An Algorithm for Varsity Swim Team Selection

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I. Introduction

In the past two decades, the introduction of mathematics into the world of sports has opened a Pandora's box of applications, combining rigorous objective statistical analysis with coaching intuition. A area that has seen significant literature development has been in drafting and team selection. The novel methods in these areas combine budget optimization with player performance prediction to draft a team that is able to compete at the highest level.

In this contribution, we introduce a new method for selecting the International School of Beijing varsity swim team. It selects individuals who are not only fast, but also those that contribute to the versatility of the team as a whole, an attribute that its predecessor lacks. The simplicity of this method (low computational complexity), which does not come at the expense of efficacy, encourages accessibility and increases transparency. Through the novel introduction of two component indices—the speciality and versatility indices—swimmers will not be penalized for being "spikey." Additionally, swimmers who specialize in many events will still be rewarded for their diverse skillset. Compared to its predecessor, this system is also less physically demanding for the athletes themselves: instead of forcing each person to swim every event during the pre-selection season, which induces lackluster performance, the proposed system adequately judges speciality and versatility through only five required events.

This contribution is organized into six sections. First, pre-selection conditions (i.e. the minimum requirements to be considered a part of the selection pool) are described. Next, the details of the algorithm are explained along wth the parameters/hyperparameters of each component. Lastly, specific examples are presented to demonstrate the efficacy of the proposed algorithm.

II. Pre-Selection Considerations

The pre-selection considerations in this section describe measures to ensure that the proposed algorithm is informed. These pre-selection conditions also ensure that the selection system adheres to ethos of the ISB swim team: diversity, camaraderie, and IM swimming.

The foremost consideration when forming the varsity team is each individual's proficiency in IM swimming. When the time comes for choosing APAC events, proficiency in IM swimming makes it easier to find an optimal entry scheme. We can enforce this criterion by mandating swimmers who would like to be considered for the team to swim at least 5 APAC events,[1] one from each stroke (Freestyle, Breaststroke, Backstroke, Butterfly, and Individual Medley) during pre-selection.

Secondly, relay events compose a significant percentage of all the points scored at APAC. Thus, the algorithm should heavily weight the performance of the final team's ability in relay events (4×100 Meters Freestyle, 4×50 Meters Individual Medley, 4×50 Meters Freestyle). As a result, swimmers should be incentivized to swim in at least one of the these relay events during pre-selection to showcase their relay capabilities. These relay events should also be given more weight during team selection because they are counted twice (once as an individual event and once as a relay event) during meets.

Lastly, an important aspect of the varsity team is the camaraderie built through collective training and struggle. This will be considered during the selection process by giving bonuses for those who regularly attend school practices. The specifics of how this bonus is applied is detailed in the following sections.

III. Selection Algorithm

This section describes the selection algorithm in detail. The section is split into three subsections, each describing a crucial component of the algorithm: the versatility index, the speciality index, and the bonus system. The combination of these two distinct indices encourages breadth and depth, respectively. Each swimmer's final composite score—excluding bonus points—will be equal to the sum of these two indices:

$$\text{Composite Score} = \text{Speciality} + \text{Versatility}. \tag{1}$$

A larger composite score is considered worse than a smaller composite score.

A. Speciality Index, σ

The speciality index, σ, is determined by summing the ranks of each swimmer's top events. Suppose that each swimmer i is associated with a collection of ranks $R_i = \{R_{i,j}\}_{j\geq 1}$ of all the events they have swam, where $R_{i,j}$ represents the jth rank of the collection associated with swimmer i and $|R_i|$ represents total number of events swimmer i has swam. The rank of a swimmer in a given event is found by the position of the first occurrence of the target swimmer's time in an ascending list of all the times in that event (see Table I).

*This work was completed during the author's sophomore/junior/senior year at the International School of Beijing.

[1] 50 Meters Freestyle, 100 Meters Freestyle, 200 Meters Freestyle, 400 Meters Freestyle; 50 Meters Breaststroke, 100 Meters Breaststroke; 50 Meters Backstroke, 100 Meters Backstroke; 50 Meters Butterfly, 100 Meters Butterfly; 100 Meters Individual Medley, 200 Meters Individual Medley.

