# FINM 36702 1 Portfolio Credit Risk: Modeling and Estimation
# TA Session 4

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# Part I. Homework 3 Review

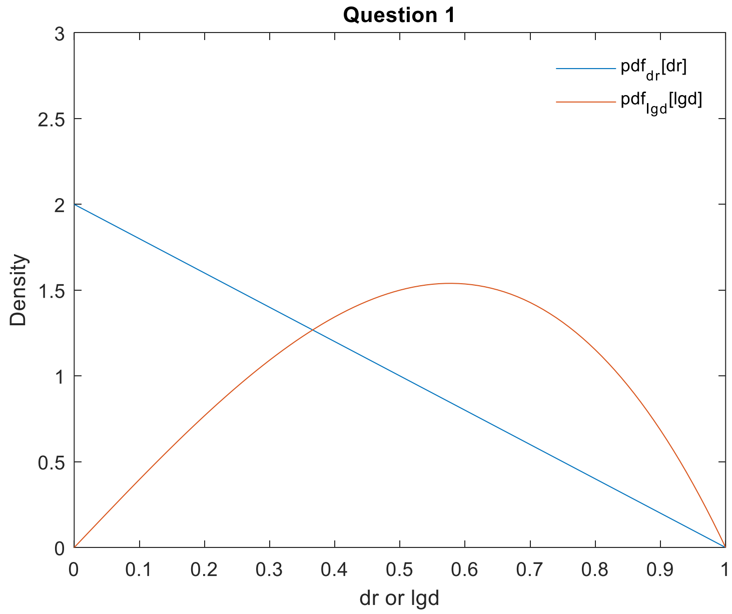
# Q1. LGD PDF Derivation & Plotting

- Suppose that the default rate of a portfolio has the triangular distribution: $pdf_{dr}[dr] = 2 - 2dr$. Suppose that in this portfolio $lgd$ is a function of $dr$: $lgd[dr] = dr^{1/2}$.

- Part 1. Derive and state the function $pdf_{lgd}[lgd]$, L2.S30–38, change of variable.

  - Step 1: Since $lgd = dr^{1/2}$, we have $dr = lgd^2$
  - Step 2: Then $\frac{\partial dr}{\partial lgd} = \frac{\partial lgd^2}{\partial lgd} = 2lgd$
  - Step 3: Apply change of variable (or the chain rule) to derive PDF of $lgd$ based on PDF of $dr$, $pdf_{lgd}[lgd] = \left|\frac{\partial dr}{\partial lgd}\right| pdf_{dr}[dr] = 2lgd(2 - 2dr) = 2lgd\left(2 - 2lgd^2\right) = 4lgd - 4lgd^3$
  - Sanity check (not required): The area under the PDF of LGD, $pdf_{lgd}[lgd] = \int_0^1 \left(4u - 4u^3\right)du = 1$.

- Part 2. Plot two functions: $pdf_{dr}[dr]$ and $pdf_{lgd}[lgd]$ on [0, 1]

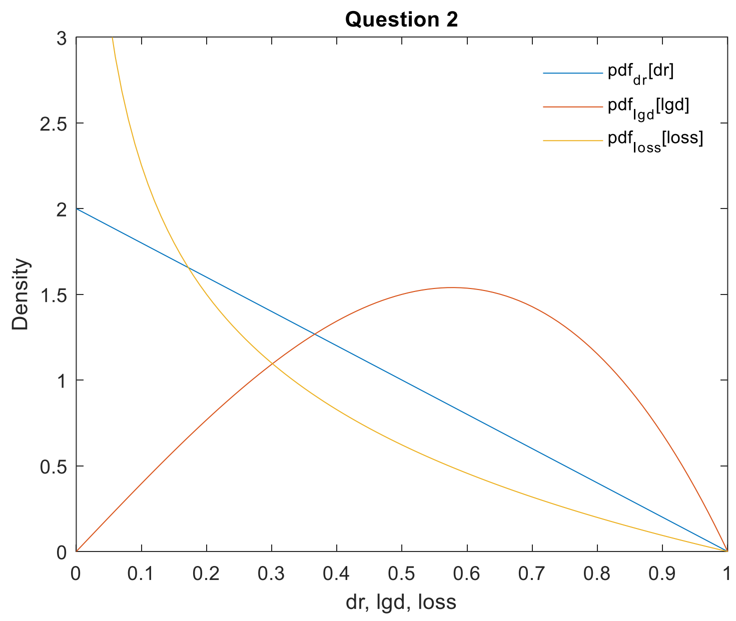


# Q2. Loss PDF Derivation & Plotting

- Same as Q1. Suppose that the default rate of a portfolio has the triangular distribution: $pdf_{dr}[dr] = 2 - 2dr$. Suppose that in this portfolio $lgd$ is a function of $dr$: $lgd[dr] = dr^{1/2}$.

