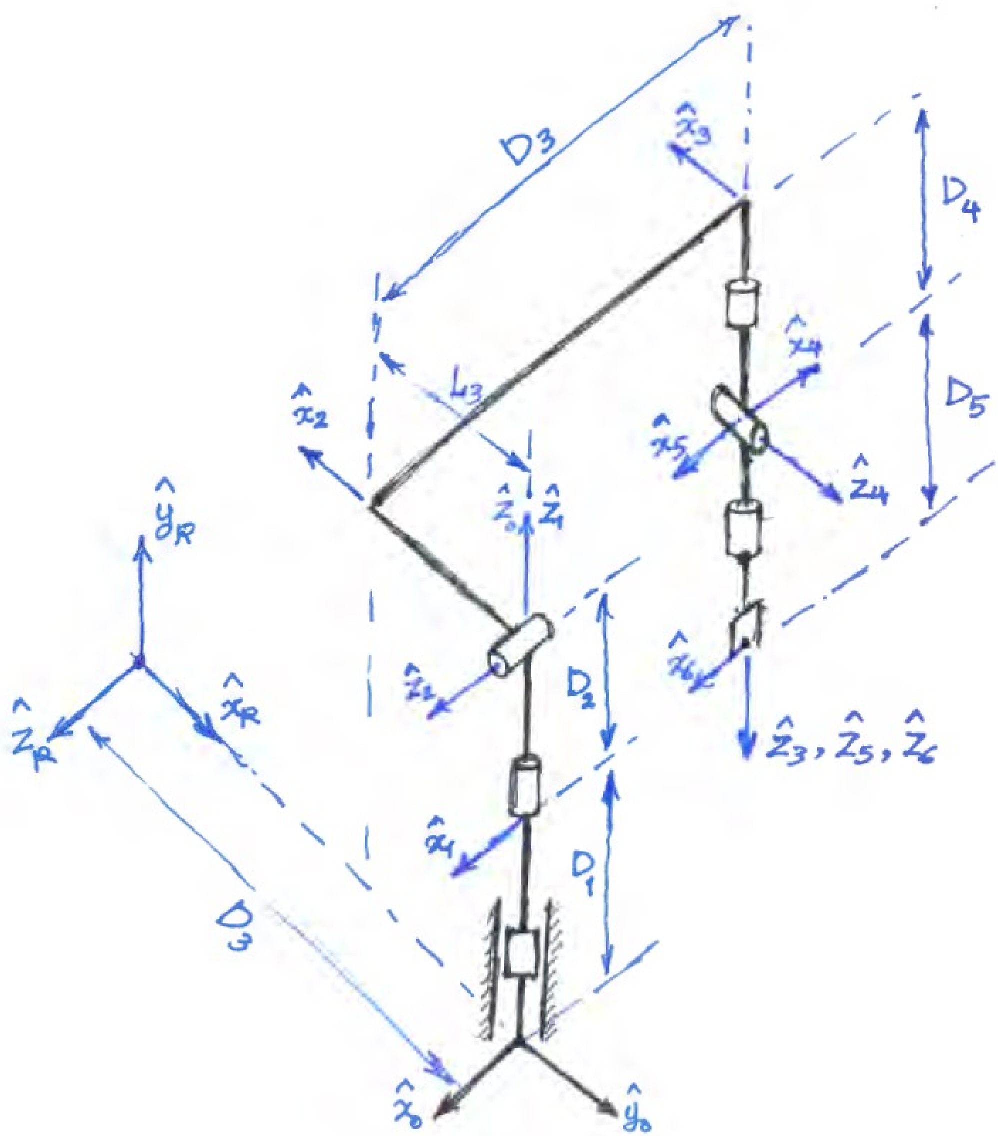
AER525 - Mid-term Test Solutions , 2014

Q1:

a) Pieper's Theorem: For a 6 d.o.f. manipulator , closed-form solutions are guaranteed if three adjacent joint axes intersect at a point. This includes manipulators with three consecutive parallel axes (since they meet at infinity.)

b) Redundant Manipulator: It is a manipulator that possesses degrees of freedom more than what is required to provide the desired end-effector position and orientation (or more than the required task space dimension.)

Q2:



Q3:

a) $d_B \hat{q}$: Correct. Differential of vector $\hat{q}$ with respect to fram $\{B\}$. ($d_B \hat{q}$ is a vector.) 2.5

b) $d_B({}^A q)$: Incorrect. ${}^A q$ is not a vector, but the expression of $\hat{q}$ in frame $\{A\}$. Hence, d_B for a set of scalars does not make sens

2.5

c) ${}^A\dot{q}$: Correct. It is the time derivative of vector $\hat{q}$ with respect to fram $\{A\}$ and expressed in frame $\{A\}$. (${}^A\dot{q}$ is a set of three scalars.)

