Tables comparing observed network statistics at different time points with 95% intervals for bootstrap/jackknife differences and 95% intervals estimating the statistic using sampling by degree sequence

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7/20/2021

Friendship Network

Time points 1 and 2

| Statistic | T1 | T2 | T2 - T1 | 95% difference interval |
|-------------------|-----------------------|-----------------------|---------|-------------------------|
| Transitivity | 0.508 | 0.561 | 0.053 | boot: [-0.01, 0.163] |
| Transitivity | DSE: [0.185, 0.323] | DSE: [0.321, 0.417] | 0.055 | jack: [0.009, 0.084] |
| Mean Degree | 4.846 | 6.538 | 1.692 | boot: [2.462, 3.077] |
| Mean Degree | 4.040 | 0.000 | 1.092 | jack: [1.16, 2.1] |
| Edge Density | 0.194 | 0.262 | 0.068 | boot: [0.098, 0.123] |
| Edge Delisity | 0.134 | 0.202 | 0.000 | jack: [0.048, 0.088] |
| Coreness, $k > 1$ | 25 | 25 | 0 | boot: [2, 4] |
| Coreness, K > 1 | DSE: [24, 25] | DSE: [24, 25] | U | jack: [-1.375, 2] |

Table 1: Note: DSE indicates the 95% C.I. of the statistic as estimated by resampling the network 1000 times using its degree sequence.

Time points 2 and 3

| Statistic | T2 | T3 | T3 - T2 | 95% difference interval |
|-------------------|-----------------------|-----------------------|---------|-------------------------|
| Transitivity | 0.561 | 0.554 | -0.007 | boot: [0.011, 0.125] |
| Transitivity | DSE: [0.321, 0.417] | DSE: [0.418, 0.497] | -0.007 | jack: [-0.036, 0.039] |
| Mean Degree | 6.538 | 7.462 | 0.923 | boot: [1.769, 2.385] |
| Mean Degree | 0.000 | 1.402 | 0.923 | jack: [0.42, 1.4] |
| Edge Density | 0.262 | 0.298 | 0.037 | boot: [0.071, 0.095] |
| Edge Density | 0.202 | 0.290 | 0.037 | jack: [0.017, 0.058] |
| Coreness, $k > 1$ | 25 | 23 | -2 | boot: [-1, -1] |
| Coronoss, K > 1 | DSE: [24, 25] | DSE: [22, 23] | | jack: [-3, 0] |

Time points 3 and 4

| Statistic | T3 | T4 | T4 - T3 | 95% difference interval |
|-------------------|-----------------------|----------------------|---------|--------------------------|
| Transitivity | 0.554 | 0.447 | -0.108 | boot: [-0.196, -0.082] |
| Transitivity | DSE: [0.418, 0.497] | DSE: [0.32, 0.409] | -0.108 | jack: [-0.128, -0.082] |
| Maan Dagraa | 7.462 | 6.615 | -0.846 | boot: [-2.692, -2.152] |
| Mean Degree | 1.402 | 0.010 | -0.040 | jack: [-1.21, -0.32] |
| Edma Danaitus | 0.298 | 0.265 | -0.034 | boot: [-0.108, -0.086] |
| Edge Density | 0.298 | 0.200 | -0.034 | jack: [-0.05, -0.013] |
| Command le > 1 | 23 | 25 | 2 | boot: [1, 1] |
| Coreness, $k > 1$ | DSE: [22, 23] | DSE: [25, 25] | 2 | jack: [1, 3] |

Time points 1 and 3 $\,$

| Statistic | T1 | T3 | T3 - T1 | 95% difference interval |
|-------------------|-----------------------|-----------------------|---------|-------------------------|
| m | 0.508 | 0.554 | 0.047 | boot: [0.067, 0.224] |
| Transitivity | DSE: [0.185, 0.323] | DSE: [0.418, 0.497] | 0.047 | jack: [0.015, 0.074] |
| Mean Degree | 4.846 | 7.462 | 2.615 | boot: [4.538, 5.154] |
| Mean Degree | 4.040 | 1.402 | 2.015 | jack: [1.99, 3.06] |
| Edge Density | 0.194 | 0.298 | 0.105 | boot: [0.182, 0.206] |
| Edge Density | 0.194 | 0.296 | 0.103 | jack: [0.083, 0.127] |
| Coreness, $k > 1$ | 25 | 23 | -2 | boot: [1, 3] |
| Coreness, k > 1 | DSE: [24, 25] | DSE: [22, 23] | -2 | jack: [-3, 0] |

