Projection onto k-dimensional subspaces

Consider an n-dimensional vector space V with the dot product at the inner product and a subspace U of V. With basis vectors $\boldsymbol{b}_1, \ldots, \boldsymbol{b}_k$ of U, we obtain the **orthogonal projection** of any vector $\boldsymbol{x} \in V$ onto U via

$$\pi_U(\boldsymbol{x}) = \boldsymbol{B}\boldsymbol{\lambda}\,, \quad \boldsymbol{\lambda} = (\boldsymbol{B}^\top \boldsymbol{B})^{-1}\boldsymbol{B}^\top \boldsymbol{x}$$
$$\boldsymbol{B} = (\boldsymbol{b}_1 | \cdots | \boldsymbol{b}_k) \in \mathbb{R}^{n \times k}$$

where $\boldsymbol{\lambda}$ is the **coordinate vector** of $\pi_U(\boldsymbol{x})$ with respect to the basis $\boldsymbol{b}_1, \ldots, \boldsymbol{b}_k$ of U.
The **projection matrix $\boldsymbol{P}$** is

$$\boldsymbol{P} = \boldsymbol{B}(\boldsymbol{B}^\top \boldsymbol{B})^{-1}\boldsymbol{B}^\top$$

such that

$$\pi_U(\boldsymbol{x}) = \boldsymbol{P}\boldsymbol{x}$$

for all $\boldsymbol{x} \in V$.