

5.

10. Let  $X = \mathbf{R}$ ,  $\mathcal{S}$  be the Borel sets, and let  $\mu$  be the Lebesgue measure. Let  $E_k$  be the following set:

$$E_k = \bigcup_{x \in \mathbf{Q}} \left( x - \frac{1}{k}, x + \frac{1}{k} \right).$$

Then, it follows that for any  $E_k$ ,  $\mu(E_k) = \infty$ . Since  $\bigcap_{k=1}^{\infty} E_k = \mathbf{Q}$ ,  $\mu(\bigcap_{k=1}^{\infty} E_k) = 0$ . However,  $\lim_{k \rightarrow \infty} \mu(E_k) = \infty$ . Thus, the hypothesis that  $\mu(E_1) < \infty$  is necessary.