Problem Set 2

FINM 37301 - 2023

UChicago Financial Mathematics

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In [1]:

```
import pandas as pd
import numpy as np
import math
from IPython.display import Markdown, display
def printmd(string):
    display(Markdown(string))

from scipy.optimize import fsolve

pd.options.display.float_format = '{:.4f}'.format

from matplotlib import pyplot as plt
import warnings

warnings.filterwarnings("ignore")
pd.set_option("display.precision", 4)
```

11) (This problem counts for 4 points) Relative volatility of spot, forward and points

Let $\boldsymbol{S_t}$ be the spot FX rate at time $\boldsymbol{t}$. For this exercise we assume $\boldsymbol{t}$ is measured in years, and t refers to the "trade date", i.e., the time rates are observed in the market, not the "value date".

Let $\boldsymbol{F_t}$ be the 5-month forward rate. Specifically, for any time t, $\boldsymbol{F_t}$ is the quoted rate for a 5-month forward (i.e., $\boldsymbol{F_t}$ is not a specific contract, but rather a rate observed each day in the market.)

We want to calculate the standard deviations of log($\boldsymbol{F_t}$) and log($\boldsymbol{F_t/S_t}$) over the period [0, 1], assuming that covered interest rate parity holds. To be clear, standard deviation in this case is measuring the uncertainty in what log($\boldsymbol{F_t}$) and log($\boldsymbol{F_t/S_t}$) will be at time t=1 conditional on their values at time t=0.

Assume log($\boldsymbol{S_t}$) has an annualized standard deviation of 12% over this period [0,1], and that the 5-month tenor "variable currency" interest rate and the 5-month tenor "fixed currency" interest rate have annualized standard deviations of 0.95% (95 basis points) and 1.15%, respectively.

(And for simplicity treat these interest rates as continuously compounded with 0.42 as the appropriate year fraction.)

If we assume the interest rates and log($\boldsymbol{S_t}$) are all uncorrelated, then what are the annualized standard deviations of log($\boldsymbol{F_t}$) and log($\boldsymbol{F_t/S_t}$) over the period?c

In [2]:

```
0.42*np.sqrt(0.0095**2 + 0.0115**2)
```

Out[2]:

```
0.006264902233874044
```

$F_t = S_t * e^{[(r_d - r_f)*(T-t)]}$
$F_t / S_t = e^{[(r_d - r_f)*(T-t)]}$
$ln(F_t / S_t) = (r_d - r_f) * (T - t)$

$T = 1 t = 0$
$var[ln(F_t / S_t)] = var[(r_d - r_f) * (T - t)]$
$var[ln(F_t / S_t)] = (var[(r_d)] + var[(r_f)]) * (T - t)^2$
$var[ln(F_t / S_t)] = 0.42^2 * ([sd(r_d)]^2 + [sd(r_f)]^2)$

$var[ln(F_t/S_t)] = 0.42^2 * (0.0095^2 + 0.0115^2)$
$var[ln(F_t/S_t)] = 0.42^2 * (0.0002225)$
$sd[ln(F_t/S_t)] = 0.006265$
The standard deviation of the $ln(F_t/S_t)$ is 0.006265.

In [4]:

```
np.sqrt(0.12**2 + (0.42*0.0095)**2 + (0.42*0.0115)**2)
```

Out[4]:

```
0.12016342621613284
```

Similarly,
$var[ln(F_t)] = var[ln(S_t)] + var[(r_d - r_f) * (T - t)]$
$var[ln(F_t)] = var[ln(S_t)] + (T - t)^2 * (var[(r_d)] + var[(r_f)])$
$var[ln(F_t)] = sd[ln[S_t]]^2 + (T - t)^2 * (sd[(r_d)]^2 + sd[(r_f)]^2)$
$var[ln(F_t)] = 0.12^2 + 0.42^2 * (0.0095^2 + 0.0115^2)$
$sd[ln(F_t)] = 0.1202$
The standard deviation of the $ln(F_t)$ is 0.1202.

12) Assume there are 181 days between spot and the 6-month forward date and assume the following rates (with bid offer listed). Assume all deposit rates are quoted ACT/360:

USDCHF	0.9305 / 0.9307
6mo CHF deposit	1.20% / 1.30%
6mo USD deposit	4.80% / 4.90%

Under covered interest rate parity, what is the bid and offer for 6-month USDCHF forward points?