TABLE I

THE RANKINGS OF BOYS 50 METERS FREESTYLE. HERE, EACH SWIMMER IS RANKED BY THE FASTEST TIME SWAM DURING PRE-SELECTION, WITH EACH SWIMMER CONTRIBUTING ONE TIME. THE POSITIONS AND INDICES ARE ASSIGNED IN ASCENDING ORDER.

Name	Time	Rank
Curtis Wong	24.03	1
Alan Sun	24.19	2
Bernard Ip	25.25	3
Alan Wang	25.36	4
Aaron Wu	25.75	5
Jerry Zheng	26.08	6
Alex Leung	26.71	7
Guan Chen Wang	26.98	8
Eddie Su	27.68	9
Evan Lin	27.95	10
Oscar Hamada	28.04	11
Kelly Zhu	28.38	12

Sort this collection in ascending order, so that $R_{i,j+1} \geq R_{i,j}$ for all j. If we use n to denote the total number of swimmers in the pre-selection pool, we can define the speciality index σ_i for the swimmer i as:

$$\sigma_i = n \cdot \max(0, h - |R_i|) + \sum_{j=1}^{\min(h,|R_i|)} R_{i,j}, \tag{2}$$

Here, h is a hyperparameter defined by the coaching staff each season, providing some variability as the goals and performance of the team fluctuate year-to-year. Intuitively, h represents the "minimum required size" of each swimmer's niche; in other words, h is the number of events each swimmer is expected to specialize in. For the 2018 and 2019 season $h = 12$, while for the 2020 season $h = 6$. The first term on the right-hand side of Eq. 2 penalizes swimmers who have not swam at least h events by assuming that they will finish last in the events they missed, while the second term simply sums the swimmer's top h ranks.

We will now provide some sample calculations of the speciality index. Suppose that two swimmers have the following ranks, as shown in Table II.

TABLE II

SWIMMER A AND SWIMMER B'S RESPECTIVE EVENTS AND RANKS. THE RANKS AND THEIR ASSOCIATED EVENTS ARE SORTED IN ASCENDING ORDER.

Swimmer A		Swimmer B	
Event	Rank	Event	Rank
50m Breaststroke	1	50m Freestyle	1
100m Butterfly	2	100m Freestyle	1
100m Individual Medley	2	100m Backstroke	1
50m Butterfly	5	50m Breastroke	2
100m Backstroke	5	100m Individual Medley	3
400m Freestyle	7		
200m Individual Medley	8		

Assuming that $h = 5$ and $n = 20$, the speciality index of swimmer A, σ_a, is

$$\sigma_a = 1 + 2 + 2 + 5 + 5 = 15,$$

and that the speciality index of swimmer B, σ_b, is

$$\sigma_b = 1 + 1 + 1 + 2 + 3 = 8.$$

Given that $\sigma_a > \sigma_b$, we affirm that the swimmer B is more specialized than A based on the scope of five events. On the other hand, if $h = 7$, then the speciality of swimmer A is

$$\sigma_a = 1 + 2 + 2 + 5 + 5 + 7 + 8 = 30.$$

However, swimmer B did not swim seven events. In this case, their available 5 ranks are summed, and we assume that they finished last (i.e. n^{th} place) in the two empty events:

$$\sigma_b = 1 + 1 + 1 + 2 + 3 + 2 \cdot 20 = 48.$$

As a result, swimmer B is less specialized than swimmer A based on the scope of seven events.

It should be noted that as h increases, it loses its value as a "speciality" constant, as it demands swimmers to exhibit peak performance for a wider and wider array of events. The author has found through meticulous experimentation that the speciality and versatility index synergize the best when $h = 4$ or $h = 5$.