2.5

d) ${}^A(d_B \hat{q})$: Correct. It is the differential of vector $\hat{q}$ with respect to frame $\{B\}$ and expressed in fram $\{A\}$. (It is a set of three scalars.)

2.5

Q4:

${}^{A}\dot{p}$ is the time derivative of vector $\hat{p}$ w.r.t. frame $\{A\}$ and expressed in $\{A\}$, and ${}^{B}\dot{p}$ is the time derivative of vector $\hat{p}$ w.r.t. $\{B\}$ and expressed in $\{B\}$.

The derivatives of a vector with respect to (viewed from) different frames are related through the Coriolis theorem:

$$\frac{d_A}{dt}\hat{p} = \frac{d_B}{dt}\hat{p} + \hat{\omega}_{AB} \times \hat{p}$$

2.5

The expressions of a vector in defferent coordinate frames are related through the rotation matrix:

$${}^{A}q = {}^{A}R_B\,{}^{B}q$$

2.5

Therefore,

$${}^{A}\left(\frac{d_A}{dt}\hat{p}\right) = {}^{A}R_B\,{}^{B}\left(\frac{d_B}{dt}\hat{p}\right) + {}^{A}\tilde{\omega}_{AB}\,{}^{A}p$$

$$\boxed{{}^{A}\dot{p} = {}^{A}R_B\,{}^{B}\dot{p} + {}^{A}\tilde{\omega}_{AB}\,{}^{A}p}$$

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where $\hat{\omega}_{AB}$ is the angular velocity (vector) of $\{B\}$ relative to $\{A\}$, and ${}^{A}\tilde{\omega}_{AB}$ is the 3x3 skew-symmetric (tilt) form of the expression of $\hat{\omega}_{AB}$ in $\{A\}$.

Q5:

a)

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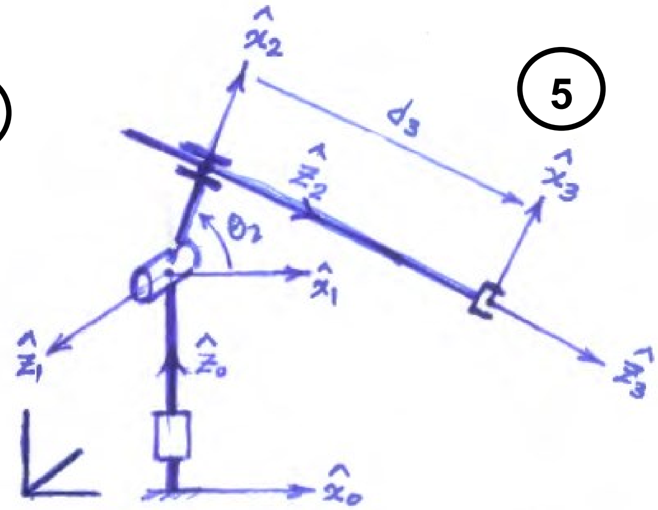
i	a_i	α_i	d_i	θ_i
1	0	90°	l_1	$\theta_1(0)$
2	l_2	90°	0	$\theta_2(\Theta_2)$
3	0	0	$d_3(D_3)$	0



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$$
{}^0T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^1T_2 = \begin{bmatrix} c_2 & 0 & s_2 & l_2c_2 \\ s_2 & 0 & -c_2 & l_2 s_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3 = \begin{bmatrix} c_1c_2 & s_1 & c_1s_2 & d_3c_1s_2 + l_2c_1c_2 \\ s_1c_2 & -c_1 & s_1s_2 & d_3s_1s_2 + l_2s_1c_2 \\ s_2 & 0 & -c_2 & -d_3c_2 + l_2s_2 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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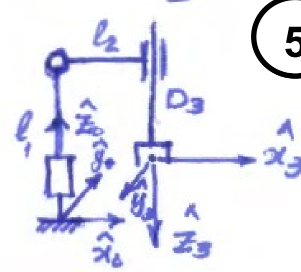
$$
\text{CHECK: } \begin{cases} \theta_1 = 0 \\ \theta_2 = 0 \\ d_3 = D_3 \end{cases} \Rightarrow {}^0T_3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -D_3 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



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b)

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$$
{}^3x_0 = {}^3R_0\,{}^0x_0 = \left({}^0R_3\right)^T {}^0x_0 = \begin{bmatrix} c_1c_2 & s_1c_2 & s_2 \\ s_1 & -c_1 & 0 \\ c_1s_2 & s_1s_2 & -c_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^3x_0 = \begin{bmatrix} c_1c_2 \\ s_1 \\ c_1s_2 \end{bmatrix}
$$

