Foreign Exchange: Markets, Products, and Pricing

Winter Quarter 2023

Week #3

FX options

What is an FX option contract?

- Option contract
  - The right, but not the obligation, to exchange
    - *N* units of foreign currency for
    - *N*K* units of domestic currency
  - at a future date, *T*

- The right to buy foreign currency is a foreign currency call
- The right to sell foreign currency is a foreign currency put
- Long option means owning the right to exchange, otherwise short
- European-style: exercise only at maturity, *T*

Notation

XXXYYY

- XXX is the “foreign” or “base” currency
  - For example, EUR for EURUSD
- YYY is the “domestic” or “terms” currency
  - For example, JPY for USDJPY
- The “domestic” or “terms” currency is the <u>numeraire</u>

To specify an option contract, you need…

- Call or put
- Buy or sell
- Expiry date and time
- Strike price
  - Fixed FX rate at which currencies may be exchanged
- Notional
  - Amount of currency XXX and amount of currency YYY
  - The strike rate will determine one of these given the other

Call or put

- Every FX option is <u>both</u> a call and a put!

- "XXX call YYY put" means the owner has the right to buy XXX and sell YYY at a fixed rate
- Foreign currency call = domestic currency put:
  - The right to buy foreign currency, and sell domestic currency

- "XXX put YYY call" means the owner has the right to sell XXX and buy YYY at a fixed rate
- Foreign currency put = domestic currency call:
  - The right to sell foreign currency, and buy domestic currency

Expiry date, cutoff time, and delivery date

- Trade date
- Spot date (date when premium is delivered)
- Expiry date,
  - Exercise time, *“Cutoff time”* or *“Cut”*
  - Typically, 10am NY time
- Delivery date
  - Date when currency is delivered if the option is exercised

Expiry date conventions

- Market convention for determining “regular” dates (e.g., 1-week, 3-months)
  - Days and weeks: Count from today (trade date) to expiry
  - Months and years: Count from spot to delivery

European style option

- European-style option
  - Exercise only at maturity
  - Can be sold early, but not exercised early
- American-style option, Bermudan-style (“mid-Atlantic”) option
  - Value of European option <= Value of American option
  - For call options on non-dividend paying stocks, =
  - For put options, and FX options in general, < ≠
- Most FX options are European-style
  - American-style FX options are extremely uncommon
  - Except for exchange-traded, options on futures contracts

Put-call parity

Put-call parity for FX options

- Consider the position: +1 foreign currency call, -1 foreign currency put

  where the call and put have the same strike, *K*, and maturity, *T*

- Cash flows at maturity will be:

  + 1 (of foreign), - K (of domestic) with certainty

- Which are the same cash flows as a forward with contract rate *K*

- The value at time *T* of *+1 (of foreign)* is *F(t,T) (of domestic)*

- So, taking domestic present values:

  Call premium - Put premium = P^d * [$F(t,T) - K$]

Put-call parity for FX options (cont'd)

- When call and put have the same strike, K, and maturity, T

  $$\textit{Call premium} - \textit{Put premium} = P^d * [\, F(t,T) - K \,]$$

- Written differently,

  Call - Put = PV [Forward contract at option strike]

- What if Strike = Forward?

  Call premium = Put premium

Valuing FX options

FX option contract: Calculating value

Example: long a foreign currency call

- Cash flow at maturity is the maximum of:

  (a) *zero*, or (b) *+1 (of foreign) - K (of domestic)*

- Domestic value of the cash flows at maturity equals

  $max(\ 0,\ S_T - K\ )$

- Since the value of cash flows at maturity is stochastic, the option value will depend on the random nature of the underlying, S_T

Calculating value (continued)

- To calculate prices of options on S_t we need:
  - Assumptions on the stochastic nature of S_t
  - Create a “risk-free” hedge portfolio, in order to find a governing PDE for the option value
  - Solve the PDE directly, with appropriate boundary conditions
- Later we will see that the governing PDE leads to an equivalent risk-neutral probability measure

The Black-Scholes Economy

Assumptions

- No transaction costs
- No probability of counterparty default
- No arbitrage
- Short selling is possible
- Continuous-time trading
- Risk-free rate is constant (for both foreign and domestic currency)
- Spot rate follows geometric Brownian motion with constant volatility

Stochastic Differential Equations

- Assume S exhibits geometric Brownian motion
  - $dS = \mu S\, dt + \sigma S\, dW$
  - and we assume μ and σ are constant (can be time-varying)
  - Then, $dS^2 = \sigma^2 S^2\, dt$
- Itô's Lemma:
  - If S is an Itô process,
  - and $V(.)$ is any twice-differentiable function of S *and* t
  - Then, $dV = \frac{1}{2} V_{SS}\, dS^2 + V_S\, dS + V_t\, dt$
- So, when S is geometric Brownian motion we have:
  - $dV = (\frac{1}{2} \sigma^2 S^2 V_{SS} + V_t)\, dt + V_S\, dS$

Risk-free portfolio for a non-dividend paying stock

- Hedge portfolio, where S is a non-dividend paying stock
  - $\Pi \;=\; \{\; +1 \text{ unit of } V,\ -V_S \text{ units of } S \;\}$
  - $d\Pi \;=\; dV - V_S\, dS$

    $= (\, \tfrac{1}{2}\, \sigma^2\, S^2\, V_{SS} \;+\; V_t \,)\, dt \;+\; V_S\, dS \;-\; V_S\, dS$

    $= (\, \tfrac{1}{2}\, \sigma^2\, S^2\, V_{SS} \;+\; V_t \,)\, dt$ *which is non-stochastic, i.e., "risk free"*
  - *Since* Π *is "risk free" it must earn the risk-free rate:*

    $d\Pi = r\Pi = (\, rV - rV_S S \,)\, dt$
  - *So,* $(\, \tfrac{1}{2}\, \sigma^2\, S^2\, V_{SS} \;+\; V_t \,)\, dt \;=\; (\, rV - rV_S S \,)\, dt$

Black-Scholes PDE

(B-S) $\quad \boldsymbol{\tfrac{1}{2}\, \sigma^2\, S^2\, V_{SS} \;+\; r\, S\, V_S \;-\; r\, V \;+\; V_t \;=\; 0}$

The spot rate is not an investible asset!

