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Climate Change and Uncertainty: An Asset Pricing Perspective

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Abstract. Climate change and uncertainty about its potential consequences has become a central concern for economists, investors, and policymakers alike. I use a stochastic, dynamic general equilibrium model where final output is produced using a mix of cheap, dirty inputs and expensive, clean inputs and preferences incorporate aversion to climate model misspecification to analyze the implications of climate change and climate model uncertainty on economic and financial market outcomes. I find that climate change leads to increased clean input production and reduced emissions and that there is a negative price of climate risk that is significantly amplified and increasing in magnitude as climate change increases due to aversion to climate model uncertainty. Existing empirical estimates are consistent with the model implications, highlighting the potentially significant influence of climate model uncertainty on macroeconomic and asset pricing outcomes.

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Keywords: climate change • model uncertainty • production-based asset pricing

> The envelope of uncertainty in climate projections has not narrowed appreciably over the past 30 years, despite tremendous increases in computing power, in observations, and in the number of scientists studying the problem … foreseeable improvements in the understanding of physical processes, and in the estimation of their effects from observations, will not yield large reductions in the envelope of climate sensitivity.
>
> Roe and Baker (2007)

> The scientific community has had difficulty narrowing its uncertainty range far beyond the prescient initial estimate by Charney et al. (1979), which was based on very limited information.
>
> Sherwood et al. (2020)

1. Introduction

Anthropogenic emissions of greenhouse gases are one of the major concerns of the 21st century given their role as a key contributor to climate change.[1] As climate change becomes more pronounced, the potential economic and financial consequences, among numerous other possible impacts, could be drastic. However, there is no clear consensus, scientific or otherwise, about how severe future climate change and climate damages will be. The focus of this paper is to provide a parsimonious and rigorous examination of the potential implications of climate change, accounting for climate model uncertainty, on economic and financial outcomes.

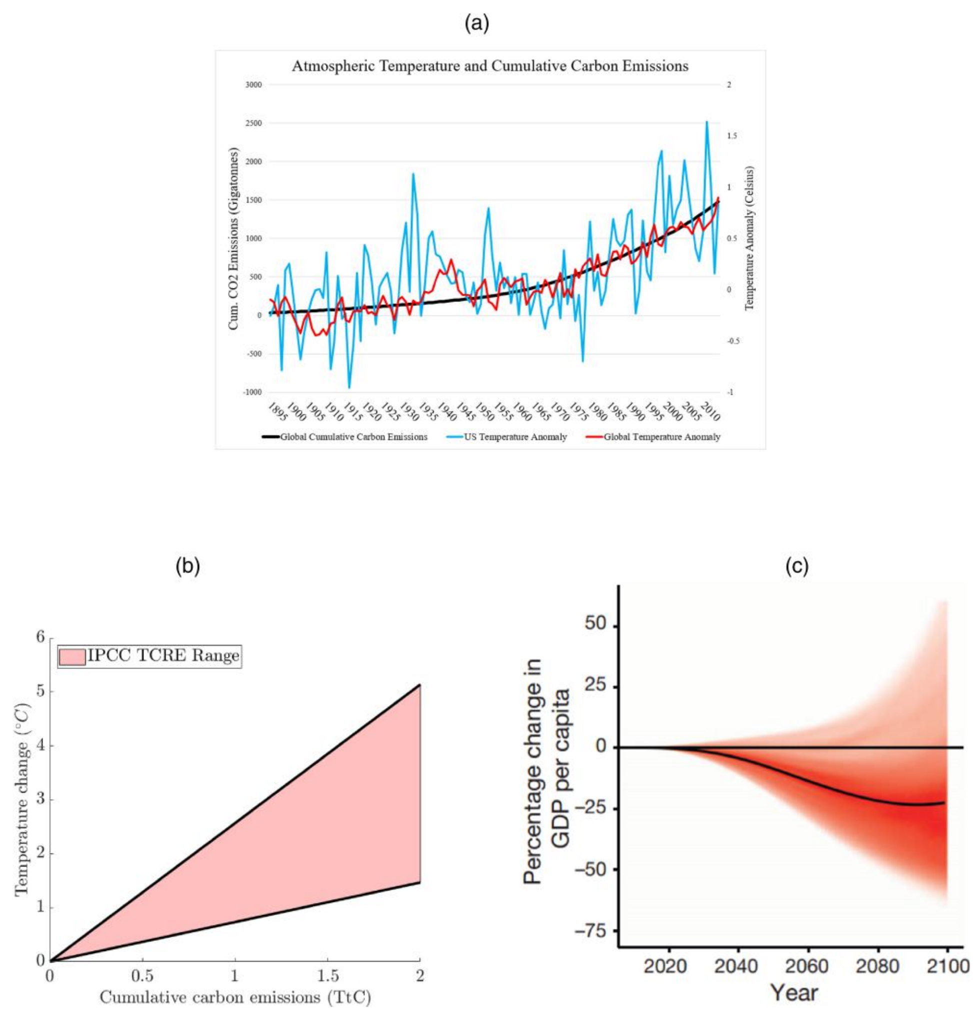
Numerous studies have noted the substantial uncertainty that exists for the geosciencific and economic components of models of the coupled climate-economy dynamics. From the geosciences perspective, Roe and Baker (2007), Matthews et al. (2012), Knutti and Sedláček (2013), Freeman et al. (2015), Knutti et al. (2017), and Proistosescu and Wagner (2020) have noted continuing and persistent uncertainty in climate change dynamics, despite decades of significant research efforts, particularly with respect to the equilibrium climate sensitivity (ECS). Recently, Sherwood et al. (2020) provide evidence that the initial uncertainty range established by Charney et al. (1979) can be reduced, but the reduction they find is relatively marginal and pertains only to the lower bound of the distribution of ECS values, meaning expected consequences are potentially worse than previously thought.[2] From the economic analysis point of view, Stern (2007), Pindyck (2013), and Pindyck (2017) have identified key areas of currently irreducible uncertainty relevant for policy decisions and social valuations of climate change consequences. The seminal work of Martin

Weitzman and coauthors, including the "Dismal Theorem" outlined in Weitzman (2009a) and the analysis of uncertainty induced fat-tailed distributions of climate change costs in Weitzman (2012), has demonstrated how the substantial uncertainty associated with climate change, and particularly damage functions, can drastically impact characterizations and social valuations of climate change consequences.

Figure 1 illustrates the extent of these geoscientific and economic uncertainties. A simple comparison of the time-series pathways for atmospheric carbon concentration and surface temperatures, shown in Figure 1(a), suggests a clear correlation between emissions and temperature over time. However, as is often the case, the correlation found here is not sufficient for characterizing the full causal relationship. Figure 1(b) plots the emissions-to-temperature relationship for various climate models as given by Allen et al. (2018), demonstrating the magnitude of geoscientific uncertainty that exists in the carbon-climate dynamics. Figure 1(c) plots implied economic damages from climate change for different models and scenarios as estimated by Burke et al. (2015), displaying the degree of uncertainty that exists about the severity of future economic damages from climate change.[3]

My analysis directly engages these geoscientific and economic uncertainties by integrating dynamic decision theory into an economic decision problem to examine the impact of climate change and climate model uncertainty on macroeconomic and asset pricing outcomes. In particular, I analyze a dynamic general equilibrium model that features aversion to model uncertainty in the preferences and a multisector production economy. The model's production component includes a final output

Figure 1. Climate Change and Model Uncertainty



Notes. Cumulative emissions-temperature correlation and the uncertainty in models of transient climate response to carbon emissions and climate change damage estimates. (Top) Cumulative emissions-temperature correlation for U.S. and global mean temperatures based on data from NASA-GISS and NOAA. (Bottom left) Climate sensitivity uncertainty based on the spread in estimated results noted by Allen et al. (2018). (Bottom right) Damage function uncertainty based on the spread in estimated economic damages from climate change from figure 5(a) of Burke et al. (2015).

sector, a dirty input sector, and a clean input sector. The final output good requires labor and some combination of the dirty and clean inputs to produce. The dirty input is cheaper than the clean input but generates carbon emissions that affect the climate, leading to increased atmospheric temperature. Climate change impacts the economy by "damaging" consumption. Labor, the other final output production input and the only input for clean and dirty input production, is in limited supply and must be split between these three sectors. This production-based multisector framework allows me to study the heterogeneous impact climate change has on different sectors.

Model uncertainty in this framework pertains to the evolution of productivity and climate change. The planner with aversion to this uncertainty uses potential worst-case models for these states to inform their optimal decisions so that they are robust to possible misspecification. As a result, the inclusion of model uncertainty and a preference for robustness to this uncertainty can substantially influence how climate change affects model outcomes. Because of the substantial uncertainty that exists about climate change implications, accounting for the impacts from model uncertainty could be crucial for capturing the full observed and expected future impacts of climate change.

I use a calibrated version of my model to demonstrate potential future implications of climate change and model uncertainty for the macroeconomy and financial markets. From these results I show how different levels of concern about model uncertainty impact results and how expected future outcomes depend on these counterfactuals. I start by focusing on the social planner's welfare maximizing setting for simplicity. I then explore alternative climate change and climate policy scenarios where there is only a partial internalization of the impact of climate change and climate model uncertainty to show how model results differ for alternative specifications.

Critically, the model produces two sets of implications. First, climate change concerns shift the economy from dirty inputs toward clean inputs. This impact is quite modest if the climate externality is only partially internalized, even with concerns about model uncertainty, but is somewhat enhanced, as is the impact of model uncertainty, when the internalization of the climate externality is larger. The model also shows that there is a state-dependent, negative price of climate risk. Although the risk price for climate change is always negative, it is significantly amplified when the climate externality is only partially internalized, even with only moderate climate model uncertainty. Because increasing climate change is a negative outcome, it leads to increased marginal utility and therefore aggregate exposure to climate change risk is negative. As a result, the negative climate risk price in the model implies that the aggregate climate risk premium, or the compensation required for holding securities with climate change risk exposure, is positive and increasing in temperature.

I provide additional insight into the model mechanisms by examining various model extensions. The first set of extensions focus on model features that add additional realism and quantitative richness, including climate damages to economic growth, climate change disasters, directed technical change in green and brown sectors, and a multidimensional climate model. I show that each of these frameworks lead to qualitatively similar results and argue that they also introduce additional aspects of model uncertainty that could potentially amplify the model uncertainty aversion mechanism highlighted in the baseline setting in significant ways. I also explore extensions with recursive preferences and with learning. In the recursive preferences setting, the model generates essentially identical results under fairly large risk aversion. As other papers in asset pricing and decision theory have highlighted, model uncertainty is one likely justification for high risk aversion in such settings, and thus the results further confirm the importance of the model uncertainty aversion mechanism. The setting with learning introduces an amplification effect to the negative climate risk price, but the effect is dynamically different from the model uncertainty aversion framework. This result highlights that my main results are not simply mechanical and common to all climate change risk settings. In addition, the results demonstrate the importance of considering my model results in the context of the empirical climate finance literature.

Finally, I compare the model implications to estimates in the empirical climate finance literature. I find that the negative and state-dependent price of climate risk found in the model is consistent with the results found by much of the work in this important and growing area. The consistency of the estimates in the empirical climate finance literature with the model implications highlights the importance of incorporating model uncertainty when analyzing the consequences of climate change.

2. Related Literature

This paper builds on numerous important areas of research to provide a novel characterization of the impacts of climate change and climate model uncertainty from an asset pricing perspective. Important work has been done modeling the economic implications of climate change, starting with the dynamic integrated climate-economy (DICE) first developed in Nordhaus (1992) and recently updated in Nordhaus (2017). Since then, important developments have provided insight about the broader economic implications and concerns related to climate change (Stern 2007), optimal carbon taxation (Golosov et al. 2014), directed technical change for clean and dirty technologies (Acemoglu et al. 2016), and climate disasters (Pindyck and Wang 2013). Recent contributions have focused on understanding optimal climate policy in models with recursive preferences, risk, and multilayered climate dynamics (Cai and Lontzek 2019, Daniel et al. 2019, Hambel et al. 2021).

My framework extends the DICE-style integrated assessment modeling setting to a multisector production framework that characterizes climate impacts using asset prices and allows for impacts from climate model uncertainty. This type of modeling framework follows the seminal papers in production-based asset pricing such as Brock (1982), Cochrane (1991), and Jermann (1998). These papers, as well as more recent work (Gomes et al. 2003, Gala 2010, Papanikolaou 2011, Kogan and Papanikolaou 2014), have sought to explain questions about the equity and value premium puzzles and link them to real economic frictions, fluctuations, and investment and production decisions. Other recent papers (Segal et al. 2015, Ai and Kiku 2016, Dou 2017) have focused on firm-level asset pricing impacts from various risk components. By incorporating clean and dirty inputs, climate change, and climate model uncertainty into such a framework, I am able to analyze the time series and cross-sectional pricing impacts of climate change and model uncertainty in connection with the macroeconomic production decisions.

Empirical studies in climate economics have focused on estimating the economic consequences of climate change. These estimates have focused on sector- and season-dependent costs (Deschenes and Greenstone 2007, Colacito et al. 2018), as well as impacts on the level and growth rate of regional and global output based on variation and differences in short- and long-term temperature trends (Dell et al. 2012, Burke et al. 2015, Hsiang et al. 2017, Burke et al. 2018, Newell et al. 2021). Substantial heterogeneity and uncertainty in terms of the estimated severity of economic damages from climate change has been found in this literature. In addition, the damage functions used in theoretical models that must postulate damages for unobserved magnitudes of climate change can at times be in direct tension in terms of magnitudes and convexity with the estimates of this empirical literature which relies on historical measurements. My analysis confronts this tension and uncertainty directly in a production-based dynamic general equilibrium setting and generates predictions about how climate change and climate model uncertainty influence outcomes to compare with empirical estimates.

