REFERENCES

[1] W. B. Davenport, "An experimental study of speech-wave probability distributions," *J. Acoust. Soc. Amer.*, vol. 24, pp. 390–399, July 1952.
[2] M. D. Paez and T. H. Glisson, "Minimum mean square-error quantization in speech," *IEEE Trans. Commun.*, vol. COM-20, pp. 225–230, Apr. 1972.
[3] L. Rabiner and R. Schafer, *Digital Processing of Speech Signals*. Englewood Cliffs, NJ: Prentice-Hall, 1978.
[4] H. K. Dunn and S. D. White, "Statistical measurements on conversational speech," *J. Acoust. Soc. Amer.*, vol. 11, pp. 278–288, Jan. 1940.
[5] P. Noll, "A comparative study of various schemes for speech encoding," *Bell Syst. Tech. J.*, vol. 54, pp. 1597–1614, Nov. 1975.
[6] P. P. Prezas and J. L. LoCicero, "Variable rate encoding using adaptive delta modulation," in *Proc. 17th Annu. Allerton Conf. Commun., Cont., Comp.*, Oct. 1979, pp. 294–302.
[7] J. L. LoCicero and P. P. Prezas, "Doubly adaptive delta modulation for bandwidth compression of speech signals," in *Proc. IEEE Nat. Telecommun. Conf.*, Dec. 1980, pp. 36.5.1–36.5.5.
[8] A. Papoulis, *Probability, Random Variables, and Stochastic Processing*. New York: McGraw-Hill, 1965.
[9] H. O. Lancaster, "The structure of bivariate distributions," *Ann. Math. Stat.*, pp. 719–736, 1958.
[10] J. F. Barrett and D. G. Lampard, "An expansion of some second-order probability distribution and its application to noise problems," *IRE Trans. Inform. Theory*, vol. IT-1, pp. 10–15, 1955.
[11] P. Noll, "Non-adaptive and adaptive DPCM of speech signals," *Polytech. Tijdschr. Ed. Elektrotech/Elektron* (The Netherlands), no. 19, 1972.

Bandwidth Limitations on Noiseless Optical Channel Capacity

STANLEY A. BUTMAN, JOSEPH KATZ, AND JAMES R. LESH

Abstract—**Even in noiseless optical channels one must take into account the fact that the time resolution available is finite. An optimization scheme under the constraint of a given information rate (in nats/second) and minimum time-slot resolution is presented. It is shown that system efficiencies in excess of tens of nats/photon will be extremely difficult to achieve due to fundamental time resolution limitations.**

I. INTRODUCTION

It has been known for some time that the channel capacity of an optical communications link utilizing direct photon detection can be substantially larger than the heterodyne detection quantum limit [1] and that in the limit of a noiseless[1] channel the capacity per received photon can even be infinite [2]–[5]. To achieve such capacity/energy ratios one can utilize pulse position modulation and either allow the bandwidth to exponentially approach infinity or the throughput to approach zero [6]. In most communications systems the real constraint is to deliver a certain throughput capacity measured in nats/second (or proportionately in bits/second) and, for that throughput, to make the most efficient use of the available signal power. Furthermore, it is recognized that one really cannot allow the bandwidth to grow without bound.

The purpose of this correspondence is to show that due to physical limitations on the minimum time resolution one can achieve, and for any given throughput capacity in nats/second, there exists an optimum PPM word size which maximizes the "power efficiency capacity" expressed in nats/photon. Such an optimization will allow the system designer to minimize the amount of power needed to achieve the desired throughput without violating his bandwidth constraint.

Throughout this correspondence we will assume a noiseless (as explained earlier) optical channel. This channel model is known to be a quite practical first-cut model for many optical space communications situations. The model does break down in the cases where certain parameters (e.g., the bandwidth) are allowed to approach infinity. However, our time resolution constraints will preclude this from happening.

II. LIMITS ON TIME RESOLUTION

There are several factors which affect ones ability to resolve time in an optical communication system. The first, and most fundamental, is the ultimate limit imposed by the quantum mechanical uncertainty principle

$$\Delta E \cdot \Delta T \geqslant h.$$

Here, ΔE is the change in energy of the system, ΔT is the resolution with which we can measure such a change and h is Planck's constant. Since we are interested in detecting individual photons and since each photon at a frequency ν has energy $h\nu$, then

$$\Delta T \geqslant \frac{1}{\nu}.$$

This statement is equivalent to saying that measurement of the arrival of a photon to less than a period of its radiation frequency is not possible.[2] At optical frequencies this resolution limit is approximately 10^{-15} s.