- S_t, the foreign exchange spot rate, is not investable,
- Money is invested as cash in a currency deposit, i.e., money market account
- Domestic deposit $D^d(t)$ *(value in domestic currency)*

  $$dD^d = r^d * D^d \, dt$$

- Foreign deposit $D^f(t)$ *(value in foreign currency)*

  $$dD^f = r^f * D^f \, dt \quad \text{(values in foreign currency)}$$

  - Value in domestic currency = $S*D^f$
  - Stochastic process in domestic currency

  $$d(S*D^f) = D^f \, dS + S* r^f * D^f \, dt$$

Risk-free portfolio for foreign exchange

The hedged portfolio

- *Π = { +1 unit of V , -V_S units of foreign currency deposit }*
  - *$d\Pi = dV - V_S\, d(S^* D^f)$ [Note that $D^f(t) = 1$, but is not a constant]*

    $= (\frac{1}{2} \sigma^2 S^2 V_{SS} + V_t)\, dt + V_S\, dS - V_S\, (dS + S^* r^f * dt)$

    $= (\frac{1}{2} \sigma^2 S^2 V_{SS} + V_t - r^f V_S S)\, dt$ *which is non-stochastic*
  - *So Π must earn the risk-free rate, $d\Pi = r^d \Pi = (r^d V - r^d V_S S)\, dt$*
  - *So, $(\frac{1}{2} \sigma^2 S^2 V_{SS} + V_t - r^f V_S S)\, dt = (r^d V - r^d V_S S)\, dt$*

Garman-Kohlhagen PDE

(G-K) $\frac{1}{2} \boldsymbol{\sigma}^2 \mathbf{S}^2 \mathbf{V}_{SS} + (\mathbf{r}^d - \mathbf{r}^f) \mathbf{S}\, \mathbf{V}_S - \mathbf{r}^d \mathbf{V} + \mathbf{V}_t = 0$

Garman-Kohlhagen formula (Black-Scholes formula for FX)

- Under the geometric Brownian motion assumption for S_t
  - The G-K PDE can be solved directly

    with boundary conditions $V(S_T,T) = \max(0,S_T-K)$, $V(0,t)=0$
- So, for an FX option, its domestic currency value at time *t* is given by:

  $\mathbf{Bl}(S_t, t, T, K, P^d, P^f, \sigma, \omega) = P^d\, \omega\, [\, F\, \Phi(\omega d_1) - K\, \Phi(\omega d_2)\,]$

  where $\omega = 1$ denotes call, and -1 denotes put

  $\Phi(*)$ denotes the standard normal cumulative density function

  and,

  $d_1 = \log(F/K) / \sigma\sqrt{(T-t)} + 0.5*\sigma\sqrt{(T-t)}$

  $d_2 = \log(F/K) / \sigma\sqrt{(T-t)} - 0.5*\sigma\sqrt{(T-t)}$

Risk-neutral measure

- There is a connection between the existence of
  - a self-financing trading strategy replicating the final value of the contingent claim, and
  - the existence of an equivalent martingale measure.
- They both guarantee that an arbitrage-free price of the contingent claim is unequivocally determined
- The arbitrage-free price can be calculated as
  - the current value of the related replica strategy, or alternatively as
  - the expected value of the discounted final payoff of the claim, under the risk-neutral probability measure

Risk-neutral measure for foreign exchange

- If V satisfies

  $\frac{1}{2}\sigma^2 S^2 V_{SS} + (r^d - r^f) S V_S - r^d V + V_t = 0$

- then using the Feynman-Kac equation (plus some additional derivation)
- There is a risk-neutral (domestic currency) measure, Q

  where

  $$V(S_t,t) = P^d * E^Q[\ V(S_T,T) \mid S_t\ ]$$

  and S obeys the SDE

  $$dS = (r^d - r^f)\, S\, dt + \sigma\, S\, dW^Q$$

The risk-neutral process (under Black-Scholes assumptions)

- *"Real-world process", "P-measure"* $dS = \mu S\, dt + \sigma S\, dW^P$
- *"Risk-neutral process", "Q-measure"* $dS = (r^d - r^f)\, S\, dt + \sigma S\, dW^Q$

- Forward rates:
  - For a general forward contract: $PV[\, F(t,T,R)\,] = P^d * E^Q[S_T - R]$
  - To set $PV = 0$, we need $R = E^Q[S_T\,]$, so $R = S_t * P^f / P^d$

- Options:
  - PV = $P^d * E^Q\,[\, max(0,\omega(S_T - K))\,]$
  - ***Bl***$(S_t,\, t,\, T,\, K,\, P^d,\, P^f,\, \sigma,\, \omega) = P^d\, \omega\, [\, F\, \Phi(\omega d_1) - K\, \Phi(\omega d_2)\,]$

Theoretical Value

Terminology

- Theoretical Value "TV": we assume r^d, r^f and σ are constant
  - Note that the assumption of constant volatility is much too restrictive and does not correspond to market prices of FX options. However, the Black-Scholes formula is used to express option markets in terms of quoted volatility instead of quoted prices
  - Theoretical value "TV" refers to the constant volatility level that an FX options dealer might quote that when used in the Black-Scholes formula gives the corresponding quoted option's price

A very useful approximation (when K=F)

What is the **Black-Scholes** option price when $K = F$?

- Call and put must have the same value, by put-call parity
- Under Black-Scholes, the value is:

$$P^d * F [\Phi(d_1) - \Phi(d_2)] = P^d * F [\Phi(0.5*\sigma\sqrt{(T-t)}) - \Phi(-0.5*\sigma\sqrt{(T-t)})]$$

- As an approximation, the quantity in brackets is roughly $0.4*\sigma\sqrt{(T-t)}$

- So, the value of an "at-the-money" option is roughly $P^d * 0.4 * F * \sigma\sqrt{(T-t)}$
- Or, about 40% of one standard deviation

Option price and premium

Option Price and Premium

- Price – the value of an option based on 1 unit notional
  - There are four methods to quote price
    - Unit notional can be either domestic or foreign currency
    - Price can be expressed in either domestic or foreign currency
- Premium – specific value of an option based on a specific notional amount
  - Two methods to quote premium
    - In domestic currency or in foreign currency
    - By the “Law of One Price”, these must be related by the <u>spot</u> <u>rate</u>
  - Option buyer will pay premium to the option seller (typically T+2)

Relationships between option price quotes

In this illustration:

- Foreign currency ("f") = EUR
- Domestic currency ("d") = USD

	Price:	In USD units	In EUR units
Option notional:	EUR 1	$P_{d/f}$	$P_{f\%}$
	USD 1	$P_{d\%}$	$P_{f/d}$

x Strike

x Spot

Four ways to express option prices (1-2)

- XXXYYY is the currency pair, XXX is the foreign (base) currency, YYY is the domestic (terms) currency, which is the numeraire

Using EURUSD as an example –

(1) Domestic currency per unit of foreign currency ($P_{d/f}$)

- Also called “USD pips”
- Premium will be paid in USD, and will equal *EUR notional* * $P_{d/f}$

(2) Foreign currency percentage ($P_{f\%}$)

