**Scoring is 2 points for each question. Question 25 will be scored at 4 points.**

24) For the GBP put / USD call with a strike of 1.1700 listed in problem #22 of problem set #3, calculate the Pips Spot Delta and the Premium-Included Delta. (Note that these deltas are from the USD investor's viewpoint.) Then calculate the Percentage Spot Delta using the formula at the bottom of slide #19 in the week #4 lecture packet to confirm that the Premium-Included Delta and Percentage Spot Delta are equal

The market information is listed below, and is the same as in problem #22

| | |
|---|---|
| Spot rate | 1.2140 |
| Trade date | 23-Feb-2023 |
| Expiry date | 23-Aug-2023 |
| Spot date | 27-Feb-2023 |
| Delivery date | 25-Aug-2023 |
| USD deposit rate | 4.75% |
| GBP deposit rate | 3.75% |
| Implied volatility | 11.35% |

***NOTE: Use ACT/360 when working with USD and ACT/365 when working with GBP interest rates. Also note that when working with "tau", the trade date to expiry date period, money market conventions do not apply. For that period use ACT/365.***

Pips Spot Delta can be calculated from the Black-Scholes formula similar to the "USD pips" price of the option. The formula for Pips Spot Delta is as follows:

$\Delta_{S,pips} = \Delta_{S,f/d} = \omega\, P^f\, \Phi(\omega d_1)$

$\Delta_{S,pips} = -(1 / (1+3.75\%*179/365))\ \Phi(-d_1)$

$\Delta_{S,pips} = -0.981942\ \Phi(-0.565912)$

$\Delta_{S,pips} = -0.2806$

For the Premium-Included Delta we subtract the "GBP %" price of the option. (The amount is subtracted since the option premium would represent a liability, i.e., a GBP-amount that the option owner would have to pay if premium were denominated in GBP.)

$\Delta^{PI} = \Delta_{S,pips} - P_{\%GBP} = -0.2806 - 0.0146 = -0.2952$

Percentage Spot Delta can also be calculated from the Black-Scholes formula, as follows:

$\Delta_{S,percentage} = \Delta_{S,f\%} = \omega\ (K/S)\ P^d\ \Phi(\omega d_2)$

$\Delta_{S,percentage} = -\ (1.1700/1.2140)*\ 0.9769\ *\ \Phi(-0.485986)$

$\Delta_{S,percentage} = -0.2952$

Note that this amount is equal to the Premium-Included Delta.

**Scoring is 2 points for each question. Question 25 will be scored at 4 points.**

25) (This problem counts for 4 points.) Delta-Neutral Strike
There is a strike rate where a call and put have offsetting deltas, $P_S + C_S = 0$

This is called the "delta-neutral" strike, since a combination of long call and long put both at this strike will have a net delta of zero (i.e., will be insensitive to the spot rate).

Use the Black-Scholes formula for Call + Put, differentiate with respect to the spot rate, set the result = 0 and derive a formula for the strike. Then use the market information below to calculate the strike:

| | |
|---|---|
| Currency pair | NZDUSD (NZD is the New Zealand dollar) |
| Spot rate | 0.6257 |
| Trade date | 2-Mar-2023 |
| Expiry date | 4-Sep-2023 |
| Spot date | 6-Mar-2023 |
| Delivery date | 6-Sep-2023 |
| USD deposit rate | 4.75% |
| AUD deposit rate | 4.40% |
| Implied volatility | 12.15% |

***NOTE: Use ACT/360 when working with USD and ACT/365 when working with NZD interest rates. Also note that when working with "tau", the trade date to expiry date period, money market conventions do not apply. For that period use ACT/365.***

Using the Black-Scholes formula for a call and put

$$Call + Put = P^d\,[\,F\,\Phi(d_1) - K\,\Phi(d_2)\,] + P^d\,[\,K\,\Phi(-d_2) - F\,\Phi(-d_1)\,]$$

$$= P^d\,[\,F\,(\Phi(d_1)-\Phi(-d_1)) - K\,(\Phi(d_2)-\Phi(-d_2))\,]$$

We differentiate the right-hand side of the equation with respect to spot and set equal to 0.

