Equity Basket Option Pricing Guide

FinPricing

Summary

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Equity Basket Option Introduction

- A basket option is a financial contract whose underlying is a weighted sum or average of different assets that have been grouped together in a basket.
- A basket option can be used to hedge the risk exposure to or speculate the market move on the underlying stock basket.
- Because it involves just one transaction, a basket option often costs less than multiple single options.
- The most important feature of a basket option is its ability to efficiently hedge risk on multiple assets at the same time.
- Rather than hedging each individual asset, the investor can manage risk for the basket, or portfolio, in one transaction.
- The benefits of a single transaction can be great, especially when avoiding the costs associated with hedging each and every individual component.

The Use of Basket Options

- A basket offers a combination of two contracdictory benefits: focus on an investment style or sector, and diversification across the spectrum of stocks in the sector.
- Buying a basket of shares is an obvious way to participate in the anticipated rapid appreciation of a sector, without active management.
- An investor bullish on a sectr but wanting downside protection may favor a call option on a basket of shares from that sector.
- A trader who think the market overestimates a basket’s volatility may sell a butterfly spread on the basket.
- A relatively risk averse investor may favor a basket buy or write.
- A trader who anticipates that the average correlation among different shares is going to increase might buy a basket option.

Equity Basket Option Payoffs

- In a basket option, the payoff is determined by the weighted average prices of the underlying stocks in a basket.
- Trading desks use this type of option to construct the payoff structures in various Equity Linked Notes.
- The payoff for a basket call option is given by

$$Payoff_C(T) = N \cdot P \cdot \max(R - K, 0)$$

- The payoff for a basket put option is given by

$$Payoff_P(T) = N \cdot P \cdot \max(K - R, 0)$$

Equity Basket Option Payoffs (Cont)

where

$R = \sum_{i=1}^{n} w_i S_i / F_i$ the weighted average of the basket return

N the notional amount

P the option participation rate

w_i the weight for asset ,

F_i the InitialFixing for asset ,

K the basket percentage strike

S_i the spot price for asset at time T

Valuation

- The Asian basket option payoff function can be solved either analytically or using Monte Carlo simulation
- In this paper, we focus on the analytical solution. It assumes that the basket price can be approximated by a lognormal distribution with moments matched to the distribution of the weighted sum of the individual stock prices.
- The model includes two- and three-moment matching algorithms.
- The model also can be used to price an Asian basket option by including a period of dates in the averaging schedule.
- The payoff types covered by the model include calls and puts, as well as digital calls and digital puts.

Valuation (Cont)

- It is well known that the sum of a series of lognormal random variables is not a lognormal random variable. The weighted summation R is approximated by a shifted lognormal random variable (SLN).

$$R \sim SLN = c + d \cdot exp\left(\frac{Z-a}{b}\right)$$

where $Z \sim N(0,1)$ follows a standard normal distribution.

- We solve for a, b, c, d by matching central moments between.
- The central moments of SLN are

$$M_1 = c + d \cdot exp\left(\frac{1}{2b^2} - \frac{a}{b}\right)$$

$$M_2 = d^2 exp\left(\frac{1}{b^2} - \frac{2a}{b}\right)\left(exp\left(\frac{1}{b^2}\right) - 1\right)$$

$$M_3 = b^3 exp\left(\frac{3}{2b^2} - \frac{3a}{b}\right)\left(exp\left(\frac{1}{b^2}\right) - 1\right)^2\left(exp\left(\frac{1}{b^2}\right) + 2\right)$$

Valuation (Cont)

- The solved a, b, c, d are given by

$$b = \left[ln \left(\sqrt[3]{1 + \frac{\theta}{2} + \sqrt{\theta + \frac{\theta^2}{4}}} + \sqrt[3]{1 + \frac{\theta}{2} - \sqrt{\theta + \frac{\theta^2}{4}}} - 1 \right) \right]^{-0.5}$$

$$\theta = \frac{M_3^2}{M_2^3}$$

$$d = sign\left(\frac{M_2}{M_3}\right)$$

$$a = b \cdot ln\left(\frac{M_2}{M_3} \cdot d \cdot exp\left(\frac{1}{2b^2}\right) \left(exp\left(\frac{1}{b^2}\right) - 1\right) \left(exp\left(\frac{1}{b^2}\right) + 2\right) \right)$$

$$c = M_1 - d \cdot exp\left(\frac{1}{2b^2} - \frac{a}{b}\right)$$

Valuation (Cont)

- After some math, we get the present value of a call basket option as

$$PV_C = (c - K)\left(1 - \Phi\left(b \cdot ln\left(\frac{K-c}{d}\right) + a\right)\right)D$$

$$+d \cdot exp\left(-\frac{a}{b} + \frac{1}{2b^2}\right)\left(1 - \Phi\left(b \cdot ln\left(\frac{K-c}{d}\right) + a - \frac{1}{b}\right)\right)D$$

where D is the discount factor.

Practical Guide

- This model assumes that the basket price can be approximated by a lognormal distribution with moments matched to the distribution of the weighted sum of the individual stock prices.
- The asset value can be accurately expressed using a volatility skew model. This represents best market practice.
- Interest rates are deterministic.
- The model can be easily extended to price an Asian basket option by including a period of dates in the averaging schedule, i.e.,

$$R = \sum_{j=1}^{m} \sum_{i=1}^{n} w_i W_j S_{ij}/F_i$$

where W_j is the weight for schedule time ,

A Real World Example

Face Value	**87.5**
Currency	USD
Digital Rebate	1
Maturity Date	6/16/2017
Call or Put	Call
Buy or Sell	Sell
Position	-21800
Underlying Assets	Initial Fixing
CTXS.O	87.5
LOGM.O	87.5

Thank You

You can find more details at

https://finpricing.com/lib/EqOption.html