Effects of Ingredient Concentrations in Cake Mixes on Taste

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Introduction

The purpose of this study is to determine what effects of varying concentrations of ingredients have on cake taste. This would allow both individuals and cake mix companies to feel confident in knowing how tweaking their recipes will affect the appeal of their cakes to most people. More broadly, this relatively accessible study should illustrate elements of proper statistical design that people should use when conducting their own experiments, taste testing or otherwise.

Material & Methods

The data to be used comes from Box, Bisgaard, and Fung [1 p. 124], who use the data as an example of Taguchi's experimental design, although they do not actually present any methods or statistical analysis of the data. They present the results of cakes scored by a taste panel, with values ranging from 1 to 7. Each cake was made by varying the 3 ingredients of flour, shortening, and eggs at what was considered either a low or high level. The experiment was replicated five times.

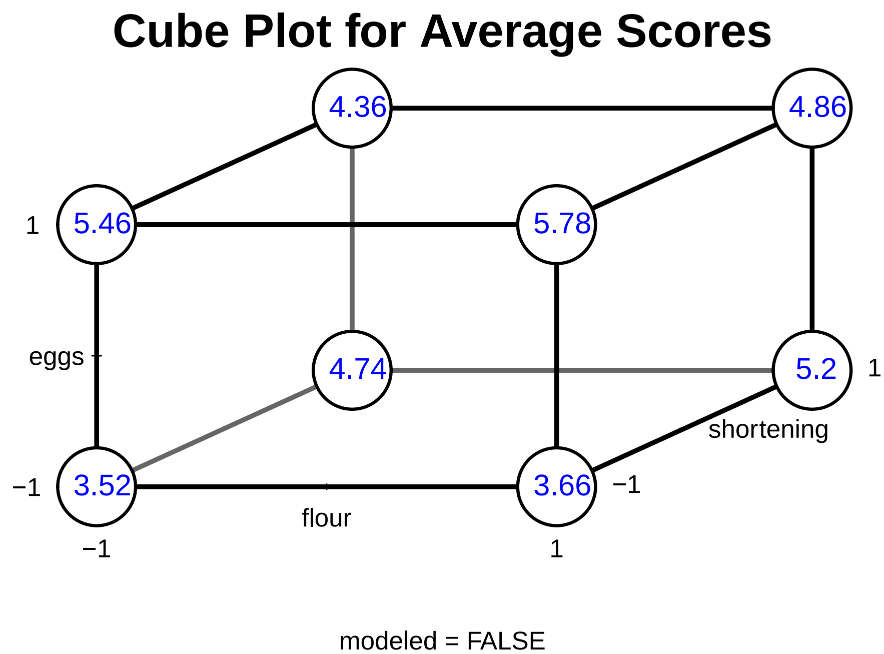
Because we have data at each factor level combination and there are 2 levels for each of 3 factors, we will employ a full 2^3 factorial design. This will allow us to precisely examine the effect of varying each ingredient because we can get an estimate for each main and interaction effect, which is the main purpose of our study.

The low level of an ingredient is indicated by a -1 and high level by a 1. The data is shown below, where each Rep represents the taste score for that replication.

Table 1: Data from cake ingredient experiment

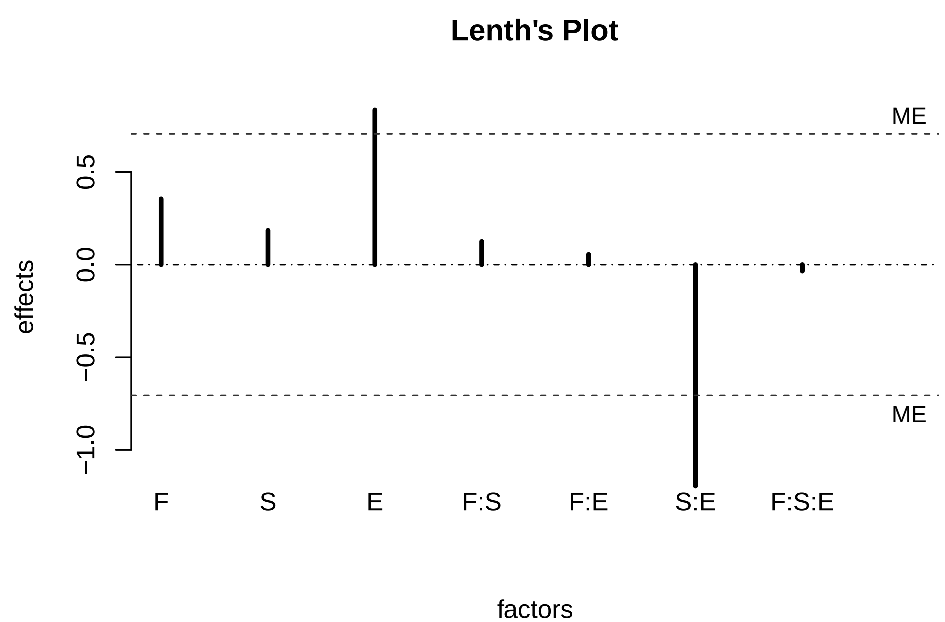
Flour	Shortening	Eggs	Rep 1	Rep 2	Rep 3	Rep 4	Rep 5
-1	-1	-1	3.1	1.1	5.7	6.4	1.3
1	-1	-1	3.2	3.8	4.9	4.3	2.1
-1	1	-1	5.3	3.7	5.1	6.7	2.9
1	1	-1	4.1	4.5	6.4	5.8	5.2
-1	-1	1	6.3	4.2	6.8	6.5	3.5
1	-1	1	6.1	5.2	6.0	5.9	5.7
-1	1	1	3.0	3.1	6.3	6.4	3.0
1	1	1	4.5	3.9	5.5	5.0	5.4

We can get an idea of the average effects of each factor with the following cube plot:



It shows the average taste score across the 5 replications for each factor level combination. For example, the average score at a low level of all 3 ingredients is 3.52. At first glance, it appears as if a higher level of flour is always better, while only one of shortening or eggs should be high to produce the best tasting cake.

