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The Risk of Stocks in the Long Run: The Barnstable College Endowment

In early 1999, Ms. Colette Adams was evaluating an interesting set of proposals by a New York securities firm. As Vice President of Finance and Administration, Ms. Adams managed the approximately $50 million endowment of Barnstable College, a small but well known Massachusetts liberal arts institution. On matters relating to the endowment, Adams reported to an investment committee of the college's board of trustees.

Barnstable College was fortunate in that it had a large endowment relative to the size of its student body. The college's policy was to spend between 4% and 5% of the endowment each year. These funds enabled it to offer significant amounts of merit scholarship money as well as to keep tuition at modest levels. In turn, this helped Barnstable fulfill its mission of attracting and educating students of the highest caliber, but with broadly diverse economic and other backgrounds.

The endowment was managed according to a fairly simple philosophy. In the long-run, the investment committee believed, stocks would outperform safer asset classes such as bonds and Treasury bills. Since the short-run demands on the endowment were relatively small, there seemed no point in sacrificing long-run returns by holding bonds and other assets that offered greater near-term safety. Thus, the endowment was managed according to a policy of staying fully invested in equities at all times. With the phenomenal rise in U.S. equity market values over the last 16 years, this policy had served Barnstable College extremely well.

Currently, the endowment was invested in three commingled pools managed by a Boston bank: An S&P 500 index fund (40%), an actively-managed portfolio of U.S. stocks (30%), and an actively-managed portfolio of non-U.S. stocks (30%). These proportions were maintained with periodic rebalancing.

The Risk of Stocks in the Long-Run

There was considerable historical evidence, and also well-accepted theory, that the risk of holding stocks, as measured by the probability of a low return, diminished as the holding period lengthened. In theory, the effect of time horizon on risk derives from the law of large numbers. For example, it is less risky to take two equally-sized but uncorrelated risks than to take on twice the amount of a single risk. If stock market movements are random, the day-to-day or month-to-month risks are sequentially or "serially" uncorrelated, in the same way that the outcome of one coin toss is unrelated to the outcome of another. These uncorrelated movements cancel each other to some degree.

Professor André F. Perold prepared this case as the basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative situation.

Mathematically, the law of large numbers translates into the dispersion of returns growing only with the square root of time rather than proportionally with time. Specifically, if σ is the standard deviation of returns for a holding period of one year, then the standard deviation of returns for a holding period of T years is $\sigma\sqrt{T}$.[1] The standard deviation of returns thus grows over time, but not as fast as the expected return, which grows proportionally with time. For example, the standard deviation of *four-year* returns is theoretically *twice* the standard deviation of *one-year* returns. On the other hand, the expected four-year return is *four times* the expected *one*-year return.[2]

The Proposals

The securities firm's proposals were directly related to the long-run behavior of stock prices. If stocks were safe when held over long periods, they should outperform bonds and other safe assets with higher and higher probability as the holding period lengthened. The securities firm illustrated this in **Exhibit 1** by assuming a lognormal distribution for stock returns (where the log of wealth is normally distributed). They further assumed that the expected total return on stocks was constant at 13% per annum, and that stock returns had an annual volatility (standard deviation) of 16% per annum. The 16% per annum volatility figure was the post-World War II standard deviation of returns for U.S. stocks; the 13% per annum expected return reflected the long-run historical risk premium of 7% per annum for stock returns over U.S. Treasury bond returns, which currently were yielding around 6% per annum. A dollar invested in stocks (with dividends reinvested) would grow at 13% per annum in expectation, and the median growth rate would be at the geometric average return of 11.7% per annum.[3] A two-sigma event would be $2x16\%x\sqrt{T}$ to either side of the median wealth level for a given holding period of length T. Two sigmas to either side captured about 95% of the cases, so that the probability of doing worse than the median wealth minus two sigmas over a holding period of length T was about 2.5%.[4]

Exhibit 1 also shows the growth of one dollar continuously compounded at the riskless rate of 6% per annum. As the holding period lengthens, the likelihood that stocks would do worse than 6% per annum declines quite dramatically. This likelihood of obtaining a shortfall below 6% per annum is shown in **Exhibit 2**, calculated on the basis of a lognormal distribution.

Also shown in **Exhibit 2** are the Black-Scholes values of European put options on the S&P 500 index for different maturities, assuming a 16% per annum volatility of returns, reinvestment of dividends, and an exercise price growing at the riskless rate of 6% per annum. Thus, the exercise price for a put option on an initial one dollar investment in the S&P 500 index is $1.062 for a one-year horizon, $1.128 for a two year horizon, etc.[5] The probability of such put options being exercised is the same as the probability that the S&P 500 with dividends reinvested would return less than 6% p.a.

The securities firm was proposing that Barnstable College take advantage of the opportunity represented by the high put prices for the low probability risk that stocks would return less than 6% per annum over the long run. There were two specific ideas. The first involved selling long horizon

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[1] This assumes returns are serially uncorrelated and that the annual standard deviation is constant.

