

Scoring is 2 points for each question. Questions 11 and 17 will be scored at 4 points each.

11) (This problem counts for 4 points) Relative volatility of spot, forward and points

Let  $S_t$  be the spot FX rate at time  $t$ . For this exercise we assume  $t$  is measured in years, and  $t$  refers to the “trade date”, i.e., the time rates are observed in the market, not the “value date”.

Let  $F_t$  be the 5-month forward rate. Specifically, for any time  $t$ ,  $F_t$  is the quoted rate for a 5-month forward (i.e.,  $F_t$  is not a specific contract, but rather a rate observed each day in the market.)

We want to calculate the standard deviations of  $\log(F_t)$  and  $\log(F_t/S_t)$  over the period  $[0, 1]$ , assuming that covered interest rate parity holds. To be clear, standard deviation in this case is measuring the uncertainty in what  $\log(F_t)$  and  $\log(F_t/S_t)$  will be at time  $t=1$  conditional on their values at time  $t=0$ .

Assume  $\log(S_t)$  has an annualized standard deviation of 12% over this period, and that the 5-month tenor “variable currency” interest rate and the 5-month tenor “fixed currency” interest rate have annualized standard deviations of 0.95% (95 basis points) and 1.15%, respectively.

(And for simplicity treat these interest rates as continuously compounded with 0.42 as the appropriate year fraction.)

If we assume the interest rates and  $\log(S_t)$  are all uncorrelated, then what are the annualized standard deviations of  $\log(F_t)$  and  $\log(F_t/S_t)$  over the period?

Answer:

Let  $r$  = variable currency interest rate, and  $R$  = fixed currency interest rate, then

$$F_t = S_t * \exp(r * 0.42 - R * 0.42)$$

$$\log F_t = \log S_t + r * 0.42 - R * 0.42$$

Let  $Std$  denote standard deviation over  $[0, 1]$ .

$$Std [\log F_t] = \sqrt{Std [\log S_t]^2 + Std [r * 0.42]^2 + Std [R * 0.42]^2}, \text{ since all pairwise correlations } = 0.$$

$$\text{So, } Std [\log F_t] = \sqrt{0.12^2 + (0.00399)^2 + (0.00483)^2} = 0.1202$$

$$\log F_t/S_t = r * 0.42 - R * 0.42, \text{ so } Std [\log F_t/S_t] = \sqrt{Std [r * 0.42]^2 + Std [R * 0.42]^2}$$

$$= \sqrt{0.00399^2 + 0.00483^2} = 0.0063$$

Notice how small the volatility of FX swaps is relative to the volatility of the spot rate!

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- 12) Assume there are 181 days between spot and the 6-month forward date and assume the following rates (with bid offer listed). Assume all deposit rates are quoted ACT/360:

USDCHF	115.30 / 115.32
6mo CHF deposit	1.20% / 1.30%
6mo USD deposit	4.80% / 4.90%

Under covered interest rate parity, what is the bid and offer for 6-month USDCHF forward points?

Answer: To use covered interest parity we construct a forward transaction through three other transactions—spot exchange, borrow and lend. The explanation below is given from the viewpoint of a market-taker:

To calculate the bid-side forward, construct a forward sale of USD:

Sell USD, buy CHF for spot at 0.9305 (at the market maker's bid)

Borrow USD for 181 days at 4.90% (at the market maker's offer)

Lend CHF for 181 days at 1.20% (at the market maker's bid)

Thus, Forward rate =  $0.9305 * (1 + 1.20\% * 181/360) / (1 + 4.90\% * 181/360) = 0.9136$

To calculate the offer-side forward, construct a forward purchase of USD:

Buy USD, sell CHF for spot at 0.9307 (at the market maker's offer)

Lend USD for 181 days at 4.80% (at the market maker's bid)

Borrow CHF for 181 days at 1.30% (at the market maker's offer)

Thus, Forward rate =  $0.9307 * (1 + 1.30\% * 181/360) / (1 + 4.80\% * 181/360) = 0.9147$

So, bid/offer on forward points is -169 / -160

- 13) If South Korean won (KRW) deposit rates are lower than USD deposit rates for a particular maturity, then must the USDKRW non-deliverable forward rate for the same maturity be lower than the USDKRW spot rate?