To account for climate model uncertainty aversion, I use the robust preferences framework from dynamic decision theory. The foundations for the development and implementation of this decision theoretic framework comes from Gilboa and Schmeidler (1989), Hansen and Sargent (2001), and Hansen et al. (2006). Additional work in this area has focused on applying these tools and related approaches to demonstrate their importance for asset prices (Tallarini 2000, Cagetti et al. 2002, Anderson et al. 2003, Maenhout 2004, Weitzman 2007) and has recently begun to characterize their implications for climate policy (Li et al. 2016; Brock and Hansen 2018; Barnett et al. 2020, 2021). Related work has explored the impact of alternative forms of uncertainty about climate change and climate damages (Kelly and Kolstad 1999, Crost and Traeger 2010, Lemoine and Traeger 2012, Rudik 2020). My paper connects concerns about climate change and climate model uncertainty to the macroeconomic and asset pricing outcomes generated by the model.

3. Model of Climate Risk and Uncertainty

I now put forward a general equilibrium, production-based model to demonstrate the role of the critical mechanisms of interest, climate change and climate model uncertainty. For simplicity, I focus first on the solution to the social planner's problem for the model. I then examine the impact of counterfactual scenarios, including partial internalization of the climate externality and parameter sensitivity comparisons. Finally, I examine various model specifications to compare alternative stories and mechanisms. The model is stylized and not designed to match quantitative outcomes perfectly. Instead, the model is used to provide quantitatively reasonable results that intuitively demonstrate the impact of model uncertainty aversion related to the climate change model and the economic consequences of climate change.

3.1. Climate Change

The climate model is based on the approximation from Matthews et al. (2009), Matthews et al. (2012), and MacDougall et al. (2017). They find that a proportional relationship between carbon emissions and temperature accurately approximates long-run outcomes of numerous complex climate system models. This linear approximation is characterized by a carbon-climate-response (CCR) or transient climate response to cumulative emissions (TCRE) parameter. Similar single equation simplifications have been used by Golosov et al. (2014), Li et al. (2016), Acemoglu et al. (2016), and others in the literature.

The key state variable for climate will therefore be atmospheric temperature in excess of the preindustrial value, T_t. Under the TCRE assumption, the dynamic evolution of T_t is given by

$$dT_t = \lambda E_t dt + \boldsymbol{\sigma}_T' d\hat{\boldsymbol{W}}_t, \tag{3.1}$$

where E_t are carbon emissions from production, λ is the scaling factor converting the dirty input use into emissions, that is, the TCRE parameter, $\boldsymbol{\sigma}_T$ is a constant volatility vector for temperature, and $\hat{\boldsymbol{W}}_t$ is a multidimensional Brownian motion.

Although the approximation is considered most applicable to time scales longer than those considered in my model, this simplification has two key benefits. First, it allows for clear intuition and tractability that can be lost in a full model of climate dynamics, which often includes many state variables to capture the multilayered carbon concentration and temperature dynamics. The second key feature is the innate, aggregated model uncertainty that exists within this framework. As can been seen in Figure 1, the distribution of estimated TCRE parameters noted in Allen et al. (2018) shows a significant spread in these values, coming from heterogeneity in the various components of the complex models these estimates are based on. These estimates provide an obvious indication for the potential scope and magnitude of model uncertainty when considering climate models.

At a high level, emissions are the result of the choice of input used in final output production, and the consequence of climate change is a negative impact on final output production. This endogenous interaction is critical for understanding the economic and financial implications of climate change. The details for the interaction between production and climate change are given in Section 3.2.

3.1.1. Remarks About the Climate Model. There are some important details about the climate model used in my analysis that I want to highlight here. First, although the linear approximation of the TCRE framework does not explicitly incorporate the logarithmic relationship between radiative forcings and temperature change first identified by Arrhenius (1896), it does implicitly. The linear relationship is consistent with the physical principles that lead the nonlinearities of the concave log dynamics of the radiative forcings-to-atmospheric temperature relationship and the convex dynamics of carbon emissions-to-radiative forcings relationship to essentially offset and produce an essentially linear emissions-to-temperature relationship. The seminal work of Matthews et al. (2009) was an important contribution in the climate sciences that estimated and identified this critical relationship. Important work by Ricke and Caldeira (2014) and MacDougall and Friedlingstein (2015) further studied and verified this approximately linear relationship. Brock and Xepapadeas (2017) provided one of the first implementations and verification in the climate-economics literature of the validity of this near linear relationship by applying a framework similar to the MacDougall and Friedlingstein (2015) analysis. Barnett et al. (2021) also examined this relationship using the pulse experiments of Joos et al. (2013) and the approximate temperature dynamics that include the Arrhenius equation and confirmed this approximately linear relationship. The recent work of Dietz et al. (2021) further demonstrates this framework as a tractable framework for incorporating climate models that are consistent with geophysical principles in climate-economics research as compared with many existing models of climate dynamics in existing integrated assesment models (IAMs).

A feature not captured directly by the TCRE model is the delayed temperature response to carbon emissions. Barnett et al. (2021) propose including these response dynamics of temperature to carbon emissions by including a state variable for the exponentially weighted average of current and past emissions. The results found by Barnett et al. (2021) using this alternative setting for climate dynamics showed very minor differences in optimal emissions choices and the social cost of carbon compared with a model using the TCRE approximation. In Section 9, I implement a very similar two-state model of carbon-climate dynamics, adapted to my model setting, and find very little differences in the main results when using this alternative climate model setting.

Finally, my analysis focuses on carbon dioxide (CO_2) emissions that result from the burning of fossil fuels like oil and coal. Pierrehumbert (2014) noted that, although short-lived climate pollutants like methane, hydrofluorocarbons, black carbon, and ozone contribute significantly to the radiative forcing that drives climate change, carbon dioxide emissions are by far the most important contributor to anthropogenic climate change. This is due in large part to the essentially irreversible climate change impact of CO_2 emissions that persists for thousands of years after they enter the atmosphere. These results are also confirmed in the IPCC Fifth Assessment Report (IPCC 2014). As a result, Pierrehumbert concluded that little is gained from mitigation of short-lived climate pollution until after CO_2 emissions have been driven to annual values near zero. Although omitting these additional greenhouse gas emissions could potentially lead to an understatement of the projected magnitude of climate change generated by my model in the numerical results, increases in emissions, climate change, and climate damages should serve to amplify the effect of the climate model uncertainty mechanism in my model. For these reasons, I view the focus on CO_2 emissions as a valuable starting point and leave the potentially important extension of my model of exploring a full set of greenhouse gas emissions for future work.

3.2. Production

There are three sectors in the economy: a final goods sector that produces output Y_t, which must be used for consumption C_t, and two input producing sectors whose goods are used as inputs for final goods production. One input sector, the dirty or "brown" sector, produces the input B_t that generates carbon emissions $E_t = B_t$ when used for production, and the other input sector, the clean or "green" sector, produces the input G_t that generates no carbon emissions. The final output production technology is given by

$$Y_t = F(L_{C,t}, B_t, G_t; A_t) = A_t L_{C,t}^{1-\alpha-\nu}[(B_t^{\omega} + G_t^{\omega})^{\frac{1}{\omega}}]^{\nu}, \quad (3.2)$$

where t denotes time, A_t is final output productivity, $L_{C,t}$ is labor used in final output production, $1 > \alpha \geq 0$ is a decreasing returns to scale parameter, ν is the input demand share of nonlabor inputs, and ω is the elasticity of substitution between green and brown inputs.

The two different input types used for final output production are produced in competitive sectors with decreasing returns to scale technologies. Input production technologies use labor as an input and are scaled by productivity factors. The input production functions are

$$B_t = A_{B,t}(L_{B,t})^{\alpha_B}, G_t = A_{G,t}(L_{G,t})^{\alpha_G}, \tag{3.3}$$

where $A_{i,t}$ is the labor productivity for sector $i = B, G$, $L_{i,t}$ is the labor used for the given type of production for $i = B, G$, and $1 > \alpha_i > 0, i = B, G$ is the decreasing returns to scale parameter for labor used in input production for $i = B, G$. I assume for simplicity that the input sectors' productivities are constants given by $A_{B,t} = \bar{A}_B, A_{G,t} = \bar{A}_G$. I also assume that A_t follows a geometric Brownian motion process:

$$dA_t/A_t = \mu_A dt + \boldsymbol{\sigma}_A' d\hat{\mathbf{W}}_t, \tag{3.4}$$

where μ_A is constant, and $\boldsymbol{\sigma}_A$ is a constant volatility vector for productivity. I omit capital as an input into the production functions for simplicity. This assumption obviously has its shortcomings and extending the model to incorporate capital can be done, but this simplification allows for greater tractability in the model by reducing the state space and allowing for more straightforward numerical solutions. Furthermore, the productivity A_t captures some of the aspects that a stochastic capital stock would, without the increased complexity of additional controls introduced by capital stocks.

3.2.1. Remarks About the Model's Production Component.

Implicit in this setup is that the brown firm uses fossil fuels to produce and that there is an infinite supply of fossil fuels. Although this assumption might be considered somewhat unrealistic, one should consider fossil fuel energy used by the dirty firm in this setting as the amalgamation of all fossil fuels including oil, natural gas, and coal. Because there is such an abundance of coal and natural gas, the case of a limited supply that could be exhausted before facing such extreme climate impacts that would force us to abandon these energy sources might be an overly strong assumption as well. Furthermore, the fact that there are regular discoveries of new reserves, often quite large, only compounds this issue. See Hassler et al. (2012) and Hassler et al. (2016) for further details on this. Rather than explicitly include fossil fuel in the brown production function, model each of these fossil fuel sources directly, and account for discoveries of new reserves, I choose this framework for simplicity and tractability.

Another important consideration for this model is how the assumed production technologies impact the magnitude of climate change generated by the model. The Cobb-Douglas specification for the final output production and the lack of additional frictions or adjustment costs for changing input choices, beyond the constant elasticity of substitution (CES) structure of green and brown inputs, make substitution away from brown inputs potentially easier than it would be in reality. As a result, the model simulations potentially understate the quantity of carbon emissions that will be produced by the economy, and thereby underestimate the magnitude of future climate change and climate damages. Although I later discuss the external estimates and direct model calibration used to choose the related model parameters and provide a parameter sensitivity analysis over ω that changes the substitutability of the clean and dirty inputs, it is important to note that adding frictions that slow the reduction in carbon emissions and generate increased climate impacts should serve to amplify the quantitative implications of climate model uncertainty.

Finally, an additional mechanism not considered in the baseline framework is the dynamic and endogenous change of productivity in the input sectors, important for precisely characterizing the quantitative dynamics of the transition to a greener economy. In Section 9, I outline a very stylized way of introducing such dynamics and discuss how these key effects should carry through in such a setting while also introducing additional uncertainty about technological innovation that could amplify the model uncertainty mechanism. Although further analysis of this mechanism is beyond the scope of this paper, I direct the reader to Acemoglu et al. (2012) and Acemoglu et al. (2016) for analysis of directed technical change and optimal carbon taxes and subsidies in a climate economics setting and to Barnett (2019), Atanasova and Schwartz (2019), Van der Ploeg and Rezai (2020), and Barnett et al. (2021) for research exploring the macroeconomic, asset pricing, and policy implications of stranded assets risk, climate policy action, and the transition to a carbon-free economy.

3.3. Climate Damages

The link between the climate and production components of the model is characterized by the damage function $\mathbb{D}(T_t)$. The damage function scales the final output production so that consumption is given by $C_t = Y_t/\mathbb{D}(T_t)$ and is such that as T_t increases consumption decreases, all else constant. In other words, defining $[\nabla\mathbb{D}](T_t)$ and $[\nabla^2\mathbb{D}](T_t)$ as the first and second derivatives of the function $\mathbb{D}(T_t)$, I assume the property that $[\nabla\mathbb{D}](T_t) > 0$.

Numerous papers have sought to empirically estimate climate damages based on the relationship between temperature and climate-related events to economics outcomes, including Deschenes and Greenstone (2007), Dell et al. (2012), Burke et al. (2018), Colacito et al. (2018), and Newell et al. (2021). Although these studies find evidence for growth rate and level impacts of climate damages, I focus on level impacts in the main model.[4]

The specific functional form I use follows from Barnett et al. (2020) and is similar to those used in Hambel et al. (2021), Golosov et al. (2014), Acemoglu et al. (2016), and Anderson et al. (2016), and the magnitude of damages are in the range of values considered by Weitzman

(2009b) and Nordhaus (2014). The functional form for climate damages is given by $\mathbb{D}(T_t) = \exp(D(T_t))$, where

$$D(T_t) = \gamma_1 T_t + \frac{\gamma_2}{2} T_t^2 + \frac{\gamma_2^+}{2}(T_t - \tau)^2 \mathbb{1}\{T_t - \tau > 0\}, \quad (3.5)$$

where γ_1, γ_2, and γ_2^+ are the damage parameters, which characterize the sensitivity of the economy to increases in temperature. The parameter γ_2^+ allows for the damage function to mimic a smoothed carbon budget, tipping point, or the more drastic damage functions proposed by Weitzmann such that damages can become quite severe after a threshold of $T_t = \tau$ is reached. For modeling purposes, it will be useful to consider the dynamic evolution of the log of damages, denoted as $Z_t = \log \mathbb{D}(T_t) = D(T_t)$, which is given by

$$dZ_t = [\nabla D](T_t)\lambda B_t dt + \frac{|\boldsymbol{\sigma}_T|^2}{2}[\nabla^2 D](T_t)dt + \boldsymbol{\sigma}_T'[\nabla D](T_t)d\hat{\boldsymbol{W}}_t. \quad (3.6)$$

Using a damage function that is consistent with empirical findings and other theoretical models allows for ease in cross-model comparison and calibration. There is tremendous uncertainty about the true economic damages of climate change, as is suggested by the results of Burke et al. (2015) in the lower right panel of Figure 1. This opens up a second key uncertainty component of the model related to climate change that I incorporate into my analysis.

