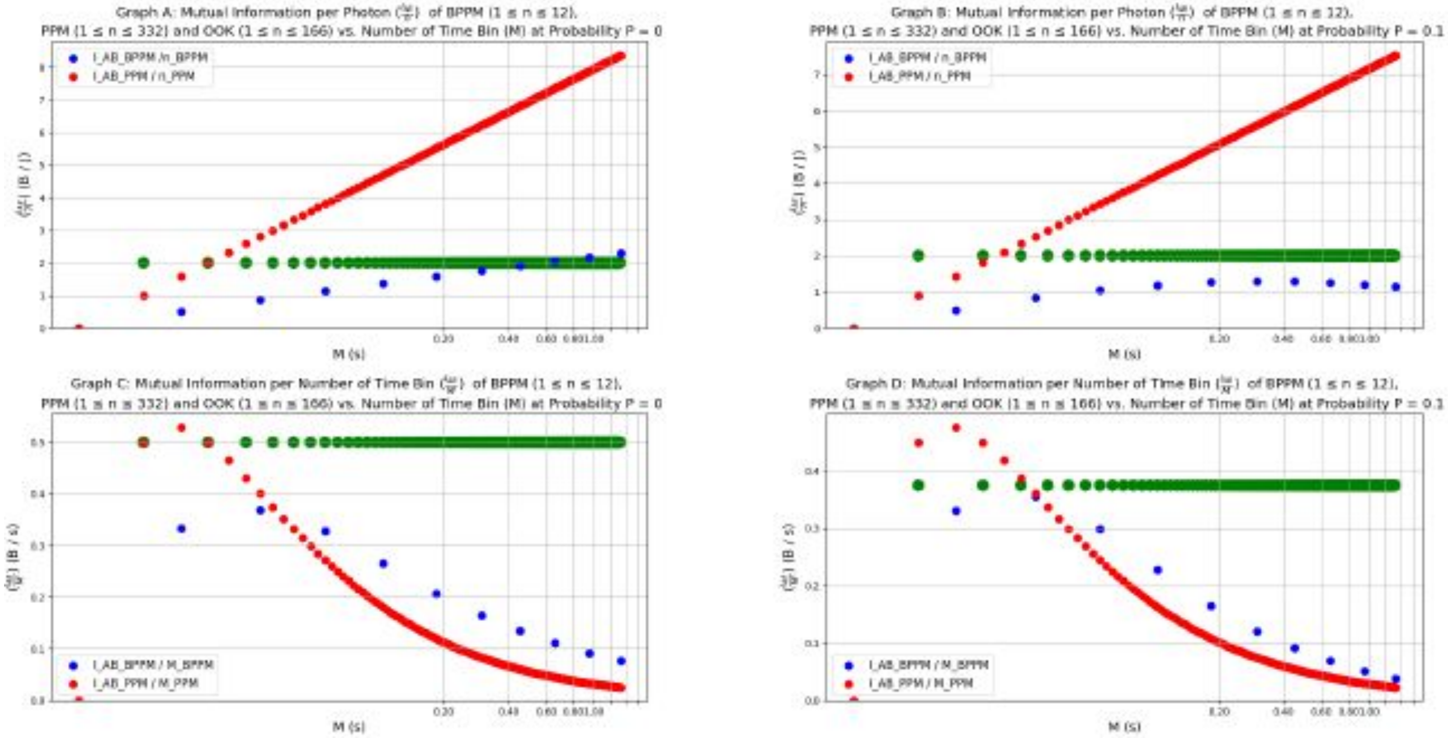
Photon Number_BPPM	Time Bin_BPPM	Transmission Rate_BPPM	Time Bin_PPM, General Protocol	Transmission Rate_PPM	Transmission Rate_G for n =2	Transmission Rate_G for n =3
3	7	0.195734765	3	0.528320834	0.528320834	0
4	14	0.156922138	4	0.5	0.646240625	0.5
5	26	0.107232168	5	0.464385619	0.664385619	0.664385619
6	46	0.070580212	6	0.430827083	0.651148433	0.720321349
7	75	0.048273247	7	0.401050703	0.627473918	0.732754717
8	113	0.034826417	8	0.375	0.600919365	0.725919365
9	165	0.025497238	9	0.352213889	0.574436111	0.710257491
10	238	0.018679263	10	0.332192809	0.54918531	0.69068906
11	332	0.014030842	11	0.314493784	0.525578156	0.669665656
12	459	0.010565952	12	0.298746875	0.50369951	0.648446643
13	610	0.008236001	13	0.284649209	0.483492478	0.627682411
14	791	0.006553243	14	0.271953923	0.464842474	0.607699617
15	1002	0.005320281	15	0.260459373	0.447616368	0.588648182
16	1259	0.00434282	16	0.25	0.431680662	0.570580189
17	1574	0.003554685	17	0.240438991	0.416909579	0.553493584
18	1947	0.002935004	18	0.2316625	0.403188213	0.537356963
19	2359	0.002469982	19	0.223575132	0.39041329	0.522123834
20	2834	0.002093354	20	0.216096405	0.37849278	0.507740905
21	3364	0.001793328	21	0.209157973	0.367345025	0.494152882
22	3909	0.001567619	22	0.202701437	0.356897684	0.481305211
23	4516	0.001376938	23	0.196676607	0.347086677	0.469145587

Protocol	R = 0.196	R = 0.0348
BPPM	n = 3, M = 7	n = 8, M = 113
PPM	n = 1, M = 23	n = 1, M = 224
General for 2 photons	n = 2, M = 53	n = 2, M = 483
General for 3 photons	n = 3, M = 84	n = 3, M = 747

