Interest Rate Capped Swap Valuation and Risk

Summary

- Capped Swap Definition
- Floored Swap Definition
- Valuation
- A real world example

Capped Swap Definition

- A capped swap is an interest rate swap with a cap where the floating rate of the swap is capped at a certain level.
- It limits the risk of the floating rate payer to adverse movements in interest rates.
- Given the optionality, an up-front fee or premium has to be paid by the floating rate payer.
- A capped swap can be decomposed as an interest rate swap plus an interest rate cap.

Floored Swap Definition

- A floored swap is an interest rate swap with a floor where the floating rate of the swap is floored at a certain level.
- It limits the risk of the floating rate receiver to adverse movements in interest rates.
- Given the optionality, an up-front fee or premium has to be paid by the floating rate receiver.
- A floored swap can be decomposed as an interest rate swap plus an interest rate floor.

Capped Swap

Valuation

- There are four types of capped or floored swaps.
  - Capped payer swap
  - Capped receiver swap
  - Floored payer swap
  - Floored receiver swap
- The present value of a capped payer swap is given by

$$PV_{CappedPayerSwap}(t) = PV_{float}(t) - PV_{fixed}(t) - PV_{cap}(t)$$

where

PV_{float} is the present value of the floating leg of the underlying swap;
PV_{fixed} is the present value of the fixed leg of the underlying swap;
PV_{cap} is the present value of the embedded cap.

Capped Swap

Valuation (Cont)

- The present value of a capped receiver swap can be expressed as

$$PV_{CappedReceiverSwap}(t) = PV_{fixed}(t) - PV_{float}(t) + PV_{cap}(t)$$

- The present value of a floored payer swap can be represented as

$$PV_{FlooredPayerSwap}(t) = PV_{float}(t) - PV_{fixed}(t) + PV_{floor}(t)$$

Where PV_{floor} is the present value of the embedded floor.

- The present value of a floored receiver swap can be computed as

$$PV_{FlooredReceiverSwap}(t) = PV_{fixed}(t) - PV_{float}(t) - PV_{floor}(t)$$

Capped Swap

Valuation (Cont)

- The present value of the fixed leg is given by

$$PV_{fixed}(t) = RN\sum_{i=1}^{n}\tau_i D_i$$

where R – the fixed rate; N – the notional; τ_i – the day count fraction for period $[T_{i-1}, T_i]$; $D_i = D(t, T_i)$ – the discount factor.

- The present value of the floating leg is given by

$$PV_{float}(t) = N\sum_{i=1}^{n}(F_i + s)\tau_i D_i$$

where s – the floating spread; $F_i = F(t; T_{i-1}, T_i) = \frac{1}{\tau_i}\left(\frac{D_{i-1}}{D_i} - 1\right)$ – the simply compounded forward rate

Capped Swap

Valuation (Cont)

- The present value of the cap is given by

$$PV_{cap}(t) = N \sum_{i=1}^{n} \tau_i D_i (F_i \Phi(d_1) - K\Phi(d_2))$$

where $d_{1,2} = \left(\ln\left(\frac{F_i}{K}\right) \pm 0.5\sigma_i^2 T_i\right) / (\sigma_i \sqrt{T_i})$ and Φ – the cumulative normal distribution function.

- The present value of the floor is given by

$$PV_{cap}(t) = N \sum_{i=1}^{n} \tau_i D_i \left(K\Phi(-d_2) - F_i \Phi(-d_1)\right)$$

Capped Swap

A real world example

Cap/Floor specification		Underlying swap specification			
		Leg 1		Leg 2	
Buy Sell	Buy				
Cap Floor	Floor	Currency	USD	Currency	USD
Strike	0.001	Day Count	dcAct360	Day Count	dcAct360
Trade Date	11/3/2016	Leg Type	Fixed	Leg Type	Float
Start Date	11/4/2016	Notional	200000000	Notional	200000000
Maturity Date	11/2/2020	Payment Freq	1M	Payment Freq	1M
Currency	USD	Pay Receive	Pay	Pay Receive	Receive
Day Count	dcAct360	Star tDate	11/4/2016	Start Date	11/4/2016
Notional	200000000	End Date	11/1/2020	End Date	11/1/2020
Pay Receive	Receive	Fixed Rate	0.01043	Spread	0
Payment Freq	1M			Index specification	
Index specification				Type	LIBOR
Day count	dcAct360			Tenor	1M
Tenor	1M			Day Count	dcAct360
Type	LIBOR				

Thanks!

Reference:
https://finpricing.com/knowledge.html