3.4. Model Uncertainty

As mentioned previously, there are two key sources of uncertainty about climate change that I will account for in the model. The first is the dynamic evolution of climate change captured in dT_t, and the second is the damages from climate change captured in dZ_t. I follow the framework set forward in Anderson et al. (2003) and others[5] to examine uncertainty on the dynamic evolution of the two state variables T_t and Z_t. The uncertainties could be related to unknown climate tipping points, uncertain collection of carbon emissions by the ocean due to overacidification, or a number of other potential concerns that would impact estimates of the calibration, parameterization, and functional form related to the TCRE climate model and economic growth climate damages. I also allow for uncertainty about productivity for completeness.

The TCRE, climate damages, and productivity models given previously are the baseline models assumed by the planner, potentially based on historical data or other source of information and represent the decision maker's best guess as to what the true models are. The decision maker recognizes the baseline models may not be correct and allows for the possibility that alternative models govern the dynamics of climate change, climate damages, and economic growth. To represent possible alternative models, a drift perturbation is added to the approximating model by replacing the Brownian motion $\hat{\boldsymbol{W}}_t$ with $\boldsymbol{W}_t + \int_0^t \boldsymbol{\Lambda}_s\, ds$, where $\boldsymbol{\Lambda}_s$ is a process adapted to the filtration generated by the Brownian motion W_t. Therefore, uncertainty is incorporated by allowing for alternative models of the form

$$\begin{aligned} dA_t/A_t &= \mu_A dt + \boldsymbol{\sigma}_A'(\boldsymbol{\Lambda}_t dt + d\boldsymbol{W}_t) \\ dT_t &= \lambda B_t dt + \boldsymbol{\sigma}_T'(\boldsymbol{\Lambda}_t dt + d\boldsymbol{W}_t) \\ dZ_t &= [\nabla D](T_t)\lambda B_t dt + \frac{|\boldsymbol{\sigma}_T|^2}{2}[\nabla^2 D](T_t)dt \\ &\quad + \boldsymbol{\sigma}_T'[\nabla D](T_t)(\boldsymbol{\Lambda}_t dt + d\boldsymbol{W}_t). \end{aligned} \quad (3.7)$$

By adding model uncertainty in this way, the model distortions are disguised by the Brownian motion and are therefore hard to detect statistically using past data, as is the case with actual climate models. Also, this setup allows for the perturbation to be given without parametric form, allowing for a large class of models as alternatives to the baseline model.

3.5. Robust Preferences

The planner is assumed to have log preferences over consumption and a preference for robustness over model uncertainty. The robustness component of the preferences pertains to the uncertainty about the climate change, climate damages, and economic growth models. The planner has concerns that these models are misspecified and considers possible model distortions to those baseline approximations when making optimal decisions. These distortions are restricted to distortions that would be difficult to distinguish from the true models using past data by penalizing alternative models that are very different from the approximating models based on distance as defined by conditional relative entropy.[6] The magnitude of the penalization is determined by a parameter ϱ, collapsing a potentially infinite dimension problem into a single component. Hansen et al. (2006) gives details and complete formulas for relative entropy.[7] Although the use of relative entropy means our analysis focuses on relatively small model distortions, even these distortions can have meaningful effects on model outcomes. As such, it is important to note that the impacts of model uncertainty related to climate change and climate damage models could be even more substantial than what is shown in the numerical results.

Of particular interest in this problem is the separation of uncertainty associated with the productivity dimension and uncertainty associated with the climate dimension. Therefore, I separate these two channels by allowing for a productivity specific drift distortion ($\boldsymbol{\Lambda}_{A,t}$) and uncertainty aversion parameter (ϱ_A), as well as a climate specific drift distortion ($\boldsymbol{\Lambda}_{T,t}$) and uncertainty aversion parameter (ϱ_T). Furthermore, I will allow each channel to have its own relative entropy penalization. I detail the full problem with these two uncertainty channels next.

Preferences are adjusted to take into account model uncertainty and robust preferences by adding to flow utility the quadratic expressions $\varrho_T(1/2)\boldsymbol{\Lambda}'_{T,t}\boldsymbol{\Lambda}_{T,t}$ and $\varrho_A(1/2)\boldsymbol{\Lambda}'_{A,t}\boldsymbol{\Lambda}_{A,t}$, which are derivatives with respect to time of the relative entropy penalizations, where $\boldsymbol{\Lambda}_{A,t}dt$ and $\boldsymbol{\Lambda}_{T,t}dt$ represent the constrained worst-case model distortions. The planner's problem is given by a maxmin setup. The minimization is over possible model distortions $\boldsymbol{\Lambda}_{A,t}$ and $\boldsymbol{\Lambda}_{T,t}$, constrained by the relative entropy penalizations scaled by the uncertainty parameters ϱ_A and ϱ_T. After finding the optimal $\boldsymbol{\Lambda}^*_{A,t}, \boldsymbol{\Lambda}^*_{T,t}$, the agent maximizes utility over choices of consumption and labor, accounting for the ex ante specified constrained worst-case models. This does not mean that the decision maker believes that the true models are the worst-case models. Rather, the decision maker chooses optimal policies that are planned as a best response to the potential worst-case models so that their optimal decisions are robust to the set of possible alternative models being considered. Thus, the planner's problem is given by

$$J = \max_{C_t} \min_{\boldsymbol{\Lambda}_{A,t}, \boldsymbol{\Lambda}_{T,t}} \mathbb{E}_t\left[\int_t^\infty \exp(-\rho s)\left\{\rho \log C_s + \varrho_A \frac{1}{2}\boldsymbol{\Lambda}'_{A,s}\boldsymbol{\Lambda}_{A,s} + \varrho_T \frac{1}{2}\boldsymbol{\Lambda}'_{T,s}\boldsymbol{\Lambda}_{T,s}\right\}ds\right], \tag{3.8}$$

subject to the production technology constraints of each sector. Note that ρ is the subjective discount rate and ϱ_A and ϱ_T are the uncertainty aversion parameters mentioned previously.

4. Solution to the Social Planner's Problem

The social planner's problem is to choose socially optimal policies, which internalize any externalities, for quantities of consumption, final output production, dirty input production, clean input production, and labor allocation. Because consumption and production choices are all determined by the labor allocation choices, the equilibrium solution is determined by the optimal labor allocation for each production technology. In other words, the social planner's equilibrium solution is given by the optimal choices of labor allocation

$$\{L^*_{C,t}, L^*_{B,t}, L^*_{G,t}\},$$

which are sufficient to characterize equilibrium consumption and production outcome functions

$$\{C(L_{C,t}, L_{B,t}, L_{G,t}), Y(L_{C,t}, L_{B,t}, L_{G,t}), B(L_{C,t}, L_{B,t}, L_{G,t}), G(L_{C,t}, L_{B,t}, L_{G,t})\}.$$

These optimal policy functions are subject to the evolution of the stochastic processes for the state variables T_t, Z_t, and A_t and resource constraints. As the solutions are functions of the state variables T_t, Z_t, and A_t, the solution to the planner's problem is a recursive Markov equilibrium.

To derive the model's socially optimal outcomes, I solve the social planner's value function, characterized by a Hamilton-Jacobi-Bellman (HJB) equation representing their optimization problem in a recursive format. First-order conditions characterizing the optimal policies are derived from this HJB equation and used to solve for the value function. From the solution of the value function, (shadow) prices can be derived which arise under a decentralization mechanism, discussed later. The social planner's problem is given by maximizing social welfare

$$J(A_t, T_t, Z_t) = \max_{L_{C,t}, L_{B,t}, L_{G,t}} \min_{\boldsymbol{\Lambda}_{A,t}, \boldsymbol{\Lambda}_{T,t}} \mathbb{E}_t\left[\int_t^\infty \exp(-\rho s) \left\{\rho \log C_s + \varrho_A \frac{1}{2}\boldsymbol{\Lambda}'_{A,s}\boldsymbol{\Lambda}_{A,s} + \varrho_T \frac{1}{2}\boldsymbol{\Lambda}'_{T,s}\boldsymbol{\Lambda}_{T,s}\right\}ds\right], \tag{4.1}$$

subject to production technology constraints given by

$$\begin{aligned}
C_t &= \exp(-Z_t)Y_t = \exp(-Z_t)A_t(L_{C,t})^{1-\alpha-\nu}(B_t^\omega + G_t^\omega)^{\nu/\omega}\\
B_t &= \bar{A}_B(L_{B,t})^{\alpha_B}\\
G_t &= \bar{A}_G(L_{G,t})^{\alpha_G}\\
L_t &= L_{C,t} + L_{B,t} + L_{G,t} = 1,
\end{aligned} \tag{4.2}$$

and the stochastic processes for productivity, temperature, and climate damages

$$\begin{aligned}
dT_t &= [\lambda B_t + \boldsymbol{\sigma}'_T \boldsymbol{\Lambda}_{T,t}]dt + \boldsymbol{\sigma}'_T d\boldsymbol{W}_t\\
dZ_t &= [\nabla D](T_t)[\lambda B_t + \boldsymbol{\sigma}'_T\boldsymbol{\Lambda}_{T,t}]dt + \frac{|\boldsymbol{\sigma}_T|^2}{2}[\nabla^2 D](T_t)dt\\
&\quad + \boldsymbol{\sigma}'_T[\nabla D](T_t)d\boldsymbol{W}_t\\
dA_t/A_t &= [\mu_A + \boldsymbol{\sigma}'_A\boldsymbol{\Lambda}_{A,t}]dt + \boldsymbol{\sigma}'_A d\boldsymbol{W}_t.
\end{aligned} \tag{4.3}$$

From the social planner's problem given previously, the first key result is derived.

Proposition 1 (Social Planner's HJB Equation and Optimal Policies). *The value function for the social planner's problem $J(A_t, T_t, Z_t)$ has a quasi-analytical simplification given by*

$$J(A_t, T_t, Z_t) = \log A_t - Z_t + v(T). \tag{4.4}$$

Note that v(T) is the solution to the simplified HJB equation:

$$\begin{aligned}
\rho v(T_t) &= \rho \log[(L_C(B_t, G_t))^{1-\alpha-\nu}(B_t^\omega + G_t^\omega)^{\nu/\omega}]\\
&\quad - \frac{|\boldsymbol{\sigma}_A|^2}{2\varrho_A} - \frac{|\boldsymbol{\sigma}_T|^2[v_T - [\nabla D](T_t)]^2}{2\varrho_T}\\
&\quad + \left(\mu_A - \frac{1}{2}|\boldsymbol{\sigma}_A|^2\right) - [\nabla D](T_t)\lambda B_t\\
&\quad - \frac{1}{2}|\boldsymbol{\sigma}_T|^2[\nabla^2 D](T_t) + \lambda B_t v_T + \frac{|\boldsymbol{\sigma}_T|^2}{2}v_{TT},
\end{aligned} \tag{4.5}$$

where I have suppressed notation for function state dependence and written final output labor as a function of B_t and G_t. The worst-case model distortion is given by

$$\boldsymbol{\Lambda}^*_{A,t} = -\frac{1}{\varrho_A}\boldsymbol{\sigma_A}, \boldsymbol{\Lambda}^*_{T,t} = -\frac{1}{\varrho_T}\boldsymbol{\sigma_T}\{v_T - [\nabla D](T_t)\}, \quad (4.6)$$

and implicit equations for the optimal choices of B_t and G_t are given by

$$\begin{aligned} 0 &= \frac{(1-\alpha-\nu)}{\nu\alpha_B A_B^{1/\alpha_B}} B_t^{1/\alpha_B - \omega}(B_t^\omega + G_t^\omega) - L_{C,t} \\ &\quad - \frac{L_{C,t}(B_t^\omega + G_t^\omega)}{\nu\rho}\lambda B_t^{1-\omega}[v_T - [\nabla D](T_t)] \\ 0 &= \frac{(1-\alpha-\nu)}{\nu\alpha_G A_G^{1/\alpha_G}} G_t^{1/\alpha_G - \omega}(B_t^\omega + G_t^\omega) - L_{C,t}. \end{aligned} \quad (4.7)$$

The solutions for B_t and G_t are functions of the state variable T_t and are solved numerically, jointly with the value function $v(T_t)$, as outlined in Online Appendix E.

The proof for this proposition is given in Online Appendix A.1. The important takeaways from this proposition are the contributions of climate change for social welfare and optimal choices in the model, as well as the potentially meaningful role of climate model uncertainty given its impact on social welfare through the worst-case model distortions.

5. Asset Prices

5.1. Decentralization

To derive the asset pricing outcomes for the social planner's problem, it is necessary to discuss the decentralization of the planner's solution. I briefly characterize the decentralization mechanism that generates prices in the social planner setting. A full derivation can be found in Online Appendix A.2. In short, by implementing an optimal tax on dirty input production, the outcomes generated by the social planner's choices can be duplicated in a decentralized economy with corresponding prices that support market clearing. These are the prices I discuss later. The need for a decentralization mechanism is due to the externality of carbon emissions from the use of the dirty input leading to increased temperature and therefore increased economic damages that reduce aggregate consumption.