B. Versatility Index, β

The versatility index, β, is a measure of how well-rounded a particular swimmer is. If a swimmer sees a consistent rank over all their events, it means that they are versatile, as whatever event they swim their *relative* performance stays constant. However, if a swimmer's rank fluctuates drastically over all of the events they swam, this implies that they are not as versatile: a change in event results in a drastic change in *relative* performance. To this end, a larger versatility index indicates less versatility compared to a smaller versatility index. Again, we suppose that swimmer i is associated with an ascending collection of ranks, R_i, as defined in the previous subsection. We can determine the versatility by finding the change between all the ranks of a given swimmer's events: large, drastic change is correlated with low versatility, while gradual, slow change is associated with high versatility. These differences are then aggregated through a *weighting function*, ω. This process is shown in Eq. 3.

$$\beta_i = \sum_{j=1}^{11} \omega(j) \cdot (R_{i,j+1} - R_{i,j}). \tag{3}$$

We first calculate the difference between each neighboring ranks: $R_{i,j+1} - R_{i,j}$. In the case that does not exist $R_{i,j+1}$, it is replaced by n – as defined above, the total number of swimmers trying out during pre-selection. Since the collection is sorted in ascending order this difference should always be nonnegative. Next, this difference is multiplied by a weight—how much this difference should contribute to the versatility constant. Consider the two swimmers in Table III, if we simply just sum the differences with a uniform weighting function, both swimmers will have the same versatility constant. Swimmer C, D have the versatility constant, β_c, β_d, of

$$\begin{aligned}\beta_c &= (20-1) + (20-20) + (20-20) + \\ &\quad (20-20) + (20-20) + (20-20), \\ &= 19; \\ \beta_d &= (1-1) + (1-1) + (1-1) + \\ &\quad (1-1) + (1-1) + (20-1), \\ &= 19,\end{aligned}$$

respectively. However, intuitively, we would expect swimmer C to be less versatile compared to swimmer D, as swimmer D has already shown more consistency in their top events. Thus, as we progress further through the collection, the relative differences should matter less and less.

TABLE III
SWIMMER C AND SWIMMER D'S RESPECTIVE EVENTS AND RANKS. THE RANKS AND THEIR ASSOCIATED EVENTS ARE SORTED IN ASCENDING ORDER. HERE, WE SEE THAT, INTUITIVELY, WE WOULD CALL SWIMMER C A MUCH LESS VERSATILE SWIMMER THAN SWIMMER D, EVEN THOUGH THE SUM OF THE DIFFERENCES IN EACH PAIR OF NEIGHBORING RANKS IS THE SAME.

Swimmer C		Swimmer D	
Event	Rank	Event	Rank
50m Breaststroke	1	50m Freestyle	1
100m Butterfly	20	100m Freestyle	1
100m Individual Medley	20	100m Backstroke	1
50m Butterfly	20	100m Breastroke	1
100m Backstroke	20	100m Individual Medley	1
400m Freestyle	20	400m Freestyle	1
200m Individual Medley	20	100m Butterfly	20

This intuition is captured by the weighting function ω. This weight function is also a hyperparameter that can be adjusted by the coaching staff each season based on the team's needs. It is advised that $\omega(j) \leq 1$ for $1 \leq j \leq 12$. For the 2020 season, a linear weighting function was employed:

$$\omega(j) = \frac{11-(j-1)}{11}. \tag{4}$$

As j increases, the weights decrease linearly. One could also employ a polynomial weighting function:

$$\omega(j) = \frac{(11-(j-1))^n}{11^n}, n \in \mathbb{N}. \tag{5}$$

The graphs of these two weighting functions are shown in Fig. 1. The faster the weighting function decays, the more emphasis is placed on versatility in each swimmer's best events compared to their worst events. There are many more options for ω; a full list is included in the appendix.

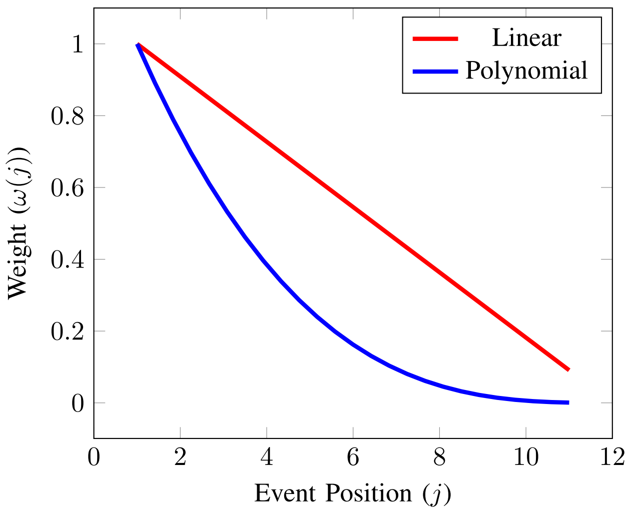

Fig. 1. The graph of a linear weighting function (in red) and a polynomial weighting function (in blue). We see that the blue curve decays faster than the red curve. The rate of change of the weighting functions determine how much we care about the differences in performances of a swimmer's top events compared to their worst performing events.