- Part 1. Derive and state the function $pdf_{loss}[loss]$, L2.S30–38, change of variable.
  - Since $loss = dr \cdot ldg = dr \cdot dr^{1/2} = dr^{3/2}$, we have $dr = loss^{2/3}$
  - Therefore, $\frac{\partial dr}{\partial loss} = \frac{\partial loss^{2/3}}{\partial loss} = \frac{2}{3} loss^{-1/3}$
  - Apply the chain rule to derive PDF of loss from PDF of dr, $pdf_{loss}[loss] = \left|\frac{\partial dr}{\partial loss}\right| pdf_{dr}[dr] = \frac{2}{3} loss^{-1/3}(2 - 2dr) = \frac{2}{3} loss^{-1/3}\left(2 - 2loss^{2/3}\right) = \frac{4}{3} loss^{-1/3} - \frac{4}{3} loss^{1/3}$
  - Sanity check (not required): the area under $pdf_{loss}[loss] = \int_0^1 \left(\frac{4}{3}u^{-1/3} - \frac{4}{3}u^{1/3}\right) du = 1$

- Part 2. Plot three functions: $pdf_{dr}[dr]$, $pdf_{lgd}[lgd]$ and $pdf_{loss}[loss]$ on [0, 1]



# Q2 Part 3. Credit Identities of a Loan

- L3.S18-25 (ELGD: Expected loss given default)

- Need to know:

$$PD = \mathbb{E}[cPD] = \mathbb{E}[dr] = \int_0^1 dr \cdot PDF_{dr} \cdot d[dr] = 0.3333$$

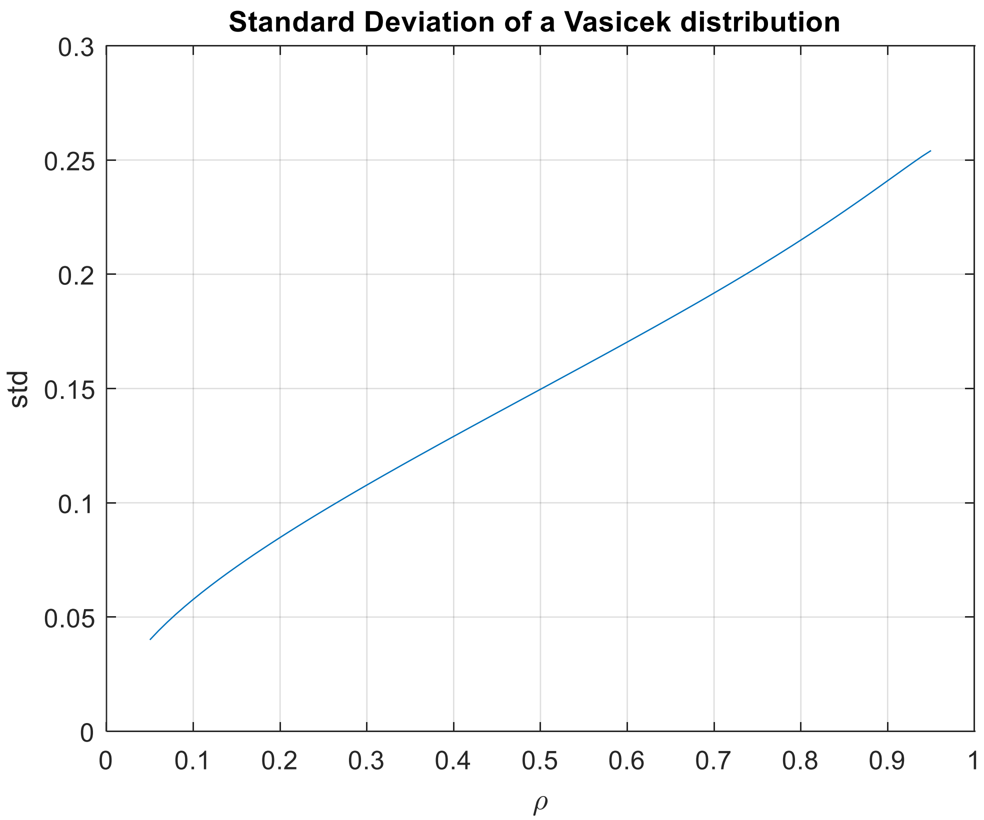
- State the values of
  - Expected loss, $EL = \mathbb{E}[closs] = \int_0^1 loss \cdot PDF_{loss} \cdot d[loss] = 0.2286$
  - Expected LGD, $ELGD = \frac{EL}{PD} = \frac{0.2286}{0.3333} = 0.6857$
  - “Time-weighted” LGD $= \mathbb{E}[clgd] = \int_0^1 lgd \cdot PDF_{lgd} \cdot d[lgd] = 0.5333$

