Convertible Bond

Introduction

Convertible bonds can be thought of as normal corporate bonds with embedded options, which enable the holder to exchange the bond asset for the issuer's stock. Having properties of both stocks and bonds, convertibles can be an attractive choice for investors and have tended to have lower risk.

A convertible bond has an embedded call option that gives bondholders the right to convert their bonds into equity at a given time for a predetermined number of shares in the issuing company. Whereas a reverse convertible bond has an embedded put option that gives the issuer the right to convert the bond's principal into shares of equity at a set date.

Convertible bonds typically have lower yields than the yields on similar bonds without the convertible option. Reverse convertible bonds usually have shorter terms to maturity and higher yields than most other bonds.

Most convertible bonds are subordinated debt of the issuer. In the event of bankruptcy, the claims of other bondholders take priority over convertible bondholders, who themselves have priority over owners of the preferred and common stock.

Issuers have several reasons to use convertible financing. By issuing convertibles they can lower their cost of debt funding compared to straight debt alone. Lower-credit companies who may not be able to access the straight debt market can often still issue convertible debt. Companies who anticipate equity appreciation can use convertibles to defer equity financing to a time when growth has been achieved.

Investors find several features of convertibles appealing. They offer greater satiability of income than common stock. They provide a yield that is often higher than the dividend yield of common stock. Finally, because they are often theoretically underpriced, they may provide a cheap source of common stock volatility. This presentation gives an overview of convertible bond and valuation model.

Valuation

The valuation of convertible or reverse convertible bonds can be quite complex because of its hybrid nature as a normal bond and as an equity call/put option. Convertible prices can only be solved by numerical methods, such as, Monte Carlo simulation, tree/lattice approaches, or partial differentiao equation (PDE) solutions.

Three sources of randomness exist in a convertible bond: the stock price, the interest rate, and the credit spread. Interest rate is assumed to be constant as the effect of a stochastic interest rate on convertible bond prices is so small that it can be neglected. Accurately modeling the equity process appears crucial.

Since convertible bonds are issued mainly by start-up or small companies (while more established firms rely on other means of financing), credit risk plays an important role in the valuation.

The value of the convertible at each node is divided into two components: a component of bond and a component of stock. The PDE of the equity component G is given by

$$\frac{\partial G}{\partial t} + 0.5\sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} + (r - q + h(1 - \varphi_s))S\frac{\partial G}{\partial S} - (r + h(1 - \varphi_s)G = 0$$

where

S	the stock price
r	the interest rate
q	the dividend
h	the hazard rate
φ_s	the equity recovery rate

The PDE of the bond component B is

$$\frac{\partial B}{\partial t} + 0.5\sigma^2 S^2 \frac{\partial^2 B}{\partial S^2} + (r - q + h(1 - \varphi_s))S\frac{\partial B}{\partial S} - \big(r + h(1 - \varphi_b)\big)B = 0$$

where φ_b is the bond recovery rate

The final conditions at maturity *T* can be generalized as

$$G_T = \begin{cases} \eta S_T & if\ \eta S_T > min\big[P_c, max\big(P_p, N + C\big)\big] \\ 0 & otherwise \end{cases}$$

$$B_T = \begin{cases} min\big[P_c, max\big(P_p, N + C\big)\big] & if\ \eta S_T \le min\big[P_c, max\big(P_p, N + C\big)\big] \\ 0 & otherwise \end{cases}$$

where

N denotes the bond principal
C denotes the coupon
P_c denotes the call price,
P_p denotes the put price
η denotes the conversion ratio.

The final conditions tell us that the convertible bond at the maturity is either a debt or an equity.

The upside constraints at time $t \in [0, T]$ are

$$\begin{cases} G = \eta S, \;\; B = 0 & if \;\; \eta S_T > min\left[P_c, max\left(P_p, \tilde{L}\right)\right] \\ G = 0, \;\; B = P_p & else\ if \quad \tilde{L} \le P_p \\ G = 0, \;\; B = P_c & else\ if \quad \tilde{L} \ge P_c \\ G = \tilde{G}, \;\; B = \tilde{B} & else \end{cases}$$

where

$\tilde{L} = \tilde{B} + \tilde{G}$ is the continuation value of the convertible bond
$\tilde{B}$ is the continuation value of the bond component
$\tilde{G}$ is the continuation value of the equity component.

Reference:
https://finpricing.com/lib/EqConvertible.html