Swimmers are ranked in ascending order based on the sum of their speciality and versatility index $\sigma+\beta$ as shown in Eq. 1.

Now, we examine the bonus points system, which incentivizes swimmers to come to practice and swim more events during pre-selection.

C. Bonus Points

Instead of assigning bonus points by subtracting off a constant from the final composite score, as was done with this system's predecessor, we argue that bonus points should be assigned by discounting a *percentage* from the final composite score. This way, the system will be more robust with respect to the various hyperparameters chosen. This method was used in the 2020 season. Now, with a Bonus% applied, the composite score is:

$$\text{Composite Score} = (1-\text{Bonus})(\text{Speciality}+\text{Versatility}). \tag{6}$$

The author recommends the maximum bonus discount given on the aggregate score to be no more than 20%. Bonuses may also be assigned for scoring in the top 2 or top 4 of relay events, as a majority of points scored at championship meets are from relays. The bonus percentages are given according to Table IV. It should be noted that swimmers may only receive individual event bonuses for a maximum of two events. For the 2020 season, an additional 2% bonus was applied for those who had perfect attendance.

TABLE IV
BONUS DISCOUNT PERCENTAGES FOR INDIVIDUAL RELAY EVENTS. THE VALUES OF THE DISCOUNTS ARE ASSIGNED BASED ON THE NUMBER OF RELAY SPOTS THERE ARE FOR EACH OF THESE "INDIVIDUAL" EVENTS.

Rank	FR50m	FR100m	BR50m	BA50m	Fly50m
1	10%	10%	10%	10%	10%
2	8%	8%	8%	8%	8%
3	6%	6%	0%	0%	0%
4	4%	4%	0%	0%	0%

Thus, if swimmer A had perfect attendance (Table II), their total bonus would be $10\% + 2\% = 12\%$.

IV. EXAMPLES

This section provides some more detailed examples, stringing together all of the components of the algorithm described previously. In these examples, we continue with the hyperparameters used in the 2020 season:

- $h = 6$
- a linear weighting function as described in Eq. 4
- bonus discount percentages for individual events as seen in Table IV
- a 2% added bonus for all of those who had perfect attendance.

The data used in this section is real historical data from the 2019 season (given in Table VII).

We determine the selection index of KYp from Table VII. Since $h = 6$, we add up the ranks of KYp's top six events. This gives us the speciality index:

$$\sigma_{\text{KYp}} = 1+2+2+2+2+3 = 12.$$

To calculate the versatility index β_{KYp} of KYp, we begin by sorting all of the ranks and then finding the differences between each neighboring rank. This process is shown in Table V.

TABLE V

WORK SHOWN FOR THE CALCULATION OF β_{KYp}. THE WEIGHTS ARE DERIVED FROM THE LINEAR WEIGHTING FUNCTION EQ. 4.

Raw		Derived	
Events	Rank	Diff	Weight
BR50m	1	1	11/11
FR100m	2	0	10/11
FLY50m	2	0	9/11
FLY100m	2	0	8/11
IM100m	2	1	7/11
FR50m	3	0	6/11
BR100m	3	2	5/11
BA100m	5	2	4/11
FR400m	7	1	3/11
FR200m	8	0	2/11
IM200m	8	12	1/11
BA50m	20	-	-

By adding up the result of the "Diff" column multiplied by the "Weight" column we obtain the versatility index:

$$\begin{aligned}\beta_{\text{KYp}} = & \ 1\cdot\frac{11}{11}+0\cdot\frac{10}{11}+0\cdot\frac{9}{11}+0\cdot\frac{8}{11}+ \\ & 1\cdot\frac{7}{11}+0\cdot\frac{6}{11}+2\cdot\frac{5}{11}+2\cdot\frac{4}{11}+ \\ & 1\cdot\frac{3}{11}+0\cdot\frac{2}{11}+12\cdot\frac{1}{11}, \\ & = 4.636.\end{aligned}$$

By adding the speciality and versatility index, we get the composite score of 16.64, without the added bonuses. Note that KYp is ranked in the top 2 for BR50m, FR100m, FLY50m as well as in the top 4 for FR50m. This results in respective bonuses of $10\%, 8\%, 8\%$, and 6%. Since each swimmer can have a maximum of two bonus discounts, not including the perfect attendance bonus, the bonus from individual events that KYp recieves is 18%. As KYp also has perfect attendance, he gets an additional 2% bonus, bringing the total to 20%.