5.2. Stochastic Discount Factor, Risk-Free Rate, Risk, and Uncertainty Prices

For log utility, the stochastic discount factor (SDF) $\hat{\pi}_t$ is given by

$$\hat{\pi}_t = exp(-\rho t)\rho C_t^{-1}. \quad (5.1)$$

However, under the assumption of robust preferences, we need to adjust this SDF for concerns about model uncertainty. Following Anderson et al. (2003), the SDF when the agent has robust preferences is given by a multiplicative function of the intertemporal marginal rate of substitution ($\hat{\pi}_t$) and adjustments for each channel of model uncertainty that are exponential martingales ($\zeta_{A,t}, \zeta_{T,t}$). Therefore, the SDF is given by

$$\pi_t = exp(-\rho t)\rho C_t^{-1}\zeta_{A,t}\zeta_{T,t}, \quad (5.2)$$

where each $\zeta_{i,t}$, $i \in \{A, T\}$ is a Radon-Nikodym derivative related to the worst-case model outcomes along that dimension that households use to account for model uncertainty. Each $\zeta_{i,t}$ is a martingale, given by

$$\begin{aligned} \zeta_{i,t} &= exp\left(-\frac{1}{2}\int_0^t (\boldsymbol{\Lambda}^*_{i,t})'(\boldsymbol{\Lambda}^*_{i,t})d\tau + \int_0^t (\boldsymbol{\Lambda}^*_{i,t})' d\boldsymbol{W_\tau}\right) \\ d\zeta_{i,t} &= \zeta_{i,t}(\boldsymbol{\Lambda}^*_{i,t})' d\boldsymbol{W_t}. \end{aligned} \quad (5.3)$$

From this, we can now derive the risk-free rate and market prices of risk and uncertainty.

Proposition 2 (Risk-Free Rate, Market Price of Risk, Market Price of Uncertainty). *The evolution of the stochastic discount factor is given by*

$$\frac{d\pi_t}{\pi_t} = -r_{f,t}dt - \boldsymbol{\sigma}'_{\boldsymbol{\pi},\boldsymbol{T}} d\boldsymbol{W_t} - \boldsymbol{\sigma}'_{\boldsymbol{\pi},\boldsymbol{Z}} d\boldsymbol{W_t} - \boldsymbol{\sigma}'_{\boldsymbol{\pi},\boldsymbol{A}} d\boldsymbol{W_t}, \quad (5.4)$$

where $r_{f,t}$ is the risk free rate and $\sigma_{\pi,T}$, $\sigma_{\pi,Z}$, $\sigma_{\pi,A}$ are the risk prices. The expression for risk-free rate is given by

$$\begin{aligned} r_{f,t} &= \rho + \left(\mu_A - \frac{1}{\varrho_A}v_A|\boldsymbol{\sigma_A}|^2\right) \\ &\quad + \left(\lambda B_t - \frac{1}{\varrho_T}[v_T - [\nabla D](T_t)]|\boldsymbol{\sigma_T}|^2\right) \\ &\quad \frac{C_T}{C} + \frac{1}{2}|\boldsymbol{\sigma_T}|^2\left[\frac{C_{TT}}{C} - 2\left(\frac{C_T}{C}\right)^2\right] + [\nabla D](T_t)\lambda B_t \\ &\quad - \frac{1}{2}|\boldsymbol{\sigma_T}|^2[\nabla^2 D](T_t) + \frac{1}{\varrho_T}[\nabla D](T_t) \\ &\quad \times [v_T - [\nabla D](T_t)]|\boldsymbol{\sigma_T}|^2 - [[\nabla D](T_t)]^2|\boldsymbol{\sigma_T}|^2, \end{aligned} \quad (5.5)$$

where the C_T and C_{TT} are derivatives of C_t with respect to T_t. The temperature, climate damage, and productivity risk prices, as well as the market price of uncertainty or contribution to the market price of risk due to model uncertainty, are given by

$$\begin{aligned} &\boldsymbol{\sigma_{\pi,T}} = \boldsymbol{\sigma_T}\left[\frac{C_T}{C} + \frac{1}{\varrho_T}v_T\right], \boldsymbol{\sigma_{\pi,Z}} = -\boldsymbol{\sigma_T}[\nabla D](T_t)\left[1 + \frac{1}{\varrho_T}\right], \\ &\boldsymbol{\sigma_{\pi,A}} = \boldsymbol{\sigma_A}\left[1 + \frac{1}{\varrho_A}\right], \end{aligned} \quad (5.6)$$

$$\boldsymbol{\Lambda}^*_{A,t} = -\frac{1}{\varrho_A}\boldsymbol{\sigma_A}, \boldsymbol{\Lambda}^*_{T,t} = -\frac{1}{\varrho_T}\boldsymbol{\sigma_T}[v_T - [\nabla D](T_t)]. \quad (5.7)$$

A full derivation of this result is given in Online Appendix A.3. Although the expression for the climate risk prices can only be derived numerically, we know they depend on the state variable for temperature. Furthermore, concerns about future uncertainty and model uncertainty magnify the impacts of climate change and climate damages. The magnitude of the effect of model uncertainty depends on ϱ_A and ϱ_T, whose inverses characterize the scalings for the additional model uncertainty contribution.

5.3. Firm Prices

To derive firm prices, I start from the standard asset pricing Euler equation, which gives the price of a 100% equity financed firm as the (stochastically) discounted value of all future dividends:

$$P_t = E_t\left[\int_t^{\infty} \mathcal{D}_s \frac{\pi_s}{\pi_t} ds\right]. \tag{5.8}$$

From this equation, the price of each firm and the market price in this model are determined.

Proposition 3 (Firm and Market Stock Prices). *The stock price of firm i P_t^i $(i = C, B, G)$ has a quasi-analytical simplification given by*

$$P_t^i = A_t D(T_t)^{-1} p^i(T_t), \tag{5.9}$$

where $p^i(T_t)$ is the solution to a sector-dependent differential equation of the form

$$\begin{aligned} 0 = &-r_{f,t}p^i + \Omega_t^i + \left(\mu_A + \frac{1}{\varrho_A}|\boldsymbol{\sigma}_A|^2\right)p^i \\ &+ \left(\lambda B_t + \frac{1}{\varrho_T}|\boldsymbol{\sigma}_T|^2 v_T\right)p_T^i + ([\nabla D](T_t)\lambda B_t \\ &+ \frac{1}{\varrho_T}[\nabla D](T_t)^2|\boldsymbol{\sigma}_T|^2]) - \boldsymbol{\sigma}'_{\boldsymbol{\pi},A}\boldsymbol{\sigma}_A p^i - \boldsymbol{\sigma}'_{\boldsymbol{\pi},T}\boldsymbol{\sigma}_T p_{\boldsymbol{T}}^i \\ &- \boldsymbol{\sigma}'_{\boldsymbol{\pi},Z_t}[\nabla D](T_t)\boldsymbol{\sigma}_T p^i + \frac{|\boldsymbol{\sigma}_T|^2}{2} p_{TT}^i, \end{aligned} \tag{5.10}$$

where sector dividends net of TFP and damages Ω_t^i are proportional to final output net of TFP and damages $\tilde{Y}_t = (L_{C,t})^{1-\alpha-\nu}(B_t^\omega + G_t^\omega)^{\nu/\omega}$:

$$\begin{aligned} \Omega_t^C &= \alpha \tilde{Y}_t \\ \Omega_t^B &= \nu(1-\alpha_B)\left(\frac{B_t^\omega}{B_t^\omega + G_t^\omega}\right)\tilde{Y}_t \\ \Omega_t^G &= \nu(1-\alpha_G)\left(\frac{G_t^\omega}{B_t^\omega + G_t^\omega}\right)\tilde{Y}_t. \end{aligned} \tag{5.11}$$

The derivation comes straight from the aforementioned asset pricing Euler equation and the dividend expression for the given firm. I solve the differential equation for a given stock price with the respective expressions for each sector $i = C, B, G$. All the quantities and prices used as inputs for the pricing partial differential equation (PDE) are given by the social planner's optimal choices and equilibrium outcomes.

5.4. Expected Returns and Risk Premium

The risk premia in the model come as a direct consequence of the previous propositions.

Proposition 4 (Risk Premia). *The risk premia for equities $i = C, B, G$ are given by*

$$\begin{aligned} E_t\left[\frac{dP_{i,t} + \mathcal{D}_{i,t}}{P_{i,t}} - r_{f,t}dt\right] &= -cov_t\left[\frac{d\pi_t}{\pi_t}, \frac{dP_{i,t}}{P_{i,t}}\right] \\ &= \boldsymbol{\sigma}'_{T,i}\boldsymbol{\sigma}_{\boldsymbol{\pi},T}dt + \boldsymbol{\sigma}'_{Z,i}\boldsymbol{\sigma}_{\boldsymbol{\pi},Z}dt \\ &\quad + \boldsymbol{\sigma}'_{A,i}\boldsymbol{\sigma}_{\boldsymbol{\pi},A}dt, \end{aligned} \tag{5.12}$$

where $\sigma_{T,i}$ and $\sigma_{A,i}$ represent the firm $i = C, B, G$ return exposures to climate and productivity risks. These exposures are given by

$$\begin{aligned} \boldsymbol{\sigma}_{T,i} &= vol_T\left(\frac{dP_t^i}{P_t^i}\right) = (\boldsymbol{\sigma}_T p_T^i)/p^i, \\ \boldsymbol{\sigma}_{Z,i} &= vol_Z\left(\frac{dP_t^i}{P_t^i}\right) = \boldsymbol{\sigma}_T[\nabla D](T_t) \\ \boldsymbol{\sigma}_{A,i} &= vol_A\left(\frac{dP_t^i}{P_t^i}\right) = \boldsymbol{\sigma}_A. \end{aligned} \tag{5.13}$$

The equity premium between any two sectors i and j is therefore given by

$$\begin{aligned} &(\boldsymbol{\sigma}_{T,i} - \boldsymbol{\sigma}_{T,j})'\boldsymbol{\sigma}_{\boldsymbol{\pi},T}dt + (\boldsymbol{\sigma}_{Z,i} - \boldsymbol{\sigma}_{Z,j})'\boldsymbol{\sigma}_{\boldsymbol{\pi},Z}dt \\ &\quad + (\boldsymbol{\sigma}_{A,i} - \boldsymbol{\sigma}_{A,j})'\boldsymbol{\sigma}_{\boldsymbol{\pi},A}dt. \end{aligned} \tag{5.14}$$

Climate change is again the key effect driving cross-sectional differences. As temperature increases, households' and firms' optimal choices change due to the carbon tax. This carries over to firm profits (and therefore dividends), thus impacting firm prices and returns. For example, increased temperature leads to a reduction in dirty input production and reduces profits for the dirty sector firms, whereas the green sector increases their output and their profits.

6. Numerical Results

Given the model solution characterizations, I can now solve the model numerically to examine the implications of climate change and climate model uncertainty for macroeconomic and asset pricing outcomes. The focus here is on outcomes for the socially optimal policy setting, where there is an optimal carbon tax and aversion to model uncertainty. Later in this section, I show results for a partial internalization equilibrium, where the climate externality is only partially internalized.

6.1. Parameters

The parameter values used for the numerical solutions are given in Table 1. These numbers are chosen based on external sources and estimates and direct model calibration. Both methods for determining parameter values focus on choosing parameters so that the model matches certain empirical targets to generate reasonable quantitative implications. The stylized nature of the model means some of the quantitative outcomes generated by the model are likely to be somewhat imprecise.

Table 1. Model Parameters

Parameter	Symbol	Value
Discount factor	ρ	0.00375
Final output DRS	α	0.05
Final output nonlabor exponent	ν	0.04
Input elasticity of substitution	ω	0.75
Brown and green firm DRS	α_B, α_G	0.95
Climate robustness parameter	ϱ_T	$\{\infty, 0.05\}$
Productivity robustness parameter	ϱ_A	0.05
Brown and green productivity	$\bar{A}_B, \bar{A}_G$	$\{62.77, 33.37\}$
Aggregate productivity drift	μ_A	0.005
Aggregate productivity volatility	$\boldsymbol{\sigma}_A'$	[0.01, 0, 0]
Climate damage curvature	$\{\gamma_1, \gamma_2, \gamma_2^+\}$	$\{0.000177, 0.0044, 0.0197\}$
Climate damage threshold	τ	2.0
TCRE parameter	λ	0.00173
Temperature volatility	$\boldsymbol{\sigma}_T'$	[0, 0.05, 0]
Initial final output productivity	A_0	19.82
Initial temperature	T_0	$1^\circ C$

Importantly, however, the parameter values and functional forms used in the model are chosen so that the results likely understate the impact of climate model uncertainty, the key mechanism of interest, on asset prices and macroeconomic outcomes. I provide next a brief summary of the parameter choices and direct the reader to Online Appendix C for full details.

The climate-related parameter of the model are all chosen based on external estimates and sources. The value for the TCRE parameter λ is based on estimates from the geosciences literature (Matthews et al. 2009, MacDougall et al. 2017). The volatility of atmospheric temperature $\boldsymbol{\sigma_T}$ is chosen to match the volatility of global mean temperature (Hambel et al. 2021). The climate damage function parameters $\gamma_1, \gamma_2, \gamma_2^+, \tau$ are chosen to be consistent with the values of Barnett et al. (2020), who calibrate this functional form to match specifications in the climate-economics literature (Nordhaus 2014, Weitzman 2010). Although the functional form for and magnitude of climate damages is highly uncertain, the functional form and parameter choices are similar to others used in the literature, and deviations from this setting would likely serve to amplify the quantitative impacts of model uncertainty on macroeconomic and asset pricing outcomes.

A number of production and preference parameters are also chosen based on external estimates and sources. The brown-green input demand share ν is chosen to match the estimated energy input share (Finn 1995). The returns-to-scale parameters $\alpha, \alpha_O, \alpha_G$ are chosen to be consistent with estimates of the returns to scale of production across sectors in the U.S. economy (Basu et al. 2006). The elasticity of substitution between brown and green inputs ω is from the range of elasticity of substitution for clean and dirty inputs in the literature (Acemoglu et al. 2012). The Cobb-Douglas specification for final output production is an implicit assumption of the elasticity of substitution, justified by estimated medium- to long-term elasticity of substitution between inputs in aggregate production (Hassler et al. 2012). The drift and volatility parameters for the growth rate of aggregate productivity μ_A and $\boldsymbol{\sigma_A}$ are chosen to fit empirical measurements for the growth rate and volatility of world gross domestic product (GDP) from the World Bank database. The subjective discount rate ρ is set to be consistent with values used in other macroeconomic and finance research (Nordhaus 2014, Acemoglu et al. 2016, Barnett et al. 2020). The parameter for the fraction of the climate externality that is internalized χ takes on two values, $\chi = 1$ for the social planner's problem with full internalization and 0.2 based on estimates from the World Bank of the fraction of emissions taxed globally for partial internalization.