# Q3. Std. of a Vasicek distribution and Plotting

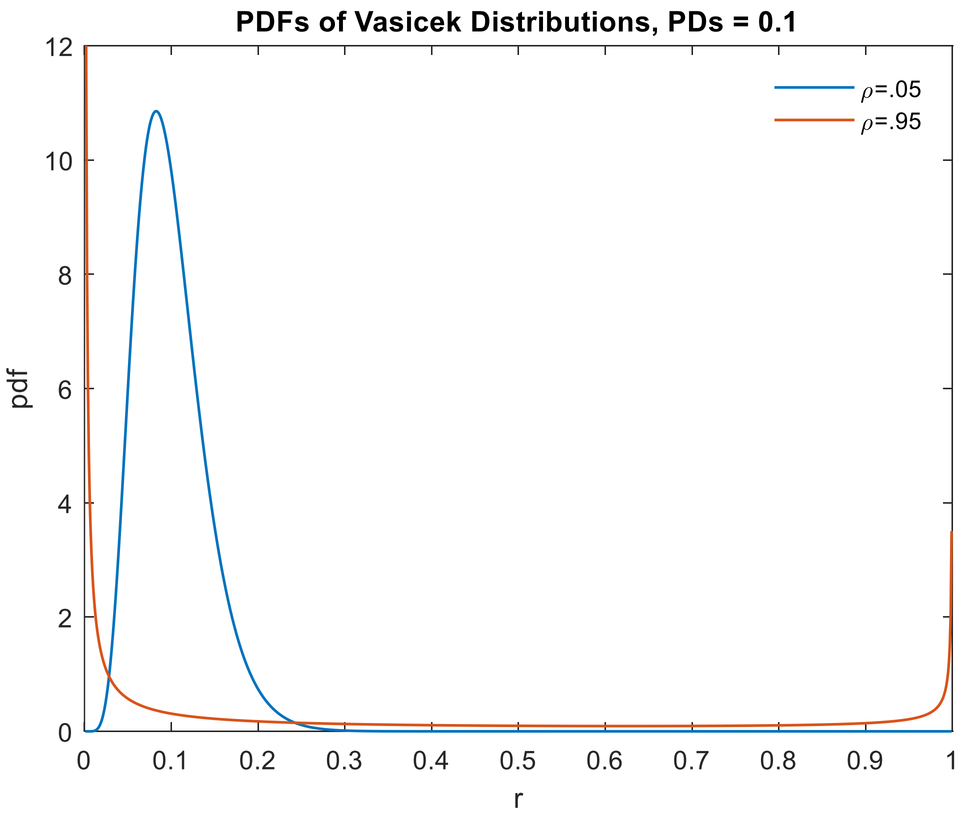
- Part 1. Express the standard deviation of a Vasicek distribution as an integral that involves the Vasicek PDF.
  - Let $f_{cPD}[r]$ denote PDF of Vasicek distribution, then
  - $\boldsymbol{f_{cPD}[r] = \frac{\sqrt{1-\rho}}{\sqrt{\rho}\,\phi[\Phi^{-1}[r]]}\,\phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[r]-\Phi^{-1}[PD]}{\sqrt{\rho}}\right]}$
  - Let $SD_{cPD}[r]$ denote the standard deviation then
  - $\boldsymbol{SD_{cPD}[r] = \sqrt{\mathbb{E}[(r-\mu)^2]} = \sqrt{\int_0^1 f_{cPD}[r]\cdot(r-\mu)^2 dr}}$, and $\boldsymbol{\mu = PD}$
  - Therefore, $\boldsymbol{SD_{cPD}[PD,\rho] = \sqrt{\int_0^1 \frac{\sqrt{1-\rho}}{\sqrt{\rho}\,\phi[\Phi^{-1}[r]]}\,\phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[r]-\Phi^{-1}[PD]}{\sqrt{\rho}}\right](r-PD)^2 dr}}$