[2] This assumes annual expected returns are constant, and ignores compounding.

[3] When stock returns are lognormally distributed, the median return is given by the geometric average of returns. The geometric mean is the arithmetic mean minus half the variance, or $\bullet - ½\bullet^2$, where • is the arithmetic mean and • is the standard deviation of returns. With an annual standard deviation of returns of • = 16% and an expected arithmetic return of • = 13% per annum, the median or geometric mean return is 11.7% per annum $(0.13 - ½(0.16)^2 = 0.117)$

[4] The wealth levels at time t corresponding to plus or minus two standard deviations around the median are given by exp $\{(\bullet - ½\bullet^2)t \pm 2\bullet\sqrt{t}\}$.

[5] With continuous compounding at rate r per annum, a dollar at time zero grows to e^{rt} over t years. If the continuously compounded rate of return is 6% per annum, the annual rate of compounding is $e^{0.06}$ = 6.184%.

put options on the S&P 500 (with dividends reinvested). For example, as shown in **Exhibit 2**, a 30-year put option was worth about 34 cents for each dollar of initial wealth, and there was only a 2.5% probability that, over 30 years, stocks would earn a total return less than 6% per annum.

The second proposal involved the creation of a trust or other suitable entity that, on the asset side, would own the stocks in the S&P 500, and on the liability side would have two classes of shares: Preference Shares and Common Shares. The trust would have a fixed life, say 30 years, during which the assets would be managed just like an S&P 500 index fund, including reinvestment of dividends. At the end of year 30, the trust would be liquidated, and the assets distributed to the liabilityholders as follows: Holders of Common Shares would receive any assets in excess of the "redemption value", while holders of Preference Shares would receive the redemption value, or the value of the assets, whichever was less. The redemption value would be equal to the initial value of the assets in the trust grown at a 6% per annum continuously compounded rate for 30 years, or $\$1.06184^{30} = \$6.05$ for each dollar of initial assets.

Barnstable College and other investors would contribute S&P 500 stocks to the trust and would in return receive a pair of Preference and Common shares for each unit so purchased. They could then sell the Preference Shares in the market place, but hold onto their Common Shares, and in so doing retain their desired exposure to the S&P 500 but raise potentially large amounts of additional cash. Since there was such a low probability of the long-run performance of the S&P 500 falling short of 6% per annum, the payoff to Preference shareholders was virtually riskless, and the Preference Shares should fetch a very high price.

Exhibit 1
Wealth Levels and Probability Ranges

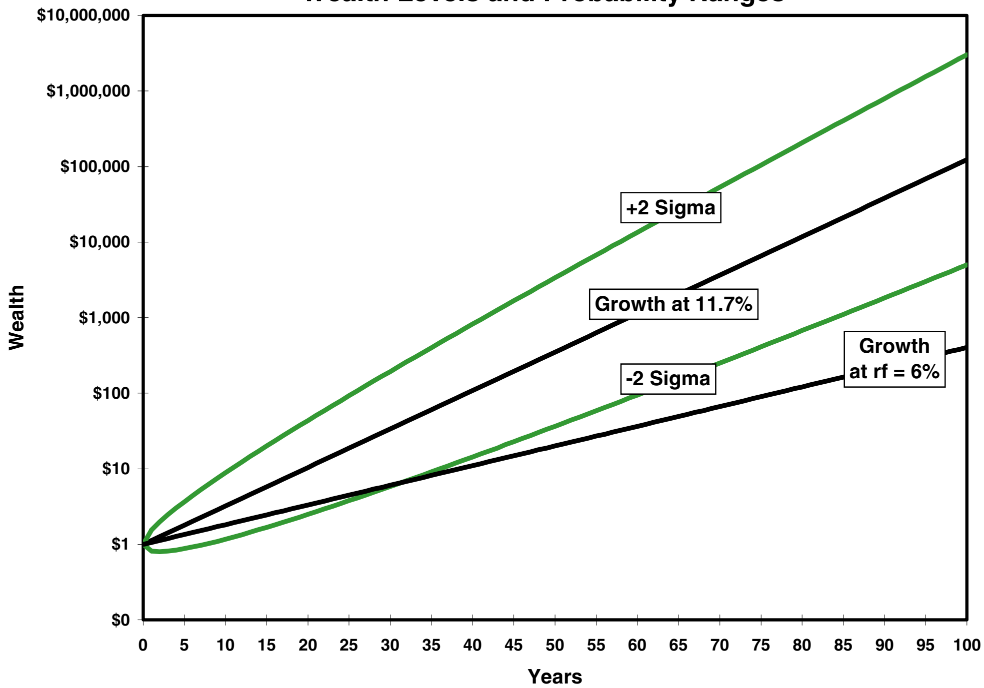

Exhibit 2
Probability of Shortfall and Value of Put Option