Visualizations of Network Statistics Over Time Using Vertex Bootstrap and Jackknife

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7/15/2021

This document contains tables and visualizations regarding distributions of the differences in several network statistics over time. The distributions are obtained using the vertex bootstrap and jackknife procedures. Two different temporal networks are examined.

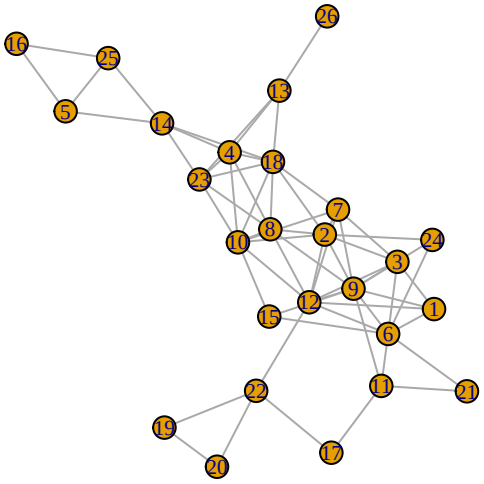
Source code (to generate distributions): https://github.com/alansun25/network-analysis/blob/main/analysis/vert-jack-viz.R

Friendship in a Dutch school class

Data source:

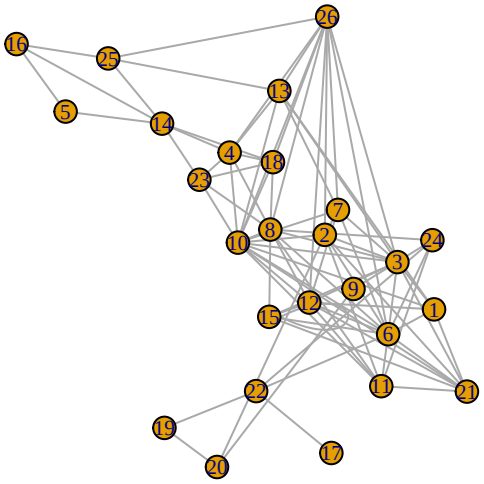
- **RSiena**
- Andrea Knecht
- https://www.stats.ox.ac.uk/~snijders/siena/tutorial2010_data.htm
- 2008

Original Friendship Networks



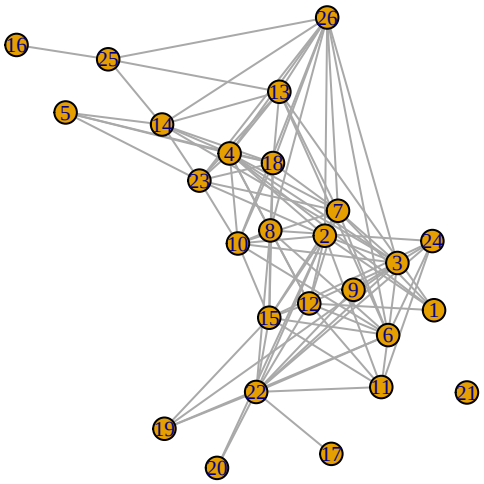
Transitivity: 0.5076923, Mean Degree: 4.8461538, Edge Density: 0.1938462, Number of nodes in a k-core where k > 1: 25

Friendships at time 2 (Three months after time 1)



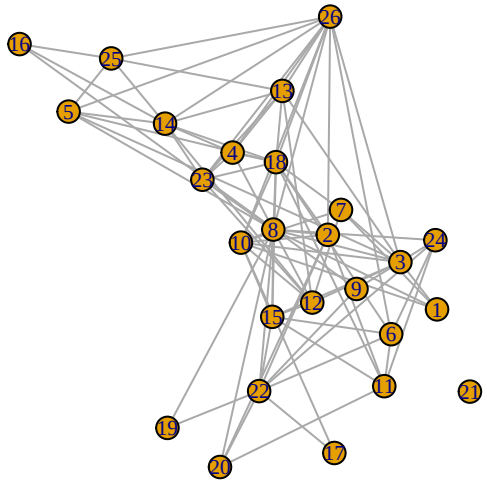
Transitivity: 0.5609756, Mean Degree: 6.5384615, Edge Density: 0.2615385, Number of nodes in a k-core where k $>$ 1: 25

Friendships at time 3 (Three months after time 2)



Transitivity: 0.5542312, Mean Degree: 7.4615385, Edge Density: 0.2984615, Number of nodes in a k-core where k $>$ 1: 23

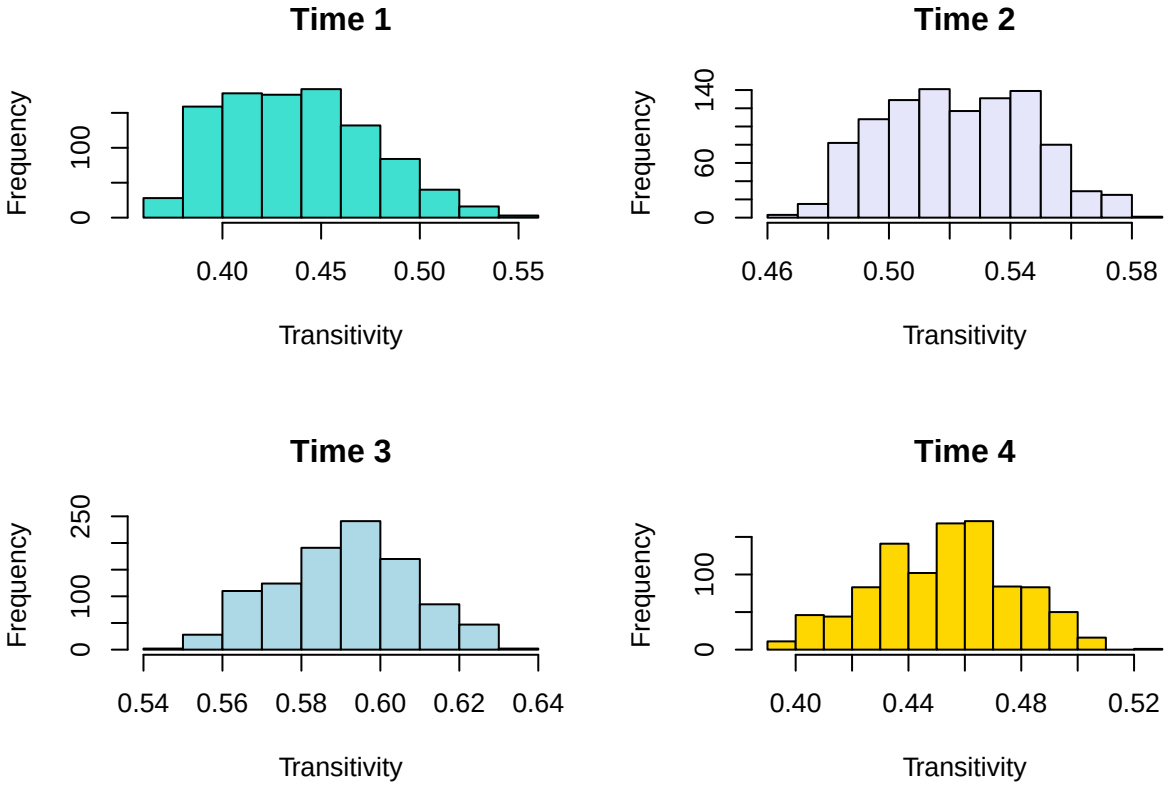
Friendships at time 4 (Three months after time 3)



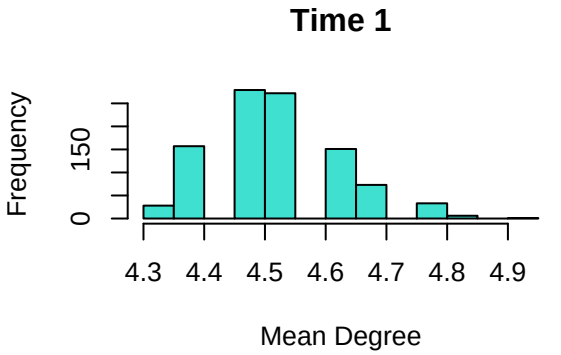
Transitivity: 0.4467085, Mean Degree: 6.6153846, Edge Density: 0.2646154, Number of nodes in a k-core where k > 1: 25

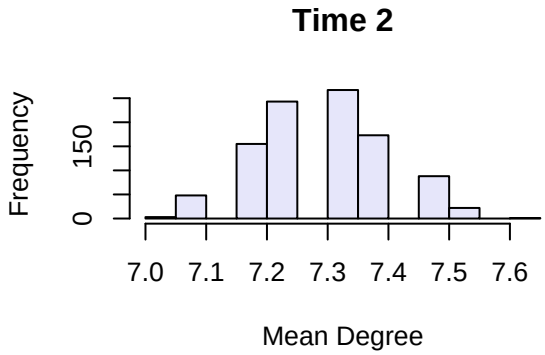
Vertex Bootstrapped Distributions of Network Statistics

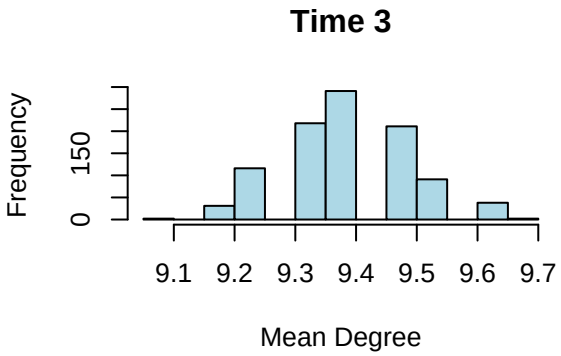
Transitivity:

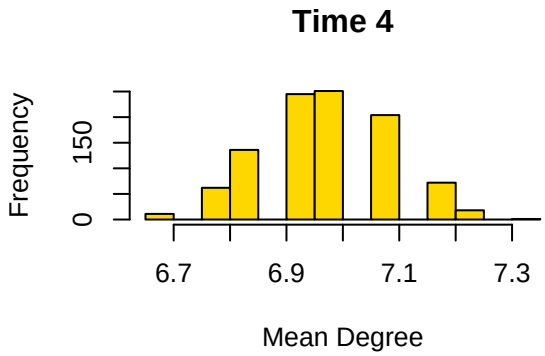


Mean Degree:

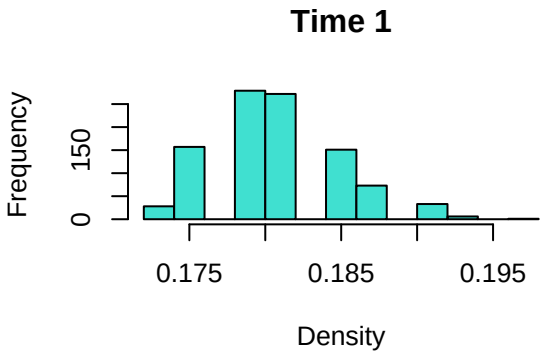


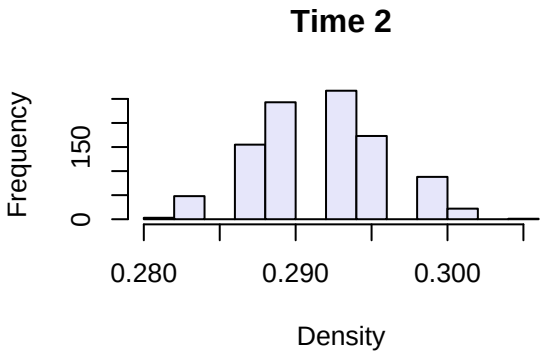


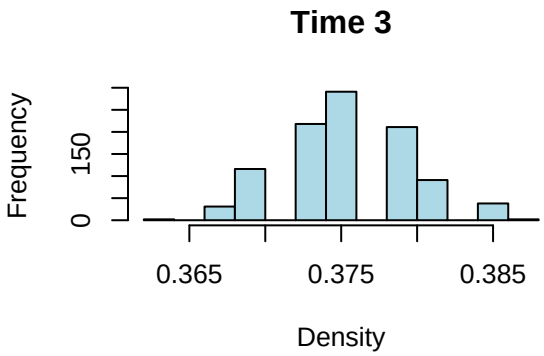


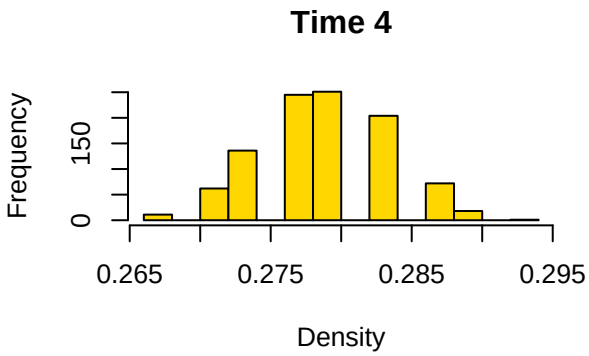


Edge Density:

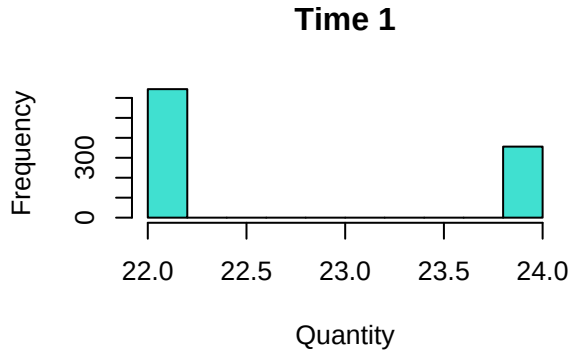


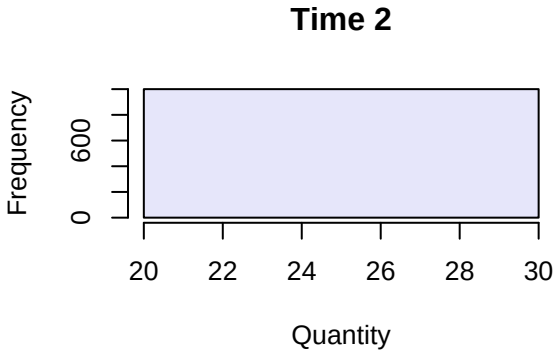






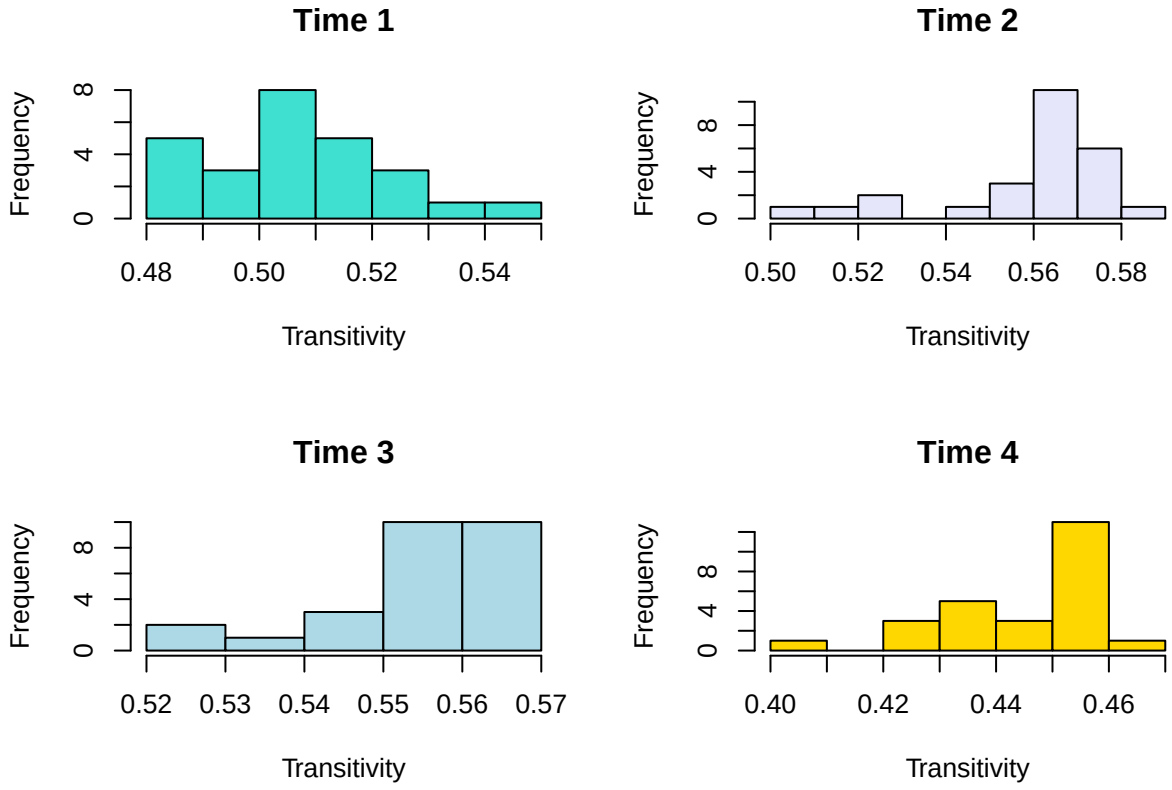
Number of nodes in k-cores where k > 1:



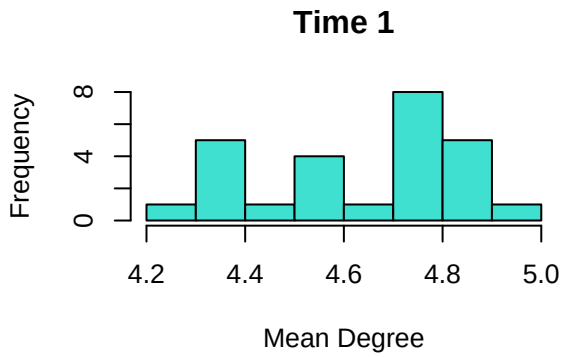


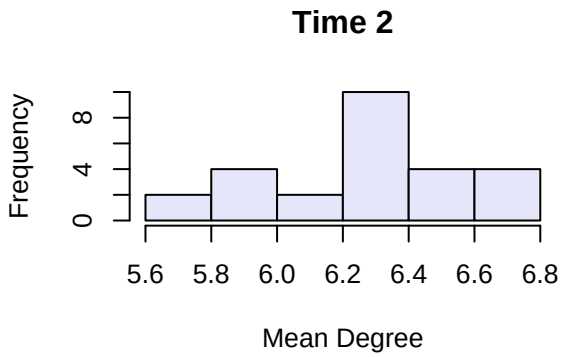
Vertex Jackknife Distributions of Network Statistics

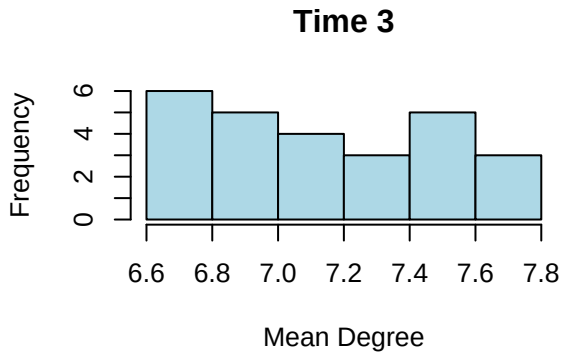
Transitivity:

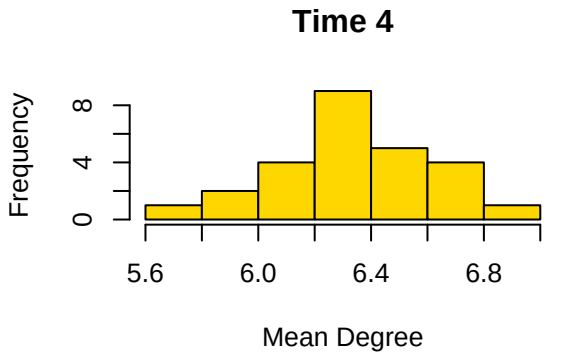


Mean Degree:

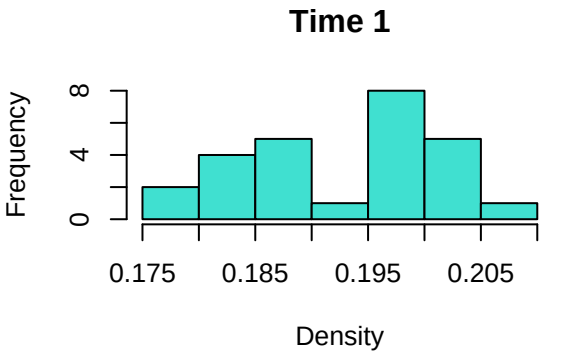


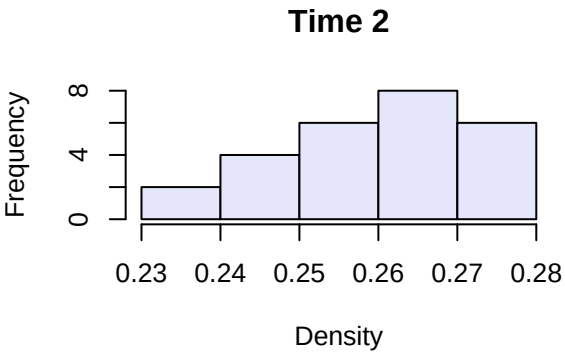


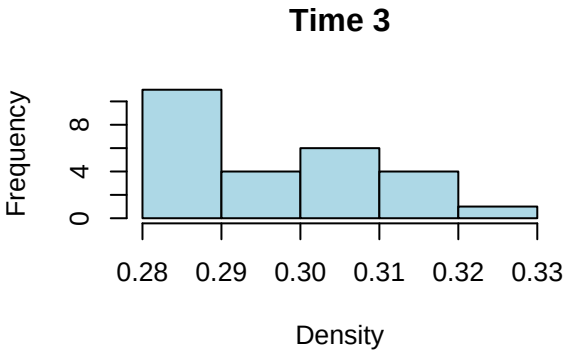


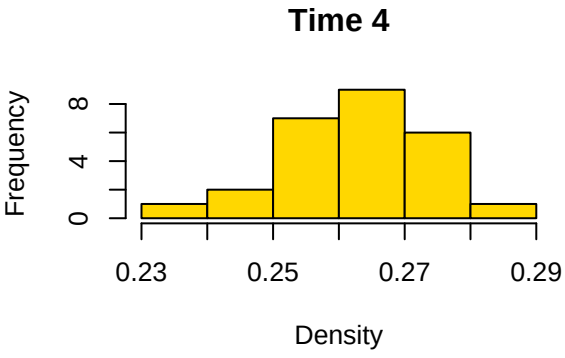


Edge Density:

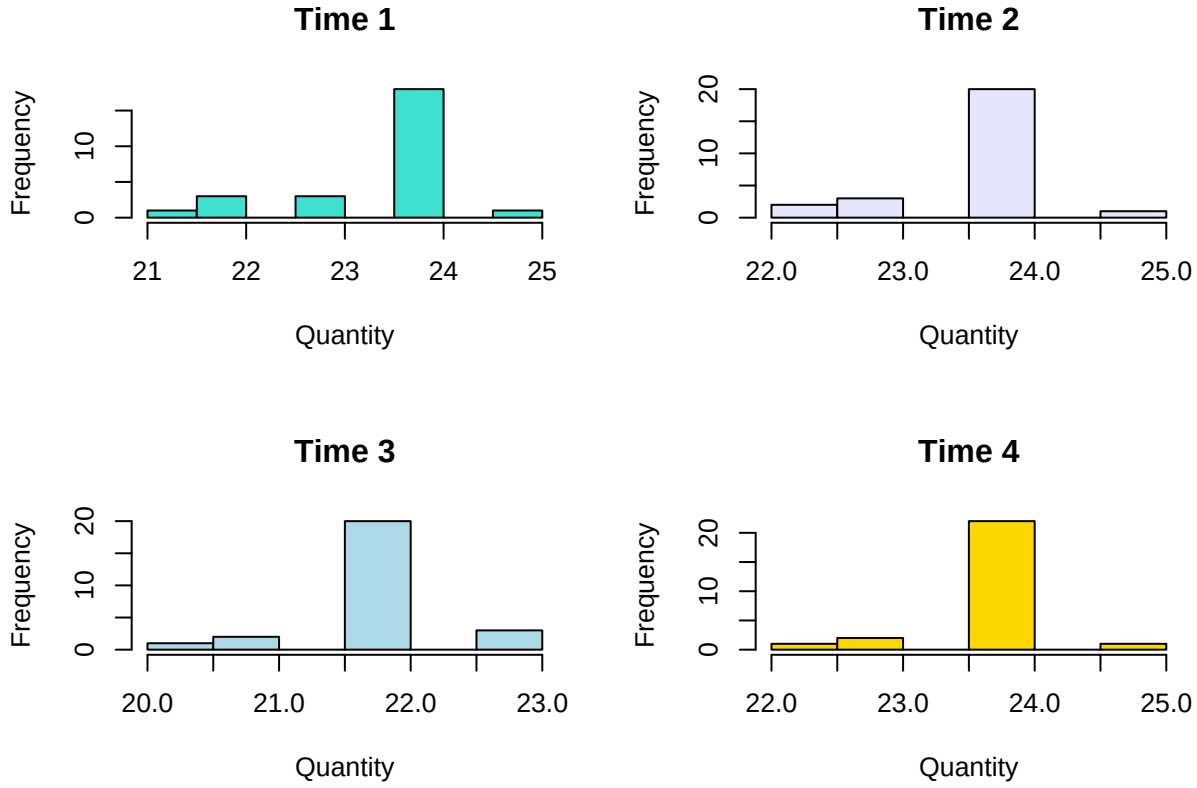








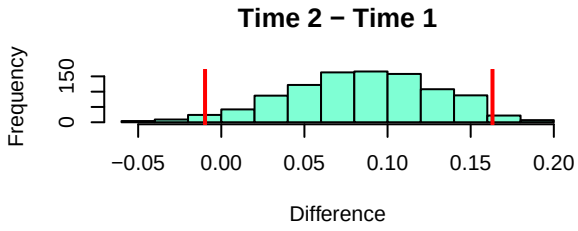
Number of nodes in k-cores where k > 1:

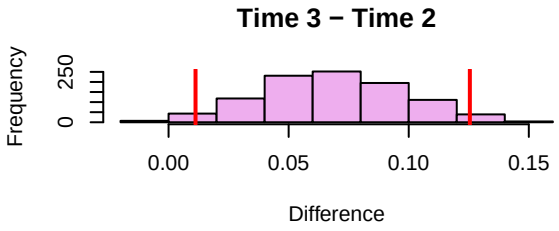


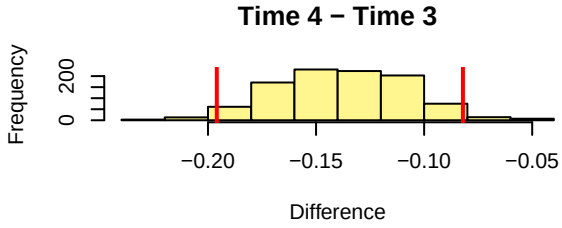
Bootstrap distributions of the differences in network statistics between time points

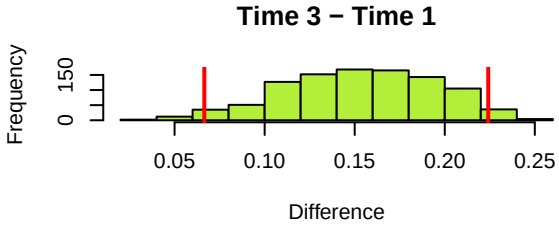
The vertical red lines indicate the middle 95% of the distributions.

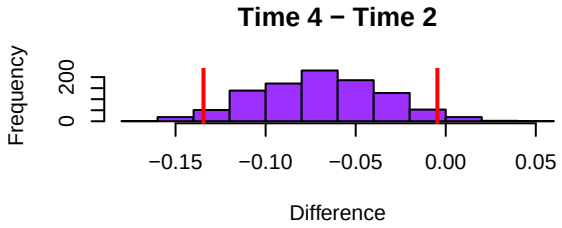
Transitivity:

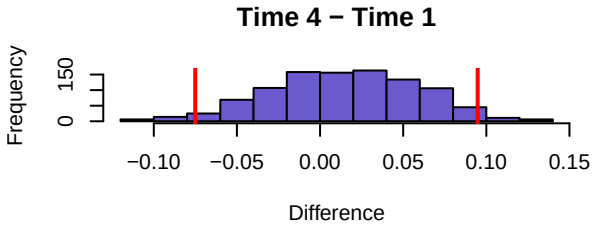




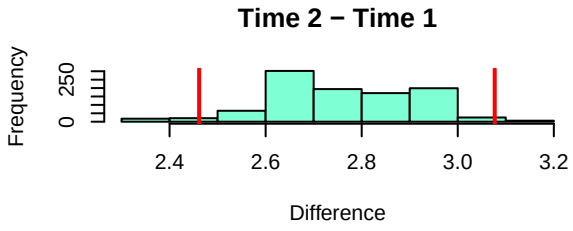


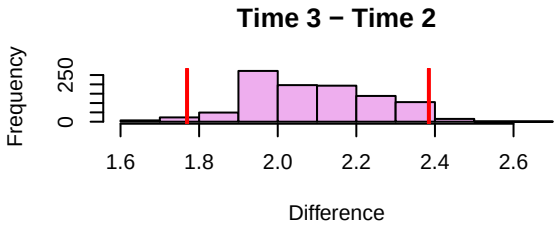


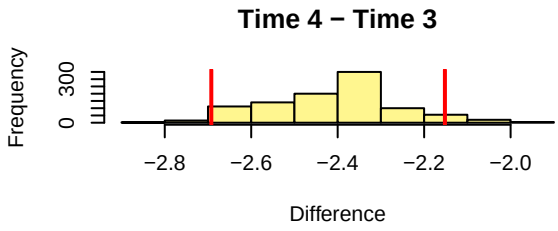


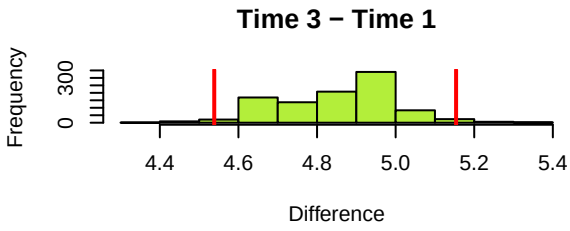


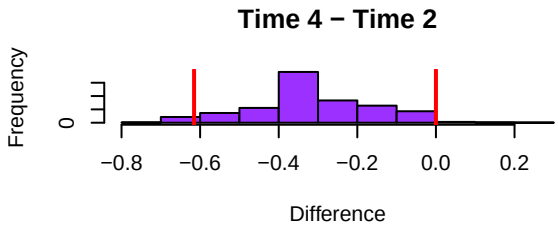
Mean Degree:

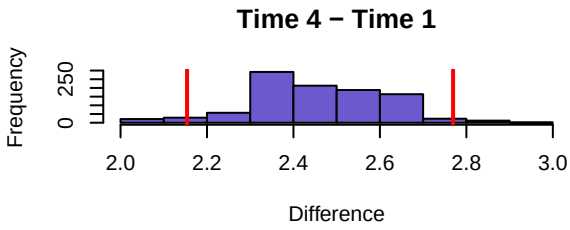




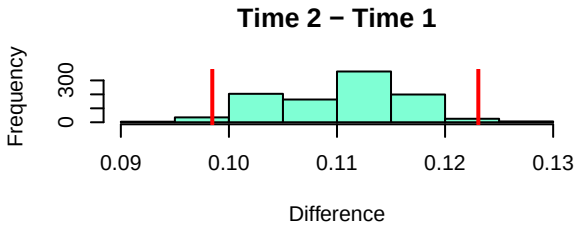


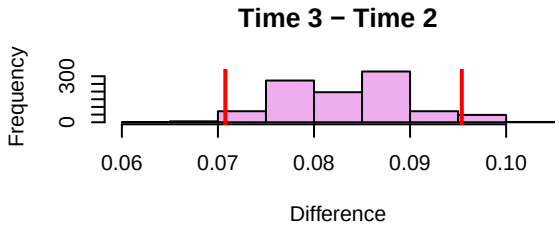


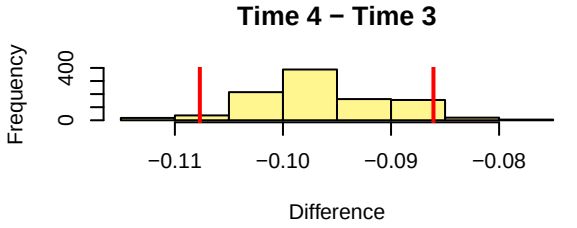


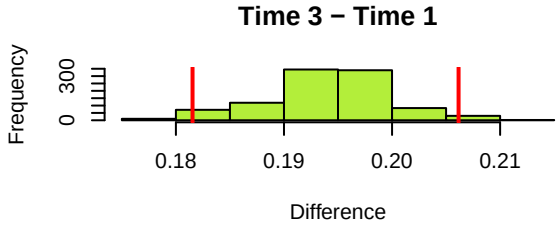


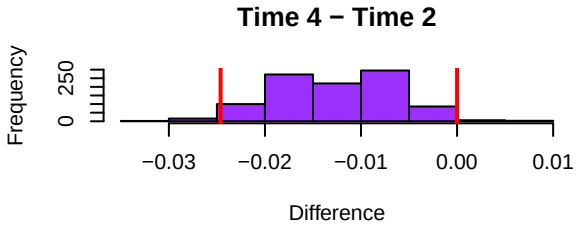
Edge Density:

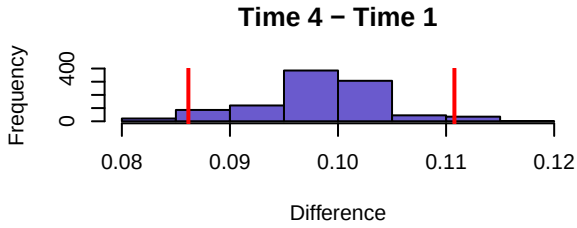




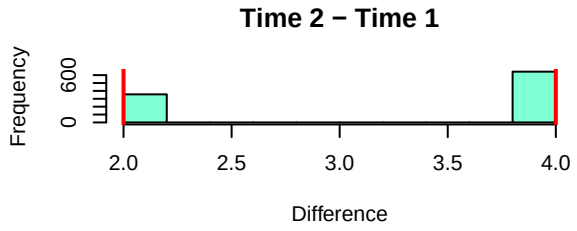


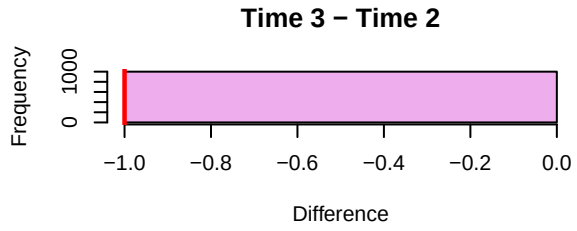


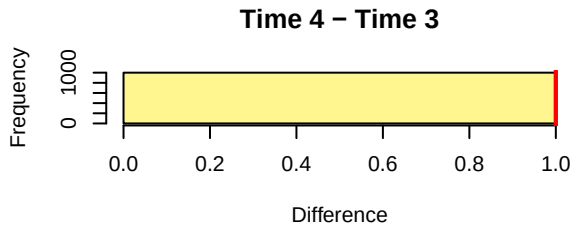


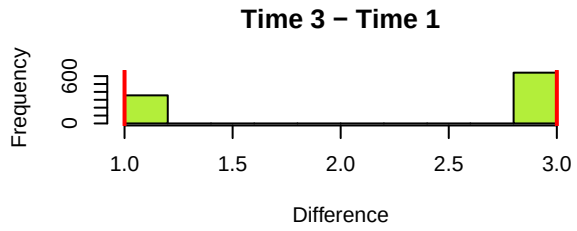


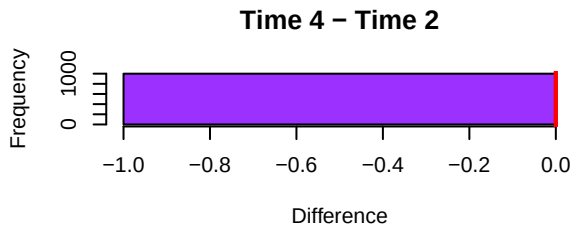
Number of nodes in k-cores where k $>$ 1:

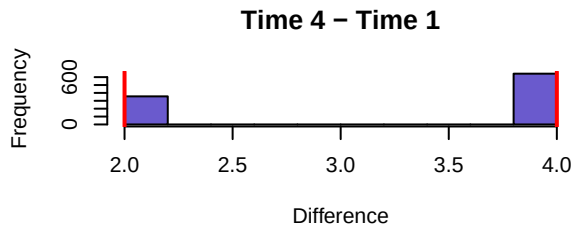








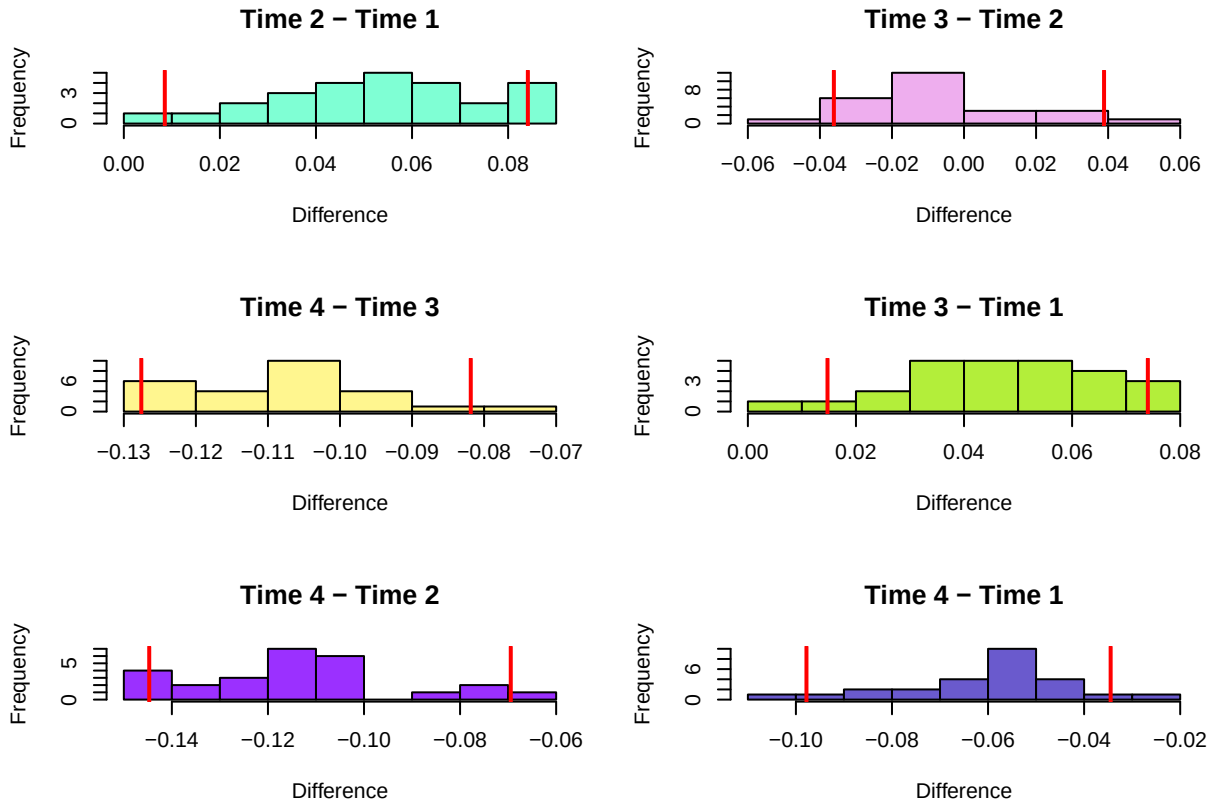




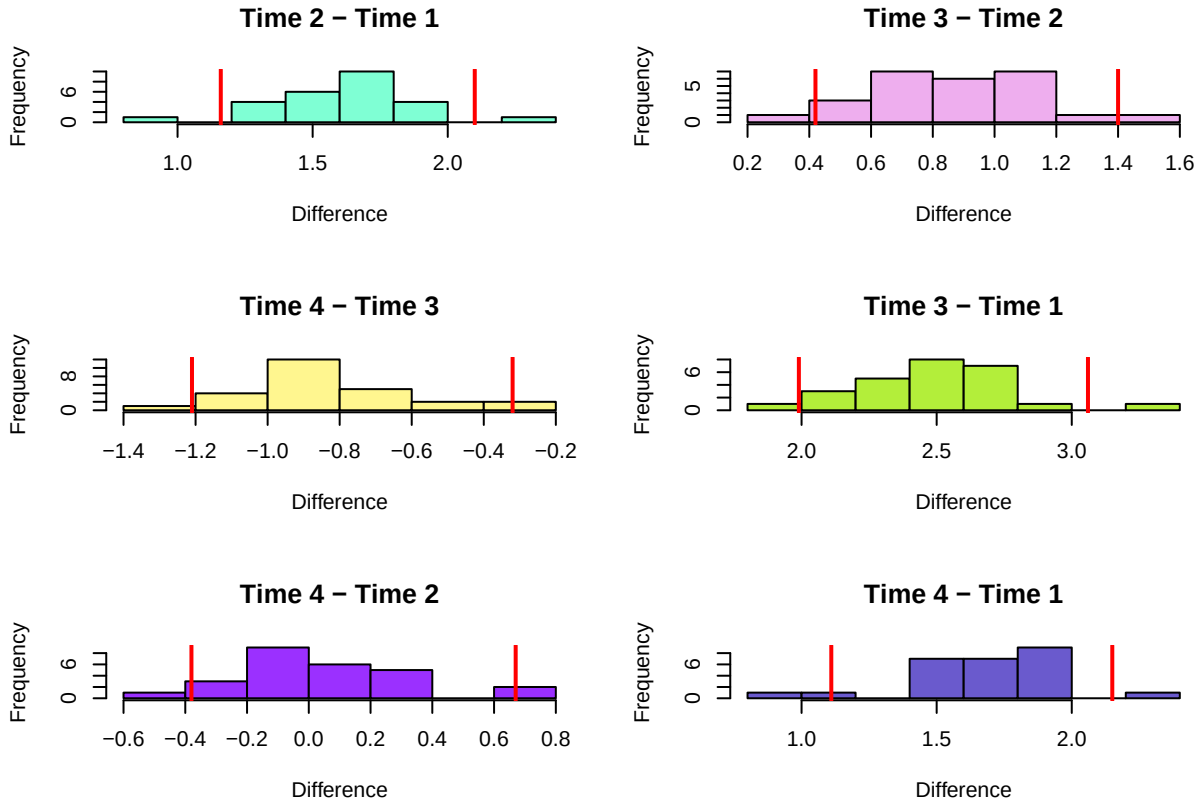
Jackknife distributions of the differences in network statistics between time points

The vertical red lines indicate the middle 95% of the distributions.

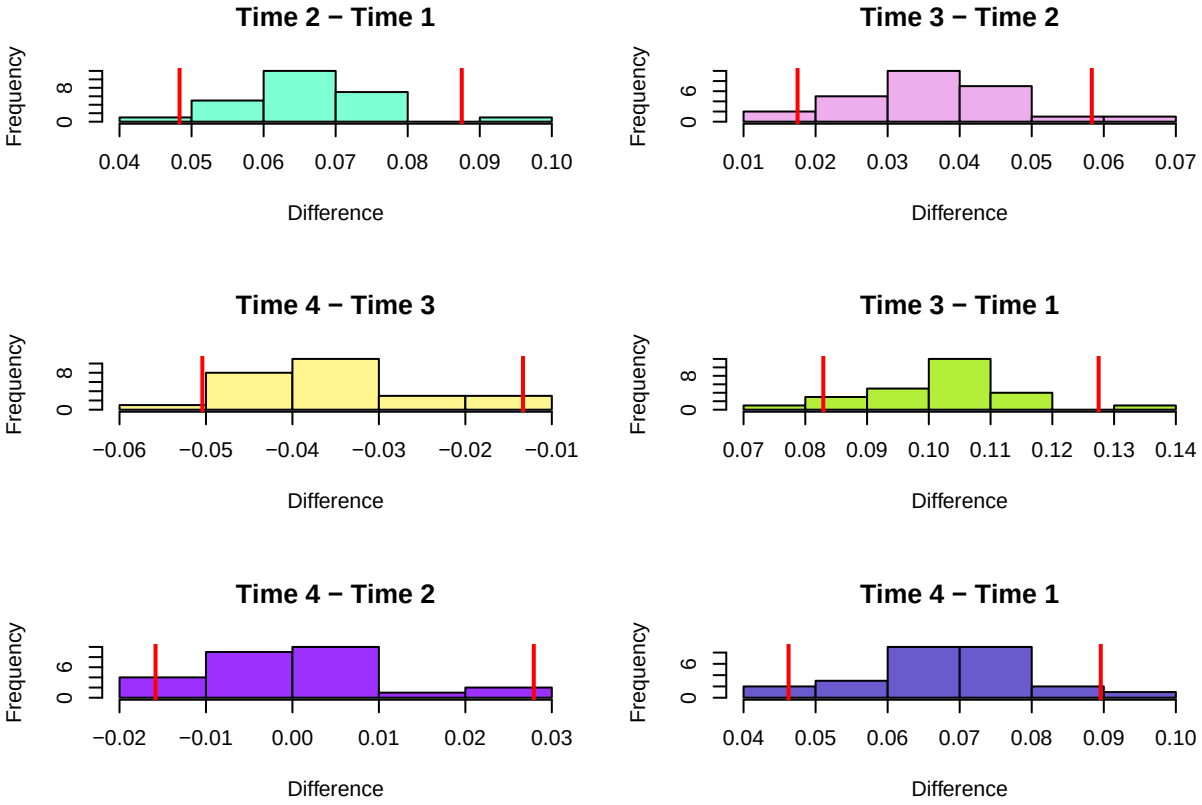
Transitivity:



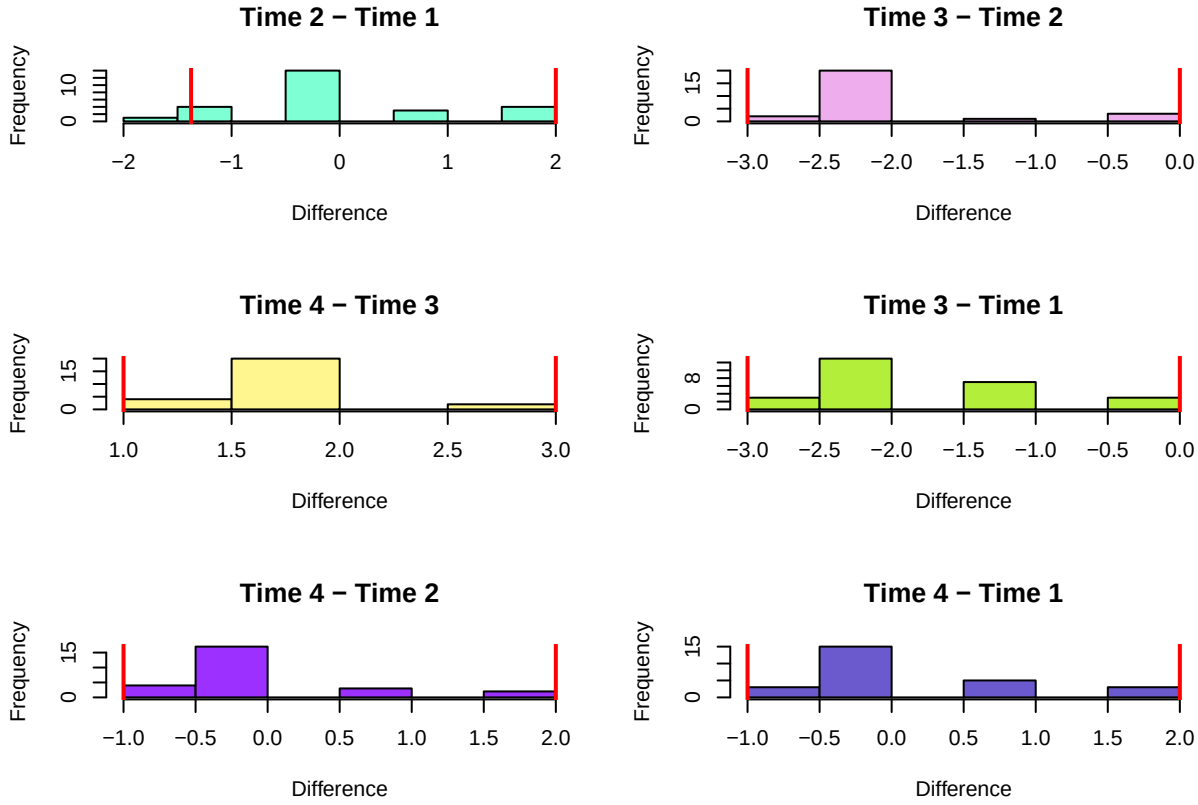
Mean Degree:



Edge Density:



Number of nodes in k-cores where k > 1:



If we assume that the distributions are normal, do we see significance in the same places?