$$0 = \partial F/\partial S * [\,(\Phi(d_1)-\Phi(-d_1))] + F * \partial/\partial S\,[\,(\Phi(d_1)-\Phi(-d_1))] - K * \partial/\partial S\,[(\Phi(d_2)-\Phi(-d_2))]$$

$$0 = (P^f/P^d) * [\Phi(d_1)-\Phi(-d_1)] + F * \partial/\partial S\,[\,(\Phi(d_1)-\Phi(-d_1))] - K * \partial/\partial S\,[\Phi(d_2)-\Phi(-d_2)] \quad \text{"Equation 1"}$$

If we focus on the last two terms on the right, we can see those must combine to zero. The reasons are as follows:

$$F * \partial/\partial S\,[\Phi(d_1)-\Phi(-d_1)] - K * \partial/\partial S\,[\Phi(d_2)-\Phi(-d_2)] =$$

**Scoring is 2 points for each question. Question 25 will be scored at 4 points.**

$= F * \Phi'(d_1)\ \partial d_1/\partial S + F * \Phi'(-d_1)\ \partial d_1/\partial S - K * \Phi'(d_2)\ \partial d_2/\partial S - K * \Phi'(-d_2)\ \partial d_2/\partial S$

Note that $\partial d_1/\partial S = \partial d_2/\partial S$. So, if we divide by $\partial d_1/\partial S$, the right side simplifies to

$F * \Phi'(d_1) + F * \Phi'(-d_1) - K * \Phi'(d_2) - K * \Phi'(-d_2) = 2*F\Phi'(d_1) - 2*K\Phi'(d_2)$

Also, $\Phi'(.)$ is symmetric, so $\Phi'(d_1) = \Phi'(-d_1)$, and $\Phi'(d_2) = \Phi'(-d_2)$, so the right side simplifies further to

$2 * F\Phi'(d_1) - 2 * K\Phi'(d_2)$

So, we have

$F * \partial/\partial S\ [\Phi(d_1)-\Phi(-d_1)] - K * \partial/\partial S\ [\Phi(d_2)-\Phi(-d_2)] = 2 * F\Phi'(d_1) - 2 * K\Phi'(d_2)$

We can relate $\Phi'(d_2)$ to $\Phi'(d_1)$ as follows: $\Phi'(d_2) = \Phi'(d_1) * (F/K)$

This is because:

$\Phi'(d_2) = (1/\sqrt{2\pi})\ exp[\ -0.5*(\ log(F/K)/\sigma\sqrt{t} - 0.5*\sigma\sqrt{t}\ )^\wedge 2]$

$= (1/\sqrt{2\pi})\ exp[\ -0.5*(\ log(F/K)/\sigma\sqrt{t} + 0.5*\sigma\sqrt{t}\ )^\wedge 2 + log(F/K)]$

$= (1/\sqrt{2\pi})\ exp[\ -0.5*(\ log(F/K)/\sigma\sqrt{t} + 0.5*\sigma\sqrt{t}\ )^\wedge 2] * exp[\ log(F/K)]$

$= (1/\sqrt{2\pi})\ exp[\ -0.5*(\ log(F/K)/\sigma\sqrt{t} + 0.5*\sigma\sqrt{t}\ )^\wedge 2] * [\ F/K\ ]$

$= \Phi'(d_1) * (F/K)$

This shows that the last two terms on the right of Equation 1 simplify to zero:

$F * \partial/\partial S\ [\Phi(d_1)-\Phi(-d_1)] - K * \partial/\partial S\ [\Phi(d_2)-\Phi(-d_2)] = 2*F\Phi'(d_1) - 2*K\Phi'(d_2)$

$2*F*\Phi'(d_1) - 2*K*\Phi'(d_2) = 2*F*\Phi'(d_1) - 2*K*(F/K)*\Phi'(-d_1) = 0$

Thus *"Equation 1"* simplifies to

$0 = (P^f/P^d)\ (\Phi(d_1) - \Phi(-d_1))$

So to find the delta-neutral strike we need $\Phi(d_1) = \Phi(-d_1)$

This can only happen when $d_1 = -d_1$, meaning that $d_1 = 0$

So, finally, to find a formula for the delta-neutral strike, we solve $d_1 = 0$ for $K$,

$[\ log(F/K) + 0.5*\sigma^2 t\ ]\ /\ \sigma\sqrt{t} = 0$

$log(F/K) + 0.5*\sigma^2 t = 0$

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*$log(K/F) = 0.5*\sigma^2 t$*

