
Homework 3

BUSN 41204 - 2023

- Aman Krishna
 - Christian Pavilanis
 - Jingwen Li
 - Yazmin Ramirez Delgado
-

```
In [ ]: knitr::opts_chunk$set(eval = FALSE)
```

Due: end of day Saturday, February 4

Submission instructions: Submit one write-up per group on [gradescope.com](https://www.gradescope.com).

IMPORTANT:

- Write names of everyone that worked on the assignment on the submission.
- Specify every member of the group when submitting on Gradescope (<https://help.gradescope.com/article/m5qz2xsnjy-student-add-group-members>)

For this homework, we will be using the case *Retention Modeling at Scholastic Travel Company*. Read:

- Case: Retention Modeling at Scholastic Travel Company (A);
- Supplement: Retention Modeling at Scholastic Travel Company (B);

which are available on Canvas.

Your goal is to help David build a model for retention.

The following code will get you started.

Load relevant libraries

```
In [ ]: library(dplyr)
library(caret)
library(glmnet)
```

Load the data

Here we will load the data from the CSV data file, examine its structure, and fix the data types incorrectly identified by R when importing from CSV.

```
In [ ]: STCdata_A<-read.csv('travelData.csv')
STCdata_A<-STCdata_A[,-1]
```

You can use the function `str` to quickly check the internal structure of an R object. Here we are using it to investigate type of data in each column of the loaded data.

```
In [ ]: str(STCdata_A)
```

```
'data.frame': 2389 obs. of 55 variables:
 $ Program.Code      : chr  "HS" "HC" "HD" "HN" ...
 $ From.Grade        : int   4 8 8 9 6 10 11 9 8 8 ...
 $ To.Grade          : int   4 8 8 12 8 12 12 9 8 8 ...
 $ Group.State       : chr   "CA" "AZ" "FL" "VA" ...
 $ Is.Non.Annual.    : int   0 0 0 1 0 0 1 0 0 0 ...
 $ Days              : int   1 7 3 3 6 4 6 8 8 4 ...
 $ Travel.Type       : chr   "A" "A" "A" "B" ...
 $ Departure.Date    : chr   "1/14/2011" "1/14/2011" "1/15/2011" "1/15/2011" ...
 $ Return.Date       : chr   "1/14/2011" "1/21/2011" "1/17/2011" "1/17/2011" ...
 $ Deposit.Date      : chr   "8/30/2010" "11/15/2009" "10/15/2010" "1/7/2011" ...
 $ Special.Pay       : chr   NA "CP" NA NA ...
 $ Tuition           : int   424 2350 1181 376 865 2025 1977 3379 2200 1428 ...
 $ FRP.Active        : int   25 9 17 0 40 9 16 10 30 51 ...
 $ FRP.Cancelled     : int   3 9 6 0 8 4 4 0 0 1 ...
 $ FRP.Take.up.percent.: num  0.424 0.409 0.708 0 0.494 0.9 0.64 0.769 0.577 0.773 ...
 $ Early.RPL         : chr   "3/29/2010" "10/20/2009" "4/29/2010" NA ...
 $ Latest.RPL        : chr   "8/12/2010" "8/10/2010" "8/16/2010" NA ...
 $ Cancelled.Pax     : int   3 11 6 1 9 3 5 1 0 1 ...
 $ Total.Discount.Pax : int   4 3 3 0 8 1 2 1 4 6 ...
 $ Initial.System.Date : chr   "3/26/2010" "10/2/2009" "1/28/2010" "10/19/2010" ...
 $ Poverty.Code      : chr   "B" "C" "C" "" ...
 $ Region            : chr   "Southern California" "Other" "Other" "Other" ...
 $ CRM.Segment       : int   4 10 10 7 10 8 8 7 5 5 ...
 $ School.Type       : chr   "PUBLIC" "PUBLIC" "PUBLIC" "CHD" ...
 $ Parent.Meeting.Flag : int   1 1 1 0 1 1 1 1 1 1 ...
 $ MDR.Low.Grade     : chr   "K" "7" "6" "" ...
 $ MDR.High.Grade    : int   5 8 8 NA 8 12 12 NA 12 8 ...
 $ Total.School.Enrollment : int  927 850 955 NA 720 939 225 NA 500 635 ...
 $ Income.Level      : chr   "Q" "A" "O" "" ...
 $ EZ.Pay.Take.Up.Rate : num  0.17 0.091 0.042 0 0.383 0.1 0.08 0 0.231 0.136 ...
 $ School.Sponsor    : int   1 0 0 0 0 0 0 0 0 0 ...
 $ SPR.Product.Type  : chr   "CA History" "East Coast" "East Coast" "East Coast" ...
 $ SPR.New.Existing  : chr   "EXISTING" "EXISTING" "EXISTING" "EXISTING" ...
 $ FPP               : int   59 22 24 18 81 10 25 13 52 66 ...
 $ Total.Pax         : int   63 25 27 18 89 11 27 14 56 72 ...
 $ SPR.Group.Revenue : int   424 2350 1181 376 865 2025 1977 3379 2200 1428 ...
 $ NumberOfMeetingswithParents : int  1 2 1 0 1 1 1 1 1 1 ...
 $ FirstMeeting      : chr   "8/12/2010" "11/17/2009" "9/13/2010" NA ...
 $ LastMeeting       : chr   "8/12/2010" "8/27/2010" "9/13/2010" NA ...
 $ DifferenceTraveltoFirstMeeting : int  155 423 124 NA 145 91 63 138 143 146 ...
 $ DifferenceTraveltoLastMeeting : int  155 140 124 NA 145 91 63 138 143 146 ...
 $ SchoolGradeTypeLow : chr   "Elementary" "Middle" "Middle" "High" ...
 $ SchoolGradeTypeHigh : chr   "Elementary" "Middle" "Middle" "High" ...
 $ SchoolGradeType    : chr   "Elementary->Elementary" "Middle->Middle" "Middle->Middle" "High->High" ...
 $ DepartureMonth     : chr   "January" "January" "January" "January" ...
 $ GroupGradeTypeLow  : chr   "K" "Middle" "Middle" "Undefined" ...
 $ GroupGradeTypeHigh : chr   "Elementary" "Middle" "Middle" "Undefined" ...
 $ GroupGradeType     : chr   "K->Elementary" "Middle->Middle" "Middle->Middle" "Undefined->Undefined" ...
 $ MajorProgramCode   : chr   "H" "H" "H" "H" ...
 $ SingleGradeTripFlag : int   1 1 1 0 0 0 0 1 1 1 ...
 $ FPP.to.School.enrollment : num  0.0636 0.0259 0.0251 NA 0.1125 ...
 $ FPP.to.PAX         : num  0.937 0.88 0.889 1 0.91 ...
 $ Num.of.Non_FPP.PAX : int   4 3 3 0 8 1 2 1 4 6 ...
 $ SchoolSizeIndicator : chr   "L" "L" "L" "" ...
 $ Retained.in.2012.  : int   1 1 1 0 0 1 0 0 1 1 ...
```

Notice that some columns are identified as numerical or integer, but really they should be factors.

For instance, we have that column `From.Grade`

```
In [ ]: n_distinct(STCdata_A$From.Grade, na.rm = FALSE) ## n_distinct is a function from dplyr package
```

11

only has 11 levels. It might be a better idea to treat it as a factor instead.

You can fix incorrectly classified data types as follows:

```
In [ ]: STCdata_A <- mutate_at(STCdata_A, vars(From.Grade), as.factor)
```

We can check that indeed the column represents a factor:

```
In [ ]: str( STCdata_A$From.Grade )

Factor w/ 10 levels "3","4","5","6",...: 2 6 6 7 4 8 9 7 6 6 ...
```

Fix other columns that are numeric at the moment, but could be converted to factors. The following line first finds numeric columns and then identifies the number of unique elements in each one.

```
In [ ]: ( unique.per.column <- sapply( dplyr::select_if(STCdata_A, is.numeric), n_distinct ) )
```

To.Grade: 11 **Is.Non.Annual.:** 2 **Days:** 12 **Tuition:** 1230 **FRP.Active:** 93 **FRP.Cancelled:** 29 **FRP.Take.up.percent.:** 476 **Cancelled.Pax:** 34
Total.Discount.Pax: 26 **CRM.Segment:** 12 **Parent.Meeting.Flag:** 2 **MDR.High.Grade:** 13 **Total.School.Enrollment:** 894
EZ.Pay.Take.Up.Rate: 371 **School.Sponsor:** 2 **FPP:** 146 **Total.Pax:** 159 **SPR.Group.Revenue:** 1230 **NumberOfMeetingswithParents:** 3
DifferenceTraveltoFirstMeeting: 343 **DifferenceTraveltoLastMeeting:** 252 **SingleGradeTripFlag:** 2 **FPP.to.School.enrollment:** 1910
FPP.to.PAX: 306 **Num.of.Non_FPP.PAX:** 26 **Retained.in.2012.:** 2

Let us convert every column that has less than 15 unique values into a factor. The following line identify names of such columns.

```
In [ ]: ( column.names.to.factor <- names(unique.per.column)[unique.per.column < 15] )

'To.Grade' · 'Is.Non.Annual.' · 'Days' · 'CRM.Segment' · 'Parent.Meeting.Flag' · 'MDR.High.Grade' · 'School.Sponsor' ·
'NumberOfMeetingswithParents' · 'SingleGradeTripFlag' · 'Retained.in.2012.'
```

From this, we can see that the columns `To.Grade`, `Is.Non.Annual.`, `Days`, `CRM.Segment`, `Parent.Meeting.Flag`, `MDR.High.Grade`, `School.Sponsor`, `NumberOfMeetingswithParents`, `SingleGradeTripFlag` can be converted to factors. We can also convert the output `Retained.in.2012.`

Convert these columns into factors.

```
In [ ]: STCdata_A <- mutate_at(STCdata_A, column.names.to.factor, as.factor)
```

Now let's take care of date columns.

```
In [ ]: date.columns = c('Departure.Date', 'Return.Date', 'Deposit.Date', 'Early.RPL', 'Latest.RPL',
                        'Initial.System.Date', 'FirstMeeting', 'LastMeeting')
STCdata_A <- mutate_at(STCdata_A, date.columns, function(x) as.Date(x, format = "%m/%d/%Y"))
```

And finally we change all the character columns to factors as well.

```
In [ ]: STCdata_A <- mutate_if(STCdata_A, is.character, as.factor)
```

Let's see what we have:

```
In [ ]: str(STCdata_A)

'data.frame': 2389 obs. of 55 variables:
 $ Program.Code      : Factor w/ 28 levels "CC","CD","CN",...: 15 6 7 12 7 6 25 5 1 7 ...
 $ From.Grade        : Factor w/ 10 levels "3","4","5","6",...: 2 6 6 7 4 8 9 7 6 6 ...
 $ To.Grade          : Factor w/ 10 levels "3","4","5","6",...: 2 6 6 10 6 10 10 7 6 6 ...
 $ Group.State       : Factor w/ 54 levels "AB","AK","AL",...: 7 5 11 49 11 20 21 29 5 47 ...
 $ Is.Non.Annual.    : Factor w/ 2 levels "0","1": 1 1 1 2 1 1 2 1 1 1 ...
 $ Days              : Factor w/ 12 levels "1","2","3","4",...: 1 7 3 3 6 4 6 8 8 4 ...
 $ Travel.Type       : Factor w/ 4 levels "A","B","N","T": 1 1 1 2 4 1 1 1 1 1 ...
 $ Departure.Date    : Date, format: "2011-01-14" "2011-01-14" ...
 $ Return.Date       : Date, format: "2011-01-14" "2011-01-21" ...
 $ Deposit.Date      : Date, format: "2010-08-30" "2009-11-15" ...
 $ Special.Pay       : Factor w/ 4 levels "", "CP", "FR", "SA": NA 2 NA NA NA NA NA 2 NA ...
 $ Tuition           : int 424 2350 1181 376 865 2025 1977 3379 2200 1428 ...
 $ FRP.Active        : int 25 9 17 0 40 9 16 10 30 51 ...
 $ FRP.Cancelled     : int 3 9 6 0 8 4 4 0 0 1 ...
 $ FRP.Take.up.percent.: num 0.424 0.409 0.708 0 0.494 0.9 0.64 0.769 0.577 0.773 ...
 $ Early.RPL         : Date, format: "2010-03-29" "2009-10-20" ...
 $ Latest.RPL        : Date, format: "2010-08-12" "2010-08-10" ...
 $ Cancelled.Pax     : int 3 11 6 1 9 3 5 1 0 1 ...
 $ Total.Discount.Pax: int 4 3 3 0 8 1 2 1 4 6 ...
 $ Initial.System.Date: Date, format: "2010-03-26" "2009-10-02" ...
 $ Poverty.Code      : Factor w/ 7 levels "", "0", "A", "B",...: 4 5 5 1 6 5 1 1 1 1 ...
 $ Region            : Factor w/ 6 levels "Dallas","Houston",...: 6 4 4 4 4 4 4 4 4 2 ...
 $ CRM.Segment       : Factor w/ 11 levels "1","2","3","4",...: 4 10 10 7 10 8 8 7 5 5 ...
 $ School.Type       : Factor w/ 4 levels "CHD","Catholic",...: 3 3 3 1 3 3 2 1 1 4 ...
 $ Parent.Meeting.Flag: Factor w/ 2 levels "0","1": 2 2 2 1 2 2 2 2 2 2 ...
 $ MDR.Low.Grade     : Factor w/ 13 levels "", "1", "10", "2",...: 12 9 8 1 8 3 11 1 8 13 ...
 $ MDR.High.Grade    : Factor w/ 12 levels "1","2","3","4",...: 5 8 8 NA 8 12 12 NA 12 8 ...
 $ Total.School.Enrollment: int 927 850 955 NA 720 939 225 NA 500 635 ...
 $ Income.Level      : Factor w/ 23 levels "", "A", "B", "C",...: 22 2 16 1 4 10 8 1 12 12 ...
 $ EZ.Pay.Take.Up.Rate: num 0.17 0.091 0.042 0 0.383 0.1 0.08 0 0.231 0.136 ...
 $ School.Sponsor    : Factor w/ 2 levels "0","1": 2 1 1 1 1 1 1 1 1 1 ...
 $ SPR.Product.Type  : Factor w/ 6 levels "CA History","Costa Rica",...: 1 3 3 3 3 3 6 3 3 3 ...
 $ SPR.New.Existing  : Factor w/ 2 levels "EXISTING","NEW": 1 1 1 1 1 2 1 1 1 1 ...
 $ FPP              : int 59 22 24 18 81 10 25 13 52 66 ...
 $ Total.Pax        : int 63 25 27 18 89 11 27 14 56 72 ...
 $ SPR.Group.Revenue : int 424 2350 1181 376 865 2025 1977 3379 2200 1428 ...
 $ NumberOfMeetingswithParents: Factor w/ 3 levels "0","1","2": 2 3 2 1 2 2 2 2 2 2 ...
 $ FirstMeeting      : Date, format: "2010-08-12" "2009-11-17" ...
 $ LastMeeting       : Date, format: "2010-08-12" "2010-08-27" ...
 $ DifferenceTraveltoFirstMeeting: int 155 423 124 NA 145 91 63 138 143 146 ...
 $ DifferenceTraveltoLastMeeting: int 155 140 124 NA 145 91 63 138 143 146 ...
 $ SchoolGradeTypeLow: Factor w/ 4 levels "Elementary","High",...: 1 3 3 2 3 2 2 2 3 3 ...
 $ SchoolGradeTypeHigh: Factor w/ 4 levels "Elementary","High",...: 1 3 3 2 3 2 2 2 3 3 ...
 $ SchoolGradeType    : Factor w/ 9 levels "Elementary->Elementary",...: 1 7 7 5 7 5 5 5 7 7 ...
 $ DepartureMonth     : Factor w/ 6 levels "April","February",...: 3 3 3 3 3 3 3 3 3 2 ...
 $ GroupGradeTypeLow  : Factor w/ 6 levels "Elementary","High",...: 3 4 4 6 4 2 2 6 4 5 ...
 $ GroupGradeTypeHigh: Factor w/ 4 levels "Elementary","High",...: 1 3 3 4 3 2 2 4 2 3 ...
 $ GroupGradeType     : Factor w/ 13 levels "Elementary->Elementary",...: 5 9 9 13 9 4 4 13 8 12 ...
 $ MajorProgramCode   : Factor w/ 4 levels "C","H","I","S": 2 2 2 2 2 2 4 3 1 2 ...
 $ SingleGradeTripFlag: Factor w/ 2 levels "0","1": 2 2 2 1 1 1 1 2 2 2 ...
 $ FPP.to.School.enrollment: num 0.0636 0.0259 0.0251 NA 0.1125 ...
 $ FPP.to.PAX        : num 0.937 0.88 0.889 1 0.91 ...
 $ Num.of.Non_FPP.PAX: int 4 3 3 0 8 1 2 1 4 6 ...
 $ SchoolSizeIndicator: Factor w/ 5 levels "", "L", "M-L", "S",...: 2 2 2 1 3 2 4 1 5 3 ...
 $ Retained.in.2012.  : Factor w/ 2 levels "0","1": 2 2 2 1 1 2 1 1 2 2 ...
```

Pretty good!!!