Thus, KYp's adjusted score is:

$$(1-0.2)\cdot 16.64 = 13.312.$$

KYp is an example of a top swimmer who is consistent across all events. For such swimmers, there are no significant discrepancies between the proposed system and its predecessor. These swimmers will also be the ones who will have their pick of events when it comes to optimizing APAC entries, because they can swim almost anything.

Next, we examine AS, who is "spikey." Table VI shows the calculation matrix of AS's selection index.

TABLE VI

WORK SHOWN FOR THE CALCULATION OF β_{AS}. AGAIN, THE WEIGHTS ARE DERIVED FROM THE LINEAR WEIGHTING FUNCTION EQ. 4.

Raw		Derived	
Events	Rank	Diff	Weight
FR50m	2	3	11/11
BR50m	5	2	10/11
FR100m	7	2	9/11
BR100m	9	0	8/11
IM100m	9	2	7/11
FR200m	11	0	6/11
BA50m	11	1	5/11
BA100m	12	0	4/11
FLY50m	12	0	3/11
IM200m	12	2	2/11
FR400m	14	1	1/11
FLY100m	15	-	-

As before, we add up AS's top six events:

$$\sigma_{\text{AS}} = 2+5+7+9+9+11 = 43.$$

We then calculate the versatility index of AS. The process is the same as previously described: we add the result of the "Diff" column multiplied by the "Weight" column in Table VI:

$$\begin{aligned}\beta_{\text{AS}} = & \ 3\cdot\frac{11}{11}+2\cdot\frac{10}{11}+2\cdot\frac{9}{11}+0\cdot\frac{8}{11}+ \\ & 2\cdot\frac{7}{11}+0\cdot\frac{6}{11}+1\cdot\frac{5}{11}+0\cdot\frac{4}{11}+ \\ & 0\cdot\frac{3}{11}+2\cdot\frac{2}{11}+1\cdot\frac{1}{11}, \\ & = 8.636.\end{aligned}$$

The composite score is then $8.636+43 = 51.64$, pre-bonus. We see that AS is top 2 in FR50m. Thus, with the individual event bonus of 8%, AS's final score is:

$$(1-0.08)\cdot(51.64) = 47.51.$$

In general, swimmers who have extreme specializations are not automatically catapulted to the top ranks, even with the added bonuses. This is a result of the distribution of the bonuses; intuitively, we would expect a top swimmer to take the majority of the bonus points, regardless of their niche. This phenomenon can be seen with AWg in Table VII. As a result, the author believes that it is unlikely for pure sprinters—those who are only able to sprint—to game the system.

V. CONCLUSION AND FUTURE WORK

In this contribution, the details of the proposed selection index was explained and specific computation examples were given. The selection index described herein partitions the two selection criterions, versatility and specialization, into distinct indices. This gives clear motivation for the selection process and makes it more systematic.

The construction of such a selection index is nontrivial, as it not only needs to be mathematically rigorous, but it also needs to be easily explained and computed. The author concedes that more effective selection criteria exist, and some of these have been considered (e.g. a selection criterion based on the strengths and weaknesses of the selected team as a whole, not only looking at the individual). Such methods involve computing a covariance matrix of scaled rankings and the comparing the niche overlap between various swimmers. The greater this niche overlap the less likely having both swimmers on the team is going to increase marginal team performance. This analysis prioritizes searching for a team that maximizes synergy based on performances.

However, these methods are comparatively harder to implement, explain, and compute. The development of a better, simpler index is left as an exercise to the reader; the author would be interested to know of better solutions to this problem.

Appendix

TABLE VII
Real historical data from the 2019 season, with the exception of the perfect attendance column; the names of the swimmers are shortened both for anonymity and ease of reference. The last column "PA" denotes if a swimmer has achieved perfect attendance, 1 for yes and 0 for no.