# Q3 Parts 2 & 3. Plotting

- Plot the standard deviation for $0.05 < \rho < 0.95$

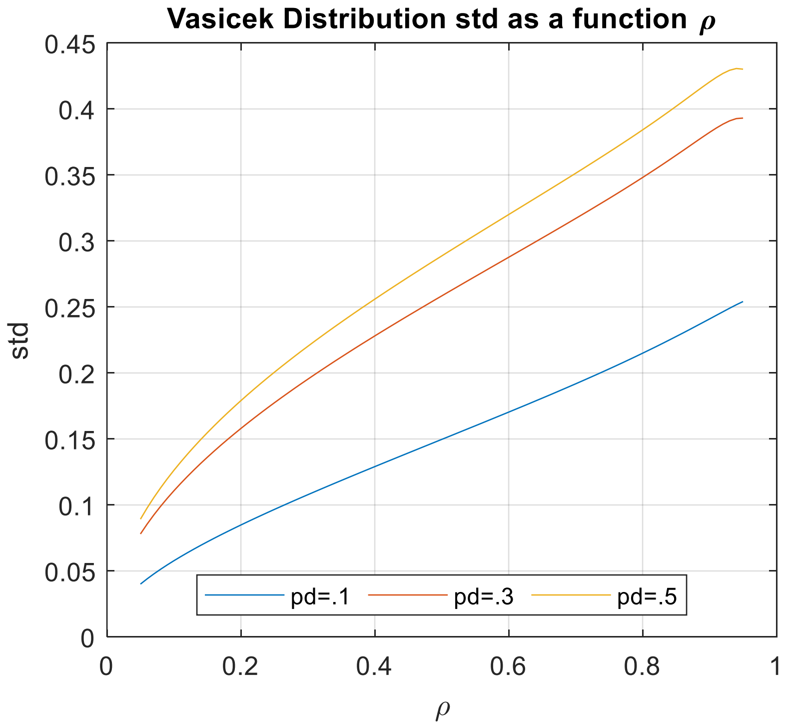


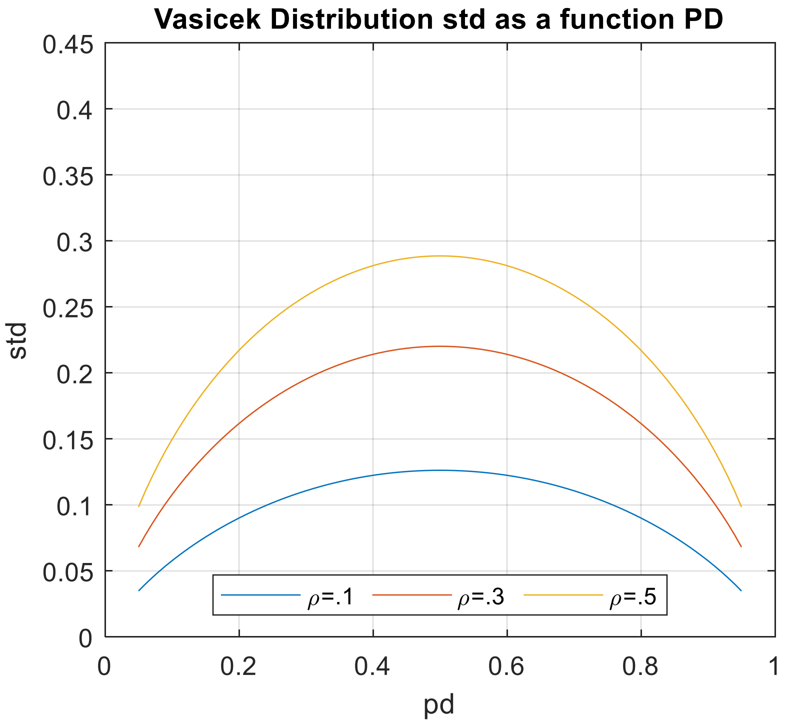

- Plot two Vasicek distributions: PD = 0.10, $\rho$ = 0.05 and PD = 0.10, $\rho$ = 0.95