The remaining time resolution limits are really imposed by technology. The first comes from the pulse characteristics of the laser source. It appears that laser pulsewidths much shorter than 0.1 ps (or equivalently laser transition linewidths in excess of 10^{13} Hz) will be very difficult to achieve. Next, there is the limitation due to the optical filter at the receiving telescope. Too narrow an optical bandwidth limits time resolution whereas too wide a filter increases susceptibility

Paper approved by the Editor for Space Communication of the IEEE Communications Society for publication without oral presentation. Manuscript received March 30, 1981; revised December 21, 1981. This work was supported by the National Aeronautics and Space Administration.

The authors are with the Communications Systems Research Section, Jet Propulsion Laboratory, Pasadena, CA 91109.

[1] By a noiseless channel we mean a channel where the background noise and thermal noise of the receiving system are negligibly small. The quantum statistical self noise of the signal is assumed to still be present.

[2] It is true that we actually count photoelectrons, not photons, and the arrival times of the former can be determined with practically arbitrary accuracy. However, time accuracies higher than the characteristic time scales of the physical processes associated with the photons (which are the actual information carriers) are meaningless.

0090-6778/82/0500-1262$00.75 © 1982 IEEE

to background radiation. For example, a 1 Å filter has a bandwidth of approximately 100 GHz which limits time resolutions to around 10 ps. Finally, and most significantly, the state of time measurement technology imposes its own limitations. Currently, this limitation (which is influenced by clock stabilities, electronic circuit bandwidths and computational complexity) is in the region of 1 ns.

The point of all this is that technology advancements may permit operation at better than 10^{-9} s resolution. However, as timing resolution improves, an increasing number of barriers are encountered and, under no circumstances, can the timing resolution be improved beyond 10^{-15} s.

III. CAPACITY ANALYSIS

Now let us apply these limits to the optical communications problem. A model for the channel is shown in Fig. 1. The Q input symbols correspond to distinct time slots in a Q-ary PPM format and the channel either reproduces the input faithfully at the output or it erases the transmission altogether. The erasure probability ϵ is given by

$$\epsilon = e^{-N_s \Delta T}$$

where N_s is the source intensity as seen by the receiver during the optical pulse (peak source intensity measured in photons/second) and ΔT is the duration of a time slot in seconds. The capacity of this channel is given by the well-known expression

$$C = (1 - e^{-N_s \Delta T}) \ln Q \qquad \text{nats/channel use.} \tag{1}$$

Since each channel use requires $T = Q\Delta T$ seconds, then the capacity per second C_T is given by

$$C_T = \frac{C}{T} = \left(\frac{1 - e^{-N_s \Delta T}}{\Delta T}\right) \frac{\ln Q}{Q} \text{ nats/s.} \tag{2}$$

The final quantity of interest is the capacity per received signal photon C_{ph}. Since each received pulse, and therefore each received channel use contains (on the average) $N_s \Delta T$ photons, then

$$C_{ph} = \frac{C}{N_s \Delta T} = \left(\frac{1 - e^{-N_s \Delta T}}{N_s \Delta T}\right) \ln Q \quad \text{nats/photon.} \tag{3}$$

We now consider the following optimization problem. We first fix the bandwidth of the system by fixing the signal slot width ΔT. Additionally, we fix the throughput capacity of the system by setting C_T to some specific value. Finally, we vary the PPM alphabet size Q in such a way that we maximize C_{ph}. This optimization procedure basically minimizes the energy required to provide a given throughput, subject to the bandwidth constraint.

It should be pointed out that (2) is effectively a constraint equation. Then, since ΔT is fixed and Q is allowed to vary, it is necessary that N_s change to maintain the constraint. Thus, N_s is really a function of Q and it is this variation which produces the efficient utilization of energy.

In Fig. 2 C_{ph} is shown as a function of Q for capacity throughput rates $C_T = 10^4$ and 10^6 nats/s. The values of ΔT selected correspond to the uncertainty principle $\Delta T = 10^{-15}$ s and for the current technological limit of $\Delta T = 10^{-9}$ s. It is clearly seen that each curve has an optimum operating point which maximizes the capacity per photon and hence minimizes the energy consumption. For example, for $C_T = 10^6$ nats/s and $\Delta T = 10^{-9}$ s, the optimum value of Q is approximately 2000 and the resulting energy efficiency is 6.5 nats/photon.