Name	Grade	FR50m	FR100m	FR200m	FR400m	BA50m	BA100m	BR50m	BR100m	FLY50m	FLY100m	IM100m	IM200m	PA
KY	JR	1	1	1	1	1	1	2	1	1	1	1	1	1
CW	SO	4	4	4	5	3	2	7	8	3	3	4	3	1
KYp	SR	3	2	8	7	20	5	1	3	2	2	2	8	1
AWg	JR	6	3	2	2	12	14	6	6	6	4	6	4	1
FZ	FR	13	8	5	6	4	4	9	7	9	7	5	2	1
AWu	SO	8	10	6	9	7	9	4	5	4	10	7	7	1
JZg	FR	14	9	3	3	6	10	10	11	10	6	12	6	0
MH	FR	16	12	7	4	2	3	16	10	14	9	8	5	1
BI	FR	7	11	13	11	4	16	8	4	5	11	11	13	0
JC	SR	9	6	9	8	10	6	12	14	11	8	13	11	1
AS	JR	2	7	11	14	11	12	5	9	12	15	9	12	0
JCw	JR	12	16	14	13	14	15	3	2	13	13	10	9	1
ASn	JR	5	5	20	20	9	20	13	20	8	5	3	10	0
MY	SR	11	15	15	12	8	7	14	13	16	14	14	16	1
SC	SR	10	14	12	15	16	11	11	12	17	16	15	15	1
ES	SO	18	17	10	10	15	8	18	16	15	12	16	14	0
EL	SO	14	13	17	17	13	13	15	17	7	20	18	18	1
PL	FR	17	18	16	16	17	17	17	15	18	17	17	17	1
TJ	SR	19	19	18	18	19	18	19	18	19	18	19	19	0
DH	FR	20	20	20	20	18	20	20	20	20	20	20	20	1

Uniform Weighting Function

A uniform weighting function assumes uniform weights across all interpolated differences. The general formulae for such a function is:

$$\omega(j) = y, \tag{7}$$

where y is a constant. This weighting function has many issues that are described in the main text. Namely, it implies that niche versatility should be weighted the same as peripheral versatility.

Linear Weighting Function

A linear weighting function decays constantly. This weighting function assumes that the importance we attribute to the interpolated differences decreases by a constant amount as we move down the list of ranks. The general formulae for this class of functions is:

$$\omega(j) = \max\left(0, \frac{r-(j-1)}{r}\right), \tag{8}$$

where r is a constant that determines how fast the function decays. As $r \to \infty$, the function becomes flatter and flatter and approaches the uniform weighting function. It is recommended that $r = 11$, as the function remains continuous and demonstrates good coverage over all 11 interpolated differences. A graph of this function and its various r values are shown in Fig. 2.

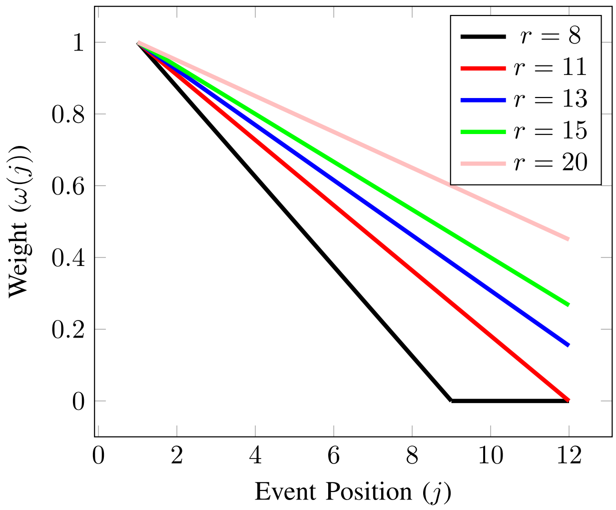

Fig. 2. Graphs of linear weighting functions with varying r, demonstrating how this hyperparameter can affect the slope of the function.

Polynomial Weighting Function

A polynomial weighting function decays quicker than a linear weighting function. The general formula for this class of functions is:

$$\omega(j) = \frac{(11-(j-1))^n}{11^n}, \tag{9}$$

where n is a constant that determines the degree of the polynomial. The shapes of this class of functions with varying degrees n is shown in Fig. 3. Notice how a larger n is associated with a faster rate of decay.