# Modeling Thinking: Parameter Tests

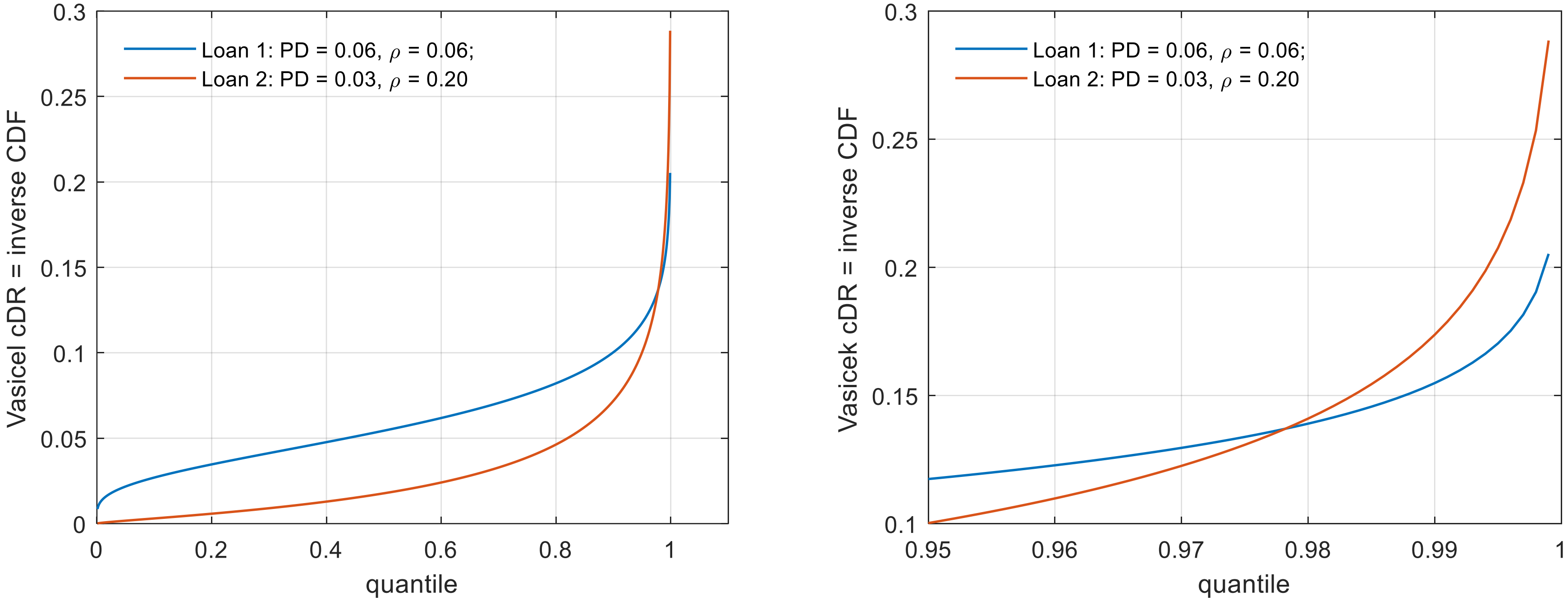
- Sensitivity analysis: Which parameter is a larger driver of <u>std</u>, *PD* or $\rho$?



# Q4. Unexpected Behaviors of cPD

- Two loans ~ Vasicek[$PD$, $\rho$]. $PD_1$ = 0.06, $\rho_1$ = 0.06; $PD_2$ = 0.03, $\rho_2$ = 0.20. Part A: Plot and compare the two inverse CDF's.




… The first loan has twice the PD, and in 98% of the years, it has a greater default rate. Only in the ~2% (=1 - 0.9782) of worst years does the other loan have greater conditional default since its $\rho$ value is much higher.