- Also called “EUR%”
- Premium will be paid in EUR, and will equal *EUR notional* * $P_{f\%}$

Four ways to express option prices (3-4)

Using EURUSD as an example –

(3) Domestic currency percentage ($P_{d\%}$)

- Also called “USD%”
- Premium will be paid in USD, and will equal *USD notional* * $P_{d\%}$

(4) Foreign currency per unit of domestic currency ($P_{f/d}$)

- Also called “EUR pips”
- Premium will be paid in EUR, and will equal *USD notional* * $P_{f/d}$

Relationships between option price quotes

(1) Domestic currency per unit of foreign currency ($P_{d/f}$)

$P_{d/f}$ = Black-Scholes formula (using domestic = numeraire)

(2) Foreign currency percentage ($P_{f\%}$)

$P_{f\%} = P_{d/f}$ / Spot

(3) Domestic currency percentage ($P_{d\%}$)

$P_{d\%} = P_{d/f}$ / Strike

(4) Foreign currency per unit of domestic currency ($P_{f/d}$)

$P_{f/d} = P_{d/f}$ / (Spot * Strike) = BS formula (using foreign = numeraire)

Example: Four ways to express option prices

3-month 1.10 EUR call USD put, with Spot = 1.07

- Assume the premium = 0.0200, given in USD per EUR
  - Then $P_{d/f}$ = 0.0200, or a trader might say "200 USD pips"
- "EUR %" would be 0.0200/1.07 = 1.87%, $P_{f\%}$ = 0.0187
- "USD %" would be 0.0200/1.10 = 1.82%, $P_{d\%}$ = 0.0182
- "EUR pips" would be 0.0200/(1.07*1.10) = 0.0170
  - Then $P_{f/d}$ = 0.0170, or a trader might say "170 EUR pips"

Option pricing screen (Bloomberg)

TS Description	EUR/USD Vanilla 20160718	
Price Date	04/16/16	16:33
Asset	EURUSD	
Spot	Mid	1.1300
Style	European	Vanilla
Direction	Client buys	Physical
Call/Put	EUR	Call
Expiry	3 months	07/18/16
Delivery	NY 10:00	07/20/16
Strike	1.1671	3.00% OTMF
Notional	EUR	1,000,000.00
Model	Black-Scholes	

More Market Data			
Vol	BGN		9.752%/10.280%
Vol Spread			0.528%
Points	BGN	Mid	31.54
Forward		Mid	1.1332...
EUR Depo	Implied	Mid	-0.731...%
USD Depo	USD OIS	Mid	0.371...%
Greeks			
Gamma		EUR	68,088.56
Vega			1,921.28

Results			
Price	USD pips	93.273 R	103.416 P
Premium	USD	9,327.27 R	10,341.58 P
Prem Date		04/20/16	
Delta	Spot	28.6211%	
Hedge		-286,211.09	

When are different quoting conventions used?

- Domestic currency per unit of foreign currency ($P_{d/f}$)
  - Used for calls and puts in currency pairs where premium is normally paid in domestic (terms) currency
- Foreign currency percentage ($P_{f\%}$)
  - Used for calls and puts in currency pairs where premium is normally paid in foreign (base) currency
- Domestic currency percentage ($P_{d\%}$)
  - Not used for calls and puts. Used for some exotic options, especially when there is no base currency notional
- Foreign currency per unit of domestic currency ($P_{f/d}$)
  - Not typically used in option quoting. Useful for some calculations

Market Quoting Conventions for Calls and Puts

- When premium is paid in ccy2 (terms currency)
  - Quote in domestic currency per unit of foreign currency ($P_{d/f}$)
- When premium is paid in ccy1 (base currency)
  - Quote in foreign currency percentage ($P_{f\%}$)
- Generally, the order or precedent is USD > EUR > other currencies
- (The information below will not be tested, no need to memorize the table!)

Currency pair	ccy1	ccy2	Premium ccy
EURUSD	EUR	USD	USD
USDJPY	USD	JPY	USD
EURJPY	EUR	JPY	EUR
USDCHF	USD	CHF	USD
EURCHF	EUR	CHF	EUR
GBPUSD	GBP	USD	USD
EURGBP	EUR	GBP	EUR
AUDUSD	AUD	USD	USD
AUDJPY	AUD	JPY	AUD
USDCAD	USD	CAD	USD
USDBRL	USD	BRL	USD
USDMXN	USD	MXN	USD

Delta

Delta (Black-Scholes Formula)

- Sensitivity of the option's value to changes in the FX spot rate
- Black-Scholes formula

$$\boldsymbol{Bl}(S, t, T, K, P^d, P^f, \sigma, \omega) = P^d \omega [F \Phi(\omega d_1) - K \Phi(\omega d_2)]$$

$d_{1,2} = \log(F/K) / \sigma\sqrt{(T-t)} +/- 0.5*\sigma\sqrt{(T-t)}$

ω = 1 denotes call, and -1 denotes put

$\Phi(*)$ denotes the standard normal cumulative density function

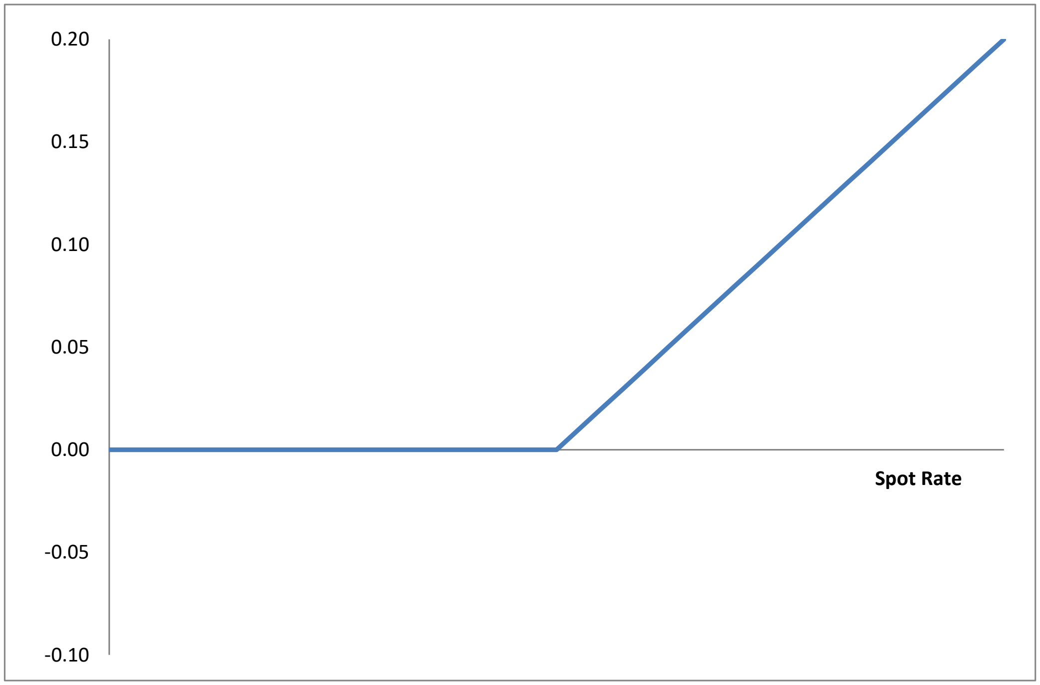
- Can re-write the formula as

$$\boldsymbol{Bl}(S, t, T, K, P^d, P^f, \sigma, \omega) = \omega P^f S \Phi(\omega d_1) - \omega P^d K \Phi(\omega d_2)$$