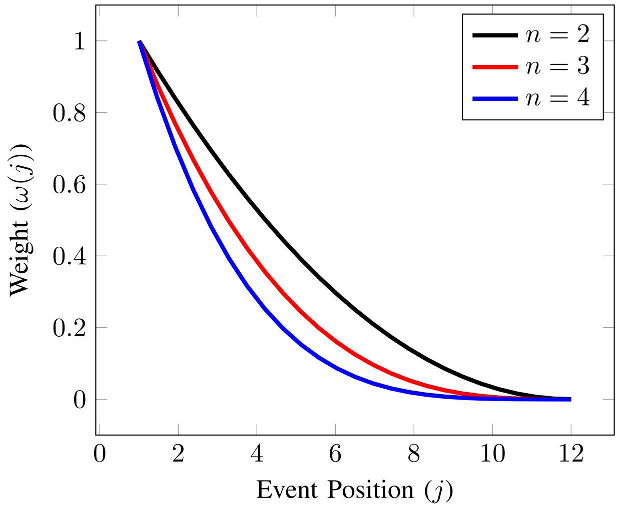

Fig. 3. Graphs of polynomial weighting functions with varying n.

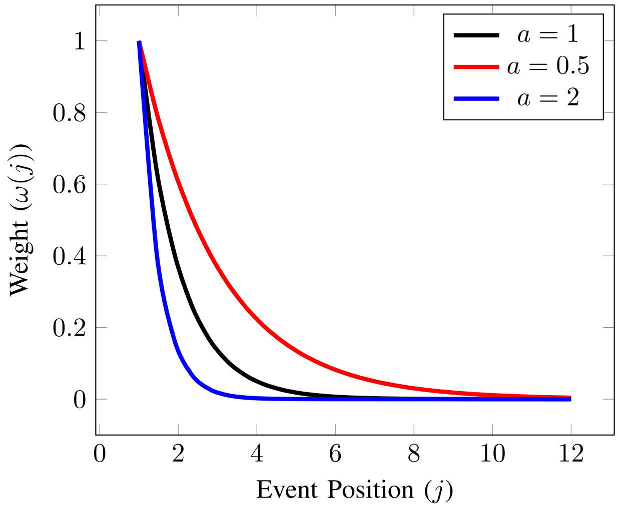

Fig. 4. Graphs of exponential weighting functions with varying a.

Exponential Weighting Function

An exponential weighting function is similar to a polynomial weighting function in that it also decays rapidly. However, generally, it is characterized by an even faster rate of change. The general formulae for this class of weighting functions is:

$$\omega(j) = e^{-a(j-1)}, \tag{10}$$

where $a \in \mathbb{R}$ represents a continuous real variable that changes the flatness of the weighting function. As $a \to 0$, the weighting function approaches a uniform weighting function. The graphs associated with various values of a is shown in Fig. 4.

Sigmoidal Weighting Function

The sigmoidal weighting function is characterized by the two flat tails at either ends and a fast rate of change in the middle of the function. The general formulae for the sigmoidal family is:

$$\omega(j) = \frac{1}{1+e^{a(j-h)}}, \tag{11}$$

where h is the hyperparameter defined in the body as the size of the required niche, and $a \in \mathbb{R}$ represents a continuous real variable that changes the flatness of the "middle rate of change." The graphs associated with this class is shown in Fig. 5.

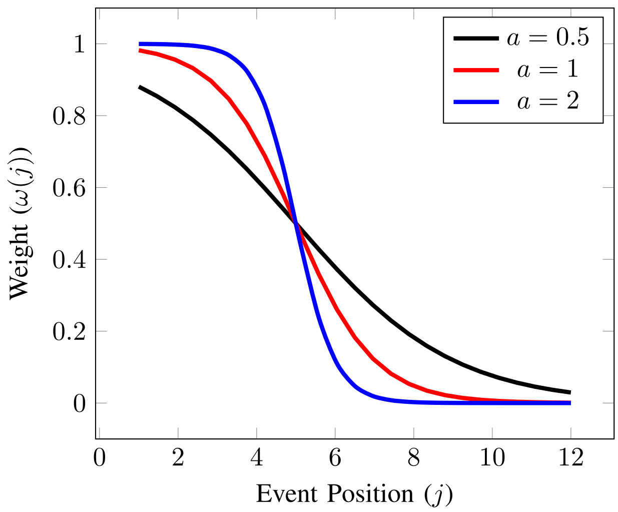

Fig. 5. Graphs of sigmoidal weighting functions with varying a. Here, for all the graphs $h = 5$. Note that as h increases, the point at which the sigmoid starts the rapidly decreases shifts to the right and vice versa.