In [5]:

```
USDCHF_spot_bid = 0.9305
USDCHF_spot_offer = 0.9307
USD_6month_bid = 0.048
USD_6month_offer = 0.049
CHF_6month_bid = 0.012
CHF_6month_offer = 0.013
```

Assuming we are the market taker side

Offer 6-month USD CHF forward points

In [6]:

```
USDCHF_forward_offer = (USDCHF_spot_bid*(1+CHF_6month_offer*181/360)/(1+USD_6month_bid*181/360) - USDCHF_spot_bid)*10000
printmd(f'**USDCHF forward offer =** {USDCHF_forward_offer:.4f}')
```

USDCHF forward offer = -159.8836

Bid 6-month USD CHF forward points

In [7]:

```
USDCHF_forward_bid = (USDCHF_spot_offer*(1+CHF_6month_bid*181/360)/(1+USD_6month_offer*181/360) - USDCHF_spot_offer)*10000
printmd(f'**USDCHF forward bid =** {USDCHF_forward_bid:.4f}')
```

USDCHF forward bid = -168.9732

13) If South Korean won (KRW) deposit rates are lower than USD deposit rates for a particular maturity, then must the USDKRW non-deliverable forward rate for the same maturity be lower than the USDKRW spot rate?

Please give a brief explanation for your answer.

Not necessarily. The relationship between deposit rates and non-deliverable forward (NDF) rates can be influenced by a variety of factors, including expectations for future exchange rate movements, interest rate differentials between the two currencies, and market supply and demand conditions. Covered Interest Rate Partiy (CIRP) does not hold for NDF currencies. The deposit rates in the NDF currency are not available to foreign investors, in general, and therefore cannot be used to calculate the forward rate.

While it's generally true that higher interest rates in one currency can lead to a stronger forward rate for that currency relative to a currency with lower interest rates, the relationship between deposit rates and NDF rates is not always direct or predictable. Other factors can come into play, such as market sentiment, economic indicators, and central bank policies.

Therefore, it's possible that the USDKRW NDF rate for a particular maturity could be higher than the spot rate, even if KRW deposit rates are lower than USD deposit rates. The key point is that the relationship between these variables is complex and can be influenced by a variety of factors beyond interest rate differentials alone.

14) Calculate the Norwegian krone (NOK) interest rates implied by the following forward rates ("implied yield"). Please calculate both bid and offer side interest rates.

USD deposit rates below are money market, ACT/360 and the implied yield should also be calculated as ACT/360 money market rates.

USDNOK spot	9.8570 / 9.8580
USDNOK 3mo fwd	9.8155 / 9.8195
3mo USD deposit	4.55% / 4.65%
Days spot to 3mo	92

In [8]:

```
USDNOK_spot_bid = 9.8570
USDNOK_spot_offer = 9.8580
USDNOK_3month_bid = 9.8155
USDNOK_3month_offer = 9.8195
USD_3month_bid_rate = 0.0455
USD_3month_offer_rate = 0.0465
days_3month = 92
```

In [9]:

```
NOK_3month_bid_implied = ((USDNOK_3month_bid/USDNOK_spot_bid)*(1+USD_3month_offer_rate*days_3month/360) - 1)*360/days_3month
printmd(f'**NOK 3 month bid implied =** {NOK_3month_bid_implied:.4f}')
```

NOK 3 month bid implied = 0.0298

2.983 / 3.004

In [10]:

```
NOK_3month_offer_implied = ((USDNOK_3month_offer/USDNOK_spot_offer)*(1+USD_3month_bid_rate*days_3month/360) - 1)*360/days_3mont
printmd(f'**NOK 3 month offer implied =** {NOK_3month_offer_implied:.4f}')
```

NOK 3 month offer implied = 0.0300

15) Given the information below, calculate the FX swap points for a long USDCHF position maturing in 3 months that needs to be rolled out to the 6 month date. Assume you are a market-taker.

USDCHF	0.9320 / 0.9322
Days spot to 3months	91
Days spot to 6months	182
3mo USD deposit	4.60% / 4.70%
6mo USD deposit	4.85% / 4.95%
3mo CHF deposit	1.10% / 1.15%
6mo CHF deposit	1.25% / 1.30%

NOTE: Assume all interest rates are money market rates using ACT/360.

In [11]:

```
USDCHF_spot_bid = 0.9320
USDCHF_spot_offer = 0.9322
days_3mo = 91
days_6mo = 182
USD_3month_bid_rate = 0.046
USD_3month_offer_rate = 0.047
USD_6month_bid_rate = 0.0485
USD_6month_offer_rate = 0.0495
CHF_3month_bid_rate = 0.011
CHF_3month_offer_rate = 0.0115
CHF_6month_bid_rate = 0.0125
CHF_6month_offer_rate = 0.013
```

In [12]:

```
f_near = USDCHF_spot_bid * (1 + CHF_3month_bid_rate*days_3mo/360) / (1 + USD_3month_offer_rate*days_3mo/360)
printmd(f'The near forward rate is {f_near:.4f}')
```

The near forward rate is 0.9236

In [13]:

```
f_far = USDCHF_spot_bid * (1 + CHF_6month_bid_rate*days_6mo/360) / (1 + USD_6month_offer_rate*days_6mo/360)
printmd(f'The far forward rate is {f_far:.4f}')
```

The far forward rate is 0.9150

In [14]:

```
printmd(f'The forward swap points are {(f_far - f_near)*10000:.4f}')
print(f'We are assuming that swap points refer to the difference between near and far rates in pips')
```

The forward swap points are -86.2633

```
We are assuming that swap points refer to the difference between near and far rates in pips
```

16) Given the information below, calculate the forward-forward FX swap points for a short AUDUSD position maturing in 2 years that needs to be shortened to the 1-year date. Assume you are a market-taker. What would the near date and far date all-in forward rates be for the FX swap?