- Formula for delta

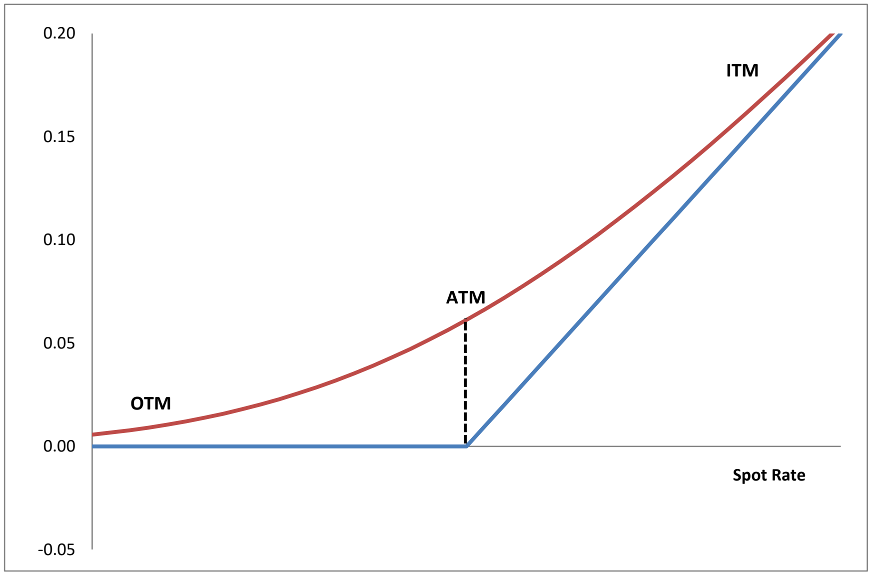
$$\boldsymbol{\Delta} = \partial \boldsymbol{Bl} / \partial S = \omega P^f \Phi(\omega d_1)$$

Delta of a call option at maturity



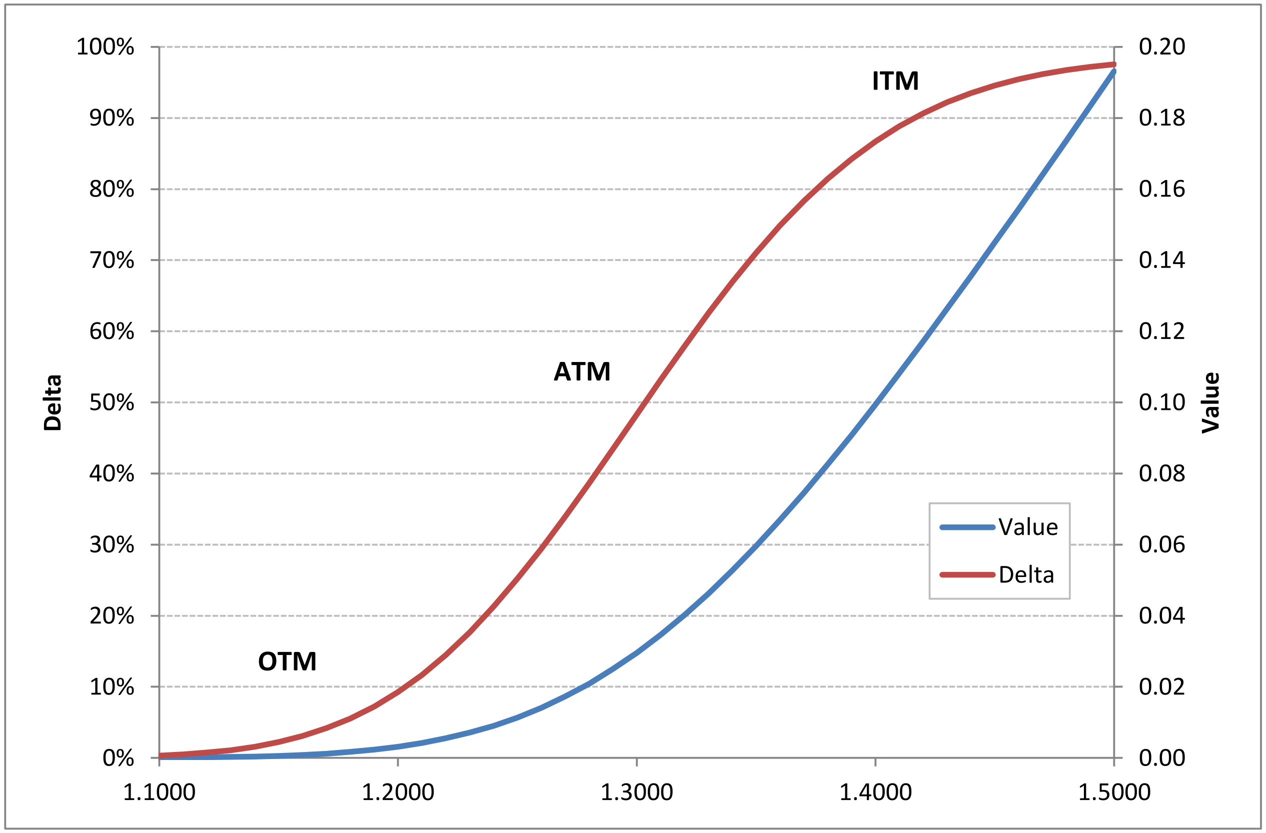
Delta of a call option prior to maturity

- EUR call USD put
- 1 year to maturity
- Volatility = 12.4%



Delta for different values of spot

- Delta generally ranges between 0% and 100%



Delta hedging in practice

- Delta gives the amount of a spot position (in the base currency) that is equivalent to a position in the option
- -1 * Delta is the amount of underlying FX that should be held to hedge the value of the option
  - For small changes in spot the gain or loss on the option position will be offset by an opposite loss or gain on the underlying (delta hedge)
- In practice delta hedging is not done continuously
  - Usually at pre-set intervals as the spot rate moves