# Part II. Perspectives and Hints for Homework 4

# Q1. Loan Identities from Data

- A loan can take one of four states as follows:

| State | A | B | C | D |
|---|---|---|---|---|
| Probability of state | 0.40 | 0.30 | 0.20 | 0.10 |
| cDR | 0.02 | 0.04 | 0.06 | 0.08 |
| cLGD | 0.10 | 0.30 | 0.50 | 0.70 |

- What is the value of
  - The expected loss of the loan (EL)?
  - The expected LGD of the loan (ELGD)?
  - The “time-weighted LGD” of the loan?
- L3.S18-25 (ELGD: Expected loss given default)

# Q2. Alternative Hypothesis for LGD Function

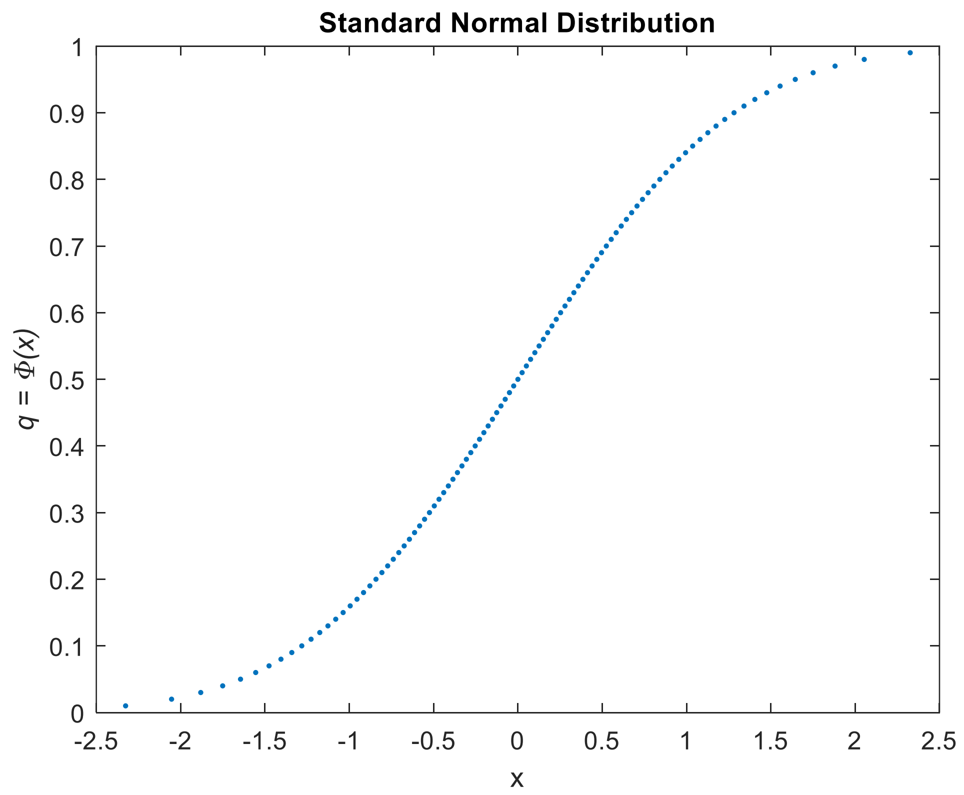
- Let PD = 5%, ELGD = 30%, and $\rho$ = 15%.
- Assume the alternative LGD function (L4.S36 – 41), and plot the function within the unit square for four values of the “a” parameter: {-2, 0, 1, 2}
  - Hint: L4.S40

# Q3. Parameters Testing

- Suppose that cPD ~ Vasicek [PD = 0.02, $\rho$ = 0.10]. Assuming that cPD and cLoss are comonotonic.
- Part 1. Plot three LGD functions for three possible distributions of cLoss:
  a. cLoss ~ Vasicek [ EL = 0.01, $\rho$ = 0.05]
  b. cLoss ~ Vasicek [ EL = 0.01, $\rho$ = 0.1]
  c. cLoss ~ Vasicek [ EL = 0.01, $\rho$ = 0.15]
  - Limit the default axis to {0, 0.5} and limit the vertical axis to {0, 1.2}.
- Part 2. Comment on the usefulness of each possible LGD function.
  - Hint: You should be plotting three cLGD functions against cPD. X-axis must cover the range [0, 0.5].

# Q3. Technical Notes

- Key formulas on L2.S31,S34
- What is a quantile, $q$?
  - $q = 0 : .01 : 1$, or
    $q \in \{0, 0.01, 0.02, \ldots, 0.99, 1\}$
- How to compute $q$?
  - Let $CDF: F_X(x) = P(X \leq x) = q$
    and is monotonic
  - Then, $x = F_X^{-1}(q)$



# Q4. ELGD = ?

- Using the assumptions of Question 3(b), what is the value of ELGD?
  - Hint: You should know the identities of a loan well and try not to over think!