Data preprocessing

The data contains a number of columns with missing values. Let's investigate. The following tells us the number of missing values in each column.

```
In [ ]: sapply(STCdata_A, function(x) sum(is.na(x)))
```

Program.Code: 0 From.Grade: 127 To.Grade: 150 Group.State: 0 Is.Non.Annual.: 0 Days: 0 Travel.Type: 0 Departure.Date: 0 Return.Date: 0 Deposit.Date: 0 Special.Pay: 1917 Tuition: 0 FRP.Active: 0 FRP.Cancelled: 0 FRP.Take.up.percent.: 0 Early.RPL: 673 Latest.RPL: 19 Cancelled.Pax: 0 Total.Discount.Pax: 0 Initial.System.Date: 8 Poverty.Code: 0 Region: 0 CRM.Segment: 4 School.Type: 0 Parent.Meeting.Flag: 0 MDR.Low.Grade: 0 MDR.High.Grade: 68 Total.School.Enrollment: 91 Income.Level: 0 EZ.Pay.Take.Up.Rate: 0 School.Sponsor: 0 SPR.Product.Type: 0 SPR.New.Existing: 0 FPP: 0 Total.Pax: 0 SPR.Group.Revenue: 0 NumberOfMeetingswithParents: 0 FirstMeeting: 337 LastMeeting: 337 DifferenceTraveltoFirstMeeting: 337 DifferenceTraveltoLastMeeting: 337 SchoolGradeTypeLow: 0 SchoolGradeTypeHigh: 0 SchoolGradeType: 0 DepartureMonth: 0 GroupGradeTypeLow: 0 GroupGradeTypeHigh: 0 GroupGradeType: 0 MajorProgramCode: 0 SingleGradeTripFlag: 0 FPP.to.School.enrollment: 91 FPP.to.PAX: 0 Num.of.Non_FPP.PAX: 0 SchoolSizeIndicator: 0 Retained.in.2012.: 0

Dealing with missing values is a challenging problem, which could occupy a quarter of its own. The purpose of this homework is not to investigate in-depth approaches to dealing with missing values, but rather to investigate classification. For that reason, we take the following simple approach.

The function `fixNAs` below fixes missing values. The function defines reactions:

- adds a new category "FIXED_NA" for a missing value of a categorical/factor variable;
- fills zero value for a missing value of a numeric variable;
- fills "1900-01-01" for a missing value of a date variable.

Then it loops through all columns in the dataframe, reads their types, and loops through all the values, applying the defined reaction to any missing data point. In addition, the function creates a surrogate dummy variable for each column containing at least one missing value (for example, `Special.Pay_surrogate`), which takes a value of 1 whenever the original variable (`Special.Pay`) has a missing value, and 0 otherwise.

```
In [ ]: # Create a custom function to fix missing values ("NAs") and
# preserve the NA info as surrogate variables
fixNAs <- function(data_frame){
  # Define reactions to NAs
  integer_reac <- 0
  factor_reac <- "FIXED_NA"
  character_reac <- "FIXED_NA"
  date_reac <- as.Date("1900-01-01")

  # Loop through columns in the data frame
  # and depending on which class the
  # variable is, apply the defined reaction and
  # create a surrogate

  for (i in 1:ncol(data_frame)) {
    if (class(data_frame[,i]) %in% c("numeric","integer")) {
      if (any(is.na(data_frame[,i]))) {
        data_frame[,paste0(colnames(data_frame)[i], "_surrogate")] <-
          as.factor(ifelse(is.na(data_frame[,i]), "1", "0"))
        data_frame[is.na(data_frame[,i]), i] <- integer_reac
      }
    } else
    if (class(data_frame[,i]) %in% c("factor")) {
      if (any(is.na(data_frame[,i]))){
        data_frame[,i]<-as.character(data_frame[,i])
        data_frame[,paste0(colnames(data_frame)[i], "_surrogate")] <-
          as.factor(ifelse(is.na(data_frame[,i]), "1", "0"))
        data_frame[is.na(data_frame[,i]),i]<-factor_reac
        data_frame[,i]<-as.factor(data_frame[,i])
      }
    } else {
      if (class(data_frame[,i]) %in% c("character")) {
        if (any(is.na(data_frame[,i]))){
          data_frame[,paste0(colnames(data_frame)[i], "_surrogate")]<-
            as.factor(ifelse(is.na(data_frame[,i]), "1", "0"))
          data_frame[is.na(data_frame[,i]),i]<-character_reac
        }
      } else {
        if (class(data_frame[,i]) %in% c("Date")) {
          if (any(is.na(data_frame[,i]))){
            data_frame[,paste0(colnames(data_frame)[i], "_surrogate")]<-
              as.factor(ifelse(is.na(data_frame[,i]), "1", "0"))
            data_frame[is.na(data_frame[,i]),i]<-date_reac
          }
        }
      }
    }
  }
}
```

```
}  
  
  return(data_frame)  
}
```

We apply the above defined function to our data frame.

```
In [ ]: STCdata_A<-fixNAs(STCdata_A)
```

We can see that the columns do not have any missing values any more.

```
In [ ]: any( sapply(STCdata_A, function(x) sum(is.na(x))) > 0)
```

FALSE

Next, we combine the rare categories. Levels that do not occur often during training tend not to have reliable effect estimates and contribute to over-fit.

Let us check for rare categories in the variable `Group.State` .

```
In [ ]: table(STCdata_A$Group.State)
```

AB	AK	AL	AR	AZ
1	5	21	10	53
Bermuda	CA	CO	CT Cayman Islands	
1	718	89	15	1
FL	GA	HI	IA	ID
62	22	9	35	14
IL	IN	KS	KY	LA
104	43	26	16	31
MA	MD	ME	MI	MN
36	15	7	71	51
MO	MS	MT	MX	NC
43	9	6	3	16
ND	NE	NH	NJ	NM
5	42	7	6	20
NV	NY	OH	OK	OR
20	19	53	33	51
PA	PR	RI	SC	SD
5	1	3	10	11
TN	TX	UT	VA	VT
38	308	9	18	1
WA	WI	WV	WY	
147	46	1	2	

Let us create a custom function to combine rare categories. The function again loops through all the columns in the dataframe, reads their types, and creates a table of counts for each level of the factor/categorical variables. All levels with counts less than the `mincount` are combined into "other." The function combines rare categories into "Other."+the name of the original variable (for example, `Other.State`). This function has two arguments:

- the name of the dataframe; and
- the count of observations in a category to define "rare."

```
In [ ]: combinerarecategories<-function(data_frame,mincount){  
  for (i in 1:ncol(data_frame)) {  
    a<-data_frame[,i]  
    replace <- names(which(table(a) < mincount))  
    levels(a)[levels(a) %in% replace] <-  
      paste("Other", colnames(data_frame)[i], sep=".")  
    data_frame[,i]<-a  
  }  
  return(data_frame)  
}
```

Let us combine categories with < 10 values in `STCdata` into "Other." Ultimately, it is going to depend on the person doing the analysis on what they decide to call ``rare".

```
In [ ]: STCdata_A<-combinerarecategories(STCdata_A,10)
```

Let us look at `Group.State` again.

```
In [ ]: table(STCdata_A$Group.State)
```


Other.Group.State	AL	AR	AZ
82	21	10	53
CA	CO	CT	FL
718	89	15	62
GA	IA	ID	IL
22	35	14	104
IN	KS	KY	LA
43	26	16	31
MA	MD	MI	MN
36	15	71	51
MO	NC	NE	NM
43	16	42	20
NV	NY	OH	OK
20	19	53	33
OR	SC	SD	TN
51	10	11	38
TX	VA	WA	WI
308	18	147	46

You can investigate other columns to see if everything looks fine.

Split the data into training and testing sets

This is a very important step, both conceptually and technically. Conceptually, because the goal of predictive modeling is not to build a model that fits well the data it trains on, but rather one that would best predict the new data. A test set is in this sense the best representation of what the "new data" may look like. Technically, to facilitate comparison between different models, we need to maintain the same IDs in the corresponding sets at all times. We will accomplish this through two "tricks":

- a random seed ensures that the random-number generator is initialized identically in each run; and
- the `inTrain` vector is created once and can then be applied anytime the data needs to be split.

By default, the code sets 500 data points in the test set, and the remainder 1,889 into the training set.

```
In [ ]: # set a random number generation seed to
# ensure that the split is the same every time
set.seed(233)

inTrain <- createDataPartition(
  y = STCdata_A$Retained.in.2012.,
  p = 1888/2389,
  list = FALSE)
df.train <- STCdata_A[ inTrain, ]
df.test <- STCdata_A[ -inTrain, ]
```

Let us check that both the training and test sets have a similar proportion of positive and negative cases.

```
In [ ]: print('Training set proportion:')
table(df.train$Retained.in.2012.) / nrow(df.train)
print('Test set proportion:')
table(df.test$Retained.in.2012.) / nrow(df.test)

[1] "Training set proportion:"
      0      1
0.3928004 0.6071996
[1] "Test set proportion:"
      0      1
0.392 0.608
```

Fitting a logistic regression model

Let us fit a logistic regression model with all the variables included on the training set.

```
In [ ]: lgfit.all <- glm(Retained.in.2012.~ .,
                        data=df.train,
                        family="binomial")
summary(lgfit.all)
```

Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"

```
Call:
glm(formula = Retained.in.2012. ~ ., family = "binomial", data = df.train)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-2.7206	-0.5092	0.2285	0.5545	3.1577

Coefficients: (44 not defined because of singularities)

	Estimate
(Intercept)	-1.699e+02
Program.CodeCD	7.495e-01
Program.CodeOther.Program.Code	4.205e-01
Program.CodeHC	2.585e-01
Program.CodeHD	2.816e-01
Program.CodeHG	-8.370e-01
Program.CodeHN	5.577e-01
Program.CodeHO	4.023e-01
Program.CodeHS	-9.511e-01
Program.CodeHVP	-7.901e-02
Program.CodeIC	-2.083e+01
Program.CodeSC	-4.916e-01
Program.CodeSG	1.934e+01
Program.CodeSK	1.633e+01
Program.CodeSM	1.672e+01
Program.CodeST	1.757e+01
From.Grade11	1.542e+00
From.Grade12	-2.486e+00
From.GradeOther.From.Grade	-1.280e+01
From.Grade4	3.236e+00
From.Grade5	2.445e+00
From.Grade6	2.174e+00
From.Grade7	2.343e+00
From.Grade8	2.767e+00
From.Grade9	2.810e-01
From.GradeFIXED_NA	2.145e+00
To.Grade11	3.170e-01
To.Grade12	-2.254e-01
To.GradeOther.To.Grade	2.718e+01
To.Grade4	-1.836e+00
To.Grade5	-1.816e+00
To.Grade6	-1.523e+00
To.Grade7	-1.094e+00
To.Grade8	-2.053e+00
To.Grade9	3.513e-01
To.GradeFIXED_NA	-1.078e+00
Group.StateAL	-2.477e-01
Group.StateAR	-1.034e+00
Group.StateAZ	-2.317e-01
Group.StateCA	6.674e-01
Group.StateCO	1.003e-01
Group.StateCT	-1.273e-01
Group.StateFL	1.053e-01
Group.StateGA	-1.002e+00
Group.StateIA	2.276e-01
Group.StateID	-1.946e+00
Group.StateIL	2.166e-01
Group.StateIN	-1.502e+00
Group.StateKS	-4.659e-01
Group.StateKY	-5.784e-01
Group.StateLA	-8.454e-01
Group.StateMA	-6.635e-01
Group.StateMD	-2.134e+00
Group.StateMI	-4.080e-01
Group.StateMN	5.950e-01
Group.StateMO	-9.632e-02
Group.StateNC	-7.515e-01
Group.StateNE	6.577e-01
Group.StateNM	1.799e-01
Group.StateNV	1.357e+00
Group.StateNY	-5.341e-01
Group.StateOH	-9.493e-01
Group.StateOK	-6.534e-02
Group.StateOR	1.818e-01
Group.StateSC	-3.473e+00
Group.StateSD	9.883e-01
Group.StateTN	-6.390e-01
Group.StateTX	7.316e-01
Group.StateVA	1.879e+00
Group.StateWA	-3.834e-01
Group.StateWI	3.284e-01
Is.Non.Annual.1	-2.936e+00
Days2	-1.337e-01
Days3	-2.684e-01
Days4	-5.705e-01
Days5	-1.257e+00
Days6	-1.015e+00
Days7	-1.938e+00
Days8	-2.652e+00
DaysOther.Days	-4.741e+00
Days11	1.339e+01
Travel.TypeB	2.628e-01
Travel.TypeOther.Travel.Type	1.651e-01
Departure.Date	-4.044e-01

Return.Date	4.154e-01
Deposit.Date	-3.985e-03
Special.PayCP	2.099e+01
Special.PayFIXED_NA	1.898e+01
Special.PayFR	1.897e+01
Special.PaySA	2.004e+01
Tuition	-3.349e-04
FRP.Active	2.981e-02
FRP.Cancelled	-3.807e-02
FRP.Take.up.percent.	-1.823e-02
Early.RPL	-1.035e-03
Latest.RPL	1.388e-03
Cancelled.Pax	3.243e-02
Total.Discount.Pax	8.370e-02
Initial.System.Date	1.078e-03
Poverty.CodeOther.Poverty.Code	-1.444e+00
Poverty.CodeA	-7.845e-01
Poverty.CodeB	-9.065e-01
Poverty.CodeC	-9.367e-01
Poverty.CodeD	-1.510e+00
Poverty.CodeE	3.821e-01
RegionHouston	-3.970e-01
RegionNorthern California	-5.908e-01
RegionOther	NA
RegionPacific Northwest	NA
RegionSouthern California	NA
CRM.Segment10	1.104e+00
CRM.Segment11	1.468e+00
CRM.Segment2	2.461e-01
CRM.Segment3	8.149e-01
CRM.Segment4	3.399e+00
CRM.Segment5	1.078e+00
CRM.Segment6	2.439e+00
CRM.Segment7	-1.231e-01
CRM.Segment8	-4.386e-01
CRM.SegmentOther.CRM.Segment	-1.558e+00
School.TypeCatholic	-7.301e-02
School.TypePUBLIC	2.718e-01
School.TypePrivate non-Christian	8.510e-01
Parent.Meeting.Flag1	3.494e+01
MDR.Low.GradeOther.MDR.Low.Grade	9.882e+00
MDR.Low.Grade3	1.100e+01
MDR.Low.Grade4	9.286e+00
MDR.Low.Grade5	9.826e+00
MDR.Low.Grade6	1.098e+01
MDR.Low.Grade7	1.112e+01
MDR.Low.Grade8	1.115e+01
MDR.Low.Grade9	5.388e+00
MDR.Low.GradeK	1.078e+01
MDR.Low.GradePK	1.070e+01
MDR.High.Grade12	-8.651e-01
MDR.High.Grade5	-1.514e+01
MDR.High.Grade6	-1.615e+01
MDR.High.Grade7	-1.311e+01
MDR.High.Grade8	-1.295e+01
MDR.High.Grade9	-4.093e-01
MDR.High.GradeFIXED_NA	NA
Total.School.Enrollment	-2.455e-05
Income.LevelA	6.971e-01
Income.LevelB	1.139e+00
Income.LevelC	4.301e-01
Income.LevelD	4.618e-01
Income.LevelE	4.157e-01
Income.LevelF	6.435e-01
Income.LevelG	1.681e-01
Income.LevelH	7.329e-01
Income.LevelI	3.560e-01
Income.LevelJ	7.167e-02
Income.LevelK	7.383e-01
Income.LevelL	3.106e-01
Income.LevelM	4.103e-01
Income.LevelN	1.388e-01
Income.LevelO	6.390e-02
Income.LevelP	1.189e+00
Income.LevelOther.Income.Level	7.638e-01
Income.LevelQ	-3.348e-01
Income.LevelZ	8.476e-01
EZ.Pay.Take.Up.Rate	1.040e-01
School.Sponsor1	-1.133e-01
SPR.Product.TypeCosta Rica	NA
SPR.Product.TypeEast Coast	1.679e-02
SPR.Product.TypeOther.SPR.Product.Type	-5.707e-01
SPR.Product.TypeInternational	6.952e+00
SPR.Product.TypeScience	-1.693e+01
SPR.New.ExistingNEW	-1.706e+00
FPP	-6.908e-03
Total.Pax	NA
SPR.Group.Revenue	NA
NumberOfMeetingswithParents1	-9.165e-02
NumberOfMeetingswithParents2	NA
FirstMeeting	-1.317e-04
LastMeeting	-7.191e-04
DifferenceTraveltoFirstMeeting	-1.460e-03

DifferenceTraveltoLastMeeting	NA
SchoolGradeTypeLowHigh	NA
SchoolGradeTypeLowMiddle	NA
SchoolGradeTypeLowUndefined	NA
SchoolGradeTypeHighHigh	NA
SchoolGradeTypeHighMiddle	NA
SchoolGradeTypeHighUndefined	NA
SchoolGradeTypeOther.SchoolGradeType	-1.978e-01
SchoolGradeTypeElementary->Middle	NA
SchoolGradeTypeHigh->High	NA
SchoolGradeTypeMiddle->High	-2.318e+00
SchoolGradeTypeMiddle->Middle	NA
SchoolGradeTypeMiddle->Undefined	NA
SchoolGradeTypeUndefined->Undefined	NA
DepartureMonthFebruary	2.289e+00
DepartureMonthOther.DepartureMonth	9.903e-02
DepartureMonthJune	-6.427e-01
DepartureMonthMarch	6.314e-01
DepartureMonthMay	3.860e-03
GroupGradeTypeLowHigh	6.263e+00
GroupGradeTypeLowK	NA
GroupGradeTypeLowMiddle	NA
GroupGradeTypeLowPK	NA
GroupGradeTypeLowUndefined	NA
GroupGradeTypeHighHigh	-1.316e+01
GroupGradeTypeHighMiddle	NA
GroupGradeTypeHighUndefined	NA
GroupGradeTypeElementary->Middle	1.498e+00
GroupGradeTypeHigh->High	NA
GroupGradeTypeK->Elementary	1.429e-01
GroupGradeTypeK->High	-5.283e-01
GroupGradeTypeK->Middle	NA
GroupGradeTypeMiddle->High	3.706e-01
GroupGradeTypeMiddle->Middle	NA
GroupGradeTypePK->Elementary	-5.049e-01
GroupGradeTypePK->High	NA
GroupGradeTypePK->Middle	NA
GroupGradeTypeUndefined->Undefined	NA
MajorProgramCodeH	-3.836e-01
MajorProgramCodeI	-1.696e+01
MajorProgramCodeS	NA
SingleGradeTripFlag1	8.017e-01
FPP.to.School.enrollment	8.266e-01
FPP.to.PAX	1.868e+00
Num.of.Non_FPP.PAX	NA
SchoolSizeIndicatorL	1.552e+00
SchoolSizeIndicatorM-L	1.301e+00
SchoolSizeIndicatorS	5.591e-01
SchoolSizeIndicatorS-M	1.634e+00
From.Grade_surrogate1	NA
To.Grade_surrogate1	NA
Special.Pay_surrogate1	NA
Early.RPL_surrogate1	-4.157e+01
Latest.RPL_surrogate1	5.639e+01
Initial.System.Date_surrogateOther.Initial.System.Date_surrogate	4.294e+01
CRM.Segment_surrogateOther.CRM.Segment_surrogate	5.486e-01
MDR.High.Grade_surrogate1	NA
Total.School.Enrollment_surrogate1	NA
FirstMeeting_surrogate1	NA
LastMeeting_surrogate1	NA
DifferenceTraveltoFirstMeeting_surrogate1	NA
DifferenceTraveltoLastMeeting_surrogate1	NA
FPP.to.School.enrollment_surrogate1	NA
Std. Error	
(Intercept)	1.307e+03
Program.CodeCD	1.070e+00
Program.CodeOther.Program.Code	1.456e+00
Program.CodeHC	1.702e+00
Program.CodeHD	1.696e+00
Program.CodeHG	2.096e+00
Program.CodeHN	1.763e+00
Program.CodeHO	2.065e+00
Program.CodeHS	1.957e+00
Program.CodeHVP	1.788e+00
Program.CodeIC	2.693e+03
Program.CodeSC	1.996e+00
Program.CodeSG	1.283e+03
Program.CodeSK	1.283e+03
Program.CodeSM	1.283e+03
Program.CodeST	1.283e+03
From.Grade11	1.154e+00
From.Grade12	1.613e+00
From.GradeOther.From.Grade	1.098e+03
From.Grade4	3.172e+00
From.Grade5	2.917e+00
From.Grade6	2.818e+00
From.Grade7	2.814e+00
From.Grade8	2.779e+00
From.Grade9	1.046e+00
From.GradeFIXED_NA	2.664e+00
To.Grade11	1.253e+00
To.Grade12	9.853e-01
To.GradeOther.To.Grade	2.639e+03

To.Grade4	3.243e+00
To.Grade5	3.035e+00
To.Grade6	2.903e+00
To.Grade7	2.803e+00
To.Grade8	2.742e+00
To.Grade9	9.735e-01
To.GradeFIXED_NA	2.606e+00
Group.StateAL	9.085e-01
Group.StateAR	1.191e+00
Group.StateAZ	6.455e-01
Group.StateCA	4.650e-01
Group.StateCO	5.293e-01
Group.StateCT	1.224e+00
Group.StateFL	6.149e-01
Group.StateGA	8.267e-01
Group.StateIA	7.929e-01
Group.StateID	1.330e+00
Group.StateIL	5.819e-01
Group.StateIN	6.923e-01
Group.StateKS	8.382e-01
Group.StateKY	1.390e+00
Group.StateLA	7.268e-01
Group.StateMA	7.948e-01
Group.StateMD	9.803e-01
Group.StateMI	6.477e-01
Group.StateMN	7.243e-01
Group.StateMO	6.553e-01
Group.StateNC	9.003e-01
Group.StateNE	7.659e-01
Group.StateNM	8.827e-01
Group.StateNV	8.468e-01
Group.StateNY	9.610e-01
Group.StateOH	6.946e-01
Group.StateOK	7.255e-01
Group.StateOR	6.281e-01
Group.StateSC	1.213e+00
Group.StateSD	9.473e-01
Group.StateTN	7.194e-01
Group.StateTX	4.907e-01
Group.StateVA	1.479e+00
Group.StateWA	4.978e-01
Group.StateWI	7.301e-01
Is.Non.Annual.1	2.487e-01
Days2	6.526e-01
Days3	1.114e+00
Days4	1.203e+00
Days5	1.303e+00
Days6	1.453e+00
Days7	1.653e+00
Days8	1.875e+00
DaysOther.Days	2.375e+00
Days11	9.916e+02
Travel.TypeB	5.016e-01
Travel.TypeOther.Travel.Type	1.184e+00
Departure.Date	2.188e-01
Return.Date	2.183e-01
Deposit.Date	2.422e-03
Special.PayCP	1.276e+03
Special.PayFIXED_NA	1.276e+03
Special.PayFR	1.276e+03
Special.PaySA	1.276e+03
Tuition	4.845e-04
FRP.Active	1.379e-02
FRP.Cancelled	4.269e-02
FRP.Take.up.percent.	4.648e-01
Early.RPL	2.632e-03
Latest.RPL	1.640e-03
Cancelled.Pax	3.334e-02
Total.Discount.Pax	7.863e-02
Initial.System.Date	2.081e-03
Poverty.CodeOther.Poverty.Code	1.402e+00
Poverty.CodeA	6.718e-01
Poverty.CodeB	6.246e-01
Poverty.CodeC	6.340e-01
Poverty.CodeD	8.452e-01
Poverty.CodeE	1.116e+00
RegionHouston	3.919e-01
RegionNorthern California	2.990e-01
RegionOther	NA
RegionPacific Northwest	NA
RegionSouthern California	NA
CRM.Segment10	4.365e-01
CRM.Segment11	1.409e+00
CRM.Segment2	6.929e-01
CRM.Segment3	1.152e+00
CRM.Segment4	8.429e-01
CRM.Segment5	4.699e-01
CRM.Segment6	9.071e-01
CRM.Segment7	9.013e-01
CRM.Segment8	9.851e-01
CRM.SegmentOther.CRM.Segment	1.139e+00
School.TypeCatholic	4.634e-01
School.TypePUBLIC	5.429e-01

School.TypePrivate non-Christian	5.022e-01
Parent.Meeting.Flag1	2.563e+02
MDR.Low.GradeOther.MDR.Low.Grade	9.235e+02
MDR.Low.Grade3	9.235e+02
MDR.Low.Grade4	9.235e+02
MDR.Low.Grade5	9.235e+02
MDR.Low.Grade6	9.235e+02
MDR.Low.Grade7	9.235e+02
MDR.Low.Grade8	9.235e+02
MDR.Low.Grade9	9.235e+02
MDR.Low.GradeK	9.235e+02
MDR.Low.GradePK	9.235e+02
MDR.High.Grade12	1.694e+00
MDR.High.Grade5	9.235e+02
MDR.High.Grade6	9.235e+02
MDR.High.Grade7	9.235e+02
MDR.High.Grade8	9.235e+02
MDR.High.Grade9	1.778e+00
MDR.High.GradeFIXED_NA	NA
Total.School.Enrollment	3.737e-04
Income.LevelA	1.786e+00
Income.LevelB	1.671e+00
Income.LevelC	1.601e+00
Income.LevelD	1.620e+00
Income.LevelE	1.591e+00
Income.LevelF	1.591e+00
Income.LevelG	1.600e+00
Income.LevelH	1.572e+00
Income.LevelI	1.569e+00
Income.LevelJ	1.572e+00
Income.LevelK	1.577e+00
Income.LevelL	1.571e+00
Income.LevelM	1.568e+00
Income.LevelN	1.575e+00
Income.LevelO	1.559e+00
Income.LevelP	1.562e+00
Income.LevelOther.Income.Level	2.503e+00
Income.LevelQ	1.570e+00
Income.LevelZ	1.695e+00
EZ.Pay.Take.Up.Rate	5.187e-01
School.Sponsor1	3.579e-01
SPR.Product.TypeCosta Rica	NA
SPR.Product.TypeEast Coast	1.429e+00
SPR.Product.TypeOther.SPR.Product.Type	1.781e+00
SPR.Product.TypeInternational	3.393e+03
SPR.Product.TypeScience	1.283e+03
SPR.New.ExistingNEW	1.945e-01
FPP	9.622e-03
Total.Pax	NA
SPR.Group.Revenue	NA
NumberOfMeetingswithParents1	3.056e-01
NumberOfMeetingswithParents2	NA
FirstMeeting	6.526e-03
LastMeeting	2.690e-03
DifferenceTraveltoFirstMeeting	6.132e-03
DifferenceTraveltoLastMeeting	NA
SchoolGradeTypeLowHigh	NA
SchoolGradeTypeLowMiddle	NA
SchoolGradeTypeLowUndefined	NA
SchoolGradeTypeHighHigh	NA
SchoolGradeTypeHighMiddle	NA
SchoolGradeTypeHighUndefined	NA
SchoolGradeTypeOther.SchoolGradeType	2.126e+00
SchoolGradeTypeElementary->Middle	NA
SchoolGradeTypeHigh->High	NA
SchoolGradeTypeMiddle->High	2.604e+00
SchoolGradeTypeMiddle->Middle	NA
SchoolGradeTypeMiddle->Undefined	NA
SchoolGradeTypeUndefined->Undefined	NA
DepartureMonthFebruary	1.105e+00
DepartureMonthOther.DepartureMonth	1.434e+00
DepartureMonthJune	5.909e-01
DepartureMonthMarch	3.535e-01
DepartureMonthMay	4.309e-01
GroupGradeTypeLowHigh	3.167e+00
GroupGradeTypeLowK	NA
GroupGradeTypeLowMiddle	NA
GroupGradeTypeLowPK	NA
GroupGradeTypeLowUndefined	NA
GroupGradeTypeHighHigh	9.235e+02
GroupGradeTypeHighMiddle	NA
GroupGradeTypeHighUndefined	NA
GroupGradeTypeElementary->Middle	1.806e+00
GroupGradeTypeHigh->High	NA
GroupGradeTypeK->Elementary	2.655e+00
GroupGradeTypeK->High	7.873e-01
GroupGradeTypeK->Middle	NA
GroupGradeTypeMiddle->High	6.303e-01
GroupGradeTypeMiddle->Middle	NA
GroupGradeTypePK->Elementary	2.663e+00
GroupGradeTypePK->High	NA
GroupGradeTypePK->Middle	NA
GroupGradeTypeUndefined->Undefined	NA

MajorProgramCodeH	1.425e+00
MajorProgramCodeI	2.400e+03
MajorProgramCodeS	NA
SingleGradeTripFlag1	4.350e-01
FPP.to.School.enrollment	1.380e+00
FPP.to.PAX	1.915e+00
Num.of.Non_FPP.PAX	NA
SchoolSizeIndicatorL	7.783e-01
SchoolSizeIndicatorM-L	7.121e-01
SchoolSizeIndicatorS	6.801e-01
SchoolSizeIndicatorS-M	6.856e-01
From.Grade_surrogate1	NA
To.Grade_surrogate1	NA
Special.Pay_surrogate1	NA
Early.RPL_surrogate1	1.062e+02
Latest.RPL_surrogate1	6.638e+01
Initial.System.Date_surrogateOther.Initial.System.Date_surrogate	8.379e+01
CRM.Segment_surrogateOther.CRM.Segment_surrogate	1.774e+00
MDR.High.Grade_surrogate1	NA
Total.School.Enrollment_surrogate1	NA
FirstMeeting_surrogate1	NA
LastMeeting_surrogate1	NA
DifferenceTraveltoFirstMeeting_surrogate1	NA
DifferenceTraveltoLastMeeting_surrogate1	NA
FPP.to.School.enrollment_surrogate1	NA
	z value
(Intercept)	-0.130
Program.CodeCD	0.701
Program.CodeOther.Program.Code	0.289
Program.CodeHC	0.152
Program.CodeHD	0.166
Program.CodeHG	-0.399
Program.CodeHN	0.316
Program.CodeHO	0.195
Program.CodeHS	-0.486
Program.CodeHVP	-0.044
Program.CodeIC	-0.008
Program.CodeSC	-0.246
Program.CodeSG	0.015
Program.CodeSK	0.013
Program.CodeSM	0.013
Program.CodeST	0.014
From.Grade11	1.336
From.Grade12	-1.541
From.GradeOther.From.Grade	-0.012
From.Grade4	1.020
From.Grade5	0.838
From.Grade6	0.771
From.Grade7	0.833
From.Grade8	0.996
From.Grade9	0.269
From.GradeFIXED_NA	0.805
To.Grade11	0.253
To.Grade12	-0.229
To.GradeOther.To.Grade	0.010
To.Grade4	-0.566
To.Grade5	-0.598
To.Grade6	-0.525
To.Grade7	-0.390
To.Grade8	-0.749
To.Grade9	0.361
To.GradeFIXED_NA	-0.413
Group.StateAL	-0.273
Group.StateAR	-0.868
Group.StateAZ	-0.359
Group.StateCA	1.435
Group.StateCO	0.190
Group.StateCT	-0.104
Group.StateFL	0.171
Group.StateGA	-1.212
Group.StateIA	0.287
Group.StateID	-1.463
Group.StateIL	0.372
Group.StateIN	-2.170
Group.StateKS	-0.556
Group.StateKY	-0.416
Group.StateLA	-1.163
Group.StateMA	-0.835
Group.StateMD	-2.177
Group.StateMI	-0.630
Group.StateMN	0.822
Group.StateMO	-0.147
Group.StateNC	-0.835
Group.StateNE	0.859
Group.StateNM	0.204
Group.StateNV	1.603
Group.StateNY	-0.556
Group.StateOH	-1.367
Group.StateOK	-0.090
Group.StateOR	0.290
Group.StateSC	-2.863
Group.StateSD	1.043
Group.StateTN	-0.888

Group.StateTX	1.491
Group.StateVA	1.271
Group.StateWA	-0.770
Group.StateWI	0.450
Is.Non.Annual.1	-11.806
Days2	-0.205
Days3	-0.241
Days4	-0.474
Days5	-0.965
Days6	-0.699
Days7	-1.173
Days8	-1.414
DaysOther.Days	-1.996
Days11	0.014
Travel.TypeB	0.524
Travel.TypeOther.Travel.Type	0.139
Departure.Date	-1.848
Return.Date	1.903
Deposit.Date	-1.645
Special.PayCP	0.016
Special.PayFIXED_NA	0.015
Special.PayFR	0.015
Special.PaySA	0.016
Tuition	-0.691
FRP.Active	2.162
FRP.Cancelled	-0.892
FRP.Take.up.percent.	-0.039
Early.RPL	-0.393
Latest.RPL	0.846
Cancelled.Pax	0.973
Total.Discount.Pax	1.064
Initial.System.Date	0.518
Poverty.CodeOther.Poverty.Code	-1.030
Poverty.CodeA	-1.168
Poverty.CodeB	-1.451
Poverty.CodeC	-1.477
Poverty.CodeD	-1.787
Poverty.CodeE	0.342
RegionHouston	-1.013
RegionNorthern California	-1.976
RegionOther	NA
RegionPacific Northwest	NA
RegionSouthern California	NA
CRM.Segment10	2.530
CRM.Segment11	1.042
CRM.Segment2	0.355
CRM.Segment3	0.708
CRM.Segment4	4.032
CRM.Segment5	2.295
CRM.Segment6	2.689
CRM.Segment7	-0.137
CRM.Segment8	-0.445
CRM.SegmentOther.CRM.Segment	-1.369
School.TypeCatholic	-0.158
School.TypePUBLIC	0.501
School.TypePrivate non-Christian	1.695
Parent.Meeting.Flag1	0.136
MDR.Low.GradeOther.MDR.Low.Grade	0.011
MDR.Low.Grade3	0.012
MDR.Low.Grade4	0.010
MDR.Low.Grade5	0.011
MDR.Low.Grade6	0.012
MDR.Low.Grade7	0.012
MDR.Low.Grade8	0.012
MDR.Low.Grade9	0.006
MDR.Low.GradeK	0.012
MDR.Low.GradePK	0.012
MDR.High.Grade12	-0.511
MDR.High.Grade5	-0.016
MDR.High.Grade6	-0.017
MDR.High.Grade7	-0.014
MDR.High.Grade8	-0.014
MDR.High.Grade9	-0.230
MDR.High.GradeFIXED_NA	NA
Total.School.Enrollment	-0.066
Income.LevelA	0.390
Income.LevelB	0.682
Income.LevelC	0.269
Income.LevelD	0.285
Income.LevelE	0.261
Income.LevelF	0.404
Income.LevelG	0.105
Income.LevelH	0.466
Income.LevelI	0.227
Income.LevelJ	0.046
Income.LevelK	0.468
Income.LevelL	0.198
Income.LevelM	0.262
Income.LevelN	0.088
Income.LevelO	0.041
Income.LevelP	0.761
Income.LevelOther.Income.Level	0.305
Income.LevelQ	-0.213

Income.LevelZ	0.500
EZ.Pay.Take.Up.Rate	0.200
School.Sponsor1	-0.317
SPR.Product.TypeCosta Rica	NA
SPR.Product.TypeEast Coast	0.012
SPR.Product.TypeOther.SPR.Product.Type	-0.320
SPR.Product.TypeInternational	0.002
SPR.Product.TypeScience	-0.013
SPR.New.ExistingNEW	-8.770
FPP	-0.718
Total.Pax	NA
SPR.Group.Revenue	NA
NumberOfMeetingswithParents1	-0.300
NumberOfMeetingswithParents2	NA
FirstMeeting	-0.020
LastMeeting	-0.267
DifferenceTraveltoFirstMeeting	-0.238
DifferenceTraveltoLastMeeting	NA
SchoolGradeTypeLowHigh	NA
SchoolGradeTypeLowMiddle	NA
SchoolGradeTypeLowUndefined	NA
SchoolGradeTypeHighHigh	NA
SchoolGradeTypeHighMiddle	NA
SchoolGradeTypeHighUndefined	NA
SchoolGradeTypeOther.SchoolGradeType	-0.093
SchoolGradeTypeElementary->Middle	NA
SchoolGradeTypeHigh->High	NA
SchoolGradeTypeMiddle->High	-0.890
SchoolGradeTypeMiddle->Middle	NA
SchoolGradeTypeMiddle->Undefined	NA
SchoolGradeTypeUndefined->Undefined	NA
DepartureMonthFebruary	2.072
DepartureMonthOther.DepartureMonth	0.069
DepartureMonthJune	-1.088
DepartureMonthMarch	1.786
DepartureMonthMay	0.009
GroupGradeTypeLowHigh	1.977
GroupGradeTypeLowK	NA
GroupGradeTypeLowMiddle	NA
GroupGradeTypeLowPK	NA
GroupGradeTypeLowUndefined	NA
GroupGradeTypeHighHigh	-0.014
GroupGradeTypeHighMiddle	NA
GroupGradeTypeHighUndefined	NA
GroupGradeTypeElementary->Middle	0.829
GroupGradeTypeHigh->High	NA
GroupGradeTypeK->Elementary	0.054
GroupGradeTypeK->High	-0.671
GroupGradeTypeK->Middle	NA
GroupGradeTypeMiddle->High	0.588
GroupGradeTypeMiddle->Middle	NA
GroupGradeTypePK->Elementary	-0.190
GroupGradeTypePK->High	NA
GroupGradeTypePK->Middle	NA
GroupGradeTypeUndefined->Undefined	NA
MajorProgramCodeH	-0.269
MajorProgramCodeI	-0.007
MajorProgramCodeS	NA
SingleGradeTripFlag1	1.843
FPP.to.School.enrollment	0.599
FPP.to.PAX	0.976
Num.of.Non_FPP.PAX	NA
SchoolSizeIndicatorL	1.994
SchoolSizeIndicatorM-L	1.827
SchoolSizeIndicatorS	0.822
SchoolSizeIndicatorS-M	2.383
From.Grade_surrogate1	NA
To.Grade_surrogate1	NA
Special.Pay_surrogate1	NA
Early.RPL_surrogate1	-0.392
Latest.RPL_surrogate1	0.849
Initial.System.Date_surrogateOther.Initial.System.Date_surrogate	0.512
CRM.Segment_surrogateOther.CRM.Segment_surrogate	0.309
MDR.High.Grade_surrogate1	NA
Total.School.Enrollment_surrogate1	NA
FirstMeeting_surrogate1	NA
LastMeeting_surrogate1	NA
DifferenceTraveltoFirstMeeting_surrogate1	NA
DifferenceTraveltoLastMeeting_surrogate1	NA
FPP.to.School.enrollment_surrogate1	NA
Pr(> z)	
(Intercept)	0.89651
Program.CodeCD	0.48347
Program.CodeOther.Program.Code	0.77274
Program.CodeHC	0.87934
Program.CodeHD	0.86813
Program.CodeHG	0.68960
Program.CodeHN	0.75177
Program.CodeHO	0.84552
Program.CodeHS	0.62703
Program.CodeHVP	0.96475
Program.CodeIC	0.99383
Program.CodeSC	0.80544

Program.CodeSG	0.98798
Program.CodeSK	0.98984
Program.CodeSM	0.98960
Program.CodeST	0.98907
From.Grade11	0.18167
From.Grade12	0.12342
From.GradeOther.From.Grade	0.99070
From.Grade4	0.30755
From.Grade5	0.40203
From.Grade6	0.44050
From.Grade7	0.40503
From.Grade8	0.31949
From.Grade9	0.78820
From.GradeFIXED_NA	0.42083
To.Grade11	0.80035
To.Grade12	0.81904
To.GradeOther.To.Grade	0.99178
To.Grade4	0.57140
To.Grade5	0.54964
To.Grade6	0.59987
To.Grade7	0.69627
To.Grade8	0.45409
To.Grade9	0.71822
To.GradeFIXED_NA	0.67924
Group.StateAL	0.78511
Group.StateAR	0.38550
Group.StateAZ	0.71966
Group.StateCA	0.15121
Group.StateCO	0.84969
Group.StateCT	0.91718
Group.StateFL	0.86401
Group.StateGA	0.22552
Group.StateIA	0.77404
Group.StateID	0.14356
Group.StateIL	0.70974
Group.StateIN	0.03004 *
Group.StateKS	0.57832
Group.StateKY	0.67740
Group.StateLA	0.24478
Group.StateMA	0.40386
Group.StateMD	0.02951 *
Group.StateMI	0.52876
Group.StateMN	0.41134
Group.StateMO	0.88314
Group.StateNC	0.40390
Group.StateNE	0.39048
Group.StateNM	0.83849
Group.StateNV	0.10902
Group.StateNY	0.57842
Group.StateOH	0.17177
Group.StateOK	0.92824
Group.StateOR	0.77218
Group.StateSC	0.00420 **
Group.StateSD	0.29683
Group.StateTN	0.37440
Group.StateTX	0.13595
Group.StateVA	0.20386
Group.StateWA	0.44109
Group.StateWI	0.65285
Is.Non.Annual.1	< 2e-16 ***
Days2	0.83762
Days3	0.80968
Days4	0.63546
Days5	0.33441
Days6	0.48474
Days7	0.24091
Days8	0.15726
DaysOther.Days	0.04595 *
Days11	0.98922
Travel.TypeB	0.60029
Travel.TypeOther.Travel.Type	0.88913
Departure.Date	0.06457 .
Return.Date	0.05706 .
Deposit.Date	0.09996 .
Special.PayCP	0.98687
Special.PayFIXED_NA	0.98813
Special.PayFR	0.98814
Special.PaySA	0.98746
Tuition	0.48943
FRP.Active	0.03059 *
FRP.Cancelled	0.37252
FRP.Take.up.percent.	0.96873
Early.RPL	0.69411
Latest.RPL	0.39754
Cancelled.Pax	0.33075
Total.Discount.Pax	0.28710
Initial.System.Date	0.60440
Poverty.CodeOther.Poverty.Code	0.30321
Poverty.CodeA	0.24291
Poverty.CodeB	0.14667
Poverty.CodeC	0.13956
Poverty.CodeD	0.07393 .
Poverty.CodeE	0.73206

RegionHouston	0.31104
RegionNorthern California	0.04817 *
RegionOther	NA
RegionPacific Northwest	NA
RegionSouthern California	NA
CRM.Segment10	0.01142 *
CRM.Segment11	0.29740
CRM.Segment2	0.72249
CRM.Segment3	0.47923
CRM.Segment4	5.52e-05 ***
CRM.Segment5	0.02174 *
CRM.Segment6	0.00717 **
CRM.Segment7	0.89139
CRM.Segment8	0.65618
CRM.SegmentOther.CRM.Segment	0.17115
School.TypeCatholic	0.87481
School.TypePUBLIC	0.61660
School.TypePrivate non-Christian	0.09015 .
Parent.Meeting.Flag1	0.89158
MDR.Low.GradeOther.MDR.Low.Grade	0.99146
MDR.Low.Grade3	0.99049
MDR.Low.Grade4	0.99198
MDR.Low.Grade5	0.99151
MDR.Low.Grade6	0.99052
MDR.Low.Grade7	0.99039
MDR.Low.Grade8	0.99037
MDR.Low.Grade9	0.99535
MDR.Low.GradeK	0.99068
MDR.Low.GradePK	0.99076
MDR.High.Grade12	0.60958
MDR.High.Grade5	0.98692
MDR.High.Grade6	0.98605
MDR.High.Grade7	0.98867
MDR.High.Grade8	0.98881
MDR.High.Grade9	0.81789
MDR.High.GradeFIXED_NA	NA
Total.School.Enrollment	0.94762
Income.LevelA	0.69630
Income.LevelB	0.49551
Income.LevelC	0.78819
Income.LevelD	0.77555
Income.LevelE	0.79394
Income.LevelF	0.68592
Income.LevelG	0.91635
Income.LevelH	0.64096
Income.LevelI	0.82049
Income.LevelJ	0.96364
Income.LevelK	0.63974
Income.LevelL	0.84328
Income.LevelM	0.79363
Income.LevelN	0.92978
Income.LevelO	0.96731
Income.LevelP	0.44652
Income.LevelOther.Income.Level	0.76025
Income.LevelQ	0.83114
Income.LevelZ	0.61706
EZ.Pay.Take.Up.Rate	0.84115
School.Sponsor1	0.75159
SPR.Product.TypeCosta Rica	NA
SPR.Product.TypeEast Coast	0.99063
SPR.Product.TypeOther.SPR.Product.Type	0.74861
SPR.Product.TypeInternational	0.99837
SPR.Product.TypeScience	0.98947
SPR.New.ExistingNEW	< 2e-16 ***
FPP	0.47281
Total.Pax	NA
SPR.Group.Revenue	NA
NumberOfMeetingswithParents1	0.76427
NumberOfMeetingswithParents2	NA
FirstMeeting	0.98390
LastMeeting	0.78924
DifferenceTraveltoFirstMeeting	0.81179
DifferenceTraveltoLastMeeting	NA
SchoolGradeTypeLowHigh	NA
SchoolGradeTypeLowMiddle	NA
SchoolGradeTypeLowUndefined	NA
SchoolGradeTypeHighHigh	NA
SchoolGradeTypeHighMiddle	NA
SchoolGradeTypeHighUndefined	NA
SchoolGradeTypeOther.SchoolGradeType	0.92586
SchoolGradeTypeElementary->Middle	NA
SchoolGradeTypeHigh->High	NA
SchoolGradeTypeMiddle->High	0.37341
SchoolGradeTypeMiddle->Middle	NA
SchoolGradeTypeMiddle->Undefined	NA
SchoolGradeTypeUndefined->Undefined	NA
DepartureMonthFebruary	0.03825 *
DepartureMonthOther.DepartureMonth	0.94495
DepartureMonthJune	0.27674
DepartureMonthMarch	0.07409 .
DepartureMonthMay	0.99285
GroupGradeTypeLowHigh	0.04799 *
GroupGradeTypeLowK	NA