Direct model calibration is used for two additional sets of economic and preference parameters. Productivity parameters A_C, A_B, and A_G are calibrated using the social planner's balanced growth path solution to the nonclimate version of the model by matching Y_t, B_t, and G_t to empirical measurements of the fraction of energy consumption from fossil fuel and renewable sources (International Energy Administration), of carbon emissions (Figueres et al. 2018) and the estimated value of World GDP (World Bank database). The parameters for aversion to misspecification ϱ_A and ϱ_T are calibrated using a two-step process. The first step calibrates ϱ_A by choosing the value that matches the market price of risk in the nonclimate version of the model to the empirically measured market price of risk for U.S. equity markets. Then, I verify that the choice of ϱ_T is statistically reasonable by ensuring an approximate detection error probability bound (Anderson et al. 2003) does not go below a standard statistical significance cutoff of 10%. The parameters for the recursive preferences specification, which I use as a comparison with the baseline results based on model uncertainty, are calibrated in a similar vein by imposing an EIS θ of unit value and setting risk aversion

ξ so that the market price of risk from the nonclimate, recursive preference version of the model matches the market price of risk as measured empirically from U.S. equity markets.

6.2. Numerical Method

I solve the PDE for the value function numerically using the method of false transient with a finite difference scheme and conjugate gradient solver. The algorithm approximates a solution to the value function while jointly solving numerically for the optimal first order conditions (FOC). I use the solution to the value function to construct all the macroeconomic outcomes of interest and most of the key asset pricing results. With these derived outcomes, I can then solve differential equations for firm prices numerically, again following a simplified false transient algorithm. With these results, I construct the remaining asset pricing outcomes from the numerical solutions. Online Appendix E provides a detailed description of the algorithm and numerical method.[8]

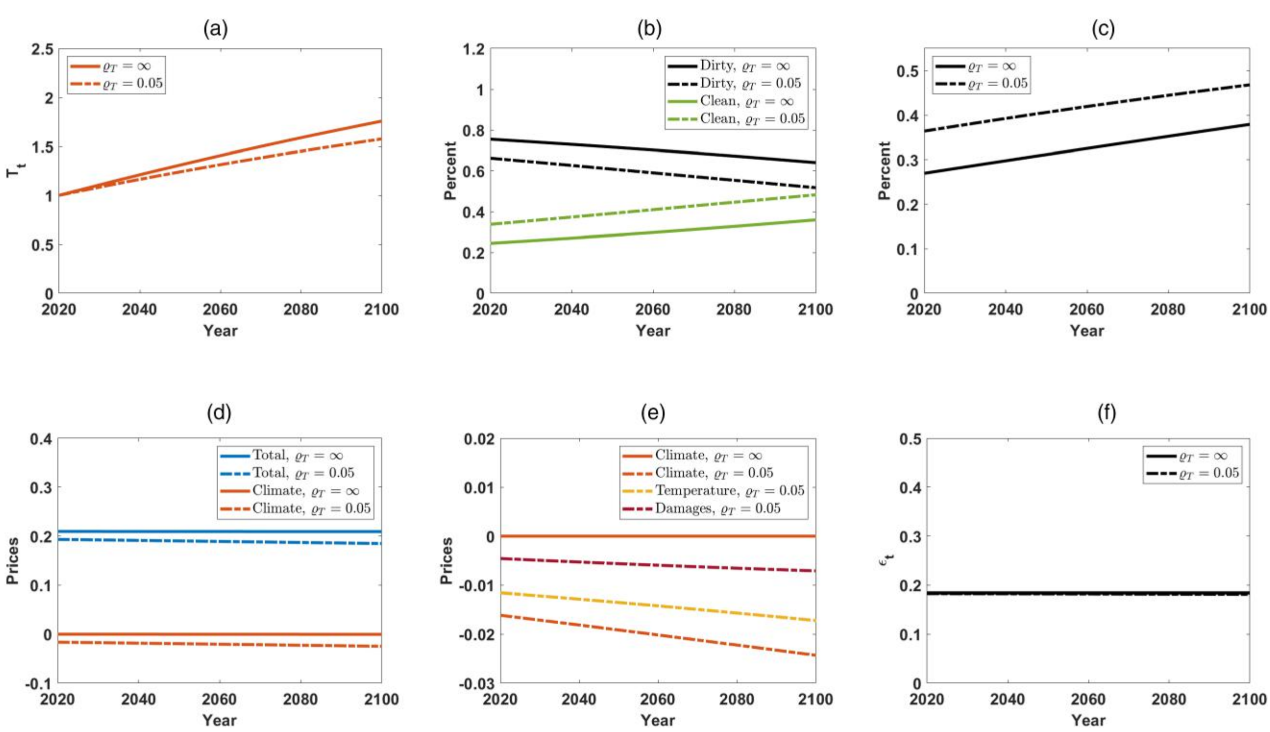
6.3. Macroeconomic and Asset Pricing Outcomes

Figure 2 shows simulated outcomes based on the model solutions. The plots show the macroeconomic and financial outcomes for two cases of uncertainty. The first is the case of high climate model uncertainty with $\varrho_T = 0.05$. The second is the case of no climate model uncertainty with $\varrho_T = \infty$. In each case, I leave economic uncertainty fixed at $\varrho_A = 0.05$. The dashed lines are for high climate model uncertainty aversion and the solid lines are for no climate model uncertainty aversion. I focus on a handful of important macroeconomic and asset pricing results: the fraction of inputs coming from the clean and dirty sectors, the carbon tax, temperature, risk and uncertainty prices, climate uncertainty prices, and detection error probability bounds.

Figure 2(a) shows the temperature pathway resulting from the two climate model uncertainty cases. In each case, temperature increases from 1°C (above the preindustrial value) to between 1.5°C and 2°C. The case with uncertainty about the climate model leads to a reduced temperature change, a reflection of the increased concern about how bad climate change and climate damages could be. Because this is the social planner's solution, these model outcomes are on the lower end of temperature change that we would expect to occur.

Figure 2(b) shows the fraction of clean (green lines) and dirty (black lines) inputs used in final output production for the two climate model uncertainty cases. There are two clear features that we can see from this figure. First, as temperature increases over time there is a transition from dirty inputs to cleaner inputs, going from around 20%−25% green in the initial period to about

Figure 2. Macroeconomic and Asset Pricing Model Outcomes—Baseline Model



Notes. Simulated outcomes for the model based on the numerical solutions. (a) Temperature pathway. (b) Fraction of inputs used in final output production coming from the clean input (green lines) and the dirty input (black lines). (c) Optimal carbon tax on dirty input production. (d) Total risk price (blue lines) and the climate risk price (orange lines). (e) Total climate uncertainty price (orange lines), temperature uncertainty price (yellow lines), and damage uncertainty price (red lines). (f) Detection error probability bound. Solid lines show results with no climate uncertainty, and dashed lines show results with climate uncertainty.

40%−50% at around 2100. The second feature is that this transition is clearly amplified by concerns about climate change uncertainty (the dashed line compared with the solid line), where the initial and final values of the green input fraction are larger. These values are initially too low when compared with the data, but this is a result of the social planner's solution. However, at least some internalization is needed to have a transition from clean to dirty as seems to be happening in the data, and so this serves as a reasonable starting point to consider.

Figure 2(c) shows the optimal tax on dirty input production in each case. Unsurprisingly, the tax is increasing with temperature and with climate model uncertainty concerns (the dashed line over time compared with the solid line over time). However, we see that the values range between 20% in the initial period without climate model uncertainty concerns up to almost 50% in 2100 for the case with climate model uncertainty concerns.

Figure 2(d) gives the total sum of risk and uncertainty prices (the orange lines) and the climate sum of the risk and prices (the blue lines). For the no climate model uncertainty case (the solid lines) the total and climate prices are essentially constant, with the climate price essentially zero due to the log utility preferences. With the inclusion of climate model uncertainty (the dashed lines) we see a decrease in the total price and a negative and increasing in magnitude climate price.

Figure 2(e) shows a detailed breakdown of the climate uncertainty price. In the case with no climate model uncertainty (the solid orange line) we see that this price is, by definition, zero. When accounting for climate model uncertainty (the dashed lines), we see more clearly the state dependent, time-varying climate risk price (the dashed orange line) that is consistent with the empirical evidence. Looking at the contribution to this climate uncertainty price coming from climate damages (the red dashed line) and from the climate model (the dashed yellow line), we see that uncertainty about the climate model itself has the biggest impact. This is likely the case because of the social planner's solution being considered, where climate damages are limited to minimize the severity of the externality and because climate damages depend importantly on the climate model.

Figure 2(f), shows the detection error probability bound. Following Anderson et al. (2003), I use an approximation of the bound for statistical detection between two models developed by Chernoff (1952) and Newman and Stuck (1979) to construct the detection error probability bound. In my setting, the value approximates the probability of the decision maker making either a type I or type II error in determining the true model for growth and climate dynamics from between the baseline model and the potential worst-case model. Online Appendix C.2.2 provides full details on the bound and its calculation. We see that the uncertainty concern is likely reasonable because the detection error probability bound is around 20%, well above the standard value used in economics for determining statistical confidence for such errors of at least 10%. Also, economic growth is producing the most significant uncertainty impact because the probability bound values are almost the same with (the dashed line) or without (the solid line) climate model uncertainty. This suggests that we are considering a modest level of climate model uncertainty in the model, and yet we still see noticeable impacts on the macroeconomic and asset pricing results of interest.

The key takeaway from these results is that accounting for climate model uncertainty has important implications for the macroeconomic and asset pricing outcomes in the model. The transition from dirty to clean is amplified by model uncertainty concerns, and the state-dependent climate risk and uncertainty price is only present when accounting for climate model uncertainty. Although some of this risk price might exist in the presence of larger risk aversion that is assumed away in a log utility model, the results also highlight that only modest uncertainty about the climate model is being imposed. Therefore, the model demonstrates that climate model uncertainty and aversion to the model uncertainty is a valuable mechanism for generating a significant, state-dependent climate risk price.

7. Partial Internalization Setting

The social planner's solution derived previously is a useful and tractable baseline for understanding the impacts of climate change and climate model uncertainty. However, an important step in providing quantitative implications of climate change and climate change uncertainty that are more realistic is to derive a solution where the climate externality is not fully internalized. Comparisons across these settings where agents vary in their internalization of the climate impacts provides the opportunity to see how production and asset pricing outcomes differ based on the assumed climate policy scenario.

As with the social planner's problem, the partial internalization equilibrium solution is given by the optimal choices of labor allocation

$$\{L^*_{C,t}, L^*_{B,t}, L^*_{G,t}\},$$

which are sufficient to characterize equilibrium consumption and production outcome functions:

$$\{C(L_{C,t}, L_{B,t}, L_{G,t}), Y(L_{C,t}, L_{B,t}, L_{G,t}), B(L_{C,t}, L_{B,t}, L_{G,t}), G(L_{C,t}, L_{B,t}, L_{G,t})\}.$$

These optimal policy functions are subject to the evolution of the stochastic process for the state variables T_t, Z_t, and A_t and resource constraints. The solution is still a recursive Markov equilibrium where optimal processes are functions of the relevant state variables A_t, T_t, Z_t, although the impacts on key outcomes differ now based on the degree of internalization of the climate externality.

For this case, I provide a proposition highlighting the key differences in the HJB equations, FOC, and pricing equations. The main differences that we will see are that the equations characterizing our equilibrium solutions are altered by removing climate implications or only accounting for a fractional amount of the true implications of emissions on climate change and its consequences.

Proposition 5 (Social Planner's HJB Equation and Optimal Policies). *The value function $J(A_t, T_t, Z_t)$ for the partial internalization setting has a quasi-analytical simplification*

$$J(A_t, T_t, Z_t) = \log A_t - Z_t + v(T), \tag{7.1}$$

where v(T) is the solution to the simplified HJB equation

$$\begin{aligned}\rho v(T_t) =& \rho \log[(L_{C,t})^{1-\alpha-\nu}(B_t^{\omega}+G_t^{\omega})^{\nu/\omega}] \\ & -\frac{1}{2\varrho}\left\{|\boldsymbol{\sigma_A}|^2 + |\boldsymbol{\sigma_T}|^2[v_T - [\nabla D](T_t)]^2\right\} \\ & + \left(\mu_A - \frac{1}{2}|\boldsymbol{\sigma_A}|^2\right) - [\nabla D](T_t)\hat{\mu}_T - \frac{1}{2}|\boldsymbol{\sigma_T}|^2[\nabla^2 D](T_t) \\ & + \hat{\mu}_T v_T + \frac{|\boldsymbol{\sigma_T}|^2}{2} v_{TT}.\end{aligned} \tag{7.2}$$

The evolution of T_t is treated as partially or entirely exogenous, and therefore the drift of T_t is given as $\hat{\mu}_T = \hat{\lambda} B_t + \bar{\mu}_T$. $\bar{\mu}_T$ is the presumed exogenous component of the temperature evolution, and $\hat{\lambda} B_t$ is the presumed endogenous component driven by anthropogenic emissions. This division of the climate impact is based on an internalization parameter $\chi \in [0,1]$, so that $\hat{\lambda} B_t = \chi \lambda B_t$ represents the fraction of the climate impact internalized and therefore endogenous, and in equilibrium $\bar{\mu}_T = (1-\chi)\lambda B_t$ is the fraction of the climate impact not internalized and therefore exogenous to the planner.

