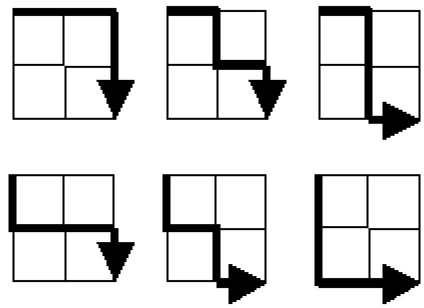
Problem 15: Lattice Paths

Description

Starting in the top left corner of a 2×2 grid, there are 6 routes (without backtracking) to the bottom right corner.



How many routes are there through a 20×20 grid?

Solution

Notice they specify that you only can move down or right! This gives us two possible moves at any given point. A BFS will be too cumbersome in a 20x20 grid situation, so let's try representing the above 2x2 grid in a series of moves. We will use R to denote right and D to denote down.

(left to right and top to bottom): RRDD, RDRD, RDDR, DRRD, DRDR, DDRR

At this point you notice that we always have to go 2 moves to the right and two moves down to reach the other end of the maze or grid. So all our paths will be of size 2n where n is the length/width of the grid. Also notice that of size 2*n, there will be n right moves, and n down moves. We know that it will be one or the other, so if we find the position of all the right moves (or down moves) we know that all the other spaces will be the opposite (i.e. if we find all places for right moves, we know that any empty spaces are down moves). So now the problem becomes choosing n positions out of 2n possible positions. If you're familiar with combinatorics, this is a binomial coefficient and can be calculated using a formula:

$$\binom{n}{k} = \frac{n!}{k!\ * (n-k)!}$$

In our case we are doing 2n choose n, so our formula will be the following:

$$\binom{2n}{n} = \frac{(2n)!}{n!\ * (2n-n)!} = \frac{(2n)!}{n! * n!}$$

This formulation in code will be incredibly quick!