```
GroupGradeTypeLowMiddle NA
GroupGradeTypeLowPK NA
GroupGradeTypeLowUndefined NA
GroupGradeTypeHighHigh 0.98863
GroupGradeTypeHighMiddle NA
GroupGradeTypeHighUndefined NA
GroupGradeTypeElementary->Middle 0.40690
GroupGradeTypeHigh->High NA
GroupGradeTypeK->Elementary 0.95708
GroupGradeTypeK->High 0.50220
GroupGradeTypeK->Middle NA
GroupGradeTypeMiddle->High 0.55652
GroupGradeTypeMiddle->Middle NA
GroupGradeTypePK->Elementary 0.84964
GroupGradeTypePK->High NA
GroupGradeTypePK->Middle NA
GroupGradeTypeUndefined->Undefined NA
MajorProgramCodeH 0.78779
MajorProgramCodeI 0.99436
MajorProgramCodeS NA
SingleGradeTripFlag1 0.06534 .
FPP.to.School.enrollment 0.54929
FPP.to.PAX 0.32917
Num.of.Non_FPP.PAX NA
SchoolSizeIndicatorL 0.04618 *
SchoolSizeIndicatorM-L 0.06768 .
SchoolSizeIndicatorS 0.41103
SchoolSizeIndicatorS-M 0.01717 *
From.Grade_surrogate1 NA
To.Grade_surrogate1 NA
Special.Pay_surrogate1 NA
Early.RPL_surrogate1 0.69542
Latest.RPL_surrogate1 0.39562
Initial.System.Date_surrogateOther.Initial.System.Date_surrogate 0.60834
CRM.Segment_surrogateOther.CRM.Segment_surrogate 0.75715
MDR.High.Grade_surrogate1 NA
Total.School.Enrollment_surrogate1 NA
FirstMeeting_surrogate1 NA
LastMeeting_surrogate1 NA
DifferenceTraveltoFirstMeeting_surrogate1 NA
DifferenceTraveltoLastMeeting_surrogate1 NA
FPP.to.School.enrollment_surrogate1 NA
---
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 2531.2 on 1888 degrees of freedom
Residual deviance: 1409.6 on 1693 degrees of freedom
AIC: 1801.6

Number of Fisher Scoring iterations: 15

The model is overfit. It has too many insignificant variables.

Let us fit a much simpler model. We will use stepwise regressions.

Recall stepwise regression from BUS 41100 Applied regression course. See, for example, [Week 9 slides](#). You can also check Section 6.1.2 of the [ISLR](#) book.

There are three approaches to running stepwise regressions: backward, forward and both. We need to specify criterion for inclusion/exclusion of variables. We will use one based on Bayesian information criteria.

Observe the process of variables being added to the model, (labeled by "+" in the output), gradual expansion of the model, and improvement of BIC.

```
In [ ]: # Start from a null model with intercept only, and add one covariate at a time until maximum BIC.
lgfit.null <- glm(Retained.in.2012 ~ 1,
                  data=df.train, family="binomial")

lgfit.selected <- step(lgfit.null,                                # the starting model for our search
                      scope=formula(lgfit.all),                  # the largest possible model that we will consider.
                      direction="forward",
                      k=log(nrow(df.train)),                      # by default step() uses AIC, but by
                                                                # multiplying log(n) on the penalty, we get BIC.
                                                                # See ?step -> Arguments -> k
                      trace=1)
```

Start: AIC=2538.74
Retained.in.2012. ~ 1

	Df	Deviance	AIC
+ SingleGradeTripFlag	1	2129.3	2144.4
+ Is.Non.Annual.	1	2236.0	2251.1
+ From.Grade	10	2196.2	2279.2
+ SPR.New.Existing	1	2265.7	2280.8
+ Total.Pax	1	2357.4	2372.5
+ FPP	1	2358.5	2373.6
+ FRP.Active	1	2387.8	2402.8
+ Total.Discount.Pax	1	2399.9	2415.0
+ Num.of.Non_FPP.PAX	1	2399.9	2415.0
+ SchoolGradeTypeHigh	3	2415.0	2445.2
+ SchoolGradeType	7	2390.5	2450.8
+ To.Grade	10	2396.0	2479.0
+ DepartureMonth	5	2446.3	2491.6
+ SchoolGradeTypeLow	3	2466.2	2496.3
+ CRM.Segment	10	2416.2	2499.2
+ Return.Date	1	2488.2	2503.3
+ Departure.Date	1	2488.7	2503.8
+ GroupGradeTypeLow	5	2459.9	2505.1
+ GroupGradeTypeHigh	3	2480.1	2510.3
+ FPP.to.PAX	1	2498.6	2513.7
+ MajorProgramCode	3	2488.3	2518.5
+ Tuition	1	2503.4	2518.5
+ SPR.Group.Revenue	1	2503.4	2518.5
+ SchoolSizeIndicator	4	2483.2	2521.0
+ DifferenceTraveltoFirstMeeting	1	2506.6	2521.7
+ School.Sponsor	1	2506.9	2521.9
+ MDR.High.Grade	7	2463.6	2523.9
+ GroupGradeType	11	2436.1	2526.6
+ SPR.Product.Type	5	2482.8	2528.1
+ Special.Pay_surrogate	1	2515.1	2530.2
+ DifferenceTraveltoLastMeeting	1	2519.0	2534.1
+ FPP.to.School.enrollment	1	2519.3	2534.4
+ Deposit.Date	1	2519.9	2535.0
+ Special.Pay	4	2498.6	2536.3
+ Total.School.Enrollment	1	2522.3	2537.4
+ Early.RPL	1	2522.6	2537.7
+ Early.RPL_surrogate	1	2522.7	2537.7
<none>		2531.2	2538.7
+ FRP.Cancelled	1	2523.7	2538.8
+ MDR.Low.Grade	10	2456.2	2539.2
+ NumberOfMeetingswithParents	2	2517.1	2539.7
+ Travel.Type	2	2517.3	2540.0
+ Poverty.Code	6	2488.7	2541.4
+ Cancelled.Pax	1	2528.6	2543.7
+ CRM.Segment_surrogate	1	2529.1	2544.2
+ Parent.Meeting.Flag	1	2529.9	2545.0
+ FirstMeeting_surrogate	1	2529.9	2545.0
+ LastMeeting_surrogate	1	2529.9	2545.0
+ DifferenceTraveltoFirstMeeting_surrogate	1	2529.9	2545.0
+ DifferenceTraveltoLastMeeting_surrogate	1	2529.9	2545.0
+ LastMeeting	1	2529.9	2545.0
+ FirstMeeting	1	2529.9	2545.0
+ School.Type	3	2514.9	2545.1
+ From.Grade_surrogate	1	2530.2	2545.3
+ MDR.High.Grade_surrogate	1	2530.3	2545.4
+ Latest.RPL_surrogate	1	2530.5	2545.6
+ Latest.RPL	1	2530.6	2545.7
+ EZ.Pay.Take.Up.Rate	1	2530.7	2545.8
+ Total.School.Enrollment_surrogate	1	2530.7	2545.8
+ FPP.to.School.enrollment_surrogate	1	2530.7	2545.8
+ FRP.Take.up.percent.	1	2530.7	2545.8
+ To.Grade_surrogate	1	2530.8	2545.8
+ Initial.System.Date_surrogate	1	2531.0	2546.1
+ Initial.System.Date	1	2531.1	2546.2
+ Region	5	2503.2	2548.4
+ Days	9	2493.5	2568.9
+ Program.Code	15	2453.9	2574.6
+ Income.Level	19	2457.9	2608.7
+ Group.State	35	2438.2	2709.8