The worst-case model distortion is given by

$$\boldsymbol{\Lambda}^*_{\boldsymbol{A},t} = -\frac{1}{\varrho_A}\boldsymbol{\sigma_A}, \boldsymbol{\Lambda}^*_{\boldsymbol{T},t} = -\frac{1}{\varrho_T}\boldsymbol{\sigma_T}[v_T - [\nabla D](T_t)], \tag{7.3}$$

and implicit equations for the optimal choices of B_t and G_t are given by

$$\begin{aligned} 0 =& \frac{(1-\alpha-\nu)}{\nu\alpha_B A_B^{1/\alpha_B}} B_t^{1/\alpha_B - \omega}(B_t^{\omega} + G_t^{\omega}) - L_{C,t} \\ & - \frac{L_{C,t}(B_t^{\omega}+G_t^{\omega})}{\nu\rho}[\chi\lambda] B_t^{1-\omega}[v_T - [\nabla D](T_t)] \\ 0 =& \frac{(1-\alpha-\nu)}{\nu\alpha_G A_G^{1/\alpha_G}} G_t^{1/\alpha_G - \omega}(B_t^{\omega}+G_t^{\omega}) - L_{C,t},\end{aligned} \tag{7.4}$$

where $\chi \in [0,1]$ represents the fraction of the climate externality internalized in this setting and thus included in the optimal policy decisions. The solutions for B_t and G_t are numerically derived jointly with the value function as outlined in Online Appendix E. Furthermore, the numerical algorithm is such that an initial guess is given for $\hat{\mu}_T$ and the solution is iterated with $\hat{\mu}_T$ updated with each iteration until $\hat{\mu}_T$ converges to the true μ_T in equilibrium as in a big K, little k macroeconomic framework.

The asset pricing equations for partial internalization solution are similar to the socially optimal results. However, the contributions from the state variable T_t for these solutions are altered by the partial internalization of the climate externality. With these results, we can now calibrate the model to provide a quantitative comparison across the different scenarios to determine how climate change and climate model uncertainty will impact asset prices and production outcomes in each scenario.

I provide two sets of comparison results for the partial internalization setting. Each case assumes an internalization parameters value of $\chi = 0.2$, based on the external evidence from the World Bank discussed in Online Appendix C. Although the choice of this parameter is based on the World Bank data does not necessarily account for implicit carbon pricing, by providing these results and the full internalization results gives idea for how alternative scenarios would play out. The partial internalization results in each case are compared the results to the main model results where I assume $\varrho_T = 0.05$.

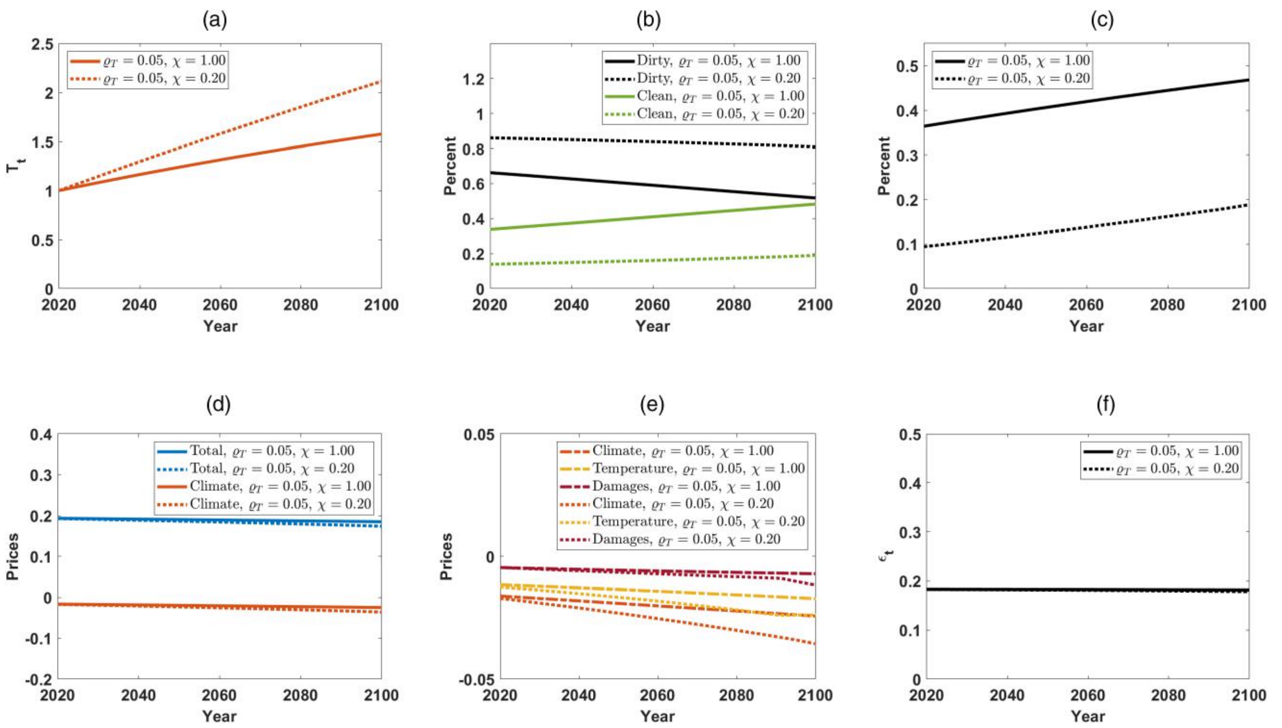
The first partial internalization case keeps all parameters, besides χ, equal to the main model results with $\varrho_T = 0.05$. There are substantial differences as a result of setting $\chi = 0.2$, as shown in Figure 3. First, the temperature increase is much larger, the transition from dirty to clean is much less, and the optimal tax is much lower. Each of these is intuitive given that the planner is not as concerned about the climate externality. On the asset pricing side, the increased temperature impact leads to larger risk and uncertainty prices for climate change and detection error probability bounds, but not dramatically so.

The second partial internalization case assumes $\chi = 0.2$ and a value of $\varrho_T = 0.035$ to show how even more severe model uncertainty can be assumed in this case while maintaining reasonable detection error probability bounds and input fraction values because of the partial internalization. These results are shown in Figure 4. As with the first case, the temperature increase is larger, the transition from dirty to clean is less, and the optimal tax is lower. However, the difference is not as stark as before because the increased concern about climate model uncertainty helps offset the reduced internalization of the climate externality. On the asset pricing side, we see much more significant impacts. In particular, the climate risk and uncertainty prices are much larger in magnitude, driving down the total risk price in a noticeable way. In addition, we see a noticeable decline in the detection error probability bound, while the value still remains at a level where we would not expect confidence in knowing the true mode. This highlights the fact that with partial internalization, we can see some transition from dirty to clean, as well as quite significant asset pricing implications because of the climate model uncertainty mechanism.

8. Parameter Sensitivity Comparison

I now highlight how the central results change for different values of the key parameters, and provide the

Figure 3. Macroeconomic and Asset Pricing Model Outcomes—Partial Internalization Case 1



Notes. Simulated outcomes for the model based on the numerical solutions. (a) Temperature pathway. (b) Fraction of inputs used in final output production coming from the clean input (green lines) and the dirty input (black lines). (c) Optimal carbon tax on dirty input production. (d) Total risk price (blue lines) and the climate risk price (orange lines). (e) Total climate uncertainty price (orange lines), temperature uncertainty price (yellow lines), and damage uncertainty price (red lines). (f) Detection error probability bound. Solid lines show results for full internalization of the climate externality, and dashed lines show results for partial internalization of the climate externality.

corresponding figures for these results in Online Appendix D. Similar types of sensitivity analysis have been done by others in the climate-economics literature, with Cai and Lontzek (2019) being a particular insightful example of providing comparisons with connections to risk-based uncertainty. This type of sensitivity analyses can also be informative in my setting as it provides additional insight not just about how quantitative effects vary across settings but also about the role of model uncertainty on the outcomes of interest not present in other papers.

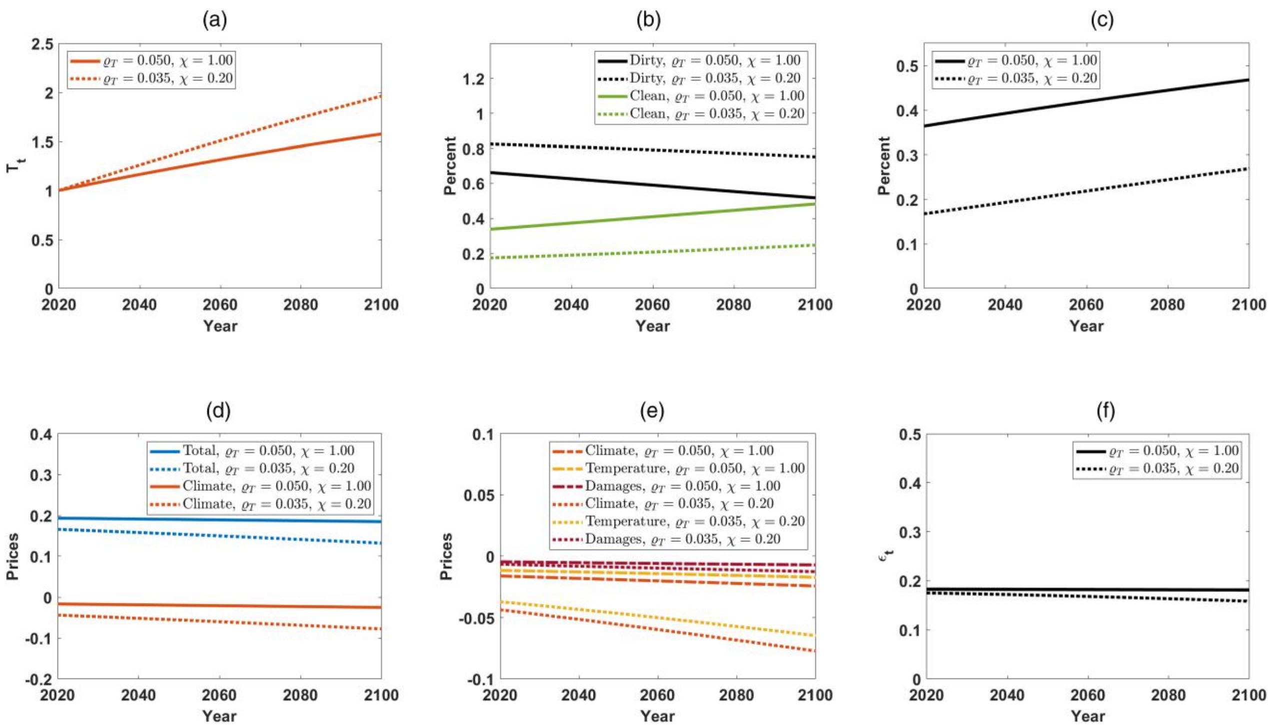
I focus on alternative values for the input elasticity of substitution (ω), the temperature volatility ($\boldsymbol{\sigma}_T$), climate damage parameters (γ_2^+, τ), and the TCRE (λ). Table 2 provides the alternative parameter values I use for these parameter sensitivity comparisons. The parameter ω is treated as known in the model, and so the exercise for this parameter is a fairly standard sensitivity analysis. The climate damage and TCRE parameters pertain to potentially misspecified climate and damage dynamics. Changing these parameters is equivalent to changing the baseline prior of the planner and directly influences the range of model distortions considered reasonable by the planner in their optimization choice, providing a sensitivity analysis of a different kind. Last, although $\boldsymbol{\sigma}_T$ is known, it explicitly scales the magnitude of the model distortion considered by the planner, and so changing this parameter provides another unique comparison that further highlights the role of model uncertainty mechanism.

For each of these comparisons, I resolve the model using parameter values greater and less than the baseline value, holding all other parameters constant at the baseline value. Importantly, this means my sensitivity analysis assumes a fixed value of $\varrho_T = 0.05$. As a result, the relative entropy values could vary substantively across cases. One could instead hold fixed the relative entropy by adjusting the ϱ_T value for each change in the parameter value. However, holding fixed ϱ_T and allowing for variations in the relative entropy produces a particularly interesting diagnostic for understanding how the uncertainty contribution varies across parameter scenarios.

Table 2. Model Sensitivity Analysis Parameters

Parameter	Symbol	Alternative values
Input elasticity of substitution	ω	$\{0.25, 0.85\}$
Climate damage curvature	γ_2^+	$\{0.00, 0.0394\}$
Climate damage threshold	τ	$\{1.5, 2.5\}$
TCRE parameter	λ	$\{0.0015, 0.002\}$
Temperature volatility	$\boldsymbol{\sigma}_T'$	$\{[0, 0.035, 0], [0, 0.06, 0]\}$

Figure 4. Macroeconomic and Asset Pricing Model Outcomes—Partial Internalization Case 2



Notes. Simulated outcomes for the model based on the numerical solutions. (a) Temperature pathway. (b) Fraction of inputs used in final output production coming from the clean input (green lines) and the dirty input (black lines). (c) Optimal carbon tax on dirty input production. (d) Total risk price (blue lines) and the climate risk price (orange lines). (e) Total climate uncertainty price (orange lines), temperature uncertainty price (yellow lines), and damage uncertainty price (red lines). (f) Detection error probability bound. Solid lines show results for full internalization, moderate climate uncertainty, and dashed lines show results for partial internalization, high climate uncertainty.

8.1. Input Elasticity of Substitution (ω)

The input elasticity of substitution, denoted by ω, pins down the ability of the planner to tradeoff between the two different energy inputs for production. I consider the following two values for ω: 0.25 and 0.85. Exploring different values of ω demonstrates the interaction between macroeconomic production choices, asset pricing outcomes, and the implications of uncertainty in an economy that is more or less constrained to using dirty inputs for final output production.