Please give a brief explanation for your answer.

No. Covered interest parity does not apply to non-deliverable forwards because the non-deliverable currency cannot be borrowed or lent by non-domestic counterparties. In particular, KRW cannot be borrowed or lent outside of South Korea. This means arbitrage is not possible, so the NDF rate may vary from rates implied by USD and KRW deposit rates

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- 14) Calculate the Norwegian krone (NOK) interest rates implied by the following forward rates (“implied yield”). Please calculate both bid and offer side interest rates.

USD deposit rates below are money market, ACT/360 and the implied yield should also be calculated as ACT/360 money market rates.

USDNOK spot      9.8570 / 9.8580  
 USDNOK 3mo fwd   9.8155 / 9.8195  
 3mo USD deposit   4.55% / 4.65%  
 Days spot to 3mo   92

Note that NOK is the “terms” or “domestic” currency in USDNOK, so we need to solve the following equation for  $r_d$ .

$$\text{Forward Rate} = \text{Spot Rate} * (1 + r_d * (92/360)) / (1 + r_f * (92/360))$$

Bid-side spot and forward is where a customer would sell USD, so these correspond to borrowing rates for USD (offer side interest rate) and lending rates in NOK (bid side interest rate). So, for the NOK bid side implied yield we solve:

$$\begin{aligned} 9.8155 &= 9.8570 * [1 + r_d * (92/360)] / [1 + 0.0455 * (92/360)] \\ (9.8155/9.8570) * [1 + 0.0455 * (92/360)] &= [1 + r_d * (92/360)] \\ 1.007623 &= [1 + r_d * (92/360)] \end{aligned}$$

So ,  $r_d = (360/92) * [1.007623 - 1]$   
 $r_d = 2.98\%$

For offer-side implied NOK interest rates, we need so solve the following for  $r_d$

$$9.8195 = 9.8580 * (1 + r_d * (92/360)) / (1 + 0.0455 * (92/360))$$

So ,  $r_d = (360/92) * [(9.8195/9.8580) * (1 + 0.0455 * 92/360) - 1]$   
 $r_d = 3.00\%$

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- 15) Given the information below, calculate the FX swap points for a long USDCHF position maturing in 3 months that needs to be rolled out to the 6 month date. Assume you are a market-taker.

USDCHF	0.9320 / 0.9322
Days spot to 3months	91
Days spot to 6months	182
3mo USD deposit	4.60% / 4.70%
6mo USD deposit	4.85% / 4.95%
3mo CHF deposit	1.10% / 1.15%
6mo CHF deposit	1.25% / 1.30%

NOTE: Assume all interest rates are money market rates using ACT/360.

A market-taker with an existing long USDCHF position would need to “sell/buy” USDCHF in order to roll the position forward. (This would establish a new long position at the far value date.)

To calculate FX swap points, we need to calculate forward rates for both the near and far dates and take their difference, in pips. We use the offer side rates for both near date and far date, since the longer leg of the FX swap has the market-taker “buying”.

Near date forward (offer-side):  $F_1 = 0.9322 * (1 + 1.15\% * 91/360) / (1 + 4.60\% * 91/360) = 0.9488$

Far date forward (offer-side):  $F_2 = 0.9322 * (1 + 1.30\% * 182/360) / (1 + 4.85\% * 182/360) = 0.9403$

FX swap points (expressed in pips) =  $166 - 81 = -85$  (or -85.2 with more precision.)

- 16) Given the information below, calculate the forward-forward FX swap points for a short AUDUSD position maturing in 2 years that needs to be shortened to the 1-year date. Assume you are a market-taker. What would the near date and far date all-in forward rates be for the FX swap?

AUDUSD spot	0.6678 / 0.6680
1yr points	+73 / +78
2yr points	+66 / +71

A market-taker with an existing short AUDUSD position would need to “sell/buy” AUDUSD in order to roll the position backward. This would establish a new short position at the 1-year date. The market-taker would “sell/buy” using the offer-side swap points.

