FINM 36702

Homework #2

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Problem 1

For the following collection of five firms, simulate 10,000 runs to find the standard deviation of the number of defaults. Simulate again to find the standard deviation of the number of defaults when all off-diagonal correlations are set equal to zero instead of the values shown.

Firm	PD_i	Correlation Matrix $\rho_{i,j}$				
1	0.5	1	0.05	0.1	0.15	0.2
2	0.4	0.05	1	0.25	0.30	0.35
3	0.3	0.10	0.25	1	0.40	0.45
4	0.2	0.15	0.30	0.40	1	0.50
5	0.1	0.20	0.35	0.45	0.50	1

Solution 1

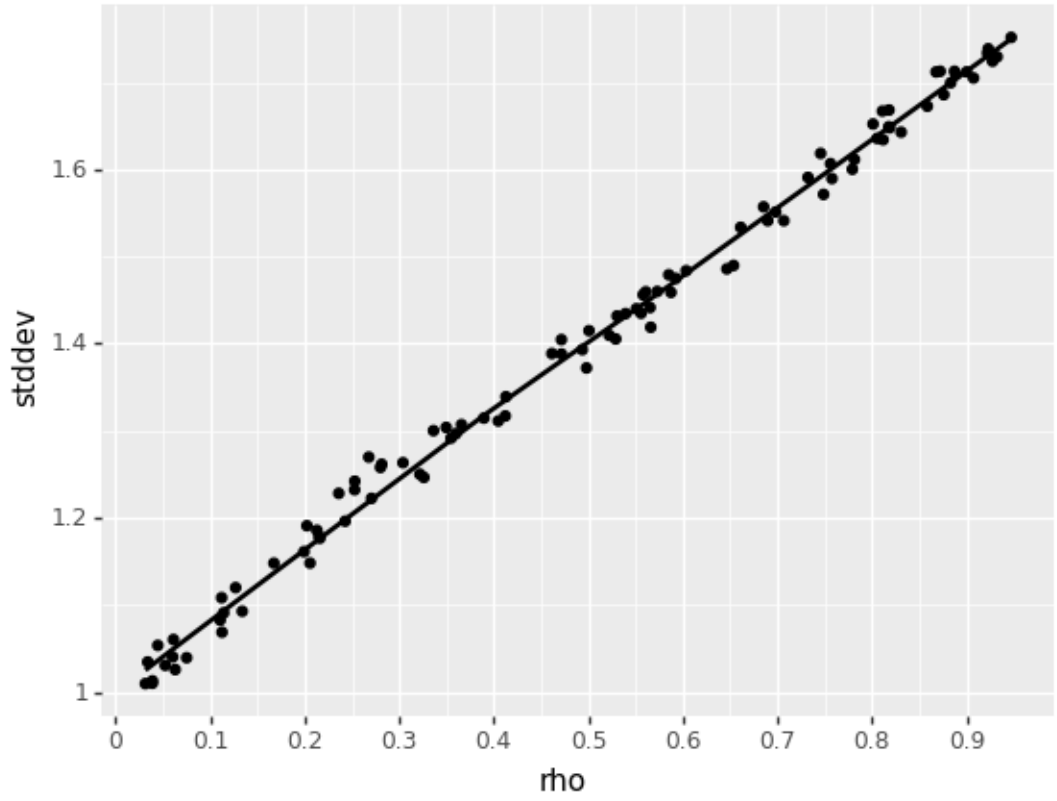
Simulating 10,000 runs to find the standard devition of number of defaults
The expected default rate is: 1.5
The standard deviation is: 1.2
When all off-diagonal correlations are set equal to zero:
The new expected default rate is: 1.5
The new standard deviation is: 0.97

Problem 2

In general, the standard deviation of the number of defaults—the risk, simply put—rises with correlation. Plot the standard deviation of the number of defaults in 1,000 simulation runs as a function of ρ in the range [0, 0.95], where every off-diagonal element in the previous matrix is replaced by the value of ρ.

Solution 2

We take 100 uniformly distributed rho values between [0,0.95] and then simulate 1000 times for each rho to get a standard deviation and then create a scatter plot of the standard deviations versus each respective rho value.
A trend can be observed that as the correlation between the firms increases the standard deviation of the number of defaults increases. This signifies the tail risk of more firms defaulting together increasing with the cross firm correlations.



Problem 3

Assume the following portfolio. Exposures are stated in USD. Questions can be answered by simulation or calculation; each method provides a check on the other.

	Correlation Matrix				
Firm	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Firm 1	1	0.15	0.2	0.25	0.3
Firm 2	0.15	1	0.25	0.3	0.35
Firm 3	0.2	0.25	1	0.35	0.40
Firm 4	0.25	0.3	0.35	1	0.45
Firm 5	0.3	0.35	0.4	0.45	1

Loan	Firm	PD	ELGD	Exposure
Loan 1	Firm 1	0.1	0.1	700
Loan 2	Firm 2	0.2	0.2	600
Loan 3	Firm 3	0.3	0.3	500
Loan 4	Firm 4	0.4	0.4	400
Loan 5	Firm 5	0.5	0.5	300
Loan 6	Firm 4	0.4	0.6	200
Loan 7	Firm 5	0.5	0.7	100

What are the values of these four quantities?

- Prob[$D_4 = 1$ and $D_5 = 1$]? (What is PDJ for these two firms?)
- Prob[$D_4 = 1$ and $D_5 = 1 |\ D3 = 1$]? (That is, what is the probability that both Firm 4 and Firm 5 default, given that Firm 3 defaults?)
- What is the portfolio expected loss rate as a fraction of the $2800 exposure?
- What is the correlation between D_3 and D_4?

Solution 3 (a)

The $P[D_4 = 1 \cap D_5 = 1] = 0.27$

Solution 3 (b)

The $P[D_4 = 1 \cap D_5 = 1 | D_3 = 1] = 0.44$

Solution 3 (c)

The portfolio expected loss rate as a fraction of $2800 = 0.11

Solution 3 (d)

Correlation between D_3 and $D_4 = 0.22$

Problem 4

Suppose that $PD_X = 0.1$, $PD_Y = 0.2$, and the latent variables responsible for default obey the 36702 distribution: $f_{x,y}[x, y] = (1 + 3x - y)/2$. What are the values of PDJ, D_{corr}, and ρ?

Solution 4

The value of PDJ using the 36702 distributions is : $PDJ = 0.025$
The value of $D_{corr} = 0.039$
The value of $\rho = 0.09$