

Final Exam

Please note the following:

- The exam is 180 points.
- You have 180 minutes to complete the exam.
- For every minute late you submit the exam, you will lose one point.
- You will upload your solution to the Final Exam assignment on Canvas, where you downloaded this.
- Your submission should be readable, (the graders can understand your answers,) and it should include all code used in your analysis in a file format that the code can be executed. (ie. .ipynb preferred, .pdf is unacceptable.)
- The exam is open-material, closed-communication.
- If you find any question to be unclear, state your interpretation and proceed. We will only answer questions of interpretation if there is a typo, error, etc.
- The exam will be graded for partial credit.

Data: Some questions in the exam rely on the data `final_exam_data.xlsx`.

- sheet “`futures (excess returns)`”: monthly excess returns on 14 commodity futures.
- sheet “`factors (excess returns)`”: monthly excess returns on factors, (MKT and UMD.)
- sheet “`forecasting (weekly)`”: weekly total returns on gold (GLD) along with two interest-rate series used as signals
- sheet “`fx (daily)`”: daily FX rate on GBP (in terms of dollars) along with the risk-free rate on dollars (SOFR) and risk-free rate on GBP (SONIA).

This data file may be found in the class github repo, in the `data` folder. For your convenience, it is also posted on Canvas, in the Midterm exam.

1 Short Answer (50pts)

1. (5pts) Consider our momentum construction of going long the biggest winners and short the biggest losers.

What is the tradeoff of focusing the long and short positions narrowly, (a single decile top and bottom) versus more broadly, (three deciles top and bottom)?

Did our empirical investigation support this theoretical tradeoff?

2. (5pts) We investigated LTCM's market exposure and found it is nonlinear. Explain this nonlinearity: does it imply LTCM has high upside, large downside, both, etc?

3. (5pts) State one reason that Mean-Variance optimization is not robust, (i.e. the solution is fragile with respect to the inputs.)

State one approach we discussed regarding how to improve the stability of our optimized portfolio.

4. (5pts) You have monthly returns from January 2001 to December 2022 for 40 portfolios of assets. You want to test the performance of some of your Linear Factor Pricing Models using Time Series and Cross Sectional regressions.

- What would be the regression sample sizes for the TS regression? How many TS regressions would you estimate?
- What would be the regression sample sizes for the CS regression? How many CS regressions would you estimate?

5. (5pts) GMO stated that they had a "contrarian" investment style. What did they mean by this? Was this seen in our investigation of the fund, GMWAX?

6. (5pts) How does Harvard make their portfolio allocation more realistic than a basic mean-variance optimization would imply? Is their approach easily implemented and computed from a numerical standpoint?

7. (5pts) If Barnstable's assumptions hold, (log iid returns, normally distributed,) then how will an investment's Sharpe ratio compare across short and long-term horizons? Explain.

8. (5pts) Does Uncovered Interest Parity (UIP) imply Covered Interest Parity (CIP)? Or vice-versa? Or neither? Explain.

9. (5pts) Name and briefly explain two reasons why we said it is very hard for investors to understand the mean returns of managed funds.

10. (5pts) Suppose we have a strategy with returns, r_t^i . If we want to hedge our position with respect to SPY, how could we calculate the optimal ratio? How would this ratio then be used to build the hedged position?

2 Value at Risk (15pts)

- Find the data on the returns of the ETF, GLD, in the sheet `forecasting (weekly)`
- 1. (5pts) Calculate the 5th percentile VaR and CVaR for GLD as of the end of the sample using the empirical CDF approach over the full sample of data.
- 2. (5pts) Calculate the 5th percentile VAR for GLD as of the end of the sample using the normal approximation, (assuming mean is zero as usual.)

Calculate it using the following methods...

- full-sample volatility.
 - rolling 150-week window for volatility.
3. (5pts) In our analysis in the course, which of the methods above did we find did best?
How did we judge which method did best?

3 Pricing Model (25 pts)

Use the provided data in the sheets

- `futures (excess returns)`,
- `factors (excess returns)`,

to test a two-factor pricing model: the market (MKT) and momentum (UMD).

$$\mathbb{E}[\tilde{r}^i] = \beta^{i,m} \mathbb{E}[\tilde{r}^m] + \beta^{i,\text{UMD}} \mathbb{E}[\tilde{r}^{\text{UMD}}] \quad (1)$$

1. (10pts) Estimate the time-series test of the pricing model.
 - (a) Report...
 - (for each asset) annualized alpha, beta, and r-squared.
 - annualized MAE of the time-series alphas,
 - mean of the r-squared statistics.
 - (b) If the pricing model worked perfectly, what would these statistics be?
2. (10pts) Estimate the cross-sectional test of the pricing model. Include an intercept in the cross-sectional regression.
 - (a) Report the...
 - annualized intercept.
 - annualized factor premia.
 - r-squared.
 - annualized mean-absolute error.
 - (b) If the pricing model worked perfectly, what would these statistics be?
3. (5pts) Compare the factor premia across the cross-sectional and time-series estimations.

4 Forecasting (50pts)

Forecast (total) returns on gold as tracked by the ETF ticker, GLD. As signals, use two interest rate signals: the yield on T-bills and the change in T-bill yields.

- Find the all data needed for this problem in the sheet **forecasting (weekly)**
- 1. (4pts) Consider the lagged regression, where the regressor, (X_t) is a period behind the target, (r_t^{GLD}) .

$$r_t^{GLD} = \alpha^{GLD, X} + (\beta^{GLD, X})' \mathbf{X}_{t-1} + \epsilon_t^{GLD, X} \quad (2)$$

Estimate (2) and report the \mathcal{R}^2 , as well as the OLS estimates for α and β . (No need to annualize these stats.)