***$K = F * exp(0.5*\sigma^2 t)$***

Using the market information above, we can calculate the specific delta-neutral strike

*$K = \underline{0.6270} * exp(0.5 * 0.1215^2 * 186/365)$*

*$K = \underline{0.6290}$*

**Scoring is 2 points for each question. Question 25 will be scored at 4 points.**

***For questions 26-29 please write a brief explanation of why you think the answer you chose is correct.***

26) The EURUSD one week outright is 1.0625. Which of these one-week options has the largest delta in absolute value?

  a) 1.0625 EUR put

  b) 1.1600 EUR put

  c) 1.0625 EUR call

  d) 1.1600 EUR call

Answer = b)

A 1.1600 EUR put is deeply in-the-money when the forward is 1.0625, so its delta approaches -1, or 1 in absolute value. The 1.1600 EUR call d) would have the lowest absolute value delta, approaching 0. Both the 1.0625 EUR put and 1.0625 EUR call would be at-the-money and the delta of either option would have absolute value near 0.5.

27) The EURJPY spot rate is 140.30. The one week outright is 140.24 and the three-month outright is 139.48 Which of these options have the highest gamma?

  a) 139.48 EUR put expiring in one hour

  b) 140.30 EUR put expiring in one hour

  c) 140.24 EUR put expiring in one week

  d) 139.48 EUR put expiring in three months

Answer = b)

Gamma is highest for an at-the-money option closest to expiry.
One way to see this is to recall that gamma is the change in delta (partial derivative of delta with respect to spot). At expiry delta is a step function taking the value 0 for Spot<Strike, and 1 for Spot>Strike. Delta is undefined for Spot=Strike and gamma has essentially infinite value there. Values of gamma are most extreme near the { expiry, Spot=Strike } point.

**Scoring is 2 points for each question. Question 25 will be scored at 4 points.**

28) An options trader wishes to take a short position in implied volatility (using a single vanilla option contract), but the trader would like to minimize the need to rebalance delta hedges as the spot market moves (i.e., to minimize any negative gamma). What can the trader do to best achieve these goals?

a) Sell a long-dated deep out-of-the-money option

b) Sell a short-dated out-of-the-money option

c) Sell a long-dated at-the-money option

d) Sell a short-dated at-the-money option

b) Using the vega-gamma relationship we know that vega equals gamma times a multiple of spot squared and time. By this a long-dated option will have low gamma relative to its vega. Since at-the-money options have the largest vega, and since vega generally increases as option time increases, a sale of a long-dated at-the-money option will create negative vega with relatively small gamma.

29) Could you ever earn positive time decay and be long volatility? Possibly, if you have the following combination of at-the-money options:

a) Long 1-month option and long 6-month option

b) Long 1-month option and short 6-month option

c) Short 1-month option and long 6-month option

d) Short 1-month option and short 6-month option

c) Generally, a longer-dated option will have greater vega than a shorter-dated option, but the longer-dated option will have smaller time decay (time decay is greatest for options close to maturity). So by buying a longer-dated option and selling a shorter-dated option it may be possible to construct a long vega position that is also earning time decay.

**Scoring is 2 points for each question. Question 25 will be scored at 4 points.**

30) Consider two options, both GBP calls, one struck at the forward rate and the other at the delta-neutral rate. Assume the implied volatility of both options is 11.55%, and assume the following market information:

| | |
|---|---|
| Currency pair | GBPUSD |
| Spot rate | 1.2030 |
| Tau | 0.62 |
| GBP deposit rate | 4.15% |
| USD deposit rate | 4.60% |

Assume both deposit rates are in continuously compounded terms, and that tau is valid for both time to option maturity and the appropriate discount factors.

What are the vega, volga and vanna of each option?

Two options:

i) K = Forward Rate = 1.2064 = 1.2030 * exp( 0.0460*0.62) / exp(0.0415*0.62)

Vega = $P^f$ * Φ'(d1) * S * sqrt(tau) = 0.3679
Vanna = $P^f$ * Φ'(d1) * d2 / σ = -0.1529
Volga = $P^f$ * S * Φ'(d1) * d1 * d2 * sqrt(tau) / σ = -0.0066

ii) K = "DNS" = 1.2080 = 1.2030 * exp( 0.5 * 0.1155^2 * 0.62)

Vega = 0.3681
Vanna = -0.2028
Volga = -0.0059