Step: AIC=2144.42
Retained.in.2012. ~ SingleGradeTripFlag

	Df	Deviance	AIC
+ SPR.New.Existing	1	1996.6	2019.2
+ Is.Non.Annual.	1	1996.7	2019.3
+ Total.Pax	1	2052.8	2075.4
+ FPP	1	2054.8	2077.5
+ Total.Discount.Pax	1	2056.0	2078.7
+ Num.of.Non_FPP.PAX	1	2056.0	2078.7
+ FRP.Active	1	2060.4	2083.0
+ SchoolGradeTypeHigh	3	2084.7	2122.4
+ SchoolGradeTypeLow	3	2086.3	2124.0
+ To.Grade_surrogate	1	2101.7	2124.3
+ From.Grade_surrogate	1	2109.3	2132.0
+ Departure.Date	1	2113.3	2135.9
+ Return.Date	1	2113.5	2136.2
+ SchoolSizeIndicator	4	2092.3	2137.5
+ DifferenceTraveltoFirstMeeting	1	2118.7	2141.3

+ Total.School.Enrollment	1	2119.7	2142.3
+ DepartureMonth	5	2089.8	2142.6
+ GroupGradeTypeHigh	3	2106.0	2143.7
+ SchoolGradeType	7	2076.3	2144.2
<none>		2129.3	2144.4
+ GroupGradeTypeLow	5	2092.2	2145.0
+ School.Sponsor	1	2123.5	2146.1
+ Special.Pay_surrogate	1	2123.8	2146.5
+ FPP.to.PAX	1	2123.8	2146.5
+ Tuition	1	2124.8	2147.4
+ SPR.Group.Revenue	1	2124.8	2147.4
+ DifferenceTraveltoLastMeeting	1	2125.4	2148.0
+ FRP.Cancelled	1	2125.5	2148.2
+ FPP.to.School.enrollment	1	2125.8	2148.4
+ Deposit.Date	1	2126.4	2149.1
+ Cancelled.Pax	1	2126.7	2149.3
+ Early.RPL	1	2127.2	2149.8
+ Early.RPL_surrogate	1	2127.2	2149.8
+ Latest.RPL_surrogate	1	2127.4	2150.1
+ Latest.RPL	1	2127.6	2150.2
+ CRM.Segment_surrogate	1	2128.3	2150.9
+ Total.School.Enrollment_surrogate	1	2129.1	2151.8
+ FPP.to.School.enrollment_surrogate	1	2129.1	2151.8
+ EZ.Pay.Take.Up.Rate	1	2129.1	2151.8
+ Initial.System.Date_surrogate	1	2129.2	2151.8
+ FRP.Take.up.percent.	1	2129.2	2151.8
+ MDR.High.Grade_surrogate	1	2129.2	2151.8
+ Initial.System.Date	1	2129.2	2151.8
+ Parent.Meeting.Flag	1	2129.3	2152.0
+ FirstMeeting_surrogate	1	2129.3	2152.0
+ LastMeeting_surrogate	1	2129.3	2152.0
+ DifferenceTraveltoFirstMeeting_surrogate	1	2129.3	2152.0
+ DifferenceTraveltoLastMeeting_surrogate	1	2129.3	2152.0
+ LastMeeting	1	2129.3	2152.0
+ FirstMeeting	1	2129.3	2152.0
+ Travel.Type	2	2122.8	2152.9
+ MajorProgramCode	3	2116.7	2154.4
+ NumberOfMeetingswithParents	2	2124.5	2154.7
+ MDR.High.Grade	7	2087.9	2155.8
+ Special.Pay	4	2112.9	2158.1
+ School.Type	3	2125.2	2162.9
+ CRM.Segment	10	2072.7	2163.2
+ From.Grade	10	2073.9	2164.4
+ Poverty.Code	6	2105.8	2166.2
+ To.Grade	10	2078.3	2168.8
+ SPR.Product.Type	5	2117.6	2170.4
+ Region	5	2117.9	2170.7
+ GroupGradeType	11	2079.5	2177.6
+ MDR.Low.Grade	10	2090.1	2180.7
+ Days	9	2108.0	2191.0
+ Program.Code	15	2100.3	2228.5
+ Income.Level	19	2090.3	2248.8
+ Group.State	35	2070.2	2349.3

Step: AIC=2019.25

Retained.in.2012. ~ SingleGradeTripFlag + SPR.New.Existing

	Df	Deviance	AIC
+ Is.Non.Annual.	1	1797.5	1827.7
+ Total.Discount.Pax	1	1949.7	1979.8
+ Num.of.Non_FPP.PAX	1	1949.7	1979.8
+ Total.Pax	1	1952.3	1982.5
+ FPP	1	1953.9	1984.1
+ FRP.Active	1	1959.0	1989.2
+ To.Grade_surrogate	1	1959.5	1989.7
+ From.Grade_surrogate	1	1965.6	1995.7
+ SchoolGradeTypeHigh	3	1952.1	1997.4
+ SchoolGradeTypeLow	3	1952.4	1997.6
+ Total.School.Enrollment	1	1984.5	2014.6
+ SchoolSizeIndicator	4	1963.4	2016.2
+ DifferenceTraveltoFirstMeeting	1	1986.8	2016.9
+ Tuition	1	1988.0	2018.1
+ SPR.Group.Revenue	1	1988.0	2018.1
+ Departure.Date	1	1988.5	2018.7
+ Return.Date	1	1988.6	2018.8
<none>		1996.6	2019.2
+ Cancelled.Pax	1	1990.3	2020.5
+ SchoolGradeType	7	1945.8	2021.2
+ Early.RPL_surrogate	1	1991.4	2021.6
+ Early.RPL	1	1991.4	2021.6
+ Travel.Type	2	1984.3	2022.0
+ FRP.Cancelled	1	1992.2	2022.3
+ DifferenceTraveltoLastMeeting	1	1992.5	2022.7
+ FPP.to.PAX	1	1994.1	2024.3
+ EZ.Pay.Take.Up.Rate	1	1995.2	2025.3
+ Initial.System.Date	1	1995.6	2025.8
+ GroupGradeTypeHigh	3	1980.8	2026.0
+ Initial.System.Date_surrogate	1	1996.0	2026.1
+ MDR.High.Grade_surrogate	1	1996.1	2026.3
+ Latest.RPL_surrogate	1	1996.1	2026.3
+ School.Sponsor	1	1996.2	2026.3
+ Latest.RPL	1	1996.2	2026.4
+ FPP.to.School.enrollment	1	1996.2	2026.4

+ Special.Pay_surrogate	1	1996.3	2026.5
+ Total.School.Enrollment_surrogate	1	1996.4	2026.6
+ FPP.to.School.enrollment_surrogate	1	1996.4	2026.6
+ CRM.Segment_surrogate	1	1996.5	2026.7
+ Deposit.Date	1	1996.6	2026.7
+ FRP.Take.up.percent.	1	1996.6	2026.8
+ Parent.Meeting.Flag	1	1996.6	2026.8
+ FirstMeeting_surrogate	1	1996.6	2026.8
+ LastMeeting_surrogate	1	1996.6	2026.8
+ DifferenceTraveltoFirstMeeting_surrogate	1	1996.6	2026.8
+ DifferenceTraveltoLastMeeting_surrogate	1	1996.6	2026.8
+ LastMeeting	1	1996.6	2026.8
+ FirstMeeting	1	1996.6	2026.8
+ DepartureMonth	5	1967.5	2027.8
+ GroupGradeTypeLow	5	1967.6	2028.0
+ NumberOfMeetingswithParents	2	1993.5	2031.2
+ MajorProgramCode	3	1987.5	2032.7
+ MDR.High.Grade	7	1961.4	2036.8
+ Special.Pay	4	1984.9	2037.7
+ School.Type	3	1993.4	2038.7
+ From.Grade	10	1941.4	2039.5
+ To.Grade	10	1945.4	2043.4
+ Region	5	1985.8	2046.2
+ SPR.Product.Type	5	1987.8	2048.1
+ Poverty.Code	6	1981.4	2049.3
+ CRM.Segment	10	1962.8	2060.8
+ MDR.Low.Grade	10	1964.4	2062.4
+ GroupGradeType	11	1958.5	2064.1
+ Days	9	1978.9	2069.4
+ Program.Code	15	1976.2	2112.0
+ Income.Level	19	1966.9	2132.9
+ Group.State	35	1939.6	2226.3

Step: AIC=1827.69

Retained.in.2012. ~ SingleGradeTripFlag + SPR.New.Existing +
Is.Non.Annual.

	Df	Deviance	AIC
+ FRP.Active	1	1743.4	1781.1
+ Total.Pax	1	1744.1	1781.8
+ FPP	1	1745.6	1783.3
+ Total.Discount.Pax	1	1746.1	1783.8
+ Num.of.Non_FPP.PAX	1	1746.1	1783.8
+ To.Grade_surrogate	1	1781.5	1819.2
+ From.Grade_surrogate	1	1783.6	1821.3
+ SchoolGradeTypeLow	3	1769.6	1822.4
+ Return.Date	1	1786.6	1824.3
+ FPP.to.PAX	1	1786.6	1824.3
+ Departure.Date	1	1786.6	1824.3
+ Tuition	1	1786.9	1824.6
+ SPR.Group.Revenue	1	1786.9	1824.6
+ SchoolGradeTypeHigh	3	1772.0	1824.8
<none>		1797.5	1827.7
+ Cancelled.Pax	1	1790.4	1828.1
+ FPP.to.School.enrollment	1	1790.6	1828.3
+ FRP.Cancelled	1	1791.0	1828.7
+ DifferenceTraveltoFirstMeeting	1	1793.5	1831.2
+ Total.School.Enrollment	1	1794.9	1832.7
+ School.Sponsor	1	1795.5	1833.2
+ Special.Pay_surrogate	1	1795.8	1833.5
+ DifferenceTraveltoLastMeeting	1	1795.8	1833.5
+ Early.RPL_surrogate	1	1796.3	1834.0
+ Early.RPL	1	1796.3	1834.0
+ Deposit.Date	1	1796.3	1834.0
+ Travel.Type	2	1789.0	1834.2
+ Initial.System.Date	1	1796.7	1834.4
+ Initial.System.Date_surrogate	1	1796.8	1834.6
+ GroupGradeTypeHigh	3	1781.8	1834.6
+ MDR.High.Grade_surrogate	1	1797.0	1834.7
+ Total.School.Enrollment_surrogate	1	1797.1	1834.8
+ FPP.to.School.enrollment_surrogate	1	1797.1	1834.8
+ FRP.Take.up.percent.	1	1797.2	1834.9
+ CRM.Segment_surrogate	1	1797.3	1835.0
+ FirstMeeting	1	1797.3	1835.0
+ LastMeeting	1	1797.3	1835.0
+ Parent.Meeting.Flag	1	1797.3	1835.0
+ FirstMeeting_surrogate	1	1797.3	1835.0
+ LastMeeting_surrogate	1	1797.3	1835.0
+ DifferenceTraveltoFirstMeeting_surrogate	1	1797.3	1835.0
+ DifferenceTraveltoLastMeeting_surrogate	1	1797.3	1835.0
+ Latest.RPL_surrogate	1	1797.3	1835.1
+ Latest.RPL	1	1797.4	1835.1
+ EZ.Pay.Take.Up.Rate	1	1797.5	1835.2
+ MajorProgramCode	3	1782.7	1835.5
+ DepartureMonth	5	1768.2	1836.1
+ SchoolSizeIndicator	4	1779.4	1839.8
+ NumberOfMeetingswithParents	2	1794.8	1840.0
+ GroupGradeTypeLow	5	1776.4	1844.3
+ School.Type	3	1792.9	1845.7
+ Special.Pay	4	1785.4	1845.7
+ MDR.High.Grade	7	1764.2	1847.2
+ SchoolGradeType	7	1766.0	1849.0
+ SPR.Product.Type	5	1785.2	1853.1

+ Region	5	1786.5	1854.4
+ Poverty.Code	6	1780.4	1855.8
+ From.Grade	10	1757.7	1863.3
+ CRM.Segment	10	1759.4	1865.0
+ To.Grade	10	1764.2	1869.8
+ Days	9	1777.7	1875.7
+ MDR.Low.Grade	10	1774.1	1879.7
+ GroupGradeType	11	1769.1	1882.2
+ Program.Code	15	1768.8	1912.2
+ Income.Level	19	1769.5	1943.0
+ Group.State	35	1743.4	2037.6

Step: AIC=1781.14

Retained.in.2012. ~ SingleGradeTripFlag + SPR.New.Existing +
Is.Non.Annual. + FRP.Active

	Df	Deviance	AIC
+ To.Grade_surrogate	1	1731.8	1777.1
+ Total.Discount.Pax	1	1733.5	1778.7
+ Num.of.Non_FPP.PAX	1	1733.5	1778.7
+ From.Grade_surrogate	1	1733.7	1778.9
<none>		1743.4	1781.1
+ Total.Pax	1	1736.5	1781.8
+ FPP	1	1737.3	1782.5
+ Return.Date	1	1737.4	1782.6
+ Departure.Date	1	1737.4	1782.7
+ FRP.Take.up.percent.	1	1739.1	1784.3
+ SchoolGradeTypeLow	3	1724.1	1784.4
+ DifferenceTraveltoFirstMeeting	1	1739.3	1784.6
+ Special.Pay_surrogate	1	1739.7	1784.9
+ Tuition	1	1740.5	1785.8
+ SPR.Group.Revenue	1	1740.5	1785.8
+ Early.RPL_surrogate	1	1741.1	1786.4
+ Early.RPL	1	1741.1	1786.4
+ SchoolGradeTypeHigh	3	1726.2	1786.5
+ DifferenceTraveltoLastMeeting	1	1741.4	1786.7
+ School.Sponsor	1	1741.5	1786.8
+ FPP.to.PAX	1	1741.7	1787.0
+ Total.School.Enrollment	1	1742.1	1787.3
+ Latest.RPL_surrogate	1	1742.1	1787.4
+ EZ.Pay.Take.Up.Rate	1	1742.2	1787.4
+ Latest.RPL	1	1742.2	1787.5
+ GroupGradeTypeHigh	3	1727.3	1787.7
+ Initial.System.Date	1	1742.5	1787.8
+ MDR.High.Grade_surrogate	1	1742.5	1787.8
+ Initial.System.Date_surrogate	1	1742.7	1788.0
+ Deposit.Date	1	1742.9	1788.1
+ FRP.Cancelled	1	1742.9	1788.2
+ Total.School.Enrollment_surrogate	1	1743.2	1788.4
+ FPP.to.School.enrollment_surrogate	1	1743.2	1788.4
+ CRM.Segment_surrogate	1	1743.3	1788.6
+ Parent.Meeting.Flag	1	1743.4	1788.6
+ FirstMeeting_surrogate	1	1743.4	1788.6
+ LastMeeting_surrogate	1	1743.4	1788.6
+ DifferenceTraveltoFirstMeeting_surrogate	1	1743.4	1788.6
+ DifferenceTraveltoLastMeeting_surrogate	1	1743.4	1788.6
+ LastMeeting	1	1743.4	1788.7
+ FirstMeeting	1	1743.4	1788.7
+ FPP.to.School.enrollment	1	1743.4	1788.7
+ Cancelled.Pax	1	1743.4	1788.7
+ Special.Pay	4	1724.5	1792.4
+ MajorProgramCode	3	1733.0	1793.3
+ Travel.Type	2	1741.7	1794.5
+ NumberOfMeetingswithParents	2	1742.1	1794.9
+ School.Type	3	1737.1	1797.5
+ SchoolSizeIndicator	4	1730.9	1798.8
+ DepartureMonth	5	1724.2	1799.7
+ GroupGradeTypeLow	5	1728.2	1803.7
+ MDR.High.Grade	7	1713.3	1803.8
+ Region	5	1734.9	1810.4
+ SchoolGradeType	7	1721.1	1811.6
+ SPR.Product.Type	5	1736.2	1811.6
+ Poverty.Code	6	1731.0	1814.0
+ CRM.Segment	10	1714.0	1827.1
+ From.Grade	10	1715.3	1828.5
+ To.Grade	10	1720.1	1833.3
+ Days	9	1729.9	1835.5
+ MDR.Low.Grade	10	1725.0	1838.2
+ GroupGradeType	11	1719.0	1839.7
+ Program.Code	15	1719.8	1870.7
+ Income.Level	19	1717.5	1898.6
+ Group.State	35	1690.5	1992.3