Figure D.1 in the online appendix shows the results for the two alternative values of ω. The first outcome to note is that changing ω has almost no impact on asset pricing outcomes, as seen by the overlapping solid and dashed lines in the plots on the bottom row. Changing ω does, however, have implications for the macroeconomic outcomes. The top left and top right plots show relatively modest changes to the temperature anomaly and carbon tax, and the input fractions for the clean and dirty inputs, given in the top middle plot, have fairly sizeable initial deviations that eventually converge to similar values. Thus, changing the input elasticity of substitution plays an important role on the quantity side, with negligible asset pricing impacts.

8.2. Climate Damage Parameters

The climate damage curvature parameter, denoted by γ_2^+, and the climate damage curvature threshold parameter, denoted by τ, are critical parameters in the climate-economic coupled dynamics. Increasing γ_2^+ and decreasing τ amplifies economic damages from climate change in important and distinct ways, either by ramping up the costs of a given temperature value or by moving forward the magnitude of climate damages on the climate change dimension. I consider the following two values for γ_2^+: 0.0 and 0.0295, and the following two values for τ: 1.5 and 2.5. Because of the central role of economic damages from climate change in the model, this comparison provides substantial insight for concerns about uncertainty on the model outcomes.

8.2.1. Climate Damage Curvature (γ_2^+). Figure D.2 in the online appendix shows the results for the two alternative values of γ_2^+ that I consider. Increasing the damage function curvature significantly increases the fraction of clean input use in final output production while conversely driving dirty input use down substantially. This is driven by a substantially higher carbon tax and is reflected in a much lower temperature anomaly. The asset pricing impacts are just as stark. The climate risk price, and in particular the temperature uncertainty component of that risk price, are quite substantial in magnitude as well. This results in a much lower total price of risk in the economy. This is at least in part driven by uncertainty, as the detection error probability bound is now lower. Conversely, without the additional damage function curvature, the macroeconomic and asset pricing effects of climate model uncertainty are all significantly muted.

8.2.2. Climate Damage Threshold (τ). Figure D.3 in the online appendix shows the results for the alternative values of τ that I consider. Shifting the climate damage threshold has a noticeable impact on macroeconomic and asset pricing outcomes, as is the case for the damage function curvature, but the magnitudes are not as extreme. Changing the threshold parameter shifts the initial input fractions either toward green, for $\tau = 1.5$, or dirty, for $\tau = 2.5$, while leaving intact the dynamic transitions. The same effect is seen in the carbon tax and temperature anomaly. The reason for this outcome is that an earlier, or later, climate damage threshold moves forward, or back, the time when damages become more severe. These same effects are also reflected in the asset pricing outcomes. The $\tau = 1.5$ climate damage threshold generates a larger, more negative climate risk price through the temperature dimension, whereas the converse holds for the $\tau = 2.5$ climate damage threshold.

8.3. TCRE (λ)

The TCRE parameter, denoted by λ, dictates the climate sensitivity. As with damages, this parameter heavily influences the coupled climate-dynamics system. I consider the following two values for λ: 0.0015 and 0.002. As this parameter dictates the rate of climate change for a given value of emissions, it is an important contributor to the uncertainty concerns motivating the model. As a result, we would expect the different values for this parameter to have meaningful impacts on macroeconomic and financial outcomes through the externality and uncertainty channels of interest.

Figure D.4 in the online appendix shows the results for the two alternative values of λ that I consider. Somewhat surprisingly, the results here are quite similar to the changes we saw for alternative values of ω. Changing λ has almost no impact on asset pricing outcomes, demonstrated by the overlapping solid and dashed lines in the plots on the bottom row of Figure D.1 in the online appendix. Changing λ does have some effect on the macroeconomic outcomes. The top left and right plots show some changes in temperature anomaly and carbon tax, whereas the input fraction for the clean and dirty inputs, given in the top middle plot, is sizeable in terms of the initial value change and the transition away from dirty inputs. Therefore, although changing the TCRE influences the macroeconomic outcomes, it plays surprisingly little role in changing the asset pricing outcomes or in altering the uncertainty implications.

8.4. Temperature Volatility ($\boldsymbol{\sigma}_T$)

The temperature volatility, denoted by $\boldsymbol{\sigma}_T$, highlights the risk component of uncertainty in the model. Importantly, beyond generating additional concerns for the planner in terms of future climate change outcomes, the temperature volatility directly influences the expression for the optimized model uncertainty contribution $\boldsymbol{\Lambda}_T$. As a result, this parameter has direct, meaningful impacts on the risk prices and the contribution of uncertainty aversion to the model outcomes. I consider the following two values for $\boldsymbol{\sigma}'_T$: [0, 0.035, 0] and [0, 0.06, 0].

Figure D.5 in the online appendix shows the results for the two alternative values of $\boldsymbol{\sigma}_T$ that I consider. As with changing the damage function curvature, there are fairly significant effects on the macroeconomic and asset pricing outcomes of the model by changing $\boldsymbol{\sigma}_T$. Increasing temperature volatility significantly increases the fraction of clean input use in final output production while conversely driving dirty input use down. This is connected to a much higher carbon tax and a noticeably lower temperature anomaly. The asset pricing impacts are just as apparent. The climate risk price, and in particular the temperature uncertainty component of that risk price, is substantially larger in magnitude. This results in a much lower total price of risk in the economy. For the smaller volatility, the transitions and climate impacts are relatively modest. The main driving force here comes entirely from the direct effect on uncertainty of changing $\boldsymbol{\sigma}_T$ without changing ϱ_T, as $\boldsymbol{\Lambda}_T$ is an explicit function of $\boldsymbol{\sigma}_T$. Thus, uncertainty clearly matters here, as can be seen by the reduced detection error probability bound, and by the fact that if concerns about climate model uncertainty are omitted from the model and the volatility is changed, there is essentially no change in outcomes, apart from the scaled adjustment in the almost negligible climate risk price because it is a direct function of $\boldsymbol{\sigma}_T$.

9. Alternative Models

The analysis thus far has focused on a simple, stylized framework to understand the impact of the climate model uncertainty mechanism on macroeconomic and asset pricing outcomes. I now briefly highlight key results from alternative models and discuss how these alternatives either deviate from or confirm the importance of the uncertainty mechanism highlighted in the baseline model. Full derivations and details of these models are given in Online Appendix B.

9.1. Climate Damages to Growth

One key alternative, which I previously mentioned, is the possibility that damages from climate change could be to economic growth rates and not to the level of output or consumption. Such damages could have significant impacts because of the permanence of the impacts. For simplicity, I solve a simplified model that assumes climate damages impact economic growth as follows:

$$dA_t/A_t = (\mu_A - \gamma T_t^{\kappa})dt + \boldsymbol{\sigma}'_A d\hat{\mathbf{W}}_t, \tag{9.1}$$

where γ and κ are the damage parameters, which indicate the sensitivity of economic growth to increases in temperature. From the derivations in this simplified setting for climate damages to economic growth, I show in Online Appendix B.1 that the functional forms for the macroeconomic and asset pricing outcomes are essentially equivalent to the baseline model. Although there could certainly be interesting quantitative differences as a result of assuming damages to economic growth, the main qualitative results, including the critical role of model uncertainty (which Burke et al. (2018) and others demonstrate is extremely relevant for these types of damages) in driving and amplifying state-dependent outcomes carry through.

9.2. Directed Technical Change

Another potentially important driver related to the economic dynamics of climate change is technical change, research and development (R&D), and adaption by firms. I explore another simplified extension to the model that allows for innovation in the clean and dirty sectors based on the optimal allocation of skilled labor. This extension adjusts the production functions for the two input types as follows:

$$\begin{aligned} B_t &= \bar{A_B}^{\varphi} m_B^{1-\varphi} (L_{B,t})^{\alpha_B} \\ G_t &= \bar{A_G}^{\varphi} m_G^{1-\varphi} (A_t L_{G,t})^{\alpha_G} \\ m_t &= m_{B,t} + m_{G,t} = 1. \end{aligned} \tag{9.2}$$

The implicit equations for the optimal choices of B_t and G_t take the same functional forms. The key difference is that the underlying technology for B_t and G_t depends on the choices for $m_{B,t}$ and $m_{G,t}$, which are given by

$$\begin{aligned} m_{B,t} &= \left\{ 1 + \left(\frac{G_t}{B_t}\right)^{\frac{1}{\alpha_B+\varphi-1}} \left(\frac{A_B}{A_G}\right)^{\frac{\varphi}{\alpha_B+\varphi-1}} \right\}^{-1} \\ m_{G,t} &= 1 - m_{B,t}. \end{aligned} \tag{9.3}$$

We can see that the innovation choice will depend on the relative productivity of each input production technology and the demand for the clean and dirty inputs. We know already that the climate and model uncertainty impacts lead B_t to decrease and G_t to increase. This same force will also shift innovation to the green sector. Thus, the innovation mechanism could potentially amplify the risk prices generated by T_t and Z_t because of the enhanced risk of transitioning from dirty to clean. This mechanism should help match long-term outcomes related to the transition from dirty to clean inputs while not changing the key qualitative outcomes seen thus far. Most importantly, this introduces an additional channel for uncertainty as it is unknown how successful R&D will be in enhancing green sector productivity, which again supports the baseline model mechanisms.

9.3. Climate Disasters

As has been noted and examined by Weitzman (2009b), Pindyck (2012), Hambel et al. (2021), Giglio et al. (2021), and Bansal et al. (2019), climate disasters are another key concept to consider in climate change. I consider another extension that follows the baseline model except the climate damage process is now given by

$$\begin{aligned} dZ_t = {} & [\nabla D](T_t)\lambda B_t dt + \frac{|\boldsymbol{\sigma}_T|^2}{2}[\nabla^2 D](T_t)dt \\ & + \boldsymbol{\sigma}_T'[\nabla D](T_t)(\boldsymbol{\Lambda}_{T,t}dt + d\boldsymbol{W}_t) + \Theta(T_t)dN_t, \end{aligned} \tag{9.4}$$

where $\Theta(T_t)$ is a general function for the size of the climate disaster that is (weakly) increasing in T_t, and $\Gamma(T_t)$ is a general function for the arrival rate of climate disaster jumps that is also (weakly) increasing in T_t. In this disaster framework, additional model uncertainty on the jump process can, and should, be used with respect to climate disasters. There are now model distortions from Brownian and jump components to consider:

$$\boldsymbol{\Lambda}^*_{A,t} = -\frac{1}{\varrho_A}\boldsymbol{\sigma}_A, \boldsymbol{\Lambda}^*_{T,t} = -\frac{1}{\varrho_T}\boldsymbol{\sigma}_T\{v_T - [\nabla D](T_t)]\}, \tag{9.5}$$

$$\Upsilon^*_t = \exp\left(-\frac{v(T_t + \Theta(T_t)) - v(T_t)}{\varrho_\Upsilon}\right). \tag{9.6}$$

This extension leads to several key asset pricing implications. In particular, in addition to the climate and productivity risk prices and market price of uncertainty connected to the Brownian shocks, there are jump risk and jump uncertainty prices. These are given by

$$\boldsymbol{\sigma}_{\boldsymbol{\pi},T} = \boldsymbol{\sigma}_T\left[\frac{C_T}{C} + \frac{1}{\varrho_T}v_T\right], \qquad \boldsymbol{\sigma}_{\boldsymbol{\pi},A} = \boldsymbol{\sigma}_A\left[1 + \frac{1}{\varrho_A}\right], \tag{9.7}$$

$$\begin{aligned} \boldsymbol{\sigma}_{\boldsymbol{\pi},Z} + \Gamma(T_t)\Upsilon_t\Psi(T_t) = {} & -\boldsymbol{\sigma}_T[\nabla D](T_t)\left[1+\frac{1}{\varrho_T}\right] \\ & + \Gamma(T_t)\exp\left(-\frac{v(T_t+\Theta(T_t))-v(T_t)}{\varrho_\Upsilon}\right)\left(1-\frac{\pi_t^+}{\pi_t}\right), \end{aligned} \tag{9.8}$$

$$\begin{aligned} & \boldsymbol{\Lambda}^*_{A,t} = -\frac{1}{\varrho_A}\boldsymbol{\sigma}_A, \ \ \boldsymbol{\Lambda}^*_{T,t} = -\frac{1}{\varrho_T}\boldsymbol{\sigma}_T[v_T - [\nabla D](T_t)], \\ & -\log(\Upsilon_t) = \frac{1}{\varrho_\Upsilon}(v(T_t+\Theta(T_t)) - v(T_t)). \end{aligned} \tag{9.9}$$

Again, this model will likely generate quantitative differences. However, the same key result that the impact of model uncertainty driving state-dependent climate change risk prices for asset pricing and implications for the transition to clean inputs will still hold.

9.4. Multidimensional Climate Model

The TCRE model used in my baseline setting does not directly account for the nonlinear, rapid rise and delayed convergence response to emissions. Such dynamics require a more general, multicomponent setting of climate dynamics. Barnett et al. (2021), in the spirit of recent work by Ricke and Caldeira (2014) and Dietz and Venmans

(2019), specifies such a setting of climate dynamics in their supplementary material by adapting the frameworks of Joos et al. (2013) and Pierrehumbert (2014). They incorporate the response dynamics of temperature to carbon emissions by including a state variable for the exponentially weighted average of current and past emissions that accommodates the initial rise in the impulse response of temperature to an emissions pulse and allows for the near-decade delay in realizing the full temperature impact of emissions in the carbon-climate dynamics. I adapt their proposed setting to demonstrate how such dynamics impact the outcomes of interest studied in my model. I provide a short summary here, with full details given in Online Appendix B.6.

