FinMath 36702 Assignment 2 Due 6pm 6 April 2023.

Lisheng will discuss strategies for solving these questions in the TA session on April 2, and he will present full solutions on April 9. Please submit homeworks as detailed in "FINM36702 Assignment Submission Instructions" located on Canvas.

State numerical answers to precision of 2 significant digits. For example, if the exact answer is $\pi/2$, then the answer to 2-digit precision is 1.6.

Question 1. For the following collection of five firms, simulate 10,000 runs to find the standard deviation of the number of defaults. Simulate again to find the standard deviation of the number of defaults when all off-diagonal correlations are set equal to zero instead of the values shown.

Firm	PD_i	Correlation Matrix $\rho_{i,j}$				
1	0.5	1	0.05	0.1	0.15	0.2
2	0.4	0.05	1	0.25	0.30	0.35
3	0.3	0.10	0.25	1	0.40	0.45
4	0.2	0.15	0.30	0.40	1	0.50
5	0.1	0.20	0.35	0.45	0.50	1

Question 2. In general, the standard deviation of the number of defaults—the *risk*, simply put—rises with correlation. Plot the standard deviation of the number of defaults in 1,000 simulation runs as a function of ρ in the range [0, 0.95], where every off-diagonal element in the previous matrix is replaced by the value of ρ.

Question 3. Assume the following portfolio. Exposures are stated in USD. Questions can be answered by simulation or calculation; each method provides a check on the other.

Loan	Firm	PD	ELGD	Exposure
Loan 1	Firm 1	0.1	0.1	700
Loan 2	Firm 2	0.2	0.2	600
Loan 3	Firm 3	0.3	0.3	500
Loan 4	Firm 4	0.4	0.4	400
Loan 5	Firm 5	0.5	0.5	300
Loan 6	Firm 4	0.4	0.6	200
Loan 7	Firm 5	0.5	0.7	100

Corrrelation matrix

	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Firm 1	1	0.15	0.2	0.25	0.3
Firm 2	0.15	1	0.25	0.3	0.35
Firm 3	0.2	0.25	1	0.35	0.4
Firm 4	0.25	0.3	0.35	1	0.45
Firm 5	0.3	0.35	0.4	0.45	1

What are the values of these four quantities?

- Prob[$D_4 = 1$ and $D_5 = 1$]? (What is PDJ for these two firms?)
- Prob[$D_4 = 1$ and $D_5 = 1 | D_3 = 1$]? (That is, what is the probability that both Firm 4 and Firm 5 default, given that Firm 3 defaults?)
- What is the portfolio expected loss rate as a fraction of the $2800 exposure?
- What is the correlation between D_3 and D_4?

Question 4. Suppose that $PD_X=0.1$, $PD_Y=0.2$, and the latent variables responsible for default obey the 36702 distribution: $f_{x,y}[x, y]= (1 + 3 x - y) / 2$. What are the values of PDJ, Dcorr, and ρ?