



# Callable Bond and Vaulation

## Summary

- ◆ Callable Bond Definition
- ◆ The Advantages of Callable Bonds
- ◆ Callable Bond Payoffs
- ◆ Valuation Model Selection Criteria
- ◆ LGM Model
- ◆ LGM Assumption
- ◆ LGM calibration
- ◆ Valuation Implementation
- ◆ A real world example

# Callable Bond

## Callable Bond Definition

- ◆ A callable bond is a bond in which the issuer has the right to call the bond at specified times (callable dates) from the investor for a specified price (call price).
- ◆ At each callable date prior to the bond maturity, the issuer may recall the bond from its investor by returning the investor's money.
- ◆ The underlying bond can be a fixed rate bond or a floating rate bond.
- ◆ A callable bond can therefore be considered a vanilla underlying bond with an embedded Bermudan style option.
- ◆ Callable bonds protect issuers. Therefore, a callable bond normally pays the investor a higher coupon than a non-callable bond.

### Advantages of Callable Bond

- ◆ Although a callable bond is a higher cost to the issuer and an uncertainty to the investor comparing to a regular bond, it is actually quite attractive to both issuers and investors.
- ◆ For issuers, callable bonds allow them to reduce interest costs at a future date should rate decrease.
- ◆ For investors, callable bonds allow them to earn a higher interest rate of return until the bonds are called off.
- ◆ If interest rates have declined since the issuer first issues the bond, the issuer is like to call its current bond and reissues it at a lower coupon.

# Callable Bond

## Callable Bond Payoffs

- ◆ At the bond maturity  $T$ , the payoff of a callable bond is given by

$$V_c(t) = \begin{cases} F + C & \text{if not called} \\ \min(P_c, F + C) & \text{if called} \end{cases}$$

where  $F$  – the principal or face value;  $C$  – the coupon;  $P_c$  – the call price;  $\min(x, y)$  – the minimum of  $x$  and  $y$

- ◆ The payoff of the callable bond at any call date  $T_i$  can be expressed as

$$V_c(T_i) = \begin{cases} \bar{V}_{T_i} & \text{if not called} \\ \min(P_c, \bar{V}_{T_i}) & \text{if called} \end{cases}$$

where  $\bar{V}_{T_i}$  – continuation value at  $T_i$

## Model Selection Criteria

- ◆ Given the valuation complexity of callable bonds, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numerical solution to price them numerically.
- ◆ The selection of interest rate term structure models
  - ◆ Popular interest rate term structure models:  
Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - ◆ HJM and LMM are too complex.
  - ◆ Hull-White is inaccurate for computing sensitivities.
  - ◆ Therefore, we choose either LGM or QGM.

## Model Selection Criteria (Cont)

- ◆ The selection of numeric approaches
  - ◆ After selecting a term structure model, we need to choose a numerical approach to approximate the underlying stochastic process of the model.
  - ◆ Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  - ◆ Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  - ◆ Therefore, we choose either PDE or lattice.
- ◆ Our decision is to use LGM plus lattice.

# Callable Bond

## LGM Model

- ◆ The dynamics

$$dX(t) = \alpha(t)dW$$

where  $X$  is the single state variable and  $W$  is the Wiener process.

- ◆ The numeraire is given by

$$N(t, X) = (H(t)X + 0.5H^2(t)\zeta(t))/D(t)$$

- ◆ The zero coupon bond price is

$$B(t, X; T) = D(T)\exp(-H(t)X - 0.5H^2(t)\zeta(t))$$



# Callable Bond

## LGM Assumption

- ◆ The LGM model is mathematically equivalent to the Hull-White model but offers
  - ◆ Significant improvement of stability and accuracy for calibration.
  - ◆ Significant improvement of stability and accuracy for sensitivity calculation.
- ◆ The state variable is normally distributed under the appropriate measure.
- ◆ The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfectly correlated.

# Callable Bond

## LGM calibration

- ◆ Match today's curve  
At time  $t=0$ ,  $X(0)=0$  and  $H(0)=0$ . Thus  $Z(0,0;T)=D(T)$ . In other words, the LGM automatically fits today's discount curve.
- ◆ Select a group of market swaptions.
- ◆ Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

# Callable Bond

## Valuation Implementation

- ◆ Calibrate the LGM model.
- ◆ Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- ◆ Calculate the payoff of the callable bond at each final node.
- ◆ Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- ◆ Compare exercise values with intrinsic values at each exercise date.
- ◆ The value at the valuation date is the price of the callable bond.

# Callabe Bond

## A real world example

Bond specification		Callable schedule	
Buy Sell	Buy	Call Price	Notification Date
Calendar	NYC	100	1/26/2015
Coupon Type	Fixed	100	7/25/2018
Currency	USD		
First Coupon Date	7/30/2013		
Interest      Accrual Date	1/30/2013		
Issue Date	1/30/2013		
Last Coupon Date	1/30/2018		
Maturity Date	7/30/2018		
Settlement Lag	1		
Face Value	100		
Pay Receive	Receive		
Day Count	dc30360		
Payment Frequency	6		
Coupon	0.015		



# Thanks!



You can find more details at

<https://finpricing.com/lib/IrCurveIntroduction.html>