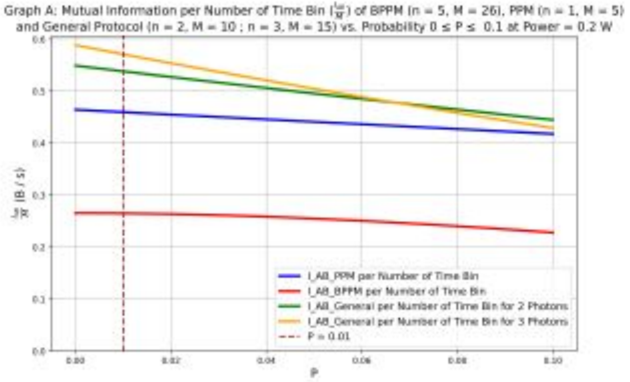

Mutual Information of BPPM (n = 3, M = 7), PPM (n = 1, M = 23) and General Protocol (n = 2, M = 53; n = 3, M = 84) vs. Probability 0 ≤ P ≤ 0.1
I_AB_PPM renormalized
I_AB_BPPM renormalized
I_AB_General for 2 Photons
I_AB_General for 3 Photons
Mutal Information, I_AB
Probability, P
Normalized Mutual Information of BPPM (n = 3, M = 7), PPM (n = 1, M = 23) and General Protocol (n = 2, M = 53; n = 3, M = 84) vs. Probability 0 ≤ P ≤ 0.1
Normalized I_AB_PPM renormalized
Normalized I_AB_BPPM renormalized
Normalized I_AB_General for 2 Photons
Normalized I_AB_General for 3 Photons
Normalized Mutal Information, I_AB
Probability, P
Mutual Information of BPPM (n = 4, M = 14), PPM (n = 1, M = 32) and General Protocol (n = 2, M = 72; n = 3, M = 114) vs. Probability 0 ≤ P ≤ 0.1
I_AB_PPM renormalized
I_AB_BPPM renormalized
I_AB_General for 2 Photons
I_AB_General for 3 Photons
Mutal Information, I_AB
Probability, P
Normalized Mutual Information of BPPM (n = 4, M = 14), PPM (n = 1, M = 32) and General Protocol (n = 2, M = 72; n = 3, M = 114) vs. Probability 0 ≤ P ≤ 0.1
Normalized I_AB_PPM renormalized
Normalized I_AB_BPPM renormalized
Normalized I_AB_General for 2 Photons
Normalized I_AB_General for 3 Photons
Normalized Mutal Information, I_AB
Probability, P

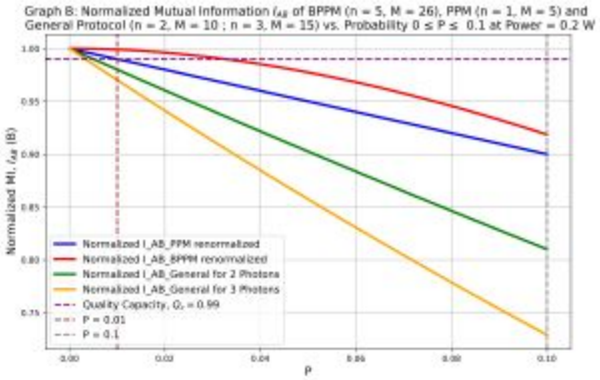
Mutual Information per Photon

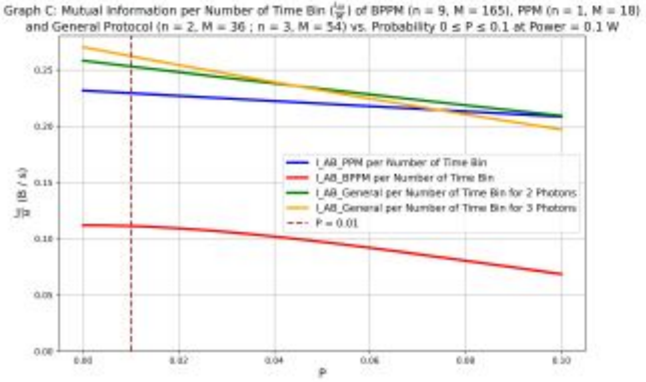
Protocol	M = 332
BPPM	$1 \leq n \leq 12$, $1 \leq M \leq 332$
PPM for 1 photon	$1 \leq M \leq 332$,
OOK	$1 \leq n \leq 166$, $1 \leq M \leq 332$

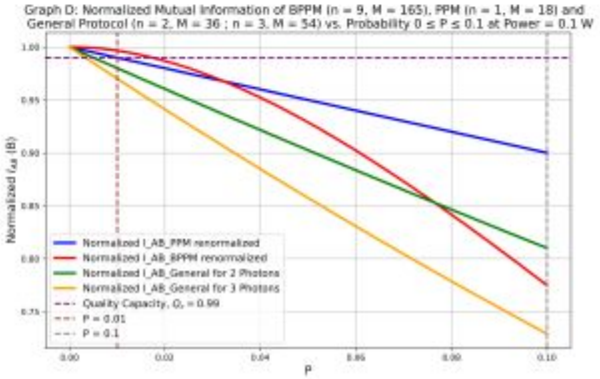


Mutual Information per Time Bin









Conclusion and Future Work

Conclusion

- Investigate information capacity in BPPM, PPM, OOK, General Protocol
- Metrics on IB, MI
- BPPM exhibits error-correcting capability
- Contain highest information quality $Q_C = 0.99$ at $P \leq 0.01$
- Slower-transmission for higher error probability

Future Work

- Develop error-correction technique: Reed-Solomon Code for BPPM

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Thank you for your participation!

Q&A