Scoring is 2 points for each question. Question 21 will be scored at 4 points.

19) The price of a EUR call/USD put struck at 1.0800 is 0.0243. If the spot rate is 1.0630 and the forward rate is 1.0760, then what is the price of an EUR put/USD call struck at 1.0800?

    Assume 0.53 years to maturity (for both options and for the forward rate) and assume the relevant USD interest rate for discounting is 4.65% (this rate is a continuously compounded zero coupon rate).

    Since the EUR put strike and EUR call strike are equal and the option maturity dates are the same, put-call parity must apply.

    So we have,

    Call price - Put price = P_{USD} ** [F - K]*

    Which means,

    *0.0243 - Put price = exp(-4.65%*0.53) * (1.0760 - 1.0800)*

    so, *0.0243 - Put price = -0.0039*

    so, *Put price = 0.0282*

20) Why does the derivation of the Garman-Kohlhagen PDE for a foreign exchange spot rate differ from the derivation of the Black-Scholes PDE for a non-dividend-paying stock?

    a) Foreign interest rates and spot FX rates are correlated, so adjustment terms must be introduced
    b) The FX spot rate is not a traded asset, so we cannot construct a risk-free hedged portfolio
    c) Foreign exchange positions must be present valued using the foreign interest rate
    d) A foreign currency position must be carried using a foreign risk-free bond

**** Please write at least a brief explanation of why you think the answer is correct.**

d) is the correct answer. Foreign currency that is held over a fixed time period earns interest and the interest rate is given by the yield on a foreign bond.

a) may or may not be true in practice, however for the Black-Scholes derivation we assume interest rates are non-stochastic.

b) is false because it <u>is</u> possible to construct the risk-free hedged portfolio.

c) is a true statement, however it does not contribute to the different derivations of option pricing formulas.

Scoring is 2 points for each question. Question 21 will be scored at 4 points.

21) (This problem counts for 4 points.) Question regarding risk-neutral valuation. It's often said that any derivative of the FX spot rate can be valued using the risk-neutral valuation theory (as in the section "Valuing FX options" of the week #3 lecture notes).

Assume the FX spot rate, S, follows a geometric Brownian motion, $dS = \mu S\, dt + \sigma S\, dW$. Let V be a very simple derivative contract based on S, where $V(S_t,t) = S_t$ at any time, t

In other words, at any time t the derivative V has value equal to the spot rate at that time.

Does the argument in the section "Valuing FX options" of the week #3 lecture notes (specifically slides 16-20) apply to the function V described above? Explain why, or why the theory does not apply in this case.

The arguments on slides 16-20 do not apply to V. The functions analyzed in slides 16-20 are initially defined at a single maturity point, T, and their values for $t<T$ are completely determined by the Garman-Kohlhagen PDE.

To explore this example further:

$V(S_t, t) = S_t$ is certainly a valid definition for a function, but the function can correspond to the value of a derivative contract only if either 1) it coincidentally satisfies the Garman-Kohlhagen PDE, or 2) we restrict the definition to a specific time T

If we apply $V(S_t, t) = S_t$ to the Garman-Kohlhagen PDE, since $V_{SS} = V_t = 0$ and $V_S = 1$, we have $(r_d - r_f)\, S - r_d\, S = 0$ So, $V(S_t, t) = S_t$ can be the value of a derivative function only if $r_f = 0$.

If we fix any time T and use the definition for V given above, we have that V has the same value at T as a forward contract with maturity T and contract rate = 0. (This is a very extreme forward contract, one that allows the owner to buy foreign currency for no cost, but one this contract does exist theoretically and can be valued).

In other words, $V(S_T, T) = S_T = S_T - 0 = F(T,T)$ where F is a forward contract with maturity T and contract rate 0

Since both V and F have the same boundary values and both obey the Garman-Kohlhagen PDE they must have the same value for all $t < T$. But we know the value of a forward contract,

$PV[\, F(t,T)\,] = S_t \exp(r_f(T-t)) - R \exp(r_f(T-t))$. So with $R=0$, $F(t,T) = S_t \exp(r_f(T-t))$

So we reach the same conclusion, that $V(S_t, t) = S_t$ could define a derivative contract only if $r_f = 0$.

It's interesting to note that if $r_f = 0$ then a forward contract with $R = 0$ as a contract rate will have the value S_t for any time t.