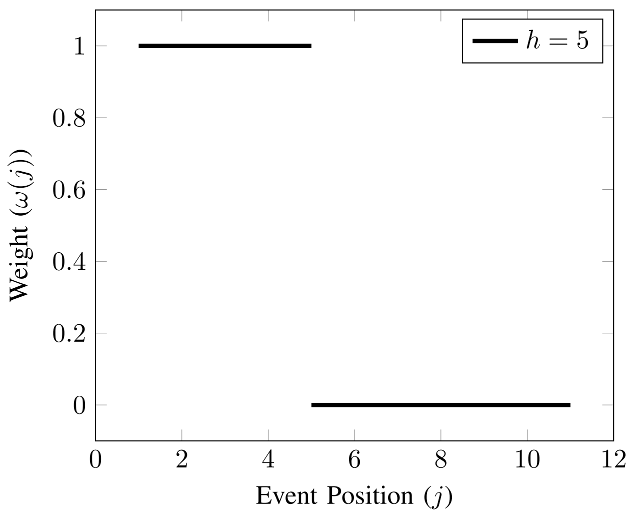

Fig. 6. The graph of a piecewise weighting functions with $h = 5$. Note how the function cuts off at $h = 5$.

Piecewise Weighting Function

The family of piecewise weighting functions is a logical continuation of the sigmoidal weighting function family. The piecewise functions are discontinuous and are more extreme. The general formulae for this family is:

$$\omega(j) = \begin{cases} y & j \leq h, \\ 0 & \text{else.} \end{cases} \tag{12}$$

Again h is the hyperparameter defined in the body denoting the size of the required niche. All of the differences in within the niche are considered, while the differences outside of the niche are not. Here, y is a constant that is constant. A graph of this function is shown in Fig. 6.

Here, the piecewise weighting function is a powerful family because it can also be used to mix any of the previously discussed functions together. Suppose that a uniform weighting function is desired for events within the niche, and a exponential weighting function is desired for events outside of the niche. This mix-and-match can be accomplished using a piecewise

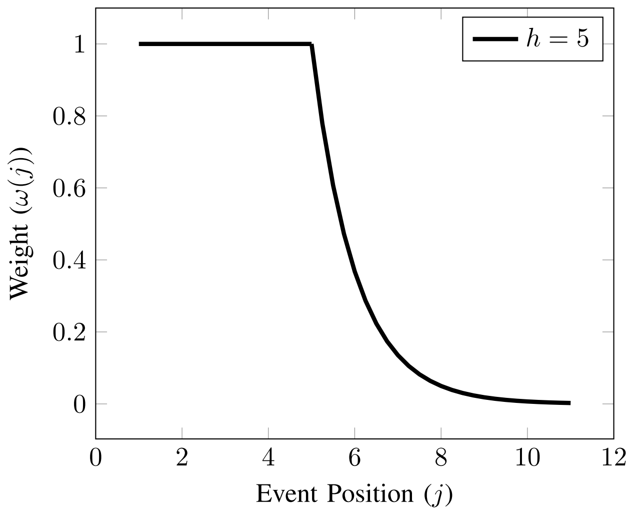

Fig. 7. The graph of a hybrid piecewise weighting functions $h = 5$. Note how the function cuts off at $h = 5$ and switches from a uniform weighting function to an exponential one.

function. The formulae of such a function is shown below:

$$\omega(j) = \begin{cases} y & j \leq h, \\ e^{-a(j-1)} & \text{else.} \end{cases} \tag{13}$$

The parameters for each component of this function is described in the previous subsections. A graph of this particular piecewise function is shown in Fig. 7.