FINM 36700 **Mark Hendricks**
Autumn 2022

Homework #1

Due on Monday, October 3, at 6:00pm.

The Harvard Management Company and Inflation-indexed Bonds [**HBS 9-201-053**].

1 HMC's Approach

Section 1 is not graded, and you do not need to submit your answers. But you are encouraged to think about them, and we will discuss them.

1. The HMC framing of the portfolio allocation problem.

   (a) Why does HMC focus on real returns when analyzing its portfolio allocation? Is this just a matter of scaling, or does using real returns versus nominal returns potentially change the MV solution?

   (b) There are thousands of individual risky assets in which HMC can invest. Explain why MV optimization across 1,000 securities is infeasible.

   (c) Rather than optimize across all securities directly, HMC runs a two-stage optimization. First, they build asset class portfolios with each one optimized over the securities of the specific asset class. Second, HMC combines the asset-class portfolios into one total optimized portfolio.

   In order for the two-stage optimization to be a good approximation of the full MV-optimization on all assets, what must be true of the partition of securities into asset classes?

   (d) Should TIPS form a new asset class or be grouped into one of the other 11 classes?

2. Portfolio constraints.

   The case discusses the fact that Harvard places bounds on the portfolio allocation rather than implementing whatever numbers come out of the MV optimization problem.

   (a) How might we adjust the stated optimization problem in the lecture notes to reflect the extra constraints Harvard is using in their bounded solutions given in Exhibits 5 and 6. Just consider how we might rewrite the optimization; don't try to solve this extra-constrained optimization.

   (b) Exhibits 5 shows zero allocation to domestic equities and domestic bonds across the entire computed range of targeted returns, (5.75% to 7.25%). Conceptually, why is the constraint binding in all these cases? What would the unconstrained portfolio want to do with those allocations and why?

   (c) Exhibit 6 changes the constraints, (tightening them in most cases.) How much deterioration do we see in the mean-variance tradeoff that Harvard achieved?

2 Mean-Variance Optimization

This section is graded for a good-faith effort by your group. Submit your write-up- along with your supporting code. Don't just submit code or messy numbers; submit a coherent write-up based on your work.

- The exhibit data that comes with the case is unnecessary–we use updated timeseries data.
- For our analysis, we use more current data found in `multi_asset_etf_data.xlsx`.[1] This data is posted in the GitHub repo for the course.
- The time-series data gives monthly returns for the 11 asset classes and a short-term Treasury-bill fund return, ("SHV",) which we consider as the risk-free rate.[2]
- The data is provided in total returns, (in which case you should ignore the SHV column,) as well as excess returns, (where SHV has been subtracted from the other columns.)
- These are nominal returns—they are not adjusted for inflation, and in our calculations we are not making any adjustment for inflation.

In the questions below, annualize the statistics you report.

- Annualize the mean of monthly returns with a scaling of 12.
- Annualize the volatility of monthly returns with a scaling of $\sqrt{12}$.
- The Sharpe Ratio is the mean return divided by the volatility of returns.[3] Accordingly, we can annualize the Sharpe Ratio with a scaling of $\sqrt{12}$.
- Note that we are not scaling the raw timeseries data, just the statistics computed from it (mean, vol, Sharpe).

We are going to analyze the problem in terms of total–not excess–returns.

- Thus, you will focus on the "Mean-Variance" section of the lecture notes, especially the formulas on slide 40.
- In using the "total returns" tab of the data, drop the column SHV. It is our proxy for the risk-free rate, which we are ignoring in our analysis of total returns.
- Thus, below, you are analyzing 11 risky assets–not SHV.

[1] The case does not give time-series data, so this data has been compiled outside of the case, and it intends to represent the main asset classes under consideration via liquid investment vehicles. For details on the specific securities/indexes, check the "Info" tab of the data.

[2] In the lecture-note we considered a constant risk-free rate. It is okay that our risk-free rate changes over time, but the assumption is that investors know it's value one-period ahead of time—making it risk-free.

[3] Technically, it is defined for the **excess** returns, but it is common to look at this ratio for total returns and excess returns.

1. Summary Statistics

    (a) Calculate and display the mean and volatility of each asset's excess return. (Recall we use volatility to refer to standard deviation.)

    (b) Which assets have the best and worst Sharpe ratios[4]?

2. Descriptive Analysis

    (a) Calculate the correlation matrix of the returns. Which pair has the highest correlation? And the lowest?[5]

    (b) How well have TIPS done in our sample? Have they outperformed domestic bonds? Foreign bonds?

    (c) Based on the data, do TIPS seem to expand the investment opportunity set, implying that Harvard should consider them as a separate asset?