Scoring is 2 points for each question. Question 21 will be scored at 4 points.

22) For a GBP put / USD call with a strike of 1.1700, calculate the option premium in the four ways listed discussed in Lecture #3. Also, assuming the notional is 85 million GBP, list the actual premium in GBP and USD amounts Use the market information listed below:

Spot rate	1.2140
Trade date	23-Feb-2023
Expiry date	23-Aug-2023
Spot date	27-Feb-2023
Delivery date	25-Aug-2023
USD deposit rate	4.75%
GBP deposit rate	3.75%
Implied volatility	11.35%

NOTE: Use ACT/360 when working with USD and ACT/365 when working with GBP interest rates. Also note that when working with "tau", the trade date to expiry date period, money market conventions do not apply. For that period use ACT/365.

We set ω = -1 and the G-K formula for premium becomes:

$$P^d * (-1) * [F\,\Phi(-d_1) - K\,\Phi(-d_2)] = P^d\,[K\,\Phi(-d_2) - F\,\Phi(-d_1)]$$

We need to keep track of two separate measures of time 1) time used for interest accrual, this measure of time determines the forward rate and USD discounting from delivery date to spot date, and 2) time to maturity which is applied to implied volatility and is measured from trade date to expiry date.

To calculate F, the forward rate, and P^d, the USD discount factor, note there are 179 days from spot to the delivery date.

$$F = Spot * (1+4.75\%*179/360) / (1+3.75\%*179/365) = 1.2202$$

$$P^d = 1 / (1+4.75\%*179/360) = 0.9769$$

Next we calculate $-d_1$ and $-d_2$

$-d_{1,2} = \log(K/F) / \sigma\sqrt{t}\ -/+\ 0.5*\sigma\sqrt{t}$ where t = 181/365, because there are 181 days to expiry

$$-d_{1,2} = \log(1.1700/1.2202) / 0.1153*\sqrt{0.496}\ -/+\ 0.5*0.1135*\sqrt{0.496}$$

$$Premium = P^d\,[K\,\Phi(-d_2) - F\,\Phi(-d_1)]$$
$$= 0.9769 * [1.1700*\Phi(-0.4860) - 1.2202*\Phi(-0.5659)] = 0.0177$$

Ways to express the premium:

1) "USD pips" would be 0.0177, or "177 USD pips", which is the actual premium when GBP notional = 1

2) For "USD %" we convert 1) to USD notional = 1, which means dividing by K,
So, 0.0177/1.1700 = 0.0151 = 1.51%

3) For "GBP pips" we convert 2) to an GBP price, which means dividing by spot,
So, 0.0151/1.2140 = 0.0125, or "125 GBP pips"

Scoring is 2 points for each question. Question 21 will be scored at 4 points.

4) For “GBP %” we convert 3) to GBP notional = 1, which means multiplying by K
So, 0.0125*1.1700 = 0.0146 = 1.46%

Finally, for GBP 85 million notional the premium would be 85,000,000*0.0177 = USD 1,505,388

And the GBP premium would be USD 1,505,388 / 1.2140 = GBP 1,240,023

23) Assume that a USD-based bank buys the following option (from a client): USD call / JPY put struck at 120.00 with a notional of USD 100 million. If the current trading day is one day before the option’s maturity and the current spot rate is 131.87, then what size of spot transaction should the bank execute in order to hedge the value of the purchased option against movements in the exchange rate.

Please write a brief explanation of why you think the answer you chose is correct.

a) Sell USD 91 million

b) Sell USD 100 million

c) Sell USD 105 million

d) Sell USD 110 million

<u>Answer = a).</u> First note that the option is so deeply in the money it is essentially a forward contract, meaning its sensitivity to the spot rate is nearly 1-to-1. It may have been tempting to answer b), but USD 100 million would be a proper hedge from a JPY-based viewpoint, not the USD-based viewpoint. (Remember that option and forward payoffs are not linear when seen from the foreign currency viewpoint.)

To understand the USD-based viewpoint we could express the option in terms of JPYUSD rates. The option is a JPY put USD call with JPY notional of JPY 12 billion. As observed before it is deeply in the money, so it is essentially a forward contract selling JPY versus USD (selling because the option is a JPY put). The appropriate spot hedge would be buying JPY 12 billion. Using the spot rate, this would be selling USD 12,000/131.87 million = <u>USD 91 million</u>.