Time points 2 and 4

| Statistic | T2 | T4 | T4 - T2 | 95% difference interval |
|-------------------|-----------------------|----------------------|---------|--------------------------|
| Transitivity | 0.561 | 0.447 | -0.114 | boot: [-0.134, -0.005] |
| Transmiving | DSE: [0.321, 0.417] | DSE: [0.32, 0.409] | -0.114 | jack: [-0.145, -0.069] |
| Moon Dogress | 6.538 | 6.615 | 0.077 | boot: [-0.615, 0] |
| Mean Degree | 0.000 | 0.013 | 0.077 | jack: [-0.38, 0.67] |
| Edga Dangitu | 0.262 | 0.265 | 0.003 | boot: [-0.025, 0] |
| Edge Density | 0.202 | 0.205 | 0.003 | jack: [-0.016, 0.028] |
| Command le > 1 | 25 | 25 | 0 | boot: [0, 0] |
| Coreness, $k > 1$ | DSE: [24, 25] | DSE: [25, 25] | U | jack: [-1, 2] |

Time points 1 and 4 $\,$

| Statistic | T1 | T4 | T4 - T1 | 95% difference interval |
|-------------------|-----------------------|----------------------|---------|--------------------------|
| Transitivity | 0.508 | 0.447 | -0.061 | boot: [-0.075, 0.095] |
| Transitivity | DSE: [0.185, 0.323] | DSE: [0.32, 0.409] | -0.001 | jack: [-0.098, -0.034] |
| Mean Degree | 4.846 | 6.615 | 1.769 | boot: [2.154, 2.769] |
| Mean Degree | 4.040 | 0.013 | 1.709 | jack: [1.11, 2.15] |
| Edge Density | 0.194 | 0.265 | 0.071 | boot: [0.086, 0.111] |
| Edge Density | 0.134 | 0.200 | 0.071 | jack: [0.046, 0.09] |
| Coreness, $k > 1$ | 25 | 25 | 0 | boot: [2, 4] |
| Coreness, k > 1 | DSE: [24, 25] | DSE: [25, 25] | 0 | jack: [-1, 2] |

Tortoise Network

1997 and 1998

| Statistic | 1997 | 1998 | 1998 - 1997 | 95% difference interval |
|-------------------|-----------------------|-----------------------|-------------|--------------------------|
| m | 0.413 | 0.533 | 0.12 | boot: [0.073, 0.193] |
| Transitivity | DSE: [0.055, 0.112] | DSE: [0.139, 0.228] | 0.12 | jack: [0.098, 0.135] |
| Mean Degree | 3.796 | 2.796 | -1 | boot: [-1.186, -1] |
| Mean Degree | 0.190 | 2.190 | -1 | jack: [-1.121, -0.879] |
| Edge Density | 0.035 | 0.026 | -0.009 | boot: [-0.011, -0.009] |
| Edge Density | 0.030 | 0.020 | -0.009 | jack: [-0.011, -0.008] |
| Coreness, $k > 1$ | 81 | 45 | -36 | boot: [-33, -27] |
| | DSE: [78, 86] | DSE: [45, 48] | | jack: [-37.325, -33] |

1998 and 1999

| Statistic | 1998 | 1999 | 1999 - 1998 | 95% difference interval |
|-------------------|-----------------------|-------------------|-------------|--------------------------|
| m | 0.533 | 0.633 | 0.000 | boot: [-0.127, 0.025] |
| Transitivity | DSE: [0.139, 0.228] | DSE: [0, 0.127] | 0.099 | jack: [0.075, 0.119] |
| Mean Degree | 2.796 | 1.167 | -1.63 | boot: [-1.241, -1.093] |
| Mean Degree | 2.130 | 1.107 | -1.03 | jack: [-1.651, -1.477] |
| Edge Density | 0.026 | 0.011 | -0.015 | boot: [-0.012, -0.01] |
| Edge Density | 0.020 | 0.011 | -0.015 | jack: [-0.016, -0.014] |
| C 1- > 1 | 45 | 27 | 10 | boot: [-24, -19] |
| Coreness, $k > 1$ | DSE: [45, 48] | DSE: [26, 33] | -18 | jack: [-20, -16] |

1997 and 1999

| Statistic | 1997 | 1999 | 1999 - 1997 | 95% difference interval |
|-------------------|-----------------------|-------------------|-------------|--------------------------|
| m ::::: | 0.413 | 0.633 | 0.219 | boot: [0.009, 0.159] |
| Transitivity | DSE: [0.055, 0.112] | DSE: [0, 0.127] | 0.219 | jack: [0.193, 0.239] |
| Moon Dograd | 3.796 | 1.167 | -2.63 | boot: [-2.352, -2.185] |
| Mean Degree | 3.190 | 1.107 | -2.03 | jack: [-2.679, -2.48] |
| Edge Density | 0.035 | 0.011 | -0.025 | boot: [-0.022, -0.02] |
| Edge Density | 0.030 | 0.011 | -0.025 | jack: [-0.025, -0.023] |
| C 1- > 1 | 81 | 27 | F 4 | boot: [-55, -47] |
| Coreness, $k > 1$ | DSE: [78, 86] | DSE: [26, 33] | -54 | jack: [-56, -50.675] |