I begin by specifying a two-state model of climate dynamics as follows:

$$dT_{1,t} = T_{2,t}dt + \boldsymbol{\sigma}_{\boldsymbol{T}}' \boldsymbol{dW}_t, \tag{9.10}$$

$$dT_{2,t} = -\vartheta T_{2,t}dt + \vartheta\lambda B_t dt, \tag{9.11}$$

where $T_{1,t}$ can be thought of as the atmospheric temperature anomaly, $T_{2,t}$ is the weighted average of emissions, and λ is the convergence rate of an emissions pulse. These nonlinear dynamics allow for a fast rise and delayed convergence of atmospheric temperature in response to carbon emissions. The altered temperature dynamics require us to update the dynamics of climate damages as

$$\begin{aligned} dZ_t = [\nabla D](T_{1,t})\lambda B_t dt + \frac{|\boldsymbol{\sigma_T}|^2}{2}[\nabla^2 D](T_{1,t})dt \\ + \boldsymbol{\sigma}_{\boldsymbol{T}}'[\nabla D](T_{1,t})d\hat{\boldsymbol{W}}_{\boldsymbol{t}}. \end{aligned} \tag{9.12}$$

The value function for the social planner's problem $J(A_t, T_{1,t}, T_{2,t}, Z_t)$ has a quasi-analytical simplification of the form:

$$J(A_t, T_{1,t}, T_{2,t}, Z_t) = \log A_t - Z_t + v(T_1, T_2), \tag{9.13}$$

a slight modification from the baseline setting that requires solving a two-state PDE. The optimal choices of model distortions and inputs are nearly identical in functional form compared with the baseline model derivations.

The temperature, climate damage, and productivity risk prices, as well as the market price of uncertainty or contribution to the market price of risk due to model uncertainty, are given by

$$\begin{aligned} \boldsymbol{\sigma}_{\boldsymbol{\pi},\boldsymbol{T}} &= \boldsymbol{\sigma}_T\left[\frac{C_{T_1}}{C} + \frac{1}{\varrho_{1,T}}v_{T_1}\right], \\ \boldsymbol{\sigma}_{\boldsymbol{\pi},\boldsymbol{Z}} &= -\boldsymbol{\sigma}_{\boldsymbol{T}}[\nabla D](T_{1,t})\left[1 + \frac{1}{\varrho_T}\right] \\ \boldsymbol{\sigma}_{\boldsymbol{\pi},\boldsymbol{A}} &= \boldsymbol{\sigma}_{\boldsymbol{A}}\left[1 + \frac{1}{\varrho_A}\right], \end{aligned} \tag{9.14}$$

$$\boldsymbol{\Lambda}^*_{\boldsymbol{A},\boldsymbol{t}} = -\frac{1}{\varrho_A}\boldsymbol{\sigma}_{\boldsymbol{A}}, \boldsymbol{\Lambda}^*_{\boldsymbol{T},\boldsymbol{t}} = -\frac{1}{\varrho_T}\boldsymbol{\sigma}_{\boldsymbol{T}}\{v_{T_1} - [\nabla D](T_t)\}. \tag{9.15}$$

With the framework in place, we can now consider the numerical results given in Figure B.2 of Online Appendix B.6. Critically, we can see that the more complex climate dynamics lead to results for the macroeconomic and asset pricing results that are essentially identical. The key takeaway from this setting is that, although it is important to match scientifically specified climate models, the TCRE model provides an approximation that is both tractable and consistent with more general climate specifications. The reason for this result is that apparently what matters to the planner is not necessarily the more transient components of the climate change dynamics, but the long-term consequences, which are consistent across the two specifications.

9.5. Learning vs. Model Uncertainty

An important alternative to consider is that of a learning model, as opposed to model uncertainty without learning. In this extension, I follow seminal work such as Pástor and Veronesi (2009) and Pastor and Veronesi (2012), assuming that the representative agent learns about state variable processes, Z_t and T_t, in our case. I assume there are unknown parameters ς for these processes for which the representative agent has a Normal prior distribution and receives a noisy signal with volatility η. Applying the standard Kalman-Bucy filtering result (described in Liptser and Shiryaev (2013)), we have that

$$\begin{aligned} d\hat{\varsigma}_t &= \hat{\boldsymbol{\sigma}}_{\boldsymbol{\varsigma},\boldsymbol{t}}'\eta^{-1}d\hat{W}_t^{\varsigma} \\ |\hat{\boldsymbol{\sigma}}^2_{\boldsymbol{\varsigma},\boldsymbol{t}}| &= \left(\frac{1}{|\boldsymbol{\sigma}(\boldsymbol{X}_t)^2|} + \frac{t}{\eta^2}\right)^{-1}. \end{aligned} \tag{9.16}$$

This will provide an additional component to the market price of climate risk, just as model uncertainty does. However, the learning component $(\partial\pi_t/\partial\hat{\varsigma}_t)|\hat{\boldsymbol{\sigma}}^2_{\boldsymbol{\varsigma},\boldsymbol{t}}| \to 0$ as $t \to \infty$. As a result, the market price of climate risk will gradually and deterministically decline over time, although some variation will exist for significant shock realizations. As I will highlight later in the paper, this is in direct contrast to empirical evidence suggesting an increasing market price of climate risk, highlighting how the model uncertainty mechanism appears to fit far better with observed outcomes than parameter learning.

9.6. Recursive Preferences

The final model extension I consider is when the representative agent has recursive preferences of the Duffie-Epstein-Zin-Weil type. Recursive preferences allow for amplified risk prices while maintaining a reasonable risk-free rate by separating risk aversion and the elasticity of intertemporal substitution. This allows for a separation of concerns about risk over time from concerns about risk across different states of nature, highlighting the importance of the resolution of uncertainty about future outcomes. Preferences take the following form:

$$h(C,J)=\begin{cases}\dfrac{\rho}{1-\theta^{-1}}\left(\dfrac{C^{1-\theta^{-1}}}{((1-\xi)J)^{\frac{\xi-\theta^{-1}}{1-\xi}}}-(1-\xi)J\right) & \text{if } \theta\neq 1\\ \rho(1-\xi)J\left(\log C-\dfrac{1}{1-\xi}\log((1-\xi)J)\right) & \text{if } \theta=1,\end{cases} \tag{9.17}$$

where ρ is the subjective discount rate, θ is the elasticity of intertemporal substitution (EIS), and ξ is risk aversion. These preferences create a handful of key differences. For the macroeconomic impact, rather than scaling the climate impact by ρ, it is scaled by $\rho(1-\xi)/\left(v^{\frac{\psi^{-1}-\xi}{1-\xi}}\right)$ when $\theta\neq 1$ so that risk aversion and the EIS play a prominent role. In addition, the climate and productivity risk prices, as well as the market price of uncertainty or contribution to the market price of risk due to model uncertainty, are now given by

$$\boldsymbol{\sigma}_{\boldsymbol{\pi},T}=-\boldsymbol{\sigma}_T\varpi_T,\quad \boldsymbol{\sigma}_{\boldsymbol{\pi},Z}=\boldsymbol{\sigma}_T[\nabla D](T_t),\quad \boldsymbol{\sigma}_{\boldsymbol{\pi},A}=\xi\boldsymbol{\sigma}_A, \tag{9.18}$$

where the expression for ϖ_T is an explicit function of risk aversion ξ, the EIS θ, the value function v, the derivative of the value function with respect to temperature v_T, consumption, and its first derivatives with respect to temperature C_t, C_T. The values for these expressions can only be determined numerically, but it is clear that concerns about the resolution of uncertainty in the future based on the EIS θ and risk aversion ξ magnify the impacts of climate risk compared with a log utility or even constant relative risk aversion (CRRA) utility. However, under an empirically based calibration, the numerical results given in Figure 1 of Online Appendix B.5 show that the level of risk aversion needed to generate similar risk prices to the model uncertainty setting in the baseline model is fairly large at $\xi=21$. What justifies this level of risk aversion? Skiadas (2003) shows that there is an equivalence between recursive preference and robust control for the case of unitary EIS. This perspective, that the significant risk aversion needed to match risk price outcomes in the recursive preference setting captures aversion to model uncertainty related to climate change, further validates the importance of the climate change model uncertainty mechanism highlighted in the baseline model.

10. Comparison with Existing Empirical Climate Finance Results

An important final question to address is whether the clear theoretical implications of the model are consistent with the estimates from the growing empirical literature on the economic and financial impacts of climate change. The central takeaway from the theoretical analysis is that climate model uncertainty has significantly amplified the effect of climate risk on asset prices, leading to an increasingly negative price of climate risk as climate change worsens. Although the model provides implications for the impacts of climate change on clean and dirty input production, these effects are less sensitive to climate model uncertainty, highlighting the value of the explicitly forward-looking nature of asset prices. For this reason, I focus here on comparing the model implications with the existing empirical literature connecting climate change to asset prices and financial markets.

In the empirical climate finance literature, an increasing number of studies have shown that the valuation of numerous types of assets has been impacted by climate change in largely negative ways.[9] Statistically significant effects for various climate change risks have been estimated in corporate and municipal bond markets (Painter 2020, Seltzer et al. 2020, Goldsmith-Pinkham et al. 2021, Huynh and Xia 2021), housing prices (Bernstein et al. 2019, Baldauf et al. 2020, Giglio et al. 2021), options markets (Ilhan et al. 2021, Kruttli et al. 2021), institutional investments (Alok et al. 2020, Krueger et al. 2020), and equity markets (Bansal et al. 2019; Barnett 2019; Choi et al. 2020; Bolton and Kacperczyk 2021a, b).

Moreover, many of these studies not only find that climate risk is priced but that the risk price has important state dependence as well. This includes amplified price impacts for bonds of longer maturity (Painter 2020, Goldsmith-Pinkham et al. 2021, Huynh and Xia 2020), as well as price impacts that have increased over time for bonds (Painter 2020, Goldsmith-Pinkham et al. 2021), house prices (Bernstein et al. 2019) and equities (Bansal et al. 2019, Barnett 2019, Bolton and Kacperczyk 2021b). Although a number of these studies link the increases in climate risk prices to increased climate change awareness, it is clear that amplified concerns about model uncertainty are necessarily linked with increased awareness and attention. The consistency between my model and the growing empirical climate finance literature highlights the validity of the theoretical analysis and the importance of accounting for climate model uncertainty when analyzing the economic and financial implications of climate change.

11. Conclusions

In this paper, I study the consequences of climate change and climate model uncertainty on quantities and asset prices. I do so by constructing a theoretical model that incorporates nonlinear climate impacts and aversion to model misspecification. These modeling elements generate a temperature state dependence in the key model results. In particular, model uncertainty and climate risk shifts the economy toward clean inputs rather than dirty inputs, but only if the climate externality is at least partially internalized. Most significantly, climate change has a negative price of risk that is significantly amplified by model uncertainty whether the climate externality is internalized or not.

There are a number of interesting extensions related to this research. One extension includes exploring the implications of alternative forms of model uncertainty, which could include smooth ambiguity preferences, structured model uncertainty, or other frameworks for analyzing uncertainty, to see how these preference specifications impact the theoretical model outcomes and how the results compare with empirical estimates. Specifically, because the uncertainties related to the interaction between climate change, economics, and finance are so substantive and the use of conditional relative entropy in this framework allows only for relatively small model distortions, the uncertainty characterization used in this paper could potentially understate the costs of climate change and the difficulty of transitioning away from dirty inputs. Thus, future work exploring alternative model uncertainty specifications that confront more severe model ambiguity or misspecification concerns and their interactions could be particularly relevant for providing further depth and insight into the critical question of the implications of climate model uncertainty on macroeconomic and asset pricing outcomes. Also, empirical work such as exploring the term structure of climate risk and the long-term implications of climate risk and climate model uncertainty could provide another important contribution to our understanding of the perceived risks associated with climate change, climate damages, and climate model uncertainty. I leave these extensions for future work.

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Endnotes
[1] IPCC Fifth Assessment Report (IPCC 2014).

[2] The very recent IPCC sixth Assessment Report (Masson-Delmotte et al. 2021) has also noted that scientific advances have allowed for a narrowing of the range of estimates for the ECS. However, as the cited scientific literature and Masson-Delmotte et al. (2021) note, meaningful uncertainties along geoscientific dimensions still exist and are expected to persist for some time.

[3] Although others, such as Newell et al. (2021), find less uncertainty in their estimates of climate damage functions based on variations and factors such as level versus growth rate damages, these differences and the unprecedented nature of potential future climate change suggest economic damages from climate change are an important channel of climate model uncertainty to account for in the analysis.

[4] I outline a model with growth rate damages from climate change in Online Appendix B.1. The main outcomes are consistent with this baseline model, although they could be even more severe given the permanent nature of growth rate damages and the magnitude of uncertainty about the severity of growth rate damages.

[5] This type of uncertainty seems particularly appropriate in the climate setting, as opposed to some form of Bayesian updating or learning, as there is very little signal that has been drawn from the data so far given the extreme long-run nature of climate dynamics and the relatively small sample of temperature and climate observations we have while facing anthropogenic impacts. In fact, this type of uncertainty coincides with the uncertainty findings of Roe and Baker (2007), Pindyck (2013), Weitzman (2012), Proistosescu and Wagner (2020), Sherwood et al. (2020), and the other geosciences and economics research cited previously.

[6] Relative entropy is the expected value of the log-likelihood ratio between two models, that is, the expected value of the log of the Radon-Nikodym derivative of the perturbed model with respect to approximating one. The value of relative entropy is zero when the models coincide and positive otherwise.

[7] Detailed explanations of robust preference problems can be found in Anderson et al. (2003), Cagetti et al. (2002), and Hansen et al. (2006), among others. Furthermore, Maccheroni et al. (2006) provide an axiomatic treatment of such formulations using penalization methods that the reader can refer to for further details.

[8] I refer the interested reader to Barnett et al. (2020) and their supplemental online notebook, which provides detailed computational information for a very similar solution method and algorithm.

[9] Murfin and Spiegel (2020) provides one clear counterpoint in the literature in that they find strong evidence of no effects of rising sea level risks on housing prices. Hong et al. (2019) is an example that finds underreaction to climate change risks in food-related stocks. The growing discussion on a possible "carbon bubble" also highlights concerns that markets may not yet fully incorporate climate risks.

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