Delta hedging in practice (continued)

- End-users will typically purchase (or sell) options with no delta hedge
- Option trades between two dealers almost always include a delta exchange
  - The dealers agree to option terms: spot, forward, volatility and therefore price
  - The delta hedge is exchanged between dealers at the referenced spot rate

Delta: Quoting Conventions

- Delta can be calculated from either the terms currency viewpoint or the base currency viewpoint
  - For calls and puts $P_{d/f}$ and $P_{f\%}$ are the relevant price quotes
- Premium Adjustment
  - Additional risk when the premium is set in a currency different from the investor's currency
- Spot Delta or Forward Delta
  - $\boldsymbol{\Delta_S} = \partial P / \partial S \quad \boldsymbol{\Delta_F} = \partial P / \partial F$

Delta: Domestic Currency Viewpoint

- The amount of base currency spot equivalent to the option
  - Terms currency investor's viewpoint. How much base currency to hold, for an option on 1 unit of base currency
  - This is the delta resulting from the Black-Scholes formula

"Pips Spot Delta" (also called "Points Spot Delta")

$$\boldsymbol{\Delta}_{S,d/f} = \partial P_{d/f} \,/\, \partial S = \partial \, \boldsymbol{BI} \,/\, \partial S = \omega \, P^f \, \Phi(\omega d_1)$$

- Example of when Pips Spot Delta is used:
  - Option on EURUSD for a USD investor, where the premium is paid in USD
  - In general, for options quoted as $P_{d/f}$ (premium paid in terms ccy)

Premium-included Delta

- Domestic currency (terms currency) viewpoint
- If premium is paid in base currency, then the premium payment represents additional FX risk from the terms currency viewpoint
- Premium-included delta

$$\boldsymbol{\Delta}^{pi} = \boldsymbol{\Delta}_{S,d/f} - P_{f\%}$$

- Example of when Premium-included Delta is relevant:
  - Option on USDJPY for a JPY investor, where the premium is paid in USD
  - In general, for options quoted as $P_{f\%}$ (premium paid in base currency)

Delta: Foreign Currency Viewpoint

- Referred to as “Percentage Spot Delta”
- The amount of base currency spot equivalent to the option
  - Base currency investor’s viewpoint. How much base currency to hold for an option on 1 unit of base currency
- Note the two differences
  - This is not simply switching to a base currency inventor’s viewpoint
  - It is also calculating how much base currency to hold and using 1 unit of base currency as the option amount

Percentage Spot Delta (definition)

- $\boldsymbol{\Delta}_{S,f\%}$ = $- S^* \partial P_{f\%} / \partial S^*$, *where* $S^* = 1/S$
- Why?
  - $\partial/\partial S^*$ because it gives the base currency investor's viewpoint
  - $\partial P_{f\%}/\partial S^*$ because it gives the amount of terms currency spot equivalent for base currency investor
  - $- S^* \partial P_{f\%}/\partial S^*$, because we want the amount of base currency in the equivalent spot position, and buying $\partial P_{f\%}/\partial S^*$ of terms currency spot means selling $S^* \partial P_{f\%}/\partial S^*$ of base currency

Percentage Spot Delta (calculation)

- $\boldsymbol{\Delta}_{\mathrm{S,f\%}} = - S^* \, \partial P_{f\%} / \partial S^*$, *where* $S^* = 1/S$

  Note that $P_{f\%} = S^* \, \boldsymbol{BI}(1/S^*)$

  $- S^* \, \partial P_{f\%} / \partial S^* = - S^* \, \partial / \partial S^* \, [S^* \, \boldsymbol{BI}(1/S^*)]$

  $= - S^* \, [\, \boldsymbol{BI}(1/S^*) - S^* \, (S^*)^{-2} \, \partial / \partial S^* \, \boldsymbol{BI}(S^*) \,]$

  $= - S^* \, [\, P_{d/f} - S \, \boldsymbol{\Delta}_{\mathrm{S,f/d}} \,]$

  $= \boldsymbol{\Delta}_{\mathrm{S,d/f}} - P_{f\%}$

- This calculation shows that $\boldsymbol{\Delta}_{\mathrm{S,f\%}} = \boldsymbol{\Delta}^{\mathrm{pi}}$

% Spot Delta = Premium-Included Delta

- Our calculations show that $\boldsymbol{\Delta}_{S,f\%} = \boldsymbol{\Delta}_{S,d/f} - P_{f\%} = \boldsymbol{\Delta}^{pi}$

Which form of delta is relevant?

- For options quoted as $P_{f\%}$ (premium paid in base ccy)
  - Terms ccy investor wants $\boldsymbol{\Delta}^{pi}$
  - Base ccy investor wants $\boldsymbol{\Delta}_{f\%}$**,** which equals $\boldsymbol{\Delta}^{pi}$
- For options quoted as $P_{d/f}$ (premium paid in terms ccy)
  - Terms ccy investor wants $\boldsymbol{\Delta}_{d/f}$
  - Base ccy investor wants $\boldsymbol{\Delta}_{f\%}$ adjusted for premium, which has a positive base currency spot equivalent (pay terms currency means receiving base currency)
    - $\boldsymbol{\Delta}_{f\%} + P_{f\%} = \boldsymbol{\Delta}_{d/f} - P_{f\%} + P_{f\%} = \boldsymbol{\Delta}_{d/f}$

Quoting Conventions

Table 3.1 Delta conventions for common currency pairs

Currency pair	ccy1	ccy2	Premium ccy	Δ convention
EURUSD	EUR	USD	USD	Pips
USDJPY	USD	JPY	USD	%
EURJPY	EUR	JPY	EUR	%
USDCHF	USD	CHF	USD	%
EURCHF	EUR	CHF	EUR	%
GBPUSD	GBP	USD	USD	Pips
EURGBP	EUR	GBP	EUR	%
AUDUSD	AUD	USD	USD	Pips
AUDJPY	AUD	JPY	AUD	%
USDCAD	USD	CAD	USD	%
USDBRL	USD	BRL	USD	%
USDMXN	USD	MXN	USD	%

Percentage Spot Delta (another calculation)

- Base currency investor's viewpoint using the Black-Scholes formula

$$\mathbf{\Delta}_{S,f\%} = -S^* K\, \partial \mathbf{BI}(\underline{S^*}, t, T, \underline{1/K}, \underline{P^f}, \underline{P^d}, \sigma, \underline{-\omega})/\partial S^*, \text{ where } S^* = 1/S$$

- Since $\partial \mathbf{BI}(\underline{S^*}, t, T, \underline{1/K}, \underline{P^f}, \underline{P^d}, \sigma, \underline{-\omega})/\partial S^* = -\omega P^d \Phi(\omega d_2)$
- $\mathbf{\Delta}_{S,f\%} = \omega (K/S) P^d \Phi(\omega d_2)$