Bootstrap distributions:

Transitivity:

```
##
##  Paired t-test
##
## data:  tboot_f2 and tboot_f1
## t = 59.485, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.08170949 0.08728438
## sample estimates:
## mean of the differences
##              0.08449693
```

```
##
##  Paired t-test
##
## data:  tboot_f3 and tboot_f2
## t = 73.38, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.06655129 0.07020852
```

```
## sample estimates:
## mean of the differences
##              0.06837991
```

```
##
##  Paired t-test
##
## data:  tboot_f4 and tboot_f3
## t = -144.96, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1402105 -0.1364652
## sample estimates:
## mean of the differences
##              -0.1383378
```

```
##
##  Paired t-test
##
## data:  tboot_f3 and tboot_f1
## t = 116.62, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1503045 0.1554492
## sample estimates:
## mean of the differences
##               0.1528768
```

```
##
##  Paired t-test
##
## data:  tboot_f4 and tboot_f2
## t = -64.355, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.07209112 -0.06782473
## sample estimates:
## mean of the differences
##             -0.06995793
```

```
##
##  Paired t-test
##
## data:  tboot_f4 and tboot_f1
## t = 10.39, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.01179303 0.01728499
## sample estimates:
## mean of the differences
##              0.01453901
```

Mean Degree:

```
##
##  Paired t-test
```

```
##
## data:  mboot_f2 and mboot_f1
## t = 565.61, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  2.756710 2.775905
## sample estimates:
## mean of the differences
##                2.766308

##
##  Paired t-test
##
## data:  mboot_f3 and mboot_f2
## t = 436.45, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  2.087036 2.105888
## sample estimates:
## mean of the differences
##                2.096462

##
##  Paired t-test
##
## data:  mboot_f4 and mboot_f3
## t = -494.11, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -2.417640 -2.398513
## sample estimates:
## mean of the differences
##               -2.408077

##
##  Paired t-test
##
## data:  mboot_f3 and mboot_f1
## t = 1031.3, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  4.853517 4.872022
## sample estimates:
## mean of the differences
##                4.862769

##
##  Paired t-test
##
## data:  mboot_f4 and mboot_f2
## t = -63.746, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.3212080 -0.3020227
## sample estimates:
## mean of the differences
```

```
##              -0.3116154
```

```
##
##  Paired t-test
##
## data:  mboot_f4 and mboot_f1
## t = 506.65, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  2.445185 2.464200
## sample estimates:
## mean of the differences
##                2.454692
```

Edge Density:

```
##
##  Paired t-test
##
## data:  dboot_f2 and dboot_f1
## t = 565.61, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1102684 0.1110362
## sample estimates:
## mean of the differences
##               0.1106523
```

```
##
##  Paired t-test
##
## data:  dboot_f3 and dboot_f2
## t = 436.45, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.08348142 0.08423550
## sample estimates:
## mean of the differences
##              0.08385846
```

```
##
##  Paired t-test
##
## data:  dboot_f4 and dboot_f3
## t = -494.11, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.09670562 -0.09594053
## sample estimates:
## mean of the differences
##             -0.09632308
```

```
##
##  Paired t-test
##
## data:  dboot_f3 and dboot_f1
```

```
## t = 1031.3, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1941407 0.1948809
## sample estimates:
## mean of the differences
##               0.1945108
```

```
##
##  Paired t-test
##
## data:  dboot_f4 and dboot_f2
## t = -63.746, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.01284832 -0.01208091
## sample estimates:
## mean of the differences
##             -0.01246462
```

```
##
##  Paired t-test
##
## data:  dboot_f4 and dboot_f1
## t = 506.65, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.09780739 0.09856799
## sample estimates:
## mean of the differences
##              0.09818769
```

Number of nodes in k-cores where k > 1:

Data are essentially constant.

Jackknife distributions:

Transitivity:

```
##
##  Paired t-test
##
## data:  tjack_f2 and tjack_f1
## t = 12.633, df = 25, p-value = 2.367e-12
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.04406672 0.06123334
## sample estimates:
## mean of the differences
##              0.05265003
```

```
##
##  Paired t-test
##
## data:  tjack_f3 and tjack_f2
## t = -1.4429, df = 25, p-value = 0.1615
```

```
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.014285086  0.002515222
## sample estimates:
## mean of the differences
##            -0.005884932

##
##  Paired t-test
##
## data:  tjack_f4 and tjack_f3
## t = -42.731, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1128502 -0.1024720
## sample estimates:
## mean of the differences
##              -0.1076611

##
##  Paired t-test
##
## data:  tjack_f3 and tjack_f1
## t = 13.553, df = 25, p-value = 5.066e-13
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.03965868 0.05387152
## sample estimates:
## mean of the differences
##               0.0467651

##
##  Paired t-test
##
## data:  tjack_f4 and tjack_f2
## t = -26.577, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1223449 -0.1047471
## sample estimates:
## mean of the differences
##               -0.113546

##
##  Paired t-test
##
## data:  tjack_f4 and tjack_f1
## t = -18.075, df = 25, p-value = 7.252e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.06783477 -0.05395722
## sample estimates:
## mean of the differences
##             -0.06089599
```

Mean Degree:

```
##
##  Paired t-test
##
## data:  mjack_f2 and mjack_f1
## t = 31.006, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  1.519576 1.735809
## sample estimates:
## mean of the differences
##                1.627692
```

```
##
##  Paired t-test
##
## data:  mjack_f3 and mjack_f2
## t = 16.671, df = 25, p-value = 4.713e-15
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.7739841 0.9921698
## sample estimates:
## mean of the differences
##               0.8830769
```

```
##
##  Paired t-test
##
## data:  mjack_f4 and mjack_f3
## t = -16.873, df = 25, p-value = 3.572e-15
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.9114576 -0.7131578
## sample estimates:
## mean of the differences
##              -0.8123077
```

```
##
##  Paired t-test
##
## data:  mjack_f3 and mjack_f1
## t = 42.761, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  2.389840 2.631699
## sample estimates:
## mean of the differences
##                2.510769
```

```
##
##  Paired t-test
##
## data:  mjack_f4 and mjack_f2
## t = 1.3559, df = 25, p-value = 0.1872
## alternative hypothesis: true difference in means is not equal to 0
```

```
## 95 percent confidence interval:
##  -0.03672164  0.17826010
## sample estimates:
## mean of the differences
##              0.07076923
```

```
##
##  Paired t-test
##
## data:  mjack_f4 and mjack_f1
## t = 30.712, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  1.584564 1.812359
## sample estimates:
## mean of the differences
##                1.698462
```

Edge Density:

```
##
##  Paired t-test
##
## data:  djack_f2 and djack_f1
## t = 31.006, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.06331565 0.07232537
## sample estimates:
## mean of the differences
##              0.06782051
```

```
##
##  Paired t-test
##
## data:  djack_f3 and djack_f2
## t = 16.671, df = 25, p-value = 4.713e-15
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.03224934 0.04134041
## sample estimates:
## mean of the differences
##              0.03679487
```

```
##
##  Paired t-test
##
## data:  djack_f4 and djack_f3
## t = -16.873, df = 25, p-value = 3.572e-15
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.03797740 -0.02971491
## sample estimates:
## mean of the differences
##             -0.03384615
```

```
##
##  Paired t-test
##
## data:  djack_f3 and djack_f1
## t = 42.761, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.09957665 0.10965412
## sample estimates:
## mean of the differences
##               0.1046154
```

```
##
##  Paired t-test
##
## data:  djack_f4 and djack_f2
## t = 1.3559, df = 25, p-value = 0.1872
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.001530068  0.007427504
## sample estimates:
## mean of the differences
##             0.002948718
```

```
##
##  Paired t-test
##
## data:  djack_f4 and djack_f1
## t = 30.712, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.06602350 0.07551497
## sample estimates:
## mean of the differences
##              0.07076923
```

Number of nodes in k-cores where k $>$ 1:

```
##
##  Paired t-test
##
## data:  kjack_f2 and kjack_f1
## t = 0.96083, df = 25, p-value = 0.3458
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.2199046  0.6045200
## sample estimates:
## mean of the differences
##               0.1923077
```

```
##
##  Paired t-test
##
## data:  kjack_f3 and kjack_f2
## t = -12.3, df = 25, p-value = 4.22e-12
## alternative hypothesis: true difference in means is not equal to 0
```

```
## 95 percent confidence interval:
##  -2.110365 -1.505020
## sample estimates:
## mean of the differences
##               -1.807692

##
##  Paired t-test
##
## data:  kjack_f4 and kjack_f3
## t = 20.278, df = 25, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  1.727756 2.118398
## sample estimates:
## mean of the differences
##                1.923077

##
##  Paired t-test
##
## data:  kjack_f3 and kjack_f1
## t = -9.666, df = 25, p-value = 6.336e-10
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.959574 -1.271195
## sample estimates:
## mean of the differences
##               -1.615385

##
##  Paired t-test
##
## data:  kjack_f4 and kjack_f2
## t = 0.76847, df = 25, p-value = 0.4494
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1938507  0.4246199
## sample estimates:
## mean of the differences
##               0.1153846

##
##  Paired t-test
##
## data:  kjack_f4 and kjack_f1
## t = 1.8732, df = 25, p-value = 0.07278
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.03061314  0.64599776
## sample estimates:
## mean of the differences
##               0.3076923
```

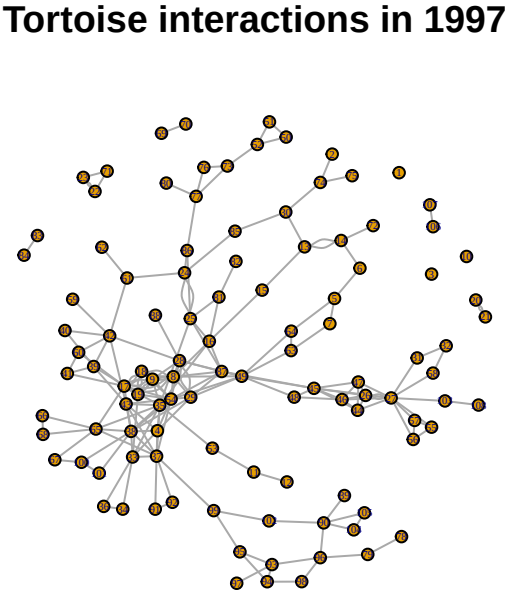
Tortoise Interactions

Data source:

- **The Network Data Repository with Interactive Graph Analytics and Visualization**
- Ryan A. Rossi and Nesreen K. Ahmed
- *AAAI*
- http://networkrepository.com/
- 2015

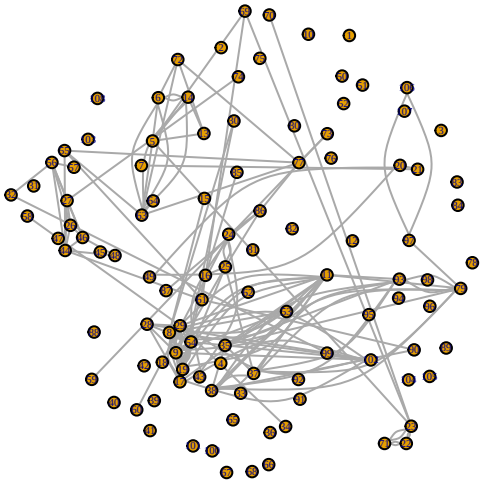
Original Tortoise Networks

Note: I omitted the network data from the year 1996 because there were only 7 data points, and because I reduced each network to only contain the intersection of nodes, using 1996 would have resulted in too few tortoises.



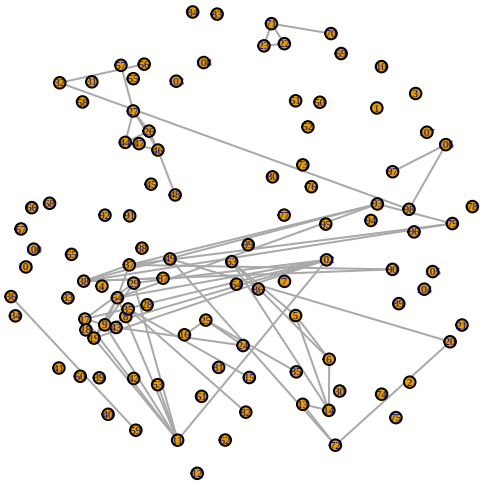
Transitivity: 0.4131148, Mean Degree: 3.7962963, Edge Density: 0.0354794, Number of nodes in a k-core where k > 1: 81

Tortoise interactions in 1998



Transitivity: 0.533101, Mean Degree: 2.7962963, Edge Density: 0.0261336, Number of nodes in a k-core where k $>$ 1: 45

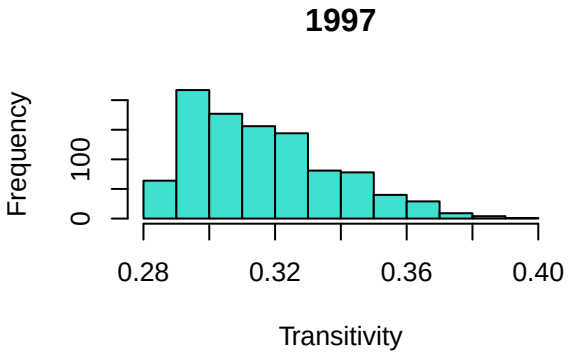
Tortoise interactions in 1999

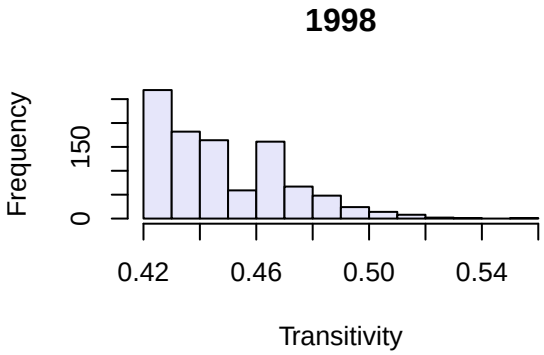


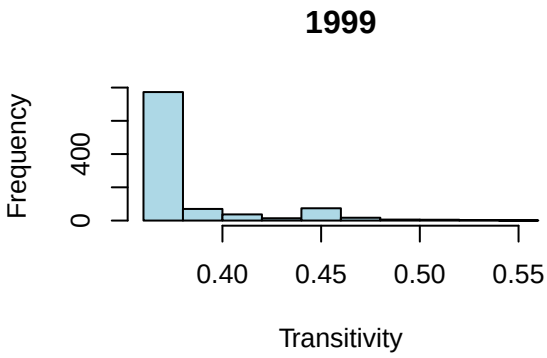
Transitivity: 0.6325301, Mean Degree: 1.1666667, Edge Density: 0.0109034, Number of nodes in a k-core where k $>$ 1: 27