Do this using both interest rate signals together. So we are estimating just one model:

- \mathbf{X} as the two regressors: Tbill rate and Tbill change.
2. (3pts) Use the forecasted GLD returns, \hat{r}_{t+1}^{GLD} , to build trading weights:

$$w_t = 0.2 + 80 \hat{r}_{t+1}^{GLD}$$

Calculate the return on this strategy:

$$r_{t+1}^x = w_t r_{t+1}^{GLD}$$

Report the first and last 5 values.

3. (3pts) For both r^x and r^{GLD} , report the following univariate stats (annualized).
 - mean, volatility, Sharpe,
 - max-drawdown

Do not worry that we are using total returns rather than excess returns for all these stats.¹

4. (5pts) Run a Linear Factor Decomposition of r^x on r^{GLD} and report the following (annualized):
 - market alpha
 - market beta
 - market Information ratio

Do not worry that we are using total returns rather than excess returns for all these stats.

5. (5pts) Suppose we were going to forecast GLD using just one of our two signals. Which of the signals would likely lead to a result where the long-term forecast compounds the effect over long horizons, as we saw for forecasting SPY using dividend-price ratios? Explain.
6. (5pts) Let's consider the out-of-sample performance of the strategy.

¹Some of these stats are typically calculated on excess returns, whereas here we are using total returns for all the stats. This is simply to save you the hassle of going back and forth between total returns and excess returns in this problem.

- Forecast values of GLD for January 2017 through Dec 2022. (So we are using the data up until January 2017 as “burn-in” data.)
- Loop through time, estimating (2) only using data through time t .
- Use the estimated parameters of (2), along with \mathbf{x}_{t+1} to calculate the out-of-sample forecast for the following period, $t + 1$.

$$\hat{r}_{t+1}^{GLD} = \hat{\alpha}_t^{GLD, \mathbf{x}} + (\boldsymbol{\beta}^{GLD, \mathbf{x}})' \mathbf{x}_t$$

Report the first 5 and last 5 values of your forecast.²

7. (10pts) Report the out-of-sample \mathcal{R}^2 , relative to a baseline forecast which is simply the mean of GLD up to the point the forecast is made.
8. (5pts) Report the correlation between the two forecasts of GLD (regression based and the baseline forecast) with the actual realized value of GLD.

Does either forecast positively correlate with what actually happens?

Between this and the OOS r-squared, does this forecast seem to be informative?

9. (10pts) Trade on the two forecasts, using the same weighting scheme as above.

For the two OOS return series, report the following univariate stats (annualized).

- mean, volatility, Sharpe,
- max-drawdown

Also report the Linear Factor Decomposition stats (annualized) when regressed on GLD:

- market alpha
- market beta
- market Information ratio

As before, do not worry that we are calculating these on total returns instead of excess returns.

²Not asking for anything fancy, just the default variable printing in Jupyter.

5 FX Carry (40pts)

We examine FX Carry for trading the British Pound, (GBP).

- Find the FX and risk-free rate data for this problem on sheet “**fx rates (daily)**”. As before, these are spot FX prices quoted as USD per GBP.
- Recall that SOFR is the risk-free rate on USD, and SONIA is the risk-free rate for GBP.
- As in Homework 8, the data is provided such that any row’s date, t , is reporting S_t^{GBP} and $r_{t,t+1}^{f,\text{GBP}}$. Both of these objects are known at time t .

1. (3pts) Transform the data to log FX prices and log interest rates, just as we did in Homework 8.

$$\begin{aligned}\mathbf{s}_t^{\text{GBP}} &\equiv \ln(s_t^{\text{GBP}}) \\ \mathbf{r}_{t,t+1}^{f,\text{GBP}} &\equiv \ln(1 + r_{t,t+1}^{f,\text{GBP}}) \\ \mathbf{r}_{t,t+1}^{f,\text{USD}} &\equiv \ln(1 + r_{t,t+1}^{f,\text{USD}})\end{aligned}$$

No other data transformation is needed.

Display the mean of all three series.

2. (3pts) If we assume the Uncovered Interest Parity to hold true, what would you expect from the (static, passive) return to GBP?
3. (3pts) Calculate the **excess** log return to a USD investor of holding GBP. Report the following annualized stats...
 - Mean
 - Volatility
 - Sharpe ratio
4. (6pts) Over the sample, was it better to be long or short GBP relative to USD?
 - Did the interest spread help on average?
 - Did the USD appreciate or depreciate relative to GBP over the sample?
5. (5pts) Assume just for this question that these daily excess log returns are normally distributed. What is the probability that holding GBP over the following 5 years will underperform the USD risk-free rate?
6. (5pts) Forecast the growth of the FX rate using the interest rate differential:

$$\mathbf{s}_{t+1}^{\text{GBP}} - \mathbf{s}_t^{\text{GBP}} = \alpha + \beta \left(\mathbf{r}_{t,t+1}^{f,\text{USD}} - \mathbf{r}_{t,t+1}^{f,\text{GBP}} \right) + \epsilon_{t+1}$$

Report the following OLS stats. (No need to annualize them.)

$$\alpha, \beta, R^2$$

7. (4pts) If we assume the Uncovered Interest Parity to hold true, what would you expect to be true of the regression estimates?
8. (4pts) Based on the regression results, if we observe an increase in the interest rate on GBP relative to USD, should we expect the USD to get stronger (appreciate) or weaker (depreciate)?
9. (4pts) If the risk free rates in USD increase relative to risk-free rates in GBP, we expect the forward exchange rate to be higher than the spot exchange rate?
10. (3pts) Based on the regression results, construct an in-sample forecast of the excess log return to holding GBP. Report the forecasted values for the first 5 and last 5 dates.