3. The MV frontier.

    (a) Compute and display the weights of the tangency portfolios: $\boldsymbol{w}^t$.

    (b) Compute the mean, volatility, and Sharpe ratio for the tangency portfolio corresponding to $\boldsymbol{w}^t$.

4. The allocation.

    (a) Compute and display the weights of MV portfolios with target returns of $\mu^p = .015$.[6]

    (b) What is the mean, volatility, and Sharpe ratio for $\boldsymbol{w}^p$?

    (c) Discuss the allocation. In which assets is the portfolio most long? And short?

    (d) Does this line up with which assets have the strongest Sharpe ratios?

5. Simple Portfolios

    (a) Calculate the performance of the equally-weighted portfolio over the sample. Rescale the entire weighting vector to have target mean $\mu^p = .015$. Report its mean, volatility, and Sharpe ratio.

    (b) Calculate the performance of the "risk-parity" portfolio over the sample. Risk-parity is a term used in a variety of ways, but here we have in mind setting the weight of the portfolio to be proportional to the inverse of its full-sample volatility estimate.

    $$w^i = \frac{1}{\sigma_i}$$

    This will give the weight vector, $\boldsymbol{w}$, but you will need to rescale it to have a target mean of $\mu^p = .015$.

    (c) How does these compare to the MV portfolio from problem 2.4?

[4]Though we are using total returns rather than excess, we'll still refer to this mean-vol ratio as the Sharpe ratio.
[5]You could get a heatmap of the correlation using the function from the "seaborn" package.
[6]This is monthly data, so the annualized target return is in a typical range.

6. Assess how much the Sharpe Ratio goes down if we drop TIPS from the investment set, (and just have a 10-asset problem.) See how much it decreases the performance statistics in 2.4. And how much worse is the performance in 3.3?

7. Out-of-Sample Performance

   Let's divide the sample to both compute a portfolio and then check its performance out of sample.

   (a) Using only data through the end of 2021, compute $\boldsymbol{w}^p$ for $\mu^p = .015$, allocating to all 11 assets.

   (b) Using those weights, calculate the portfolio's Sharpe ratio within that sample, through the end of 2021.

   (c) Again using those weights, (derived using data through 2021,) calculate the portfolio's Sharpe ratio based on performance in 2022.

3 Excess Returns

Not required to submit these, but we will discuss them.

1. Re-do the analysis for **excess** returns.

   - Subtract SHV from each total return in order to form **excess** returns, $\tilde{r}$.

   - Use the section of the notes "Excess Returns". Particularly the solutions in slide 50.

2. Long-short positions.

   (a) Consider an allocation between only Domestic Bonds (IEF) and Inflation-Protected Bonds (TIP). Drop all other return columns and recompute $\boldsymbol{w}^p$ for $\tilde{\boldsymbol{\mu}}^p = .0135$.

   (b) What is causing the extreme long-short position?

   (c) Make an adjustment to $\tilde{\boldsymbol{\mu}}^{\mathrm{TIP}}$ of -0.0015 (note the negative sign.) Recompute $\boldsymbol{w}^p$ for $\tilde{\boldsymbol{\mu}}^p = .0135$ for these two assets.

   How does the allocation among the two assets change?

   (d) What does this suggest about the statistical precision of the MV solutions?

3. Robustness

   (a) In building $\boldsymbol{w}^{\tan}$, do not use $\boldsymbol{\Sigma}$ as given in the formulas in the lecture. Rather, use a diagonalized $\boldsymbol{\Sigma}^D$, which zeroes out all non-diagonal elements of the full covariance matrix, $\boldsymbol{\Sigma}$.

   How does the allocation look now?

   (b) What does this suggest about the sensitivity of the solution to estimated means and estimated covariances?

   (c) HMC deals with this sensitivity by using explicit constraints on the allocation vector. Conceptually, what are the pros/cons of doing that versus modifying the formula with $\boldsymbol{\Sigma}^D$?

   (d) How does this diagonalized covariance approach compare to the risk-parity approach?

4. Iterate the Out-of-Sample performance every year, not just the final year. Namely,

   - Start at the end of 2014, and calculate the MV weights through that time. Apply them to the returns in the upcoming year, (2015.)

   - Step forward a year in time, and recompute.

   - Continue until again calculating the MV weights through 2021 and applying them to the returns in 2022.

   Report the mean, volatility, and Sharpe from this dynamic MV approach. Compare them to the dynamic versions of...

   (a) the equally-weighted portfolio

   (b) the "risk-parity" portfolio

   (c) the diagonalized MV portfolio.