Step: AIC=1777.07

Retained.in.2012. ~ SingleGradeTripFlag + SPR.New.Existing +
Is.Non.Annual. + FRP.Active + To.Grade_surrogate

	Df	Deviance	AIC
+ Total.Discount.Pax	1	1723.6	1776.4
+ Num.of.Non_FPP.PAX	1	1723.6	1776.4
<none>		1731.8	1777.1
+ Return.Date	1	1725.3	1778.2
+ Departure.Date	1	1725.4	1778.2

+ Total.Pax	1	1727.0	1779.8
+ FPP	1	1727.7	1780.5
+ Special.Pay_surrogate	1	1728.5	1781.3
+ DifferenceTraveltoFirstMeeting	1	1729.0	1781.9
+ FRP.Take.up.percent.	1	1729.4	1782.2
+ Tuition	1	1729.8	1782.6
+ SPR.Group.Revenue	1	1729.8	1782.6
+ School.Sponsor	1	1730.1	1782.9
+ Early.RPL_surrogate	1	1730.3	1783.1
+ Early.RPL	1	1730.3	1783.1
+ Deposit.Date	1	1730.5	1783.3
+ FPP.to.PAX	1	1730.5	1783.3
+ DifferenceTraveltoLastMeeting	1	1730.6	1783.4
+ Total.School.Enrollment	1	1730.6	1783.4
+ GroupGradeTypeHigh	3	1715.6	1783.5
+ Initial.System.Date	1	1731.0	1783.8
+ Total.School.Enrollment_surrogate	1	1731.0	1783.8
+ FPP.to.School.enrollment_surrogate	1	1731.0	1783.8
+ EZ.Pay.Take.Up.Rate	1	1731.1	1783.9
+ Initial.System.Date_surrogate	1	1731.2	1784.0
+ Latest.RPL_surrogate	1	1731.4	1784.2
+ Latest.RPL	1	1731.4	1784.2
+ FRP.Cancelled	1	1731.5	1784.3
+ MDR.High.Grade_surrogate	1	1731.5	1784.3
+ CRM.Segment_surrogate	1	1731.5	1784.3
+ Cancelled.Pax	1	1731.8	1784.6
+ FPP.to.School.enrollment	1	1731.8	1784.6
+ From.Grade_surrogate	1	1731.8	1784.6
+ FirstMeeting	1	1731.8	1784.6
+ LastMeeting	1	1731.8	1784.6
+ Parent.Meeting.Flag	1	1731.8	1784.6
+ FirstMeeting_surrogate	1	1731.8	1784.6
+ LastMeeting_surrogate	1	1731.8	1784.6
+ DifferenceTraveltoFirstMeeting_surrogate	1	1731.8	1784.6
+ DifferenceTraveltoLastMeeting_surrogate	1	1731.8	1784.6
+ MajorProgramCode	3	1718.7	1786.5
+ SchoolGradeTypeHigh	2	1726.2	1786.5
+ SchoolGradeTypeLow	3	1722.5	1790.4
+ Special.Pay	4	1715.4	1790.8
+ NumberOfMeetingswithParents	2	1730.9	1791.3
+ Travel.Type	2	1731.0	1791.3
+ School.Type	3	1726.0	1793.9
+ SchoolSizeIndicator	4	1720.2	1795.7
+ DepartureMonth	5	1713.8	1796.8
+ GroupGradeTypeLow	5	1716.0	1799.0
+ MDR.High.Grade	7	1702.2	1800.3
+ SPR.Product.Type	5	1722.3	1805.3
+ Region	5	1723.6	1806.5
+ Poverty.Code	6	1721.6	1812.1
+ SchoolGradeType	7	1720.0	1818.0
+ CRM.Segment	10	1698.0	1818.7
+ Days	9	1716.8	1830.0
+ MDR.Low.Grade	10	1712.5	1833.2
+ To.Grade	9	1720.1	1833.3
+ From.Grade	10	1714.0	1834.7
+ GroupGradeType	11	1707.4	1835.7
+ Program.Code	15	1705.4	1863.8
+ Income.Level	19	1708.1	1896.7
+ Group.State	35	1681.9	1991.2

Step: AIC=1776.43

Retained.in.2012. ~ SingleGradeTripFlag + SPR.New.Existing +
Is.Non.Annual. + FRP.Active + To.Grade_surrogate + Total.Discount.Pax

	Df	Deviance	AIC
<none>		1723.6	1776.4
+ Return.Date	1	1717.9	1778.3
+ Departure.Date	1	1718.0	1778.3
+ FPP.to.PAX	1	1719.3	1779.6
+ DifferenceTraveltoFirstMeeting	1	1721.5	1781.8
+ GroupGradeTypeHigh	3	1706.5	1782.0
+ School.Sponsor	1	1722.0	1782.4
+ Special.Pay_surrogate	1	1722.2	1782.6
+ Deposit.Date	1	1722.4	1782.7
+ Early.RPL_surrogate	1	1722.4	1782.7
+ Early.RPL	1	1722.4	1782.7
+ Tuition	1	1722.5	1782.8
+ SPR.Group.Revenue	1	1722.5	1782.8
+ Total.School.Enrollment	1	1722.7	1783.0
+ DifferenceTraveltoLastMeeting	1	1722.7	1783.1
+ Initial.System.Date	1	1722.8	1783.1
+ Total.School.Enrollment_surrogate	1	1722.8	1783.2
+ FPP.to.School.enrollment_surrogate	1	1722.8	1783.2
+ Initial.System.Date_surrogate	1	1722.9	1783.3
+ FPP.to.School.enrollment	1	1723.0	1783.3
+ FRP.Cancelled	1	1723.3	1783.6
+ CRM.Segment_surrogate	1	1723.3	1783.6
+ MDR.High.Grade_surrogate	1	1723.3	1783.7
+ Latest.RPL_surrogate	1	1723.4	1783.8
+ Latest.RPL	1	1723.4	1783.8
+ EZ.Pay.Take.Up.Rate	1	1723.5	1783.8
+ From.Grade_surrogate	1	1723.5	1783.9
+ FRP.Take.up.percent.	1	1723.5	1783.9

+ FirstMeeting	1	1723.6	1783.9
+ LastMeeting	1	1723.6	1783.9
+ Parent.Meeting.Flag	1	1723.6	1783.9
+ FirstMeeting_surrogate	1	1723.6	1783.9
+ LastMeeting_surrogate	1	1723.6	1783.9
+ DifferenceTraveltoFirstMeeting_surrogate	1	1723.6	1783.9
+ DifferenceTraveltoLastMeeting_surrogate	1	1723.6	1783.9
+ Cancelled.Pax	1	1723.6	1784.0
+ FPP	1	1723.6	1784.0
+ Total.Pax	1	1723.6	1784.0
+ SchoolGradeTypeHigh	2	1718.1	1786.0
+ MajorProgramCode	3	1710.8	1786.2
+ SchoolGradeTypeLow	3	1713.4	1788.8
+ NumberOfMeetingswithParents	2	1722.8	1790.7
+ Travel.Type	2	1723.2	1791.1
+ School.Type	3	1718.1	1793.5
+ Special.Pay	4	1711.5	1794.5
+ SchoolSizeIndicator	4	1713.0	1796.0
+ DepartureMonth	5	1706.7	1797.2
+ GroupGradeTypeLow	5	1707.9	1798.5
+ MDR.High.Grade	7	1694.7	1800.3
+ SPR.Product.Type	5	1714.3	1804.8
+ Region	5	1714.8	1805.3
+ Poverty.Code	6	1713.7	1811.7
+ CRM.Segment	10	1687.7	1815.9
+ SchoolGradeType	7	1711.0	1816.6
+ Days	9	1709.4	1830.1
+ To.Grade	9	1711.5	1832.2
+ MDR.Low.Grade	10	1704.3	1832.5
+ From.Grade	10	1705.6	1833.9
+ GroupGradeType	11	1699.0	1834.7
+ Program.Code	15	1696.8	1862.8
+ Income.Level	19	1700.0	1896.1
+ Group.State	35	1674.6	1991.4

The algorithm stops once none of the 1-step expanded models lead to a lower BIC.

This is the selected model.

```
In [ ]: summary(lgfit.selected)
```

```
Call:
glm(formula = Retained.in.2012. ~ SingleGradeTripFlag + SPR.New.Existing +
    Is.Non.Annual. + FRP.Active + To.Grade_surrogate + Total.Discount.Pax,
    family = "binomial", data = df.train)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.8150  -0.7108   0.3982   0.6079   2.7149

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept)    0.100495   0.141541   0.710 0.477699
SingleGradeTripFlag1 1.220935   0.130267   9.373 < 2e-16 ***
SPR.New.ExistingNEW -1.597414   0.129210  -12.363 < 2e-16 ***
Is.Non.Annual.1    -2.427700   0.194144  -12.505 < 2e-16 ***
FRP.Active         0.023528   0.006669   3.528 0.000419 ***
To.Grade_surrogate1 0.738475   0.235902   3.130 0.001745 **
Total.Discount.Pax 0.108888   0.039687   2.744 0.006077 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2531.2  on 1888  degrees of freedom
Residual deviance: 1723.6  on 1882  degrees of freedom
AIC: 1737.6
```

Number of Fisher Scoring iterations: 5

You can predict probabilities from this model using the following.

```
In [ ]: phat.lgfit.selected <- predict(lgfit.selected,
    newdata = df.test,
    type = "response")
```

You will use these probabilities later.

While we are investigating variable selection in logistic regression models, let us also use a more modern approach to variable selection. We will use the lasso.

If you have not seen this in BUS 41100 Applied regression course, do not worry. We will provide more details in the Week 5. You can also check Section 6.2.2 of the [ISLR](#) book.

I provide the code to fit a lasso logistic regression model. We find coefficients β that minimize the deviance loss plus the penalty: $-2 \sum_{i=1}^n \log p(y_i, x_i; \beta) + \lambda \sum_{j=1}^p |\beta_j|$. Here, λ is the user chosen penalty that controls the flexibility of the fit.

First, we need to create a model matrix that will be used as an input to the package.

```
In [ ]: X <- model.matrix(formula(lgfit.all), STCdata_A)
#need to subtract the intercept
X <- X[,-1]

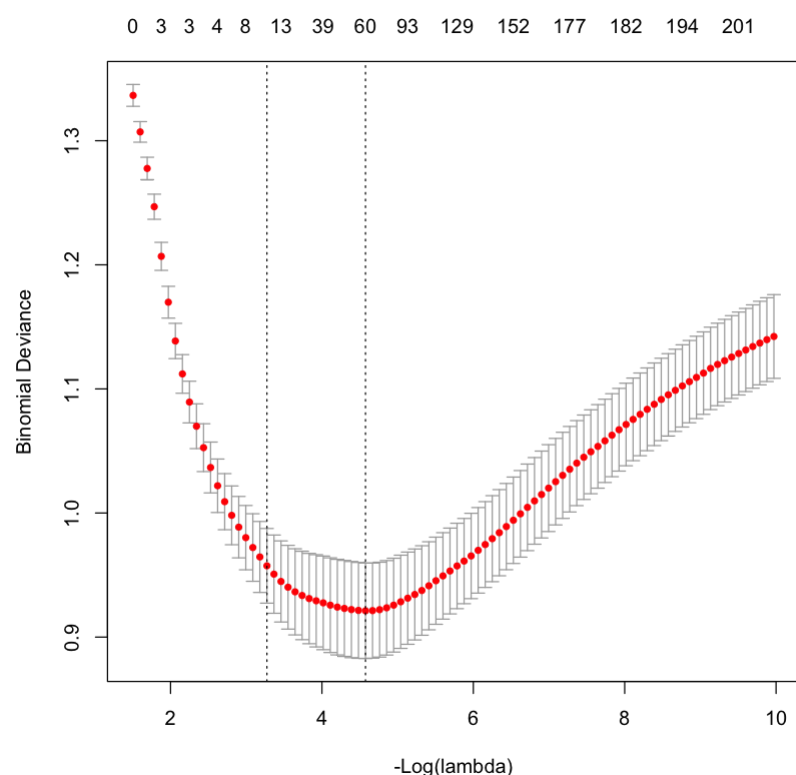
X.train = X[ inTrain, ]
X.test = X[ -inTrain, ]
```

Next, we run 5-fold cross-validation.

```
In [ ]: cv.l1.lgfit <- cv.glmnet(
  x      = X.train,
  y      = df.train$Retained.in.2012.,
  family = "binomial",
  alpha  = 1, #alpha=0 gives ridge regression
  nfolds = 5)
```

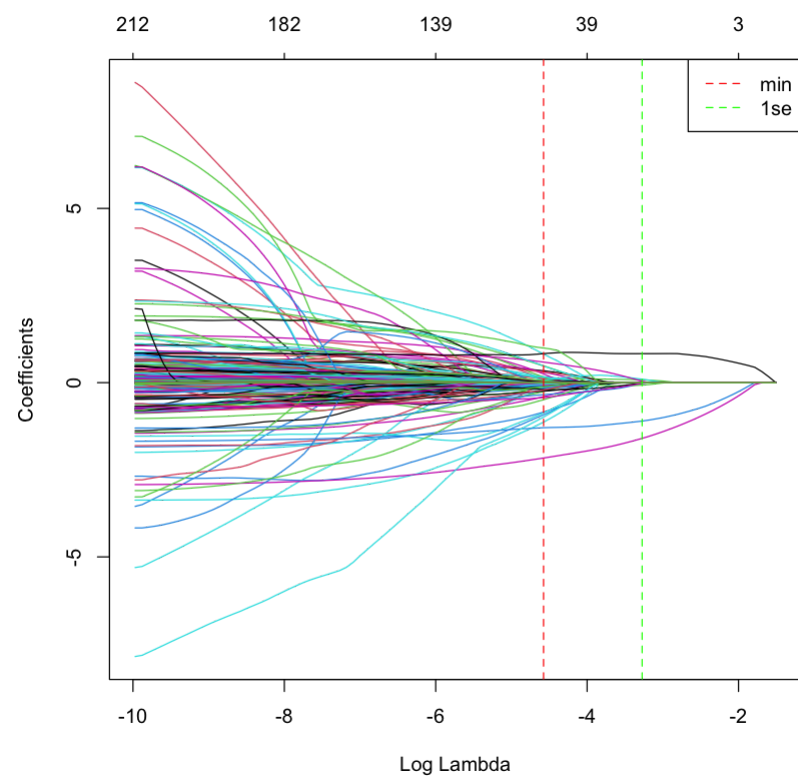
We can plot the cross-validation curve, which shows us an estimate of out-of-sample deviance as a function of the tuning parameter λ . The x-axis represents to $-\log(\lambda)$. Therefore, on the left we have large values of λ and on the right we have small values of λ . At the top, you can see the number variables that were selected into the model. The two vertical dashed lines correspond to λ values that minimize the cross-validation error and the largest value of lambda such that error is within 1 standard error of the minimum.

```
In [ ]: plot(cv.l1.lgfit, sign.lambda=-1)
```



Let us now plot the fitted coefficients as a function of λ . Note that `cv.l1.lgfit$glmnet.fit` corresponds to a fitted glmnet object for the full data.

```
In [ ]: glmnet.fit <- cv.l1.lgfit$glmnet.fit
plot(glmnet.fit, xvar = "lambda")
abline(v = log(cv.l1.lgfit$lambda.min), lty=2, col="red")
abline(v = log(cv.l1.lgfit$lambda.1se), lty=2, col="green")
legend("topright", legend=c("min", "1se"), lty=2, col=c("red", "green"))
```



For our predictive model, we will use 1 standard error λ . Below you can see the variables that are selected by the lasso.

```
In [ ]: betas <- coef(cv.l1.lgfit, s = "lambda.1se")
model.1se <- which(betas[2:length(betas)]!=0)
colnames(X[,model.1se])
```

'From.Grade8' · 'Is.Non.Annual.1' · 'FRP.Active' · 'Total.Discount.Pax' · 'CRM.Segment8' · 'MDR.High.Grade8' · 'Income.LevelIP' ·
'SPR.New.ExistingNEW' · 'Total.Pax' · 'SchoolGradeTypeHighHigh' · 'DepartureMonthJune' · 'SingleGradeTripFlag1' · 'SchoolSizeIndicatorS'

We now use our model to predict probabilities on the test set.

```
In [ ]: phat.l1.lgfit <- predict(glmnet.fit,
                                newx = X.test,
                                s = cv.l1.lgfit$lambda.1se,
                                type = "response")
```

Questions

How well does logistic regression do?

