FinMath 36702 Assignment 3

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1. Question 1. Suppose that the default rate of a portfolio has the triangular distribution: $pdf_{dr}[dr] = 2 - 2dr$. Suppose that in this portfolio lgd is a function of $dr : lgd[dr] = dr^{1/2}$. Derive and state the function $pdf_{lgd}[lgd]$. Create a single diagram containing plots of ($pdf_{dr}[dr]$ and $pdf_{lgd}[lgd]$) for variables in the range between 0 and 1.

Solution

We have the following equations

$$pdf[dr] = 2 - 2dr \tag{1}$$

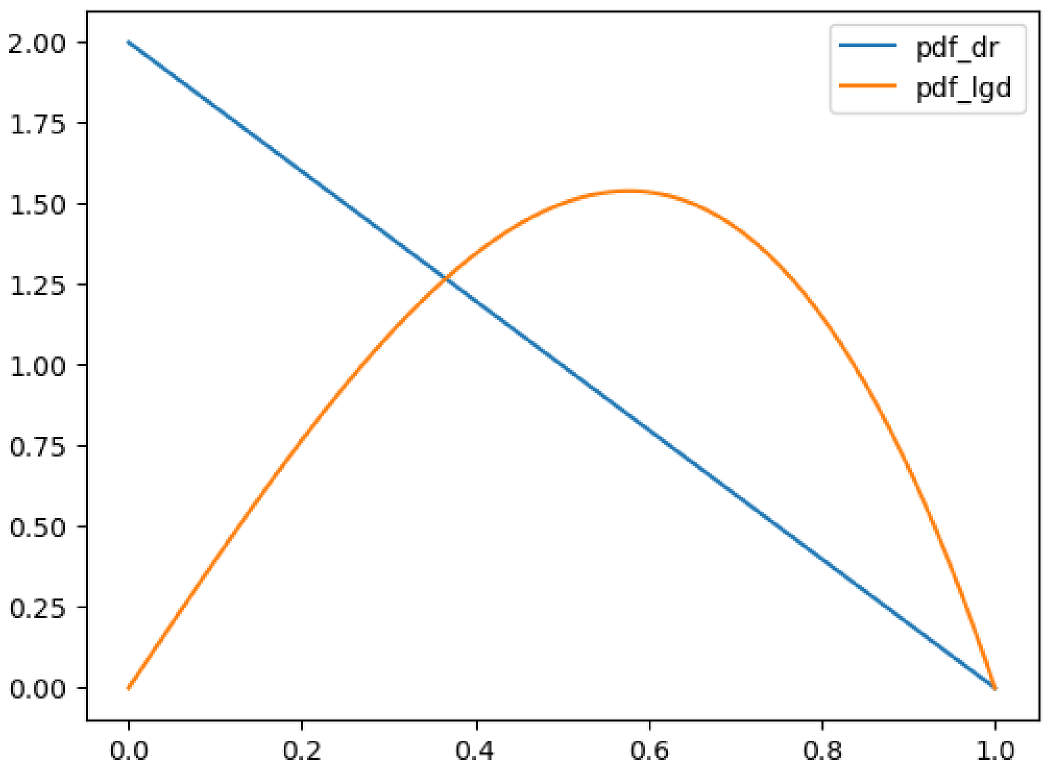
$$lgd[dr] = dr^{1/2} \tag{2}$$

Calculating the pdf of the lgd,

$pdf[lgd] = pdf[dr] * \frac{d(dr)}{d(lgd)}$

$pdf[lgd] = (2 - 2lgd^2) * 2lgd$

$pdf[lgd] = 4lgd - 4lgd^3$



2. Making the same assumptions as in Question 1, derive and state $pdf_{loss}[loss]$. Create a diagram containing the two plots from Question 1 along with the plot of $pdf_{loss}[loss]$ for variables in the range between 0 and 1; limit the vertical axis to the range from zero to 3. State the values of

- Expected loss, EL
- Expected LGD, ELGD
- "Time-weighted" LGD

Solution

The Loss equation is as follows:

$$loss = dr * lgd \tag{3}$$

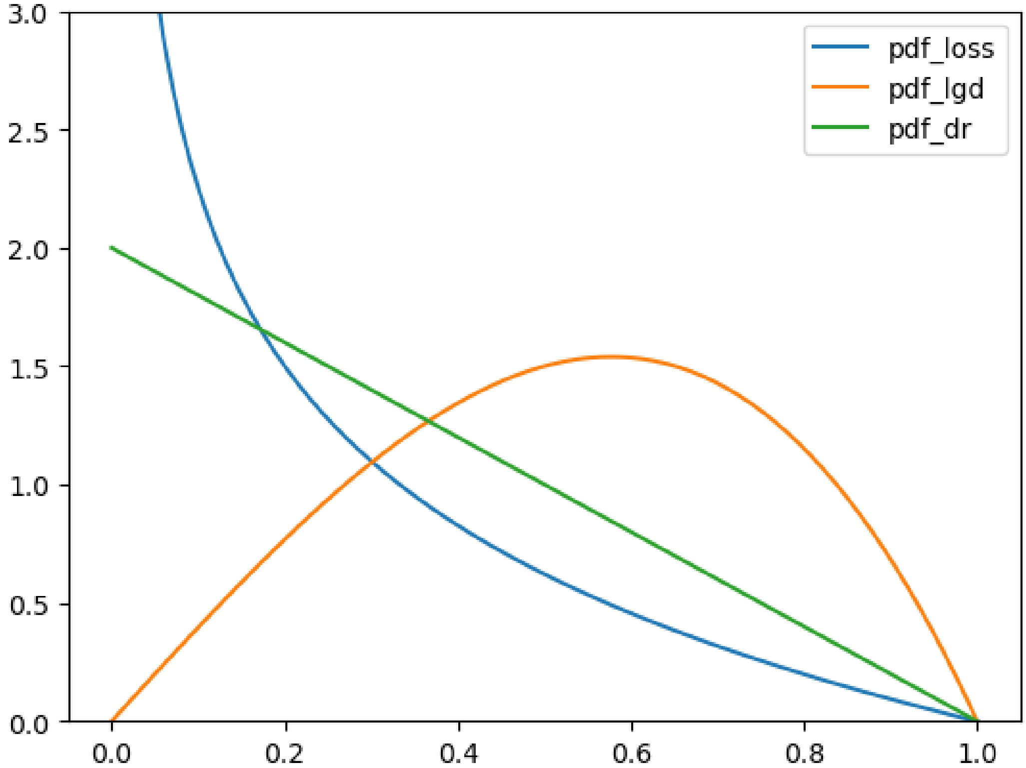
Substituting for lgd we get:

$dr = loss^{2/3}$

$pdf_loss[loss] = (2 - 2loss^{2/3}(2/3)loss^{-1/3}$

$$pdf_loss[loss] = \frac{4}{3}(loss^{(-1/3)} - loss^{(1/3))} \tag{4}$$

We plot the three PDFs:



The EL, LGD, and the cLGD are calculated using python are below:

```
#intergrate pdf_loss from 0 to 1 to get the expected loss. use scipy int
```

```
print ('Expected Loss is: ', round(ELGD*PD,3))
print ('Expected Loss Given Default is: ', round(LGD,3))
print ('Conditional Expected Loss Given Default is: ', round(CLGD,3))
```

0.0s

```
Expected Loss is:  0.178
Expected Loss Given Default is:  0.533
Conditional Expected Loss Given Default is:  0.533
```

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3. Express the standard deviation of a Vasicek distribution as an integral that involves the Vasicek PDF. For distributions with PD = 0.10, numerically integrate and plot the standard deviation for $0.05 < \rho < 0.95$. On a separate diagram, plot two Vasicek distributions: $PD = 0.10$, $\rho = 0.05$ and $PD = 0.10$, $\rho = 0.95$, limiting the vertical axis to $\{0, 0.12\}$.

Solution

4. Suppose two loans have Vasicek distributions. One loan has $PD = 0.06$, $\rho = 0.06$, the second loan has $PD = 0.03$, $\rho = 0.20$, and both loans respond to the same systematic risk factor. Plot on a single diagram the two inverse CDFs. At the lower quantiles, the first loan has greater cPD than the second. The situation is reversed at very high quantiles. Estimate the quantile at which both loans have the same value of cPD.

Solution

```
from scipy.stats import norm
pd1, rho1 = 0.06, 0.06
pd2, rho2 = 0.03, 0.20

# q at which cPD1 = cPD2
numerator = (norm.ppf(pd1)/np.sqrt(1-rho1) - norm.ppf(pd2)/np.sqrt(1-rho2
denominator = (np.sqrt(rho2)/np.sqrt(1-rho2) - np.sqrt(rho1)/np.sqrt(1-rh
q = norm.cdf(numerator/denominator)

print(f"q = {q:.2f}")
```

✓ 0.0s

```
q = 0.98
```

```
def inv_cdf(pd, q, rho):
    return norm.cdf((norm.ppf(pd) + np.sqrt(rho)*norm.ppf(q))/np.sqrt(1-
```

✓ 0.0s

```
#plot the invcdf_cpd1 and invcdf_cpd2 for q from 0 to 1
q = np.linspace(0,1,100)
plt.plot(q, inv_cdf(pd1, q, rho1))
plt.plot(q, inv_cdf(pd2, q, rho2))
plt.legend(['invcdf_cpd1', 'invcdf_cpd2'])
plt.show()
```