We can get a better idea of which factorial effects are significant with Lenth's method, which computes a margin of error (ME) associated with a pseudo standard error (PSE) of the factorial effects at some significance alpha [2]. A simultaneous margin of error (SME) can also be computed that accounts for multiple testing. These values are displayed below along with a plot of the method applied to the factors in our study.



```
##     alpha       PSE        ME       SME
## 0.0500000 0.1875000 0.7057731 1.6890576
```

It appears as if the interaction between shortening and eggs is quite significant, which matches our earlier thoughts. Flour does not seem to be a significant effect in this method however.

We fit a linear regression model to estimate the factorial effects, where a factor takes the value -1 at a low level and 1 at a high level for that run. The interactions take the product of the corresponding factors, e.g. the interaction of shortening and eggs takes 1 if both shortening and eggs are low. We model taste scores based on flour, shortening, and egg levels and their interactions.

Results

The estimates of the linear regression model are given below, along with the standard error and p-values.

```
##
## Call:
## lm.default(formula = taste ~ flour * shortening * eggs, data = pivot_longer(cake,
##     rep_1:rep_5, names_to = "rep", names_prefix = "rep_", names_transform = as.numeric,
##     values_to = "taste"))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -2.420 -1.055  0.130  0.690  2.880
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)             4.6975     0.2260  20.786   <2e-16 ***
## flour                   0.1775     0.2260   0.785   0.4380
## shortening              0.0925     0.2260   0.409   0.6850
## eggs                    0.4175     0.2260   1.847   0.0740 .
## flour:shortening        0.0625     0.2260   0.277   0.7839
## flour:eggs              0.0275     0.2260   0.122   0.9039
## shortening:eggs        -0.5975     0.2260  -2.644   0.0126 *
## flour:shortening:eggs  -0.0175     0.2260  -0.077   0.9388
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.429 on 32 degrees of freedom
## Multiple R-squared:  0.2607, Adjusted R-squared:  0.09898
## F-statistic: 1.612 on 7 and 32 DF,  p-value: 0.1678
```

Because the least squares estimate of each coefficient is 1/2 the difference in mean taste scores at each level of the factorial [2], we have to double the coefficient estimates and standard error to obtain confidence intervals for the factorial effects. We present intervals for all main and interaction effects, where main effects are the differences in mean taste scores when varying either flour, shortening, or eggs across all levels of the other factors, and interaction effects are the conditional differences when holding some levels constant.

```
##                             2.5 %     97.5 %
## (Intercept)            8.47431514 10.3156849
## flour                 -0.56568486  1.2756849
## shortening            -0.73568486  1.1056849
```

```
## eggs                      -0.08568486  1.7556849
## flour:shortening          -0.79568486  1.0456849
## flour:eggs                -0.86568486  0.9756849
## shortening:eggs           -2.11568486 -0.2743151
## flour:shortening:eggs -0.95568486  0.8856849
```

We see the only effect that does not contain 0 is the interaction between shortening and eggs, confirming our expectation of a statistically significant negative effect on taste when both are used at the same level. While we also expected a positive effect for flour, we can conclude that it was not large enough to be significant.

Conclusion

We conducted a 2^3 factorial study to determine the effects varying concentrations of flour, shortening, and eggs have on cake taste. We first examined the data graphically before fitting a linear regression model and calculating confidence intervals for each factorial effect. This yielded a statistically significant negative effect on taste for the interaction between shortening and eggs. The experiment could be replicated more times to obtain narrower confidence intervals and perhaps reveal a statistically significant positive effect for flour.

We can conclude that when making a cake, we should include a moderate combination of shortening and eggs, but not either little or a lot of both. This makes sense when we consider that shortening is a fat used to "shorten" or weaken the cake to make it light and airy [3], which applies to eggs too [4]. Our study confirms the hypothesis that having a lot of both ingredients would be redundant and might overpower the cake.

This study should also highlight the benefits of and general steps to take in analyzing a 2^3 full factorial experiment. If one had simply tried adding more egg to their cake mix when it already had a lot of shortening, they might have concluded that less egg is better without realizing the interaction effect taking place. Such an effect became clear in a 2^3 factorial experiment that tested all factor level combinations multiple times.

References

[1] George Box, Soren Bisgaard, and Conrad Fung. 1998. "An Explanation and Critique of Taguchi's Contributions to Quality Engineering." *Quality and Reliability Engineering International*, vol. 4, no. 2, pp. 123–31

[2] Nathan Taback. 2022. "Factorial Designs at Two Levels—2k Designs." *Design and Analysis of Experiments and Observational Studies Using R*, CRC Press LLC

[3] Becky Krystal. April 25, 2022. "Can you substitute butter for shortening? Here's what to consider." *Washington Post*, Retrieved April 1, 2023.

[4] "What do eggs do in a cake? We bet you under-'egg'-stimate them, don't you?" *Food Heaven Magazine*, Retrieved April 1, 2023.