Percentage Spot Delta formulas

$$\mathbf{\Delta}_{S,f\%} = \mathbf{\Delta}_{S,d/f} - P_{f\%}$$

$$\mathbf{\Delta}_{S,f\%} = (K/S)\, \omega P^d \Phi(\omega d_2)$$

Gamma

Gamma

- As illustrated on previous slides, delta changes as the spot rate changes
- With forward positions, this is not true:
  - No optionality implies no change in delta
- Gamma
  - Second derivative, or curvature, of the value function
    - Theoretical: $\partial^2 P / \partial S^2$
    - Practitioner: $\Delta^2 P / \Delta S^2$
  - Practitioner's definition: "Change in delta / small change in FX rate"
  - Most common units: "Percent change in delta / 1% move in spot"

Gamma

- Interpretation
  - Gamma indicates how much to adjust the FX rate hedge to remain "delta neutral"
  - Higher gamma implies faster changes in delta
  - Positive gamma: "increasing delta as the market increases"
    - Is this valuable?

Gamma (Black-Scholes Formula)

- Gamma $\partial^2 \boldsymbol{BI} / \partial S^2$
  - $\boldsymbol{\Gamma} = \partial^2 \boldsymbol{BI} / \partial S^2 = P^f \; \Phi'(d_1) / (S \, \sigma \sqrt{(T-t)})$

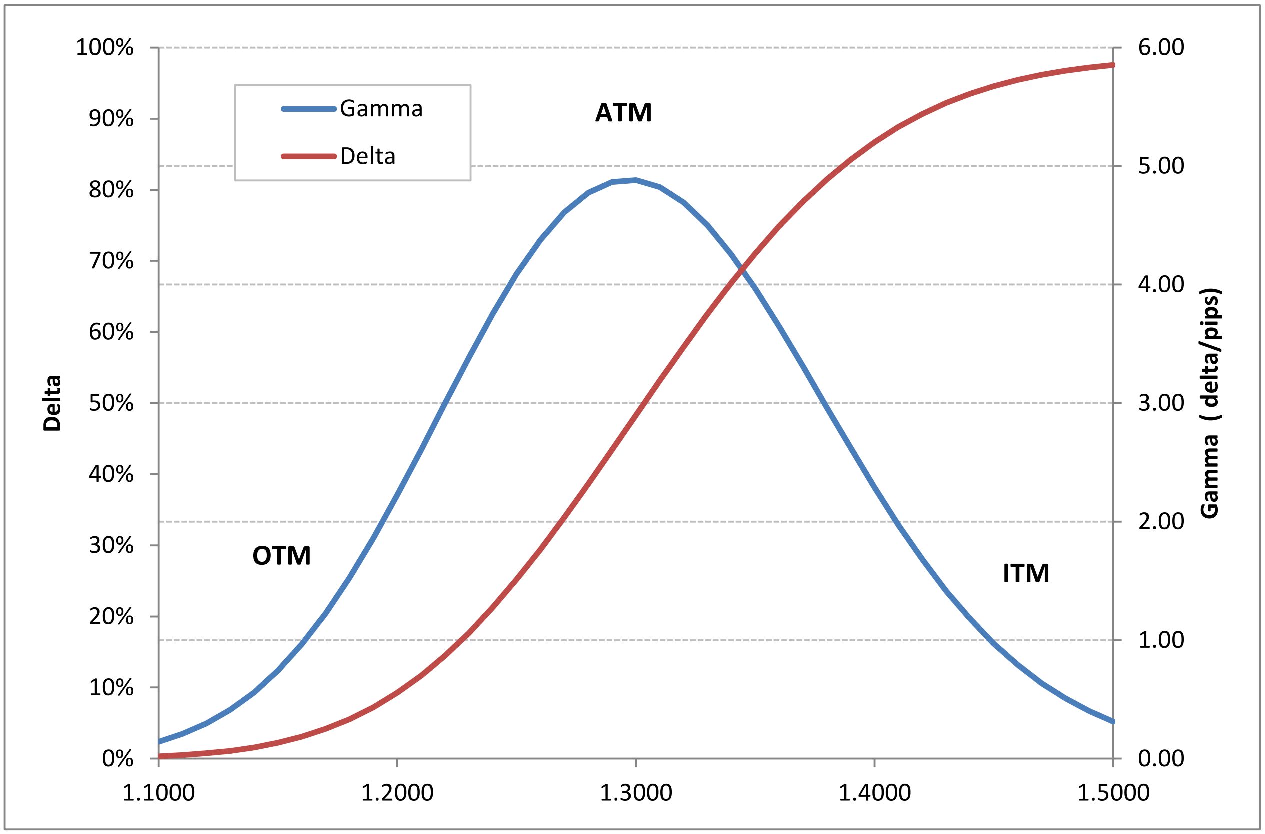
  where Φ'() is the standard normal density

- *"Trader's Gamma"*
  - *Usually scaled to a 1% move in the spot rate (i.e., change in Delta for a 1% change in Spot)*
  - $\boldsymbol{\Gamma^{tr}} = \boldsymbol{\Gamma} * (S/100)$

- Premium-included Gamma (used when premium is included in Delta, i.e., when base ccy is the viewpoint)
  - $\boldsymbol{\Gamma^{pi}} = \Gamma - \Delta / S + 2 * 0.01 * P_{f\%} / S$