1. Create a confusion matrix for two logistic regression models build above. Use probabilities `phat.lgfit.selected` and `phat.l1.lgfit` to do so.

To solve this question, you need to make a major decision. What should the cutoff or "threshold" for the probability be, above which you will label a customer as being classified as "retained?" In our case, the data is slightly unbalanced---about 60.72% of data points are in Class 1. For very unbalanced data, we would first need to balance it (over- or under-sample). In this case, the benefits of balancing are unclear, hence one can implement the average probability of being retained as a cutoff.

Predict classification using 0.6072 threshold.

What can we see from the confusion matrices?

```
In [ ]: threshold <- mean(phat.lgfit.selected)
```

```
In [ ]: get_confusion_matrix = function(y, phat, thr=0.5){
  yhat = as.factor(ifelse(phat > thr, 1, 0)) # 1 of greater than thr, 0 o.w.
  confusionMatrix(yhat, y)
}
```

```
In [ ]: get_confusion_matrix(df.test$Retained.in.2012., phat.lgfit.selected, threshold)
```

Confusion Matrix and Statistics

```

      Reference
Prediction  0   1
      0 147  66
      1  49 238

      Accuracy : 0.77
      95% CI : (0.7306, 0.8062)
No Information Rate : 0.608
P-Value [Acc > NIR] : 1.062e-14

      Kappa : 0.5248

McNemar's Test P-Value : 0.1357

      Sensitivity : 0.7500
      Specificity : 0.7829
Pos Pred Value : 0.6901
Neg Pred Value : 0.8293
Prevalence : 0.3920
Detection Rate : 0.2940
Detection Prevalence : 0.4260
Balanced Accuracy : 0.7664

'Positive' Class : 0

```

```
In [ ]: get_confusion_matrix(df.test$Retained.in.2012., phat.l1.lgfit, threshold)
```

Confusion Matrix and Statistics

```

      Reference
Prediction  0   1
      0 149  60
      1  47 244

      Accuracy : 0.786
      95% CI : (0.7474, 0.8212)
No Information Rate : 0.608
P-Value [Acc > NIR] : <2e-16

      Kappa : 0.5563

McNemar's Test P-Value : 0.246

      Sensitivity : 0.7602
      Specificity : 0.8026
Pos Pred Value : 0.7129
Neg Pred Value : 0.8385
Prevalence : 0.3920
Detection Rate : 0.2980
Detection Prevalence : 0.4180
Balanced Accuracy : 0.7814

'Positive' Class : 0

```

From the confusion matrices, we can see that the lasso model does a better job, though not by much. It is observed that the lasso model has both less false positives and false negatives, increasing the accuracy by .01% which we consider not to be a significant improvement.

1. Plot ROC curves for the two classifiers and report the area under the curve.

Note that the AUC of an error-free classifier would be 100%, and an AUC of a random guess would be 50%. For values in-between, we can think of AUC as follows:

- 90%+ = excellent,
- 80–90% = very good,
- 70–80% = good,
- 60–70% = so-so, and
- below 60% = not much value.

```
In [ ]: library(ROCR)
```

```
In [ ]: # Create a list with the 2 phat vectors
phat_list = list()
phat_list$lgfit = matrix(phat.lgfit.selected, ncol = 1)
phat_list$lasso = matrix(phat.l1.lgfit, ncol = 1)
nmethod <- length(phat_list)
```

```
In [ ]: #' @param y: should be 0/1
#' @param phat: probabilities obtained by our algorithm
#' @param wht: shrinks probabilities in phat towards .5
#' this helps avoid numerical problems --- don't use log(0)!
#' @return deviance loss
get_deviance = function(y,phat,wht=1e-7) {
  if(is.factor(y)) y = as.numeric(y)-1
  phat = (1-wht)*phat + wht*.5
}
```



```

py = ifelse(y==1, phat, 1-phat)
return(-2*sum(log(py)))
}

```

```

In [ ]: phat_best = matrix(0.0,nrow(df.test),nmethod) #pick off best from each method
colnames(phat_best) = names(phat_list)

for(i in 1:nmethod) {
  nrun = ncol(phat_list[[i]])
  lvec = rep(0,nrun)
  for(j in 1:nrun) lvec[j] = get_deviance(df.test$Retained.in.2012.,phat_list[[i]][,j])
  imin = which.min(lvec)
  phat_best[,i] = phat_list[[i]][,imin]
}

```

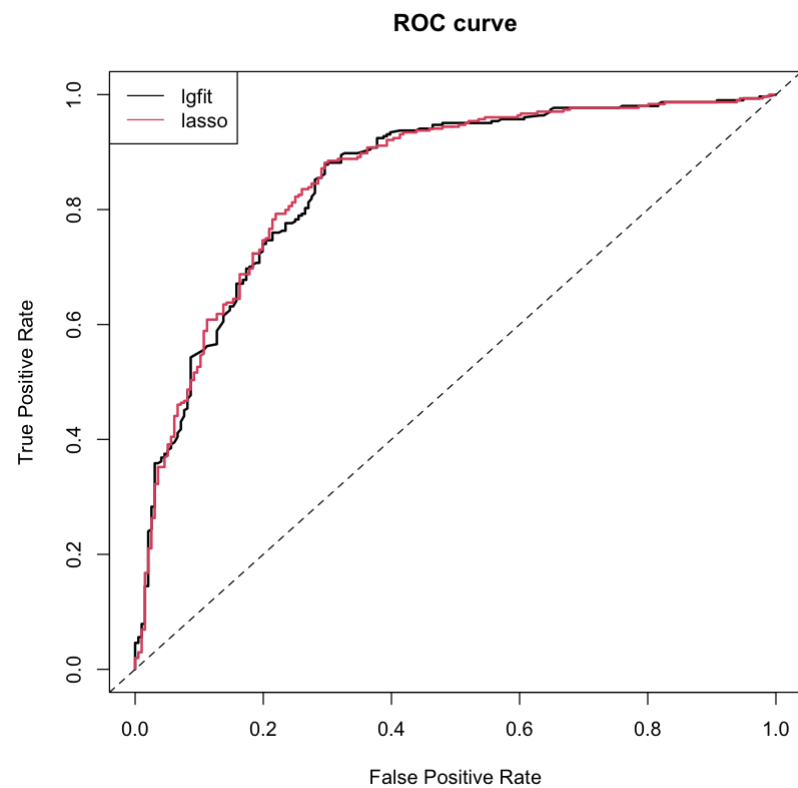
```

In [ ]: for(i in 1:ncol(phat_best)) {
  pred = prediction(phat_best[,i], df.test$Retained.in.2012.)
  perf = performance(pred, measure = "tpr", x.measure = "fpr")

  if (i == 1) {
    plot(perf, col=1, lwd=2,
         main= 'ROC curve',
         xlab='False Positive Rate',
         ylab='True Positive Rate')
  }

  else {
    plot(perf, add=T, col=i, lwd=2)
  }
}
abline(0, 1, lty=2)
legend("topleft", legend=names(phat_list), col=1:nmethod, lty=rep(1,nmethod))

```



```

In [ ]: for(i in 1:ncol(phat_best)) {
  pred = prediction(phat_best[,i], df.test$Retained.in.2012.)
  perf = performance(pred, measure = "auc")
  print(paste0("AUC ", names(phat_list)[i], " :: ", perf@y.values[[1]]))
}

```

```

[1] "AUC lgfit :: 0.849321965628357"
[1] "AUC lasso :: 0.85170515574651"

```

The AUC (Area Under the Curve) values of 0.849 and 0.852 represent the performance of two different models, "lgfit" and "lasso", in a binary classification task. An AUC of 0.8 is considered a good model performance, and an AUC value close to 1 indicates a perfect classifier. The higher the AUC value, the better the model is at distinguishing between the positive and negative class. In this case, the "lasso" model has a slightly better performance with an AUC of 0.8517 compared to the "lgfit" model with an AUC of 0.8493.

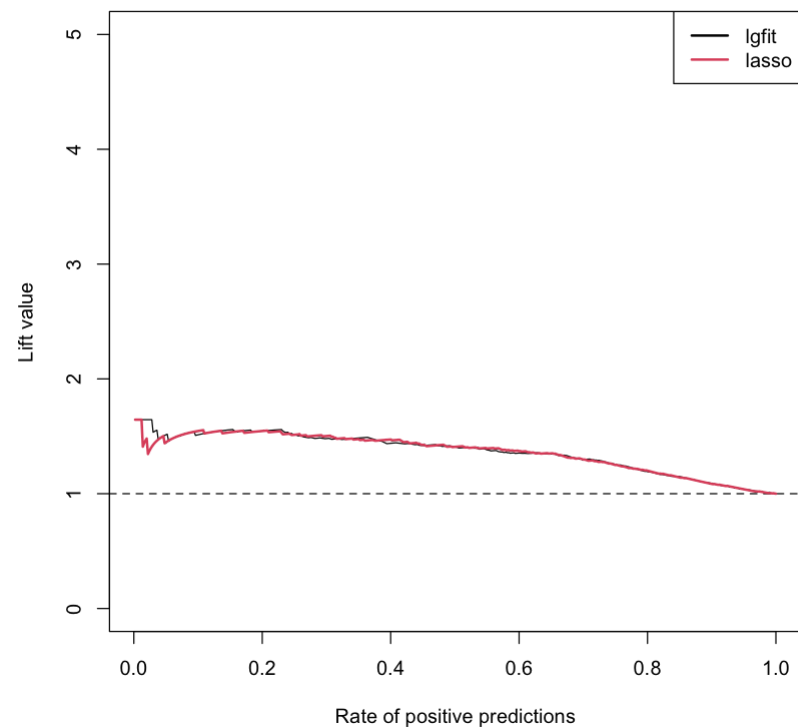
1. Plot lift curves for the two classifiers.

```

In [ ]: pred = prediction(phat_best[,1], df.test$Retained.in.2012.)
perf = performance(pred, measure="lift", x.measure="rpp", lwd=2)
plot(perf, col=1, ylim=c(0,5))
abline(h=1, lty=2)

for(i in 2:ncol(phat_best)) {
  pred = prediction(phat_best[,i], df.test$Retained.in.2012.)
  perf = performance(pred, measure="lift", x.measure="rpp")
  plot(perf, add=T, col=i, lwd=2)
}
legend("topright", legend=names(phat_list), col=1:nmethod, lty=rep(1,nmethod), lwd=2)

```



We can observe from the lift curves that they are very similar.

1. Create the profit curve (the amount of net profit vs the number of groups targeted for promotion) for the two classifiers. Suppose that the benefit of retaining a group is 100, *while the cost of a promotion is 40*.

How many groups should be targeted to maximize the profit?

How would this number change as the ratio between the benefit and cost changes?

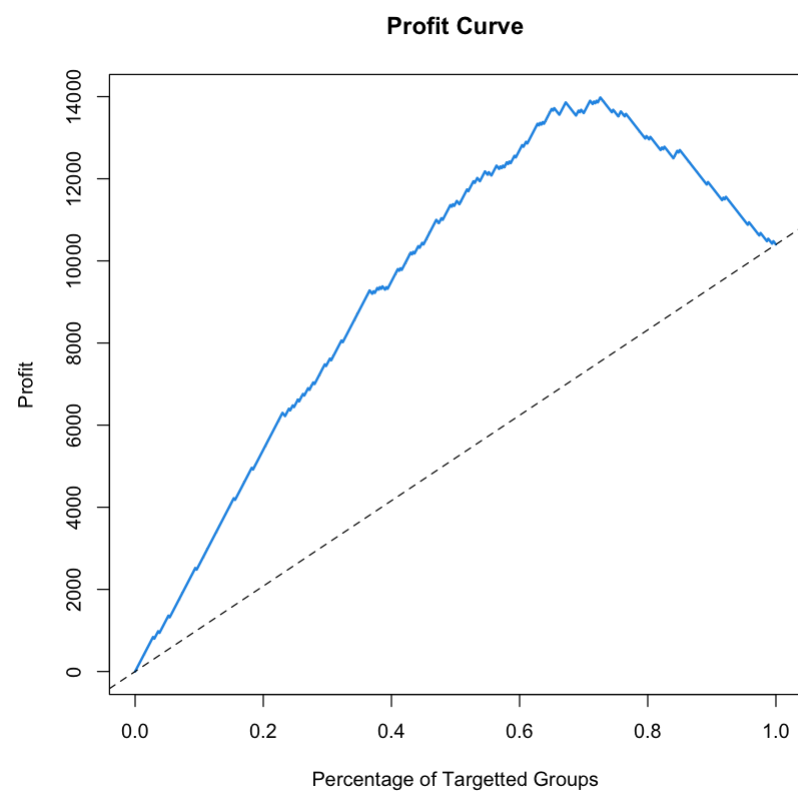
```
In [ ]: # Function to plot a profit curve
#
# Inputs:
# - benefitTP(FN/FP/TN): the net benefit for a true positive (false negative,...)
#   which is positive for a gain, and negative for a loss
# - y: vector of true labels, which has to be labeled as "0" and "1"
# - phat: vector of predicted probabilities
# Outputs:
#   the function returns the profit curve

ProfitCurve <- function(benefitTP, benefitFN, benefitFP, benefitTN, y, phat){
  if(length(y) != length(phat)) stop("Length of y and phat not identical")
  if(length(levels(y))!=2 | levels(y)[1]!="0" | levels(y)[2]!="1") stop("y should be a vector of factors, only with
n <- length(y)
  df <- data.frame(y, phat)
# Order phat so that we can pick the k highest groups for promotion
  df <- df[order(df[,2], decreasing = T),]
  TP <- 0; FP <- 0; FN <- table(y)[2]; TN <- table(y)[1]
# Initializing the x and y coordinates of the plot
  ratio.vec <- seq(0,n)/n
  profit.vec <- rep(0,n+1)
  profit.vec[1] <- FN * benefitFN + TN * benefitTN
  for(k in 1:n){
    # k is the number of groups classified as "YES"
    # In every round, we are picking one more group for promotion.
    # If this group was retained (positive), then in this round, it is classified
    # as a "YES" instead of "NO" before. The confusion matrix is updated each round
    # with one more TP, and one less FN. It's similar when the group was not retained.
    if(df[k,1]=="1"){TP <- TP + 1; FN <- FN - 1}
    else{FP <- FP + 1; TN <- TN - 1}
    # print(paste(TP, FP, TP-FP, benefitTP, benefitFP))
    profit.vec[k+1] <- TP*benefitTP + FP*benefitFP + FN*benefitFN + TN*benefitTN
  }

  # Get a matrix with profit and ratio
  profit.mat <- cbind(ratio.vec, profit.vec)

  plt <- plot(ratio.vec, profit.vec, type="l", lwd=2, col=4, main="Profit Curve",
    xlab="Percentage of Targetted Groups", ylab="Profit")
  abline(b=(profit.vec[n+1]-profit.vec[1]), a=profit.vec[1], lty=2) #Random guess
  return(profit.mat )
}
```

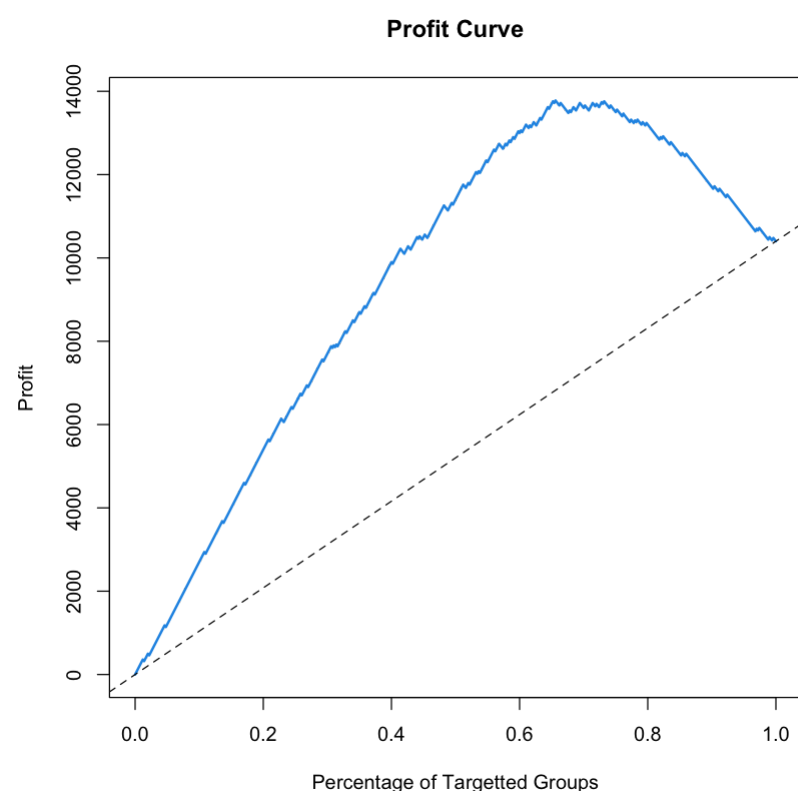
```
In [ ]: curve_1 <- ProfitCurve(60,0,-40,0,df.test$Retained.in.2012.,phat_best[,1])
```



```
In [ ]: # Get the maximum profit and the corresponding ratio with the corresponding row number
max_profit <- max(curve_1[,2])
max_ratio <- curve_1[which.max(curve_1[,2]),1]
max_row <- which.max(curve_1[,2])
print(paste("Maximum profit is", max_profit, "with ratio", max_ratio, "and the number of groups", max_row))

[1] "Maximum profit is 13980 with ratio 0.726 and the number of groups 364"
```

```
In [ ]: curve_2 <- ProfitCurve(60,0,-40,0,df.test$Retained.in.2012.,phat_best[,2])
```



```
In [ ]: # Get the maximum profit and the corresponding ratio with the corresponding row number
max_profit <- max(curve_2[,2])
max_ratio <- curve_2[which.max(curve_2[,2]),1]
max_row <- which.max(curve_2[,2])
print(paste("Maximum profit is", max_profit, "with ratio", max_ratio, "and the number of groups", max_row))

[1] "Maximum profit is 13780 with ratio 0.656 and the number of groups 329"
```

Let's suppose that the cost increases, we would expect the ratio of targeted groups to reduce as well. This is because the cost of the promotion is now higher than the benefit of retaining a group. The profit curve is a function of the ratio between the benefit and cost, and as the ratio increases, the number of groups targeted for promotion increases.