Vertex Bootstrapped Distributions of Network Statistics

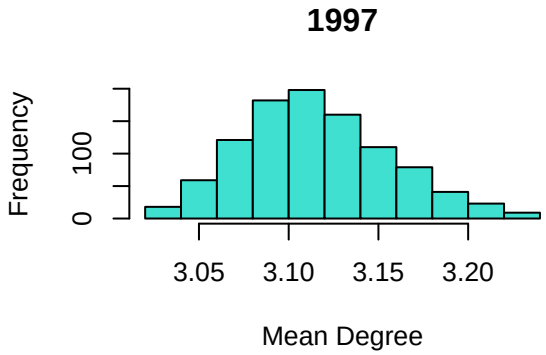
Transitivity:

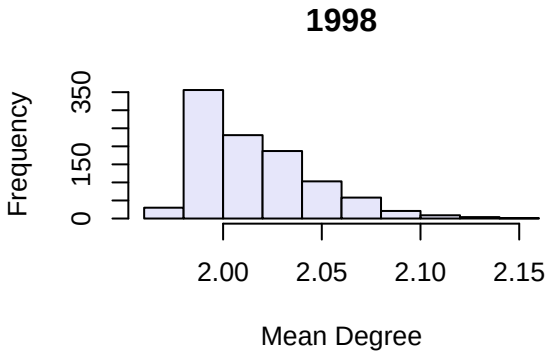


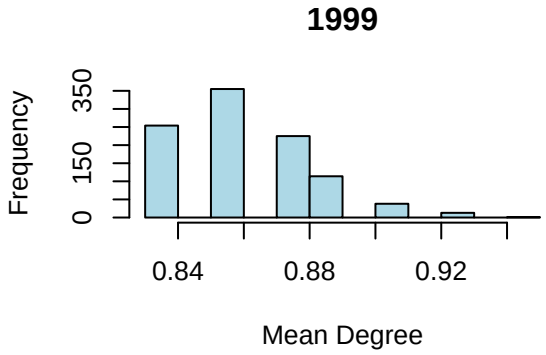




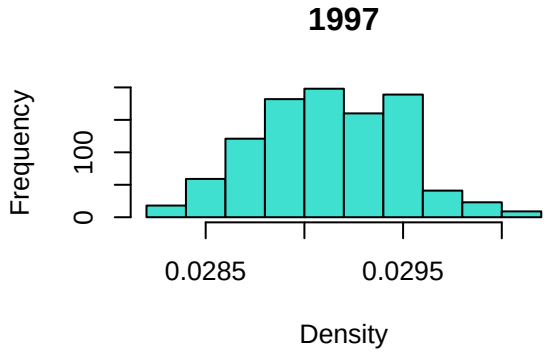
Mean Degree:

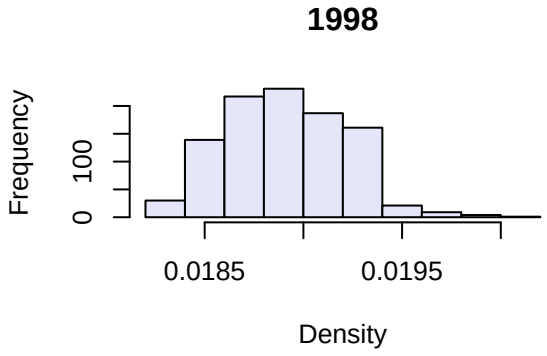


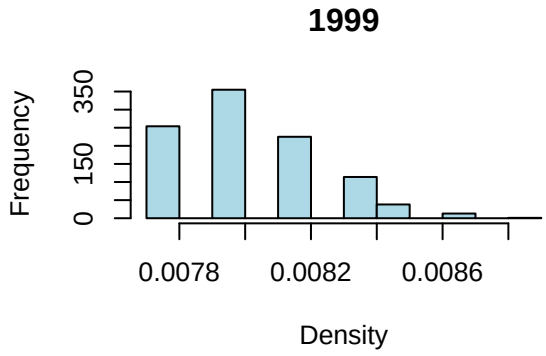




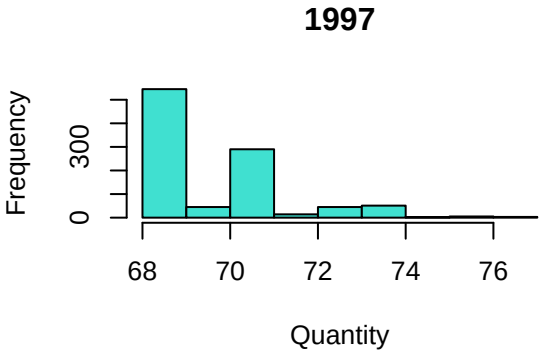
Edge Density:

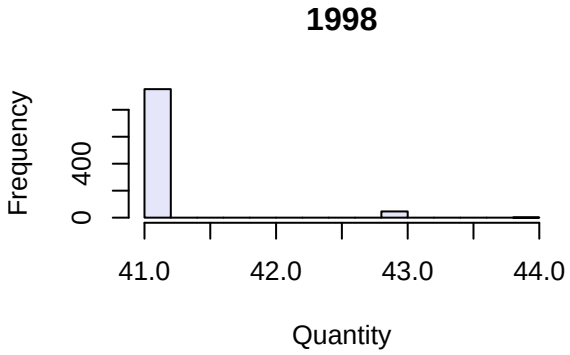


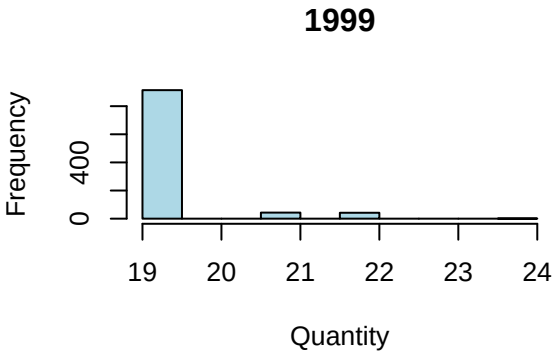




Number of nodes in k-cores where k > 1:

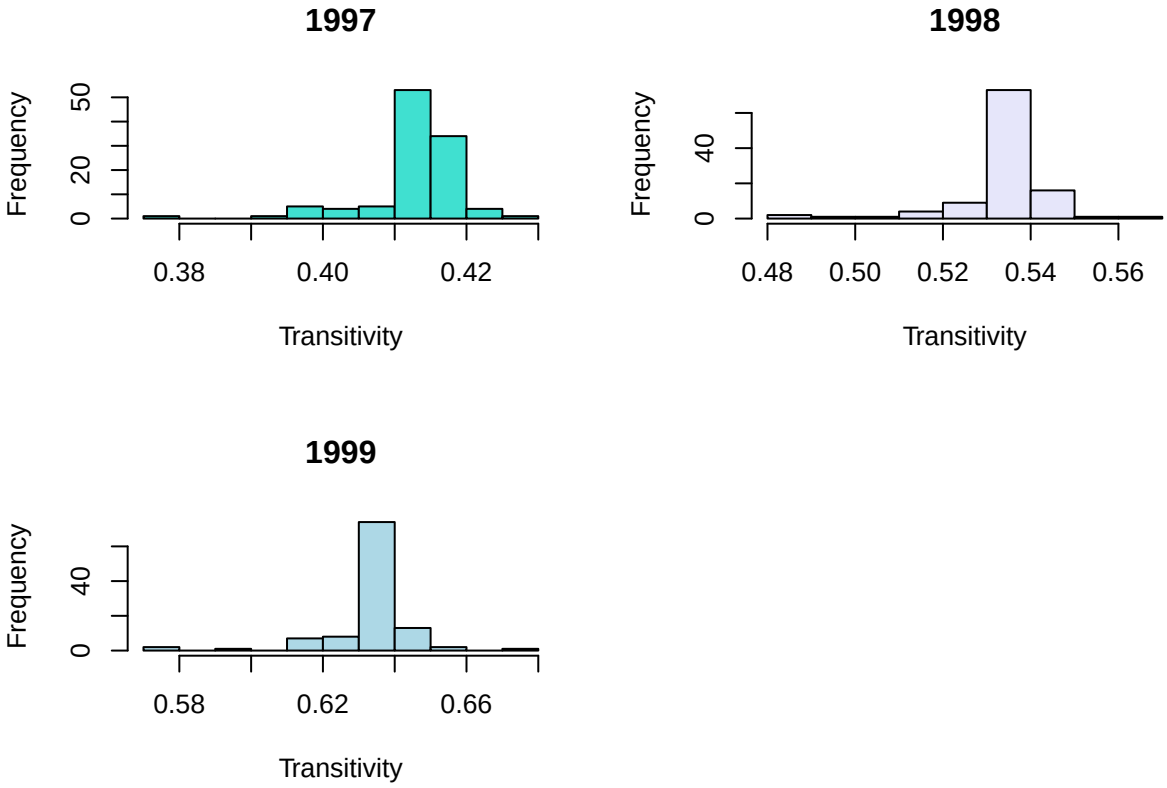




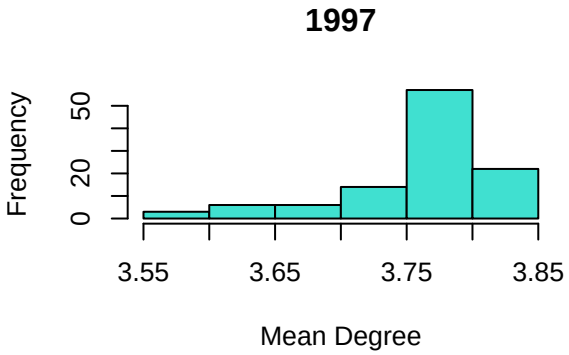


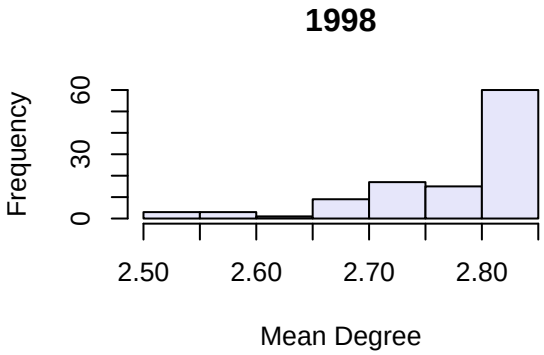
Vertex Jackknife Distributions of Network Statistics

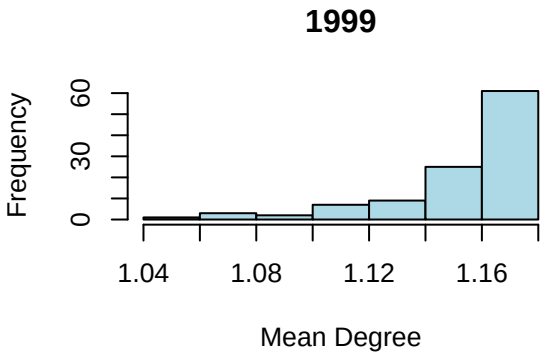
Transitivity:



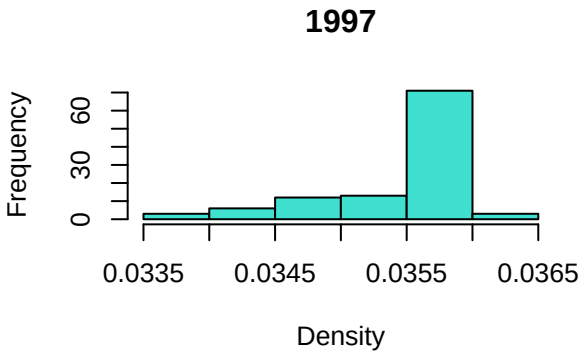
Mean Degree:

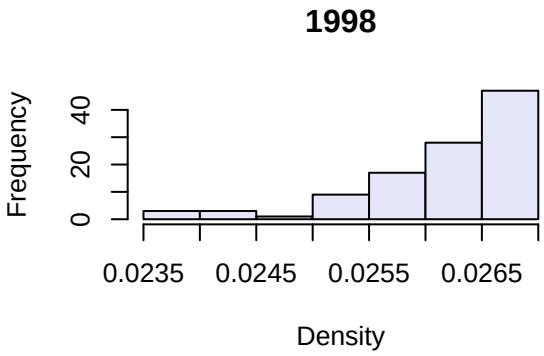


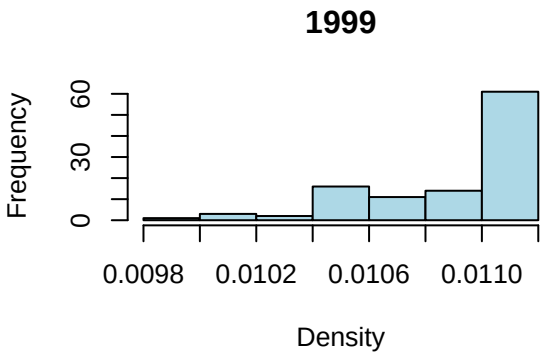




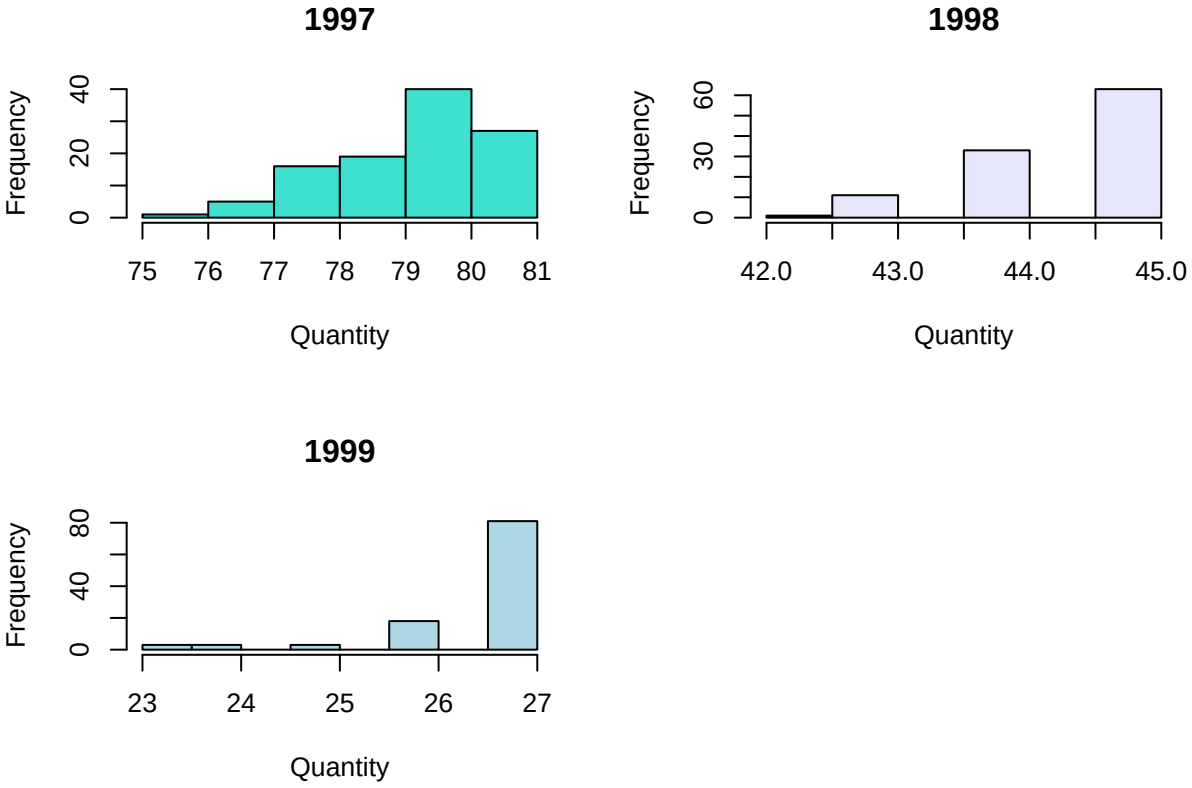
Edge Density:







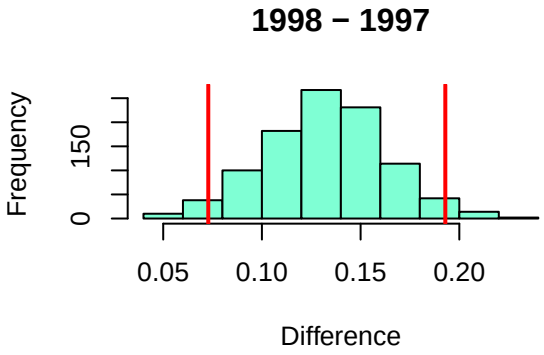
Number of nodes in k-cores where k > 1:

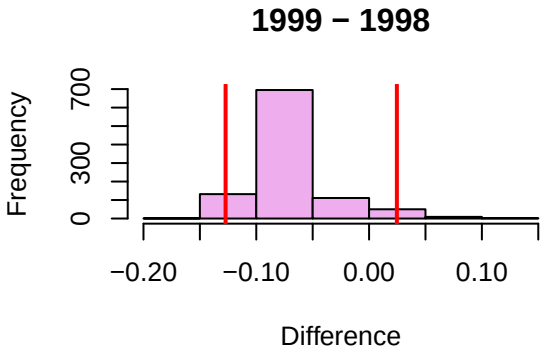


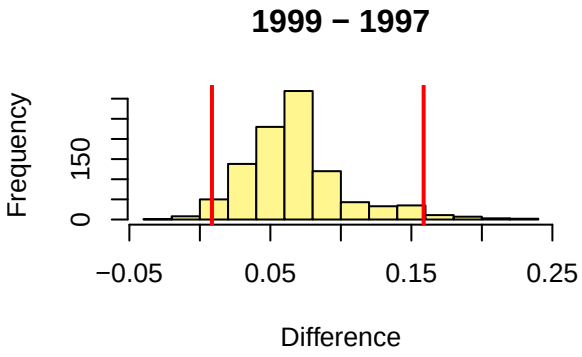
Bootstrap distributions of the differences in network statistics between time points

The vertical red lines indicate the middle 95% of the distributions.

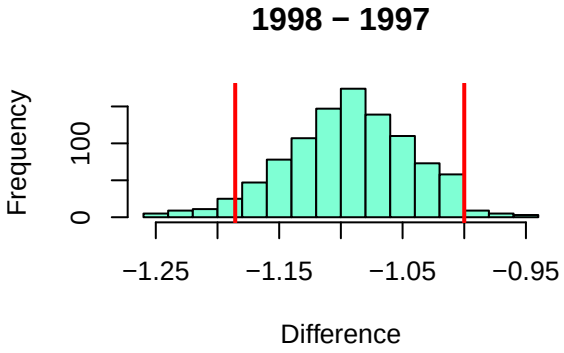
Transitivity:

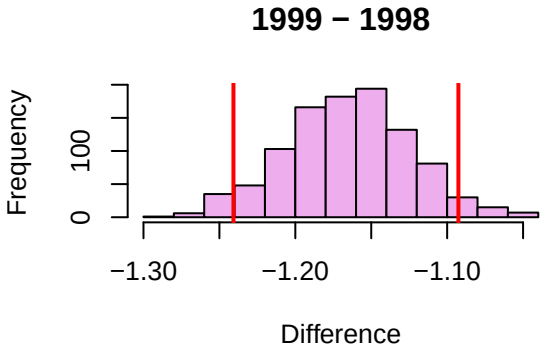


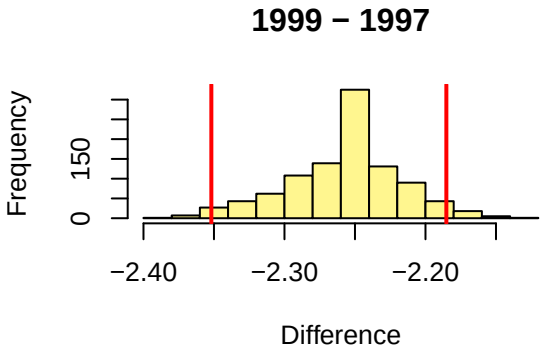




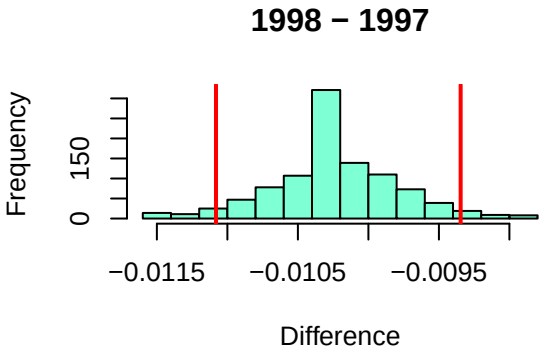
Mean Degree:

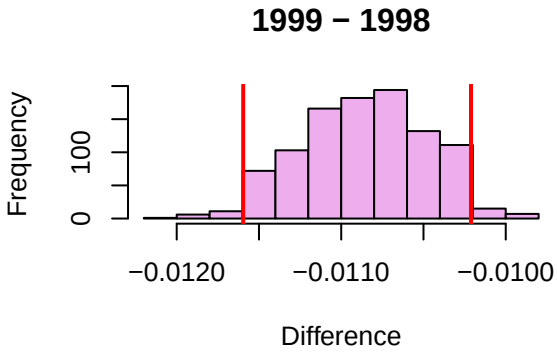


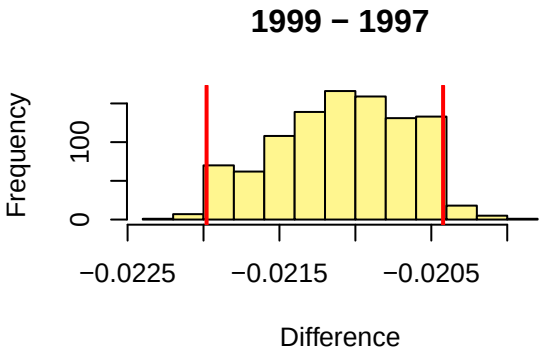




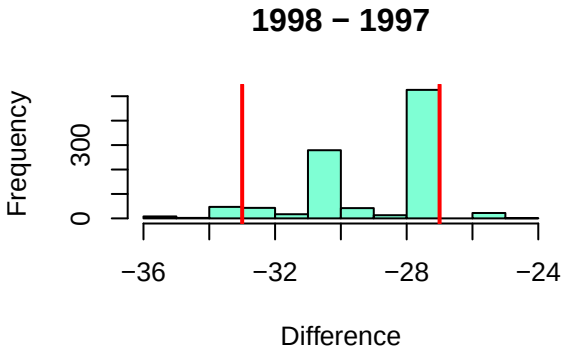
Edge Density:

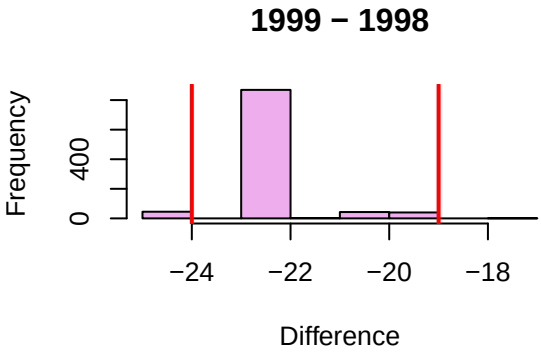


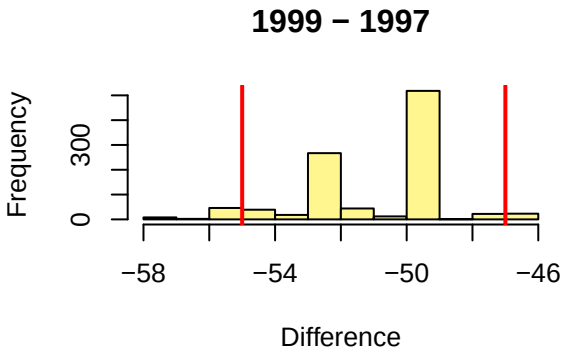




Number of nodes in k-cores where k $>$ 1:



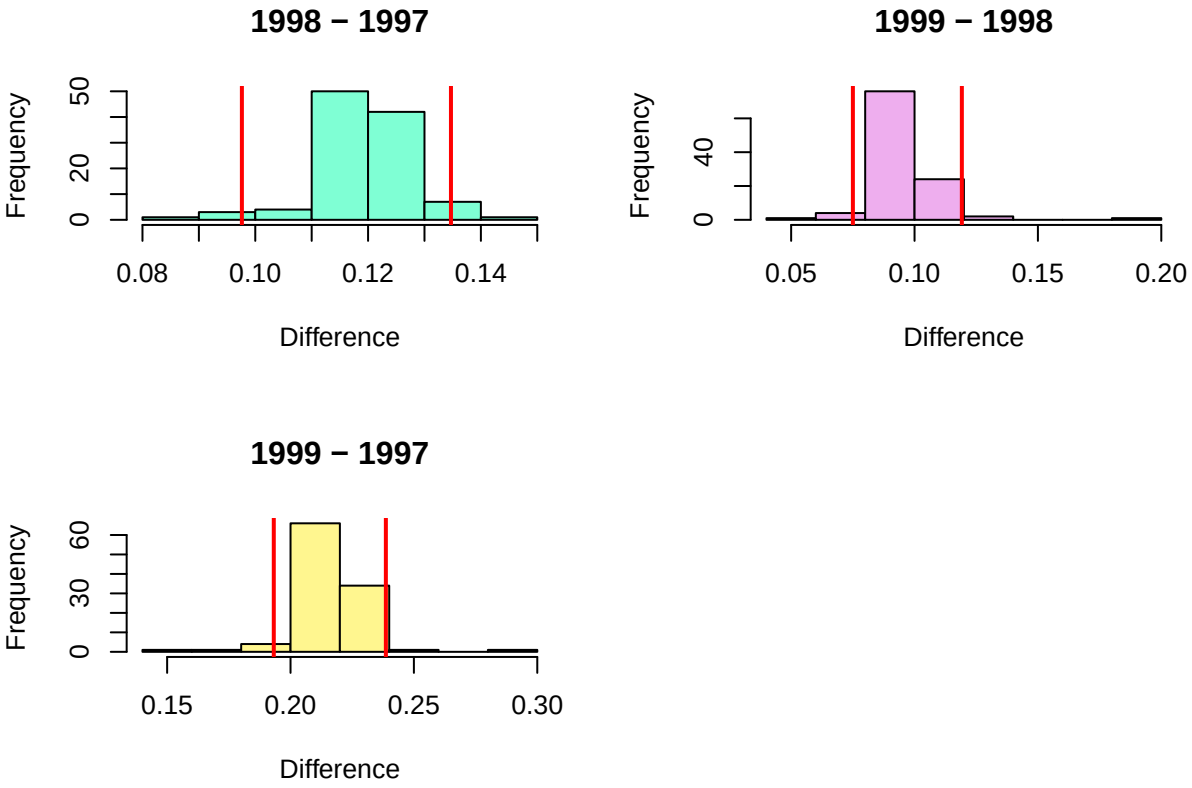




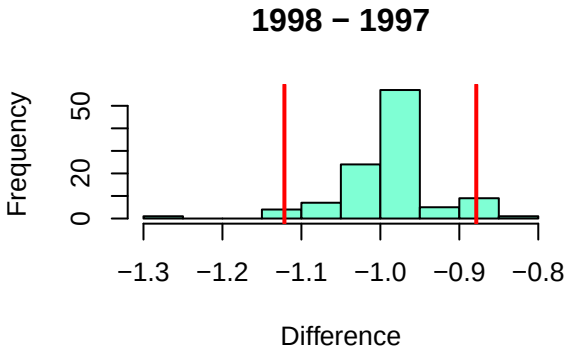
Jackknife distributions of the differences in network statistics between time points

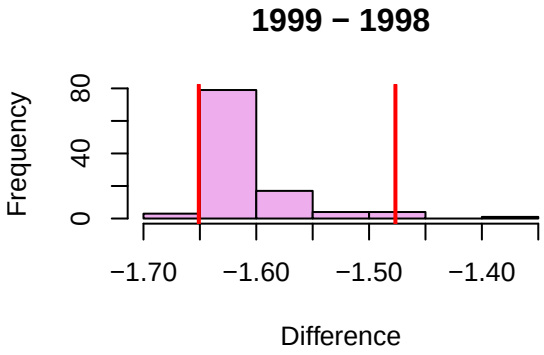
The vertical red lines indicate the middle 95% of the distributions.

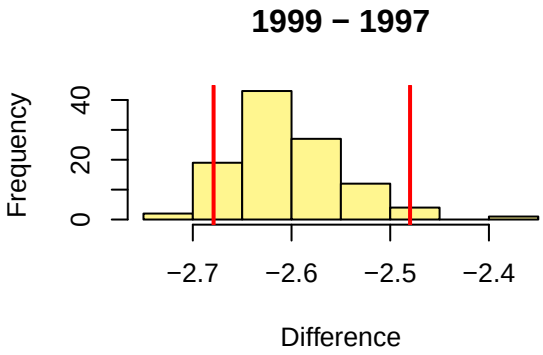
Transitivity:



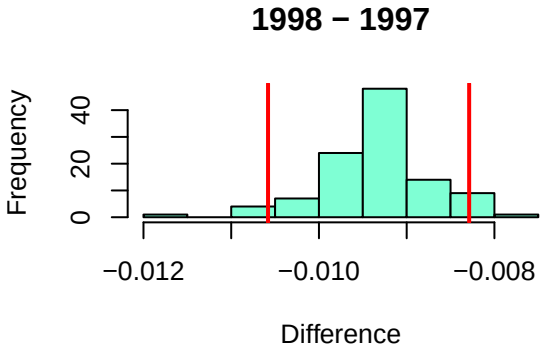
Mean Degree:

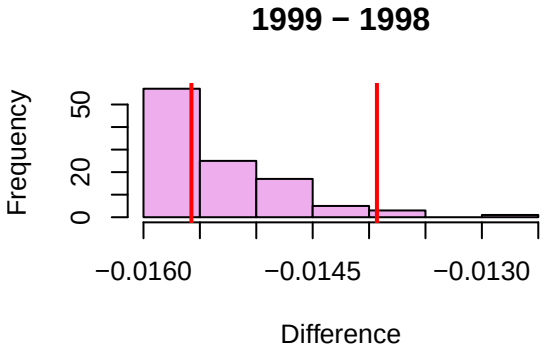


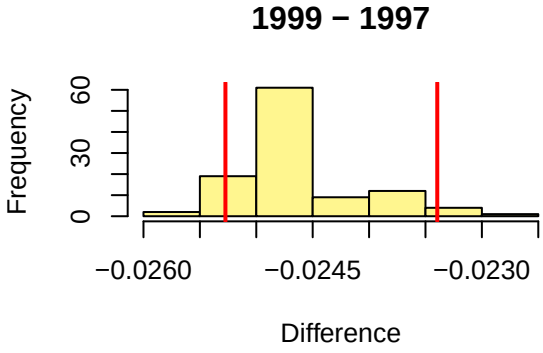




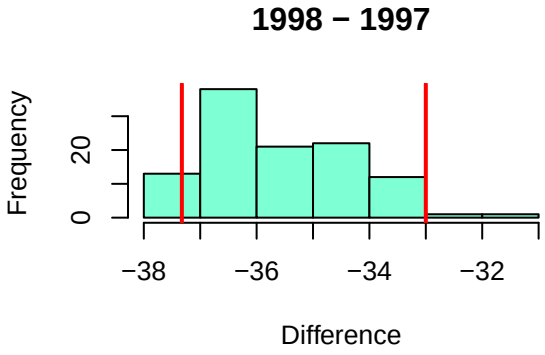
Edge Density:

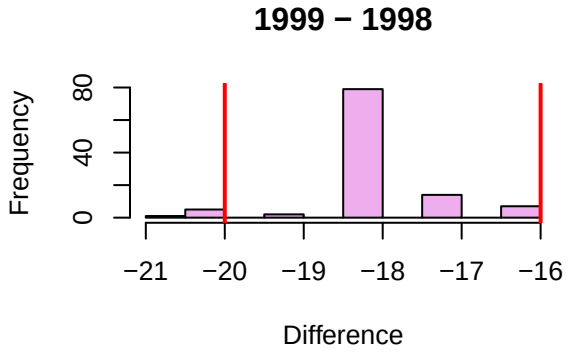


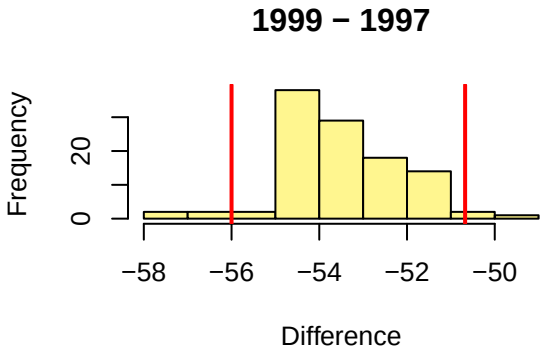




Number of nodes in k-cores where k > 1:







If we assume that the distributions are normal, do we see significance in the same places?

Bootstrap distributions:

Transitivity:

```
##
##  Paired t-test
##
## data:  tboot_t98 and tboot_t97
## t = 137.29, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1304898 0.1342741
## sample estimates:
## mean of the differences
##               0.1323819
```

```
##
##  Paired t-test
##
## data:  tboot_t99 and tboot_t98
## t = -56.01, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.06679573 -0.06227371
## sample estimates:
```

```
## mean of the differences
##             -0.06453472
```

```
##
##  Paired t-test
##
## data:  tboot_t99 and tboot_t97
## t = 59.477, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.06560874 0.07008571
## sample estimates:
## mean of the differences
##              0.06784722
```

Mean Degree:

```
##
##  Paired t-test
##
## data:  mboot_t98 and mboot_t97
## t = -712.13, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.098110 -1.092075
## sample estimates:
## mean of the differences
##               -1.095093
```

```
##
##  Paired t-test
##
## data:  mboot_t99 and mboot_t98
## t = -941.58, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.164867 -1.160022
## sample estimates:
## mean of the differences
##               -1.162444
```

```
##
##  Paired t-test
##
## data:  mboot_t99 and mboot_t97
## t = -1621.8, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -2.260269 -2.254806
## sample estimates:
## mean of the differences
##               -2.257537
```

Edge Density:

```
##
```

```
##  Paired t-test
##
## data:  dboot_t98 and dboot_t97
## t = -712.13, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.01026271 -0.01020631
## sample estimates:
## mean of the differences
##             -0.01023451
```

```
##
##  Paired t-test
##
## data:  dboot_t99 and dboot_t98
## t = -941.58, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.01088661 -0.01084133
## sample estimates:
## mean of the differences
##             -0.01086397
```

```
##
##  Paired t-test
##
## data:  dboot_t99 and dboot_t97
## t = -1621.8, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.02112401 -0.02107295
## sample estimates:
## mean of the differences
##             -0.02109848
```

Number of nodes in k-cores where k $>$ 1:

```
##
##  Paired t-test
##
## data:  kboot_t98 and kboot_t97
## t = -443.29, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -28.65931 -28.40669
## sample estimates:
## mean of the differences
##                 -28.533
```

```
##
##  Paired t-test
##
## data:  kboot_t99 and kboot_t98
## t = -809.83, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
```

```
##  -21.93101 -21.82499
## sample estimates:
## mean of the differences
##                 -21.878
```

```
##
##  Paired t-test
##
## data:  kboot_t99 and kboot_t97
## t = -753.11, df = 999, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -50.54235 -50.27965
## sample estimates:
## mean of the differences
##                 -50.411
```

Jackknife distributions:

Transitivity:

```
##
##  Paired t-test
##
## data:  tjack_t98 and tjack_t97
## t = 148.2, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.1182426 0.1214489
## sample estimates:
## mean of the differences
##               0.1198457
```

```
##
##  Paired t-test
##
## data:  tjack_t99 and tjack_t98
## t = 82.301, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.09701859 0.10180769
## sample estimates:
## mean of the differences
##              0.09941314
```

```
##
##  Paired t-test
##
## data:  tjack_t99 and tjack_t97
## t = 165.97, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.2166400 0.2218778
## sample estimates:
## mean of the differences
##               0.2192589
```

Mean Degree:

```
##
##  Paired t-test
##
## data:  mjack_t98 and mjack_t97
## t = -166.42, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.0024545 -0.9788539
## sample estimates:
## mean of the differences
##              -0.9906542
```

```
##
##  Paired t-test
##
## data:  mjack_t99 and mjack_t98
## t = -332.62, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.624021 -1.604778
## sample estimates:
## mean of the differences
##               -1.614399
```

```
##
##  Paired t-test
##
## data:  mjack_t99 and mjack_t97
## t = -490.44, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -2.615583 -2.594524
## sample estimates:
## mean of the differences
##               -2.605054
```

Edge Density:

```
##
##  Paired t-test
##
## data:  djack_t98 and djack_t97
## t = -166.42, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.009457118 -0.009234471
## sample estimates:
## mean of the differences
##            -0.009345794
```

```
##
##  Paired t-test
##
## data:  djack_t99 and djack_t98
```

```
## t = -332.62, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.01532095 -0.01513941
## sample estimates:
## mean of the differences
##             -0.01523018
```

```
##
##  Paired t-test
##
## data:  djack_t99 and djack_t97
## t = -490.44, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.02467531 -0.02447664
## sample estimates:
## mean of the differences
##             -0.02457598
```

Number of nodes in k-cores where k > 1:

```
##
##  Paired t-test
##
## data:  kjack_t98 and kjack_t97
## t = -268.25, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -35.38924 -34.87002
## sample estimates:
## mean of the differences
##               -35.12963
```

```
##
##  Paired t-test
##
## data:  kjack_t99 and kjack_t98
## t = -227.41, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -18.03549 -17.72377
## sample estimates:
## mean of the differences
##               -17.87963
```

```
##
##  Paired t-test
##
## data:  kjack_t99 and kjack_t97
## t = -390.46, df = 107, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -53.27839 -52.74013
## sample estimates:
## mean of the differences
```

```
##                -53.00926
```