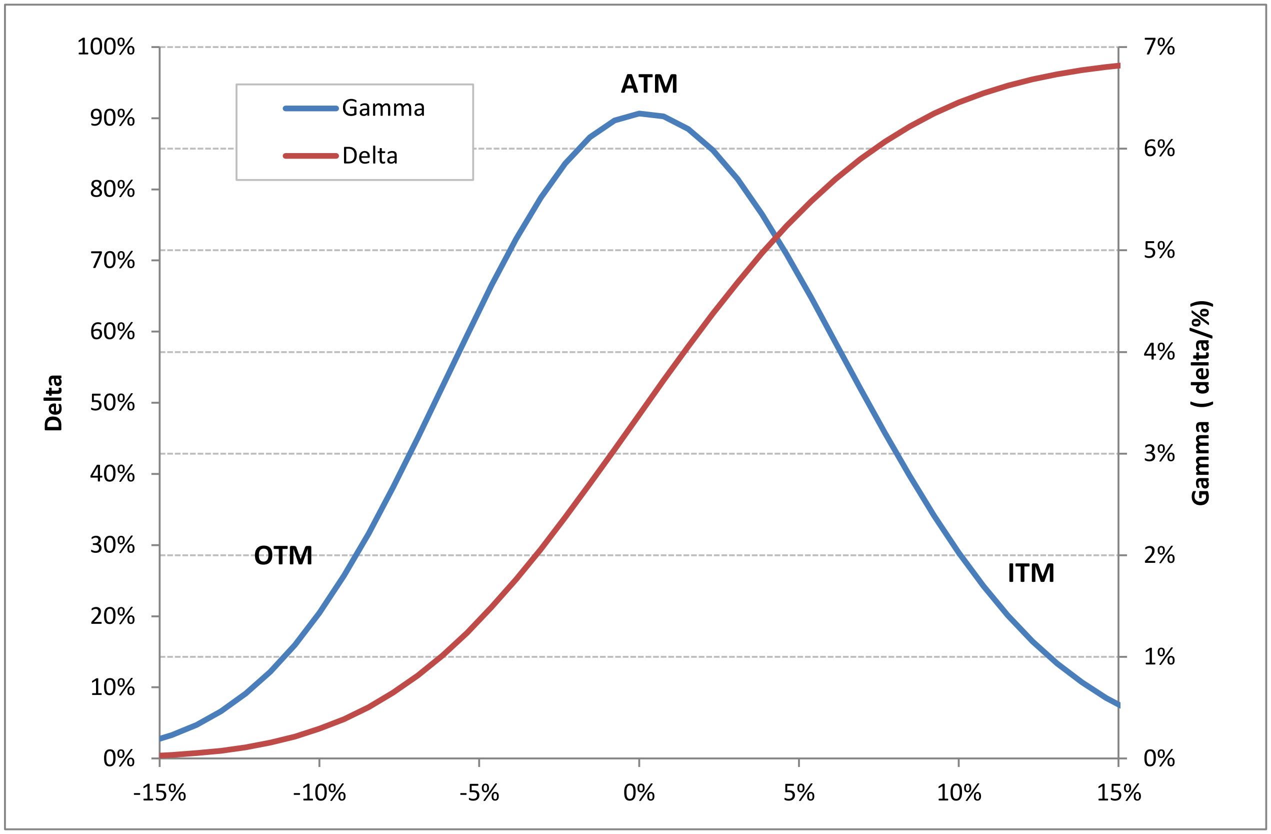
Gamma: what are the units?

- Standard formula: change in delta per “numeraire, pips” change in spot
- The value for 1 unit of base currency matters in the standard formula
- 5.00 means: ±5% in delta for ±0.01 of spot

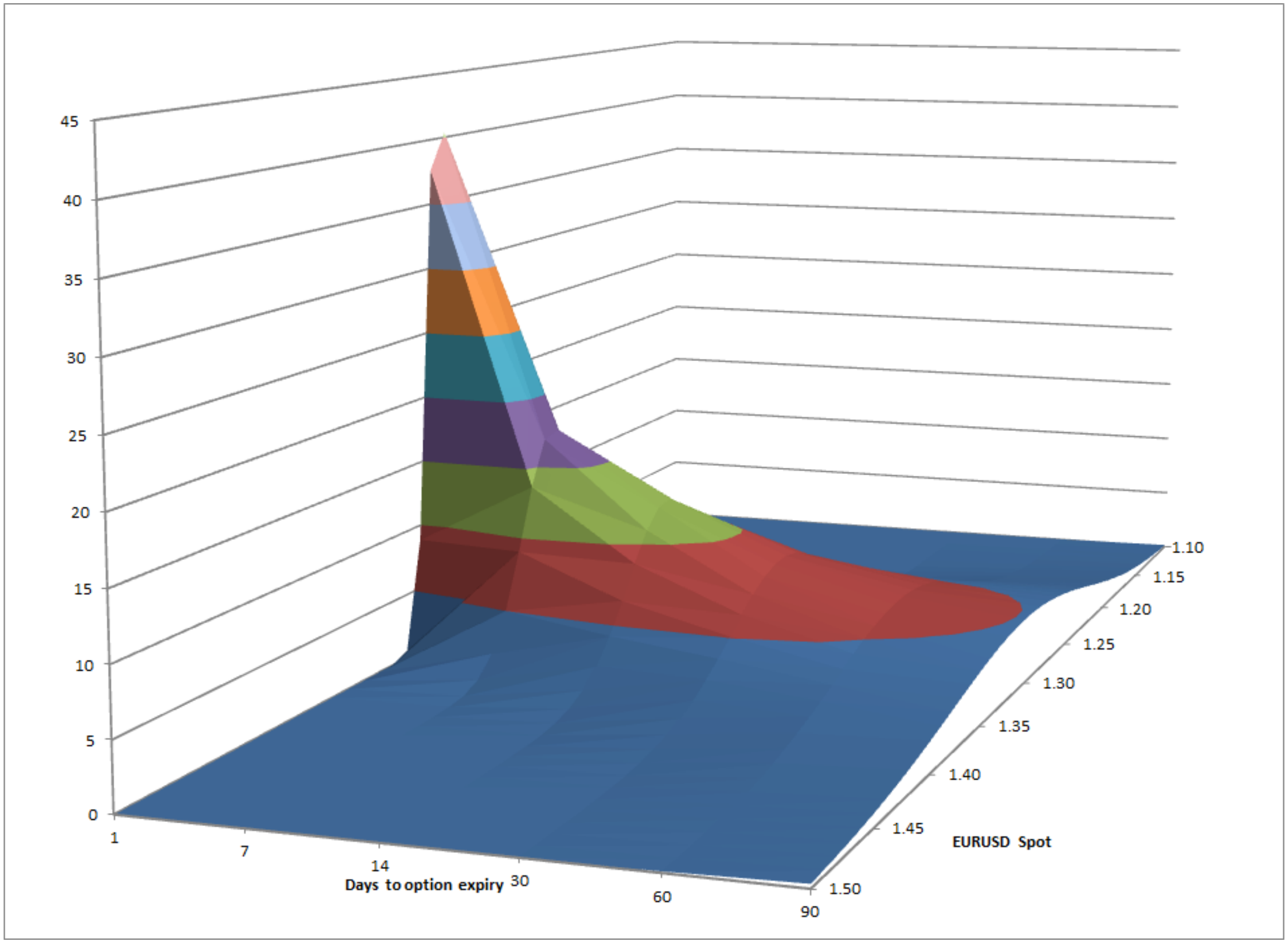


Gamma: what are the units?