1. Develop a decision tree, random forest, and a boosting model using the training data.

Report ROC, AUC, lift, and profit curves for these models.

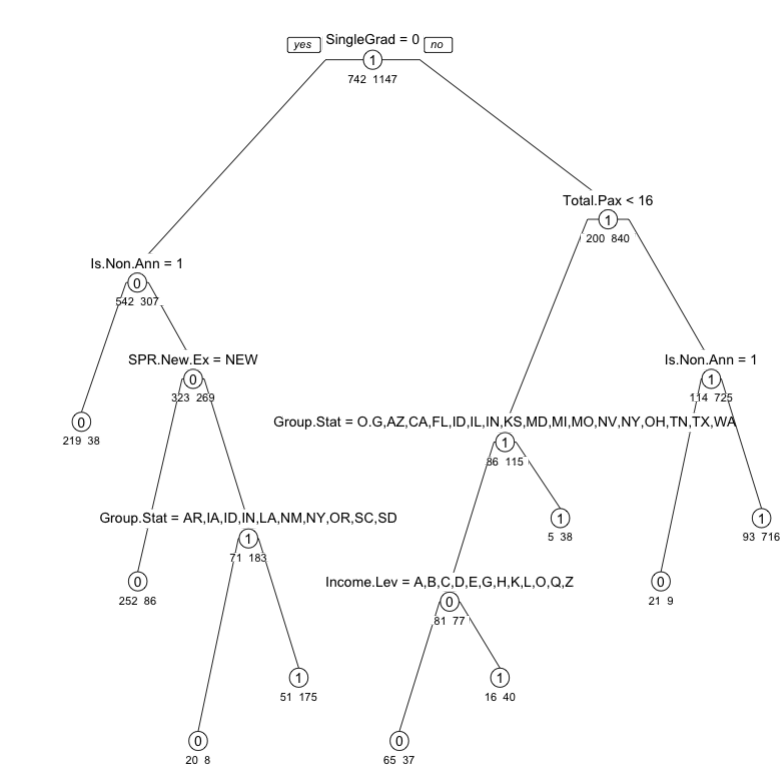
How do these methods compare to the logistic regression models?

```
In [ ]: library (ranger)
library(rpart)
library(rpart.plot)
```

Decision Tree

```
In [ ]: default.ct <- rpart(Retained.in.2012. ~ ., data = df.train, method = "class")
```

```
In [ ]: prp(default.ct, type = 1, extra = 1, under = TRUE, split.font = 1, varlen = -10)
```



```
In [ ]: deeper.ct <- rpart(Retained.in.2012. ~ ., data = df.train, method = "class", cp = 0, minsplit = 1)
```

```
In [ ]: length(deeper.ct$frame$var[deeper.ct$frame$var == "<leaf>"])
```

229

```
In [ ]: default.ct.point.pred.train <- predict(default.ct, df.train, type = "class")
deeper.ct.point.pred.train <- predict(deeper.ct, df.train, type = "class")
cm.default.train <- confusionMatrix(default.ct.point.pred.train, df.train$Retained.in.2012.)
cm.deeper.train <- confusionMatrix(deeper.ct.point.pred.train, df.train$Retained.in.2012.)
print(cm.default.train)
print(cm.deeper.train)
```

Confusion Matrix and Statistics

```

      Reference
Prediction 0  1
0  577 178
1  165 969

      Accuracy : 0.8184
      95% CI   : (0.8003, 0.8356)
No Information Rate : 0.6072
P-Value [Acc > NIR] : <2e-16

      Kappa : 0.6205

McNemar's Test P-Value : 0.517

      Sensitivity : 0.7776
      Specificity : 0.8448
Pos Pred Value : 0.7642
Neg Pred Value : 0.8545
Prevalence : 0.3928
Detection Rate : 0.3055
Detection Prevalence : 0.3997
Balanced Accuracy : 0.8112

      'Positive' Class : 0
```

Confusion Matrix and Statistics

```

      Reference
Prediction 0  1
0  742  0
1  0 1147

      Accuracy : 1
      95% CI   : (0.998, 1)
No Information Rate : 0.6072
P-Value [Acc > NIR] : < 2.2e-16

      Kappa : 1

McNemar's Test P-Value : NA

      Sensitivity : 1.0000
      Specificity : 1.0000
Pos Pred Value : 1.0000
Neg Pred Value : 1.0000
Prevalence : 0.3928
Detection Rate : 0.3928
Detection Prevalence : 0.3928
Balanced Accuracy : 1.0000

      'Positive' Class : 0
```

```
In [ ]: default.ct.point.pred.valid <- predict(default.ct, df.test, type = "class")
deeper.ct.point.pred.valid <- predict(deeper.ct, df.test, type = "class")
cm.default.valid <- confusionMatrix(default.ct.point.pred.valid, df.test$Retained.in.2012.)
cm.deeper.valid <- confusionMatrix(deeper.ct.point.pred.valid, df.test$Retained.in.2012.)
print(cm.default.valid)
print(cm.deeper.valid)
```

Confusion Matrix and Statistics

```
      Reference
Prediction 0  1
0 140  56
1  56 248
```

```
Accuracy : 0.776
95% CI : (0.7369, 0.8118)
No Information Rate : 0.608
P-Value [Acc > NIR] : 9.848e-16
```

```
Kappa : 0.5301
```

```
Mcnemar's Test P-Value : 1
```

```
Sensitivity : 0.7143
Specificity : 0.8158
Pos Pred Value : 0.7143
Neg Pred Value : 0.8158
Prevalence : 0.3920
Detection Rate : 0.2800
Detection Prevalence : 0.3920
Balanced Accuracy : 0.7650
```

```
'Positive' Class : 0
```

Confusion Matrix and Statistics

```
      Reference
Prediction 0  1
0 128  81
1  68 223
```

```
Accuracy : 0.702
95% CI : (0.6598, 0.7418)
No Information Rate : 0.608
P-Value [Acc > NIR] : 7.392e-06
```

```
Kappa : 0.3821
```

```
Mcnemar's Test P-Value : 0.3256
```

```
Sensitivity : 0.6531
Specificity : 0.7336
Pos Pred Value : 0.6124
Neg Pred Value : 0.7663
Prevalence : 0.3920
Detection Rate : 0.2560
Detection Prevalence : 0.4180
Balanced Accuracy : 0.6933
```

```
'Positive' Class : 0
```

```
In [ ]: cv.ct <- rpart(Retained.in.2012. ~ ., data = df.train, method = "class",
  cp = 0.00001, minsplit = 5, xval = 5)
printcp(cv.ct)
plotcp(cv.ct)
```



```
Classification tree:
rpart(formula = Retained.in.2012. ~ ., data = df.train, method = "class",
      cp = 1e-05, minsplit = 5, xval = 5)
```

Variables actually used in tree construction:

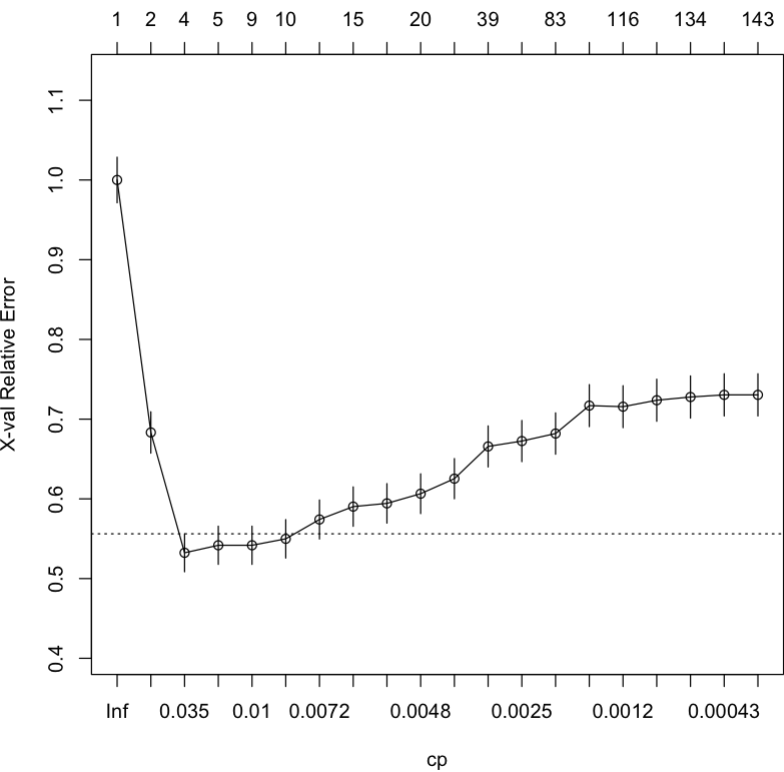
[1] CRM.Segment	Days
[3] Departure.Date	Deposit.Date
[5] DifferenceTraveltoFirstMeeting	DifferenceTraveltoLastMeeting
[7] EZ.Pay.Take.Up.Rate	Early.RPL
[9] FPP	FPP.to.PAX
[11] FPP.to.School.enrollment	FRP.Active
[13] FRP.Cancelled	FRP.Take.up.percent.
[15] FirstMeeting	From.Grade
[17] Group.State	GroupGradeType
[19] Income.Level	Initial.System.Date
[21] Is.Non.Annual.	LastMeeting
[23] Latest.RPL	MDR.High.Grade
[25] MDR.Low.Grade	Poverty.Code
[27] Program.Code	Region
[29] SPR.New.Existing	School.Sponsor
[31] School.Type	SchoolGradeType
[33] SchoolSizeIndicator	SingleGradeTripFlag
[35] Special.Pay	To.Grade
[37] Total.Pax	Total.School.Enrollment
[39] Tuition	

Root node error: 742/1889 = 0.3928

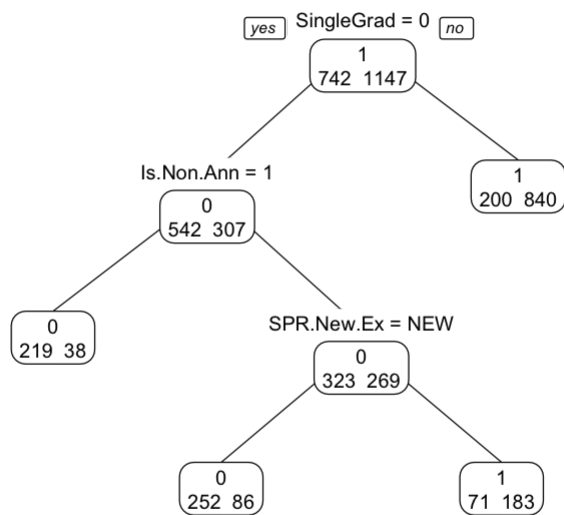
n= 1889

	CP	nsplit	rel error	xerror	xstd
1	0.31671159	0	1.000000	1.00000	0.028606
2	0.07547170	1	0.683288	0.68329	0.025956
3	0.01617251	3	0.532345	0.53235	0.023821
4	0.01347709	4	0.516173	0.54178	0.023974
5	0.00808625	8	0.462264	0.54178	0.023974
6	0.00763702	9	0.454178	0.54987	0.024104
7	0.00673854	13	0.420485	0.57412	0.024480
8	0.00539084	14	0.413747	0.59030	0.024720
9	0.00494160	16	0.402965	0.59434	0.024779
10	0.00471698	19	0.388140	0.60647	0.024953
11	0.00404313	21	0.378706	0.62534	0.025214
12	0.00269542	38	0.308625	0.66577	0.025741
13	0.00224618	71	0.207547	0.67251	0.025825
14	0.00202156	82	0.176550	0.68194	0.025940
15	0.00134771	92	0.150943	0.71698	0.026347
16	0.00112309	115	0.119946	0.71563	0.026332
17	0.00089847	121	0.113208	0.72372	0.026421
18	0.00067385	133	0.102426	0.72776	0.026466
19	0.00026954	137	0.099730	0.73046	0.026495
20	0.00001000	142	0.098383	0.73046	0.026495

size of tree



```
In [ ]: pruned.ct <- prune(cv.ct, cp = cv.ct$cpstable[which.min(cv.ct$cpstable[, "xerror"]), "CP"])
length(pruned.ct$frame$var[pruned.ct$frame$var == "<leaf>"])
prp(pruned.ct, type = 1, extra = 1, split.font = 1, varlen = -10)
```



```

In [ ]: # this is the cp parameter with smallest cv-error
index_cp_min = which.min(cv.ct$cptable[, "xerror"])

# one standard deviation rule
# need to find first cp value for which the xerror is below horizontal line on the plot
(val_h = cv.ct$cptable[index_cp_min, "xerror"] + cv.ct$cptable[index_cp_min, "xstd"])
(index_cp_std = Position(function(x) x < val_h, cv.ct$cptable[, "xerror"]))
(cp_std = cv.ct$cptable[ index_cp_std, "CP" ])

```

0.556165682910288

3

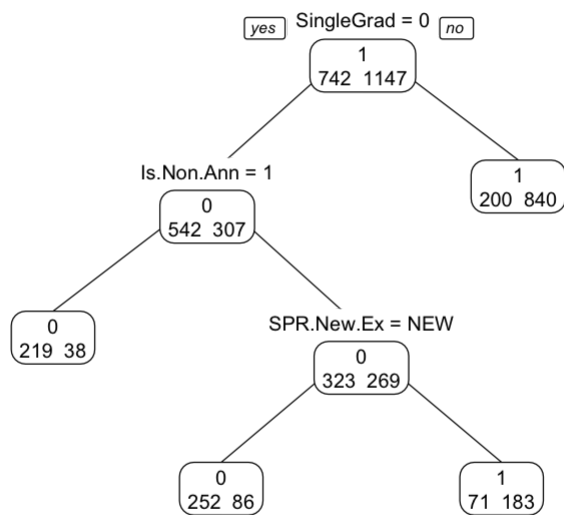
0.0161725067385445

```

In [ ]: pruned.ct <- prune(cv.ct, cp = cp_std)
length(pruned.ct$frame$var[pruned.ct$frame$var == "<leaf>"])
prp(pruned.ct, type = 1, extra = 1, split.font = 1, varlen = -10)

```

4



```

In [ ]: phat.tree <- predict(pruned.ct, df.test, type = "prob")
# Drop the first column, which is the probability of "NO"
phat.tree <- phat.tree[,2]
phat.tree <- data.frame(phat.tree)
# Add to phat_list the phat tree as a matrix 500 x 1
phat_list <- cbind(phat_list, phat.tree)

```

Random Forest

```

In [ ]: # Run a random forest model
p = ncol(df.train) - 1

grid_rf = expand.grid(
  mtry = c(p, ceiling(sqrt(p))),
  node_size = c(5, 10, 20)
)

for (i in 1:nrow(grid_rf)) {

```

```

rf = ranger(Retained.in.2012. ~ ., data = df.train, mtry = grid_rf$mtry[i],
            min.node.size = grid_rf$node_size[i], probability = TRUE)
phat.rf <- predict(rf, df.test)$predictions
phat.rf <- data.frame(phat.rf)
}

# Select the best model
phat.rf <- predict(rf, df.test)$predictions
phat.rf <- data.frame(phat.rf)
# Add to phat_list the phat rf as a matrix 500 x 1 only X1
phat_list <- cbind(phat_list, phat.rf[,1])

```

```

In [ ]: phat_best = matrix(0.0,nrow(df.test),nmethod) #pick off best from each method
colnames(phat_best) = names(phat_list)

for(i in 1:nmethod) {
  nrun = ncol(phat_list[[i]])
  lvec = rep(0,nrun)
  for(j in 1:nrun) lvec[j] = get_deviance(df.test$Retained.in.2012.,phat_list[[i]][,j])
  imin = which.min(lvec)
  phat_best[,i] = phat_list[[i]][,imin]
}

```

Error in dimnames(x) <- dn: length of 'dimnames' [2] not equal to array extent
Traceback:

```

1. `colnames<-`(`*tmp*`, value = c("lgfit", "lasso", "phat.tree",
. "phat.rf[, 1]"))

```