AUDUSD spot	0.6678 / 0.6680
1yr points	+73 / +78
2yr points	+66 / +71

In [15]:

```
AUDUSD_spot_bid = 0.6678
AUDUSD_spot_offer = 0.6680
AUDUSD_1yr_fwdpt_bid = 73
AUDUSD_1yr_fwdpt_offer = 78
AUDUSD_2yr_fwdpt_bid = 66
AUDUSD_2yr_fwdpt_offer = 71
```

In [16]:

```
AUDUSD_1yr_fwd_bid = AUDUSD_spot_offer + AUDUSD_1yr_fwdpt_offer/10000
printmd(f'The 1 year forward bid is {AUDUSD_1yr_fwd_bid:.4f}')
```

The 1 year forward bid is 0.6758

In [17]:

```
AUDUSD_2yr_fwd_bid = AUDUSD_spot_offer + AUDUSD_2yr_fwdpt_offer/10000
printmd(f'The 2 year forward bid is {AUDUSD_2yr_fwd_bid:.4f}')
```

The 2 year forward bid is 0.6751

In [18]:

```
FX_swap_points = ((-AUDUSD_1yr_fwd_bid + AUDUSD_2yr_fwd_bid)*10000)
printmd(f'The FX swap points are {FX_swap_points:.4f}')
```

The FX swap points are -7.0000

17) (This problem counts for 4 points) <u>Window forward</u>:

A client needs to buy Mexican peso (MXN) 1 billion versus USD but is unsure of the timing. The client asks you to quote a single forward rate where the client will be committed to buy MXN 1 billion, but can do so any time between the 3 month (91 days) and 6 month (183 days) forward dates.

Assuming the rates below (ignoring bid and offer, and assuming covered interest rate parity holds and that both currencies follow an ACT/360 convention) what forward rate would you quote? (Hint: you are allowed to be greedy, but not unreasonable.)

USDMXN spot	19.77
3mo USD deposit	4.60%
6mo USD deposit	4.70%
3mo MXN deposit	10.70%
6mo MXN deposit	11.20%.

In [19]:

```
USDMXN_spot_bid = 19.77
USD_3month_bid_rate = 0.046
USD_6month_bid_rate = 0.047
MXN_3month_bid_rate = 0.1070
MXN_6month_bid_rate = 0.1120
days_6mo = 183
days_3mo = 91
```

In [20]:

```
near_rate = USDMXN_spot_bid * (1 + MXN_3month_bid_rate*days_3mo/360) / (1 + USD_3month_bid_rate*days_3mo/360)
printmd(f'The near rate is {near_rate:.4f}')
```

The near rate is 20.0713

In [21]:

```
far_rate = USDMXN_spot_bid * (1 + MXN_6month_bid_rate*days_6mo/360) / (1 + USD_6month_bid_rate*days_6mo/360)
printmd(f'The far rate is {far_rate:.4f}')
```

The far rate is 20.4080

In [22]:

```
USDMXN_fwd_bid = USDMXN_spot_bid * (1 + MXN_3month_bid_rate*days_3mo/360) / (1 + USD_6month_bid_rate*days_6mo/360)
printmd(f'The forward bid is {USDMXN_fwd_bid:.4f}')
```

The forward bid is 19.8309

We give the customer the worse rate using a combination of the deposit rates such that we get the lowest USDMXN forward rate between 3 and 6 months. This would mean that we are giving him the rate which assumes MXN rates are the lowest and USD rates are the highest among the given 3mo and 6mo deposit rates.
This rate (19.8309) is lower than the actual 3 mo rate (20.4080) hence making it a bit greedy.

18) Would you be willing to offer the product in problem 6 above if the client asked to sell CAD 100 million instead of MXN 1 billion, when CAD deposit rates are equal to USD deposit rates? Why or why not? (HINT: Do CAD interest rates present a difficulty?)

If the rates are equal, the spot rates would equal the forward rates, and the same oppotunity would not be possible. This is because the numerator and the denominator would be equal in our CIRP equation.
Hence, we would not be able to benefit from the arbitrage opportunity in this particular case. and hence would not be willing to offer the product to the CAD customer.