- More useful measure is to scale Gamma by percent changes in spot
- 5% means: ±5% in delta for ±1% in spot



Gamma close to expiry

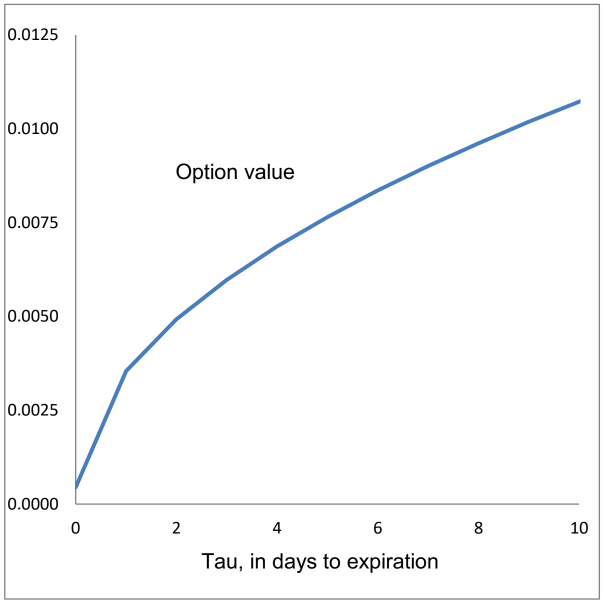


Theta

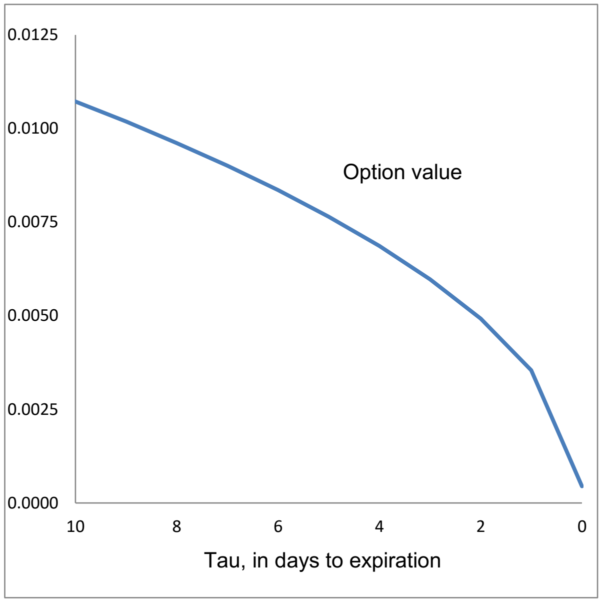
Theta

- Theta: $\partial P / \partial \tau$, where $\tau = T - t$, Practitioner's view: $\Delta P / \Delta t$
- If nothing changes except the passage of time, the value of an option drops
- The amount of option value that erodes with the passage of time
- Roughly a square root of time impact (for at-the-money options)

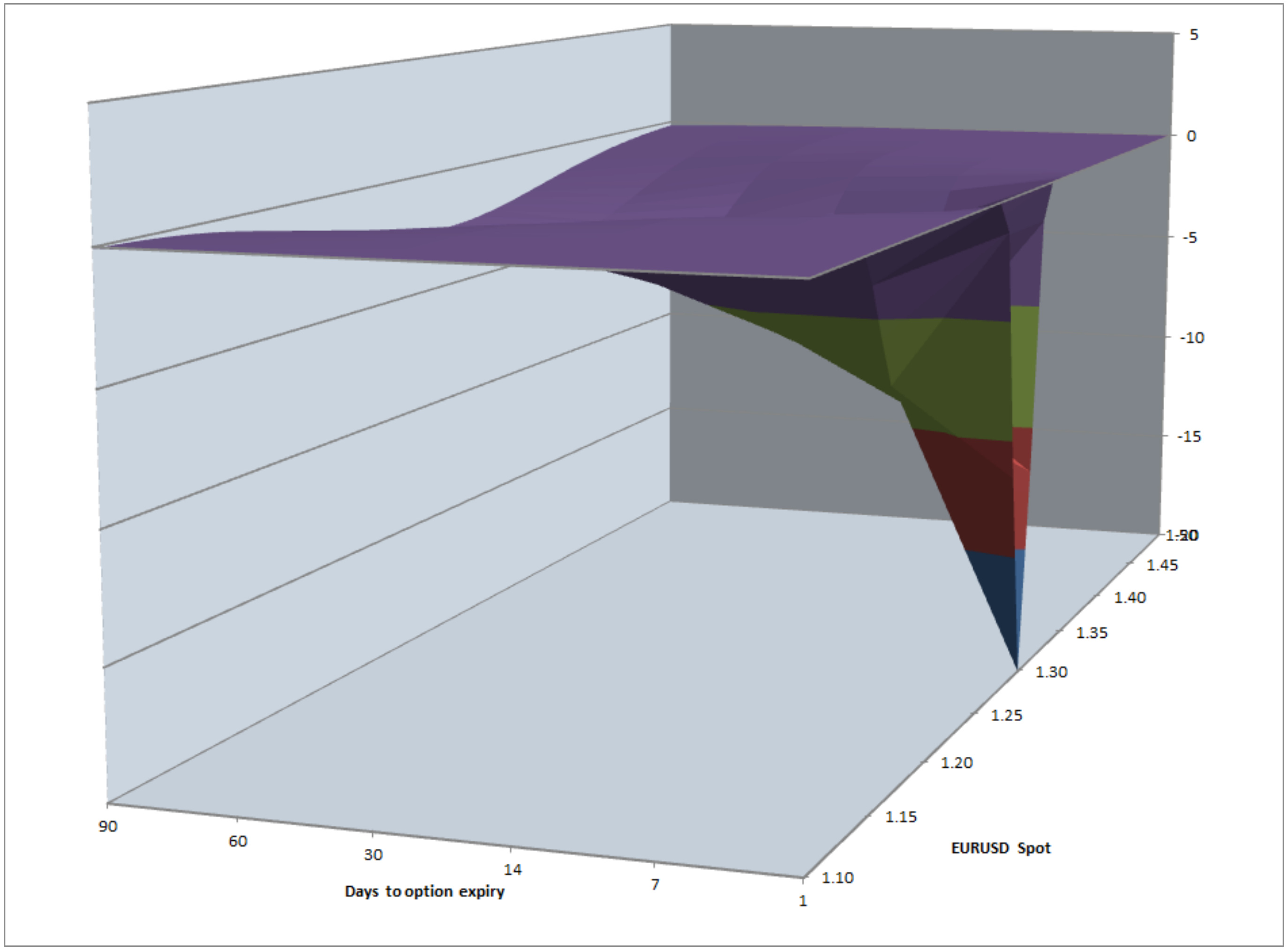
Academic view:
Time-to-expiration increasing (tau = T-t)



Trading view: calendar increasing (t)
Time-to-expiration decreasing (tau = T-t)



Theta



5
0
-5
-10
-15
-20
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
EURUSD Spot
90
60
30
14
7
1
Days to option expiry