An interesting trend emerges from these results. If one multiplies the throughput constraint C_T, the slot width constraint ΔT, and Q^*, the optimizing value of the Q, one gets for

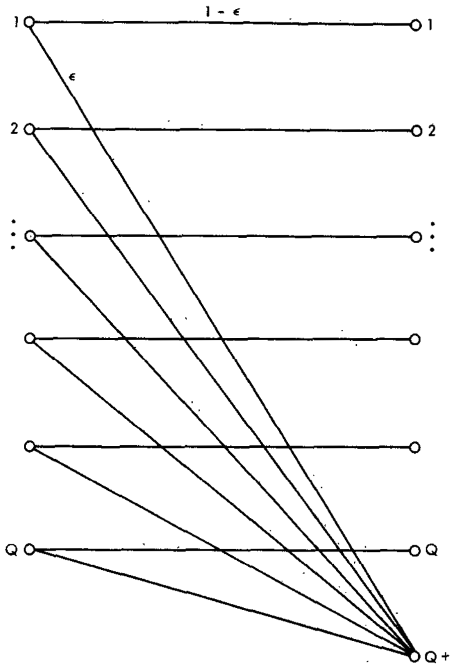

Fig. 1. Channel model for the Q-ary erasure channel.

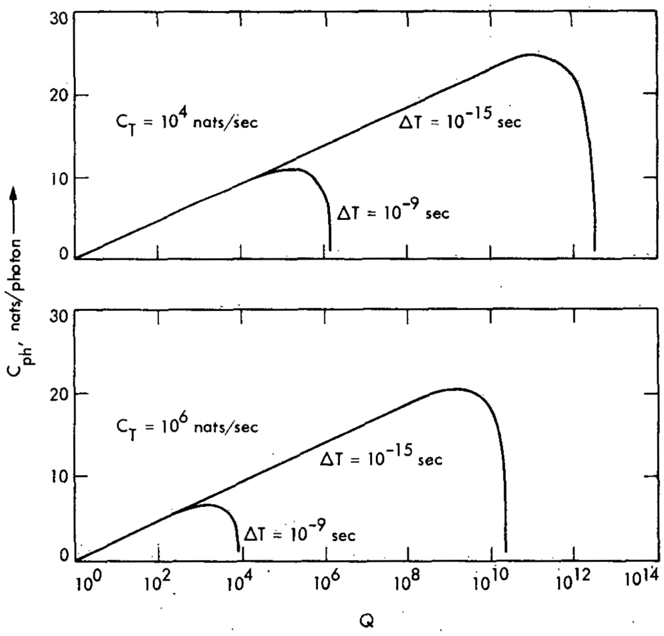

Fig. 2. Variation in channel capacity (per photon) with PPM word size for fixed throughput capacity C_T and bandwidth ΔT.

each of the curves shown

$$C_T \Delta T Q^* \approx 2.$$

Thus, it appears from these few examples that the optimizing value of Q can be easily estimated once C_T and ΔT are specified. This trend can be shown to be true more generally [7].

As a final comment we note that for a huge improvement in ΔT (6 orders of magnitude from 10^{-9} to 10^{-15} s) there is a much smaller increase (only a factor of 2 or 3) in C_{ph}. This explosive bandwidth expansion has already been observed by McEliece [6]. We note also that increasing C_{ph} beyond 20 nats/photon will be very hard indeed, except perhaps at extremely low throughput rates.

REFERENCES

[1] A. Yariv, *Introduction to Optical Electronics*, 2nd ed. New York: Holt, Rinehart and Winston, 1975.

[2] J. P. Gordon, "Quantum effects in communications systems," *Proc. IRE*, vol. 50, pp. 1898–1908, Sept. 1962.

[3] L. B. Levitin, "Photon channels with small occupation numbers," *Problemy Predachi Informatsii*, vol. 2, no. 2, pp. 60–68, 1966.

[4] J. W. S. Liu, "Reliability of quantum-mechanical communication systems," *IEEE Trans. Inform. Theory*, vol. IT-16, May 1970.

[5] J. R. Pierce, "Optical channels: Practical limits with photon counting," *IEEE Trans. Commun.*, vol. COM-26, Dec. 1978.

[6] R. J. McEliece, "Practical codes for photon communication," *IEEE Trans. Inform. Theory*, vol. IT-27, pp. 393–398, July 1981.

[7] J. R. Lesh, "Capacity limit of the noiseless, energy efficient optical PPM channel," Jet Propulsion Lab., Pasadena, CA, TDA Progr. Rep. 42–67, Feb. 1982.

Correction to the Forthcoming Special Issues List in the March Issue

The listing on page 423 of the March issue for the forthcoming Special Issue on Encryption of Analog Signals should have included the name and address of the Guest Editor:

Dr. Allen Gersho
Dep. Elec. & Comput. Eng.
Univ. California
Santa Barbara, CA 93106