Projection onto 1D subspaces

Consider a vector space V with the dot product at the inner product and a subspace U of V. With a basis vector $\boldsymbol{b}$ of U, we obtain the **orthogonal projection** of any vector $\boldsymbol{x} \in V$ onto U via

$$\pi_U(\boldsymbol{x}) = \lambda \boldsymbol{b}, \quad \lambda = \frac{\boldsymbol{b}^\top \boldsymbol{x}}{\boldsymbol{b}^\top \boldsymbol{b}} = \frac{\boldsymbol{b}^\top \boldsymbol{x}}{\|\boldsymbol{b}\|^2}$$

where λ is the **coordinate** of $\pi_U(\boldsymbol{x})$ with respect to $\boldsymbol{b}$.
The **projection matrix** $\boldsymbol{P}$ is

$$\boldsymbol{P} = \frac{\boldsymbol{b}\boldsymbol{b}^\top}{\boldsymbol{b}^\top \boldsymbol{b}} = \frac{\boldsymbol{b}\boldsymbol{b}^\top}{\|\boldsymbol{b}\|^2}$$

such that

$$\pi_U(\boldsymbol{x}) = \boldsymbol{P}\boldsymbol{x}$$

for all $\boldsymbol{x} \in V$.