The 1-year swap points (offer side) combined with spot (offer side) give the all-in 1-year forward (offer side), which is  $0.6758 = 0.6680 + 0.0078$ . Similarly, the 2-year swap points (offer side) give the all-in 2-year forward rate (offer side), which is  $0.6751 = 0.6680 + 0.0071$

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So the all-in forward rates on the FX swap would be 0.6758 and 0.6751 for the 1-year and 2-year dates, respectively. The swap points would be  $0.6751 - 0.6758 = -7$  pips.

Note that the swap points are negative when moving from 1 year to 2 years. Since this transaction would be moving an existing short AUDUSD position from 2 years to 1 year, the effective swap points would increase (near date is 7 pips higher) meaning that the transaction involves receiving the points. (The resulting rate is more favorable for a short position.)

17) (This problem counts for 4 points) Window forward:

A client needs to buy Mexican peso (MXN) 1 billion versus USD but is unsure of the timing. The client asks you to quote a single forward rate where the client will be committed to buy MXN 1 billion, but can do so any time between the 3 month (91 days) and 6 month (183 days) forward dates.

Assuming the rates below (ignoring bid and offer, and assuming covered interest rate parity holds and that both currencies follow an ACT/360 convention) what forward rate would you quote?

(Hint: you are allowed to be greedy, but not unreasonable.)

USDMXN spot	19.77
3mo USD deposit	4.60%
6mo USD deposit	4.70%
3mo MXN deposit	10.70%
6mo MXN deposit	11.20%

First, examine the two USD amounts that the client would commit to paying with either a standard 3-month or a standard 6-month forward contract:

3-month forward  $19.77 * (1 + 10.70\% * 91/360) / (1 + 4.60\% * 91/360) = 20.0713$

USD payment = MXN 1 billion / 20.0713 = USD 49,822,288

6-month forward  $19.77 * (1 + 11.20\% * 183/360) / (1 + 4.70\% * 183/360) = 20.4080$

USD payment = MXN 1 billion / 20.4080 = USD 49,000,413

Notice that the USD payment is higher when the corresponding FX rate is lower, and that the lowest FX rate for any forward in a 3-month to 6-month maturity horizon must be the 3-month rate, 20.0713. This is because all forward points are positive and increasing with longer maturities, given that MXN interest rates are clearly higher than USD interest rates.

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The potential USD payment must be greater than or equal to USD 49,822,288 since this is the USD amount for a 3-month forward, and the client could decide to deliver in exactly 3 months. In other words, the window forward rate cannot be lower than 20.0713.

Consider what would happen if the trader granted the client a delivery window, set 20.0713 as the rate, and hedged as if the client were delivering in exactly 3 months. (An offsetting hedge would be for the trader to sell USD/MXN in a 3-month forward contract.) If the client did not deliver at the 3-month date the trader would “buy/sell” USDMXN to extend the maturity of the hedge. If MXN interest rates were still higher than USD interest rates this “buy/sell” USDMXN transaction would generate additional profit for the trader.

The probability of USD interest rates rising above MXN interest rates in the next 6 months is negligible (it is essentially zero). And although the probability of the client delivering after the 3-month date is unknown, since the client is requesting the product there is at least some chance of additional profit for the trader. Giving the client a rate of 20.0713 would not be unreasonable.

Note: if you believed the risk of USD interest rates surpassing MXN interest rates in 6 months was significant enough to request additional profit (meaning to increase the 20.0713 rate slightly) then I would accept that as a solution to this problem.

- 18) Would you be willing to offer the product in problem 6 above if the client asked to sell CAD 100 million instead of MXN 1 billion, when CAD deposit rates are equal to USD deposit rates? Why or why not?

(HINT: Do CAD interest rates present a difficulty?)

No. Offering this product for USDCAD would entail an unhedgable risk of expected loss.

With USDCAD the trader could not be sure if the FX swap points in the future would be positive or negative. With real risk of the FX swap points changing parity, there is a real risk that anticipated additional profit could become actual loss if the client delivered unexpectedly.