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c)

From Forward Kinematics ($^0T_3$):

$$\begin{cases} P_x = d_3 c_1 s_2 + l_2 c_1 c_2 = c_1(d_3 s_2 + l_2 c_2) & ① \\ P_y = d_3 s_1 s_2 + l_2 s_1 c_2 = s_1(d_3 s_2 + l_2 c_2) & ② \\ P_z = -d_3 c_2 + l_2 s_2 + l_1 \Rightarrow P_z - l_1 = -d_3 c_2 + l_2 s_2 & ③ \end{cases}$$

$① \,\&\, ②$: $\tan\theta_1 = \dfrac{P_y \cdot (d_3 s_2 + l_2 c_2)}{P_x \cdot (d_3 s_2 + l_2 c_2)}$

For a general case: $d_3 s_2 + l_2 c_2 \neq 0$ (the special case will be considered in (e)). However, note that although the term can be cancelled from both numerator and denominator, it makes their sign ambiguous. It means that theoretically θ_1 can be in either the first or the third quadrant. Therefore, there exist 2 possible solutions for θ_1 theoretically:

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$$\theta_1 = \mathrm{ATAN2}\left(\frac{P_y}{P_x}\right) \quad \text{OR} \quad \theta_1' = \theta_1 + 180^\circ$$

$$①^2 + ②^2 + ③^2 \Rightarrow P_x^2 + P_y^2 + (P_z - l_1)^2 = d_3^2 + l_2^2$$

$$d_3 \geqslant 0 \Rightarrow d_3 = \sqrt{P_x^2 + P_y^2 + (P_z - l_1)^2 - l_2}$$

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Having d_3, ③ is a simple trigonemtric equation for θ_2 (case c). Therefore, 2 possible solutions exist for θ_2:

$$\theta_2 = \mathrm{ATAN2}\left(\frac{d_3}{l_2}\right) - \mathrm{ATAN2}\left(\frac{-P_z + l_1}{+\sqrt{P_x^2 + P_y^2}}\right) \quad \text{(corresponding to } \theta_1\text{)}$$

OR

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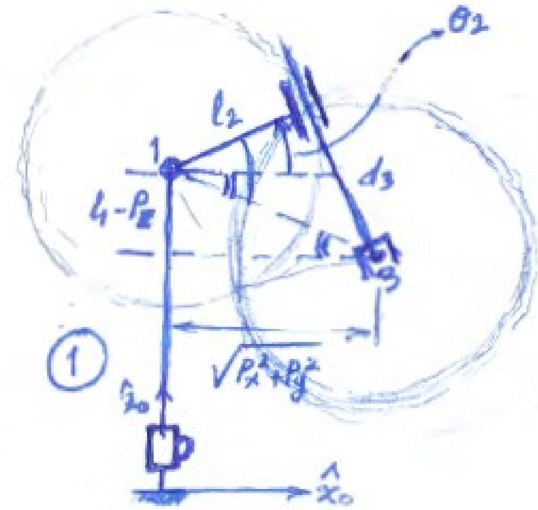
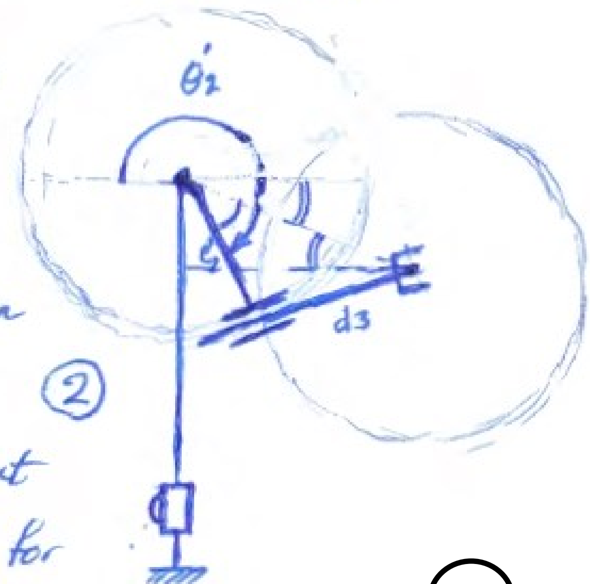
$$\theta_2' = \mathrm{ATAN2}\left(\frac{d_3}{l_2}\right) - \mathrm{ATAN2}\left(\frac{-P_z + l_1}{-\sqrt{P_x^2 + P_y^2}}\right) \quad \text{(corresponding to } \theta_1'\text{)}$$

c) Discussion of I.K. solutions :

The problem has two possible solutions, as shown in the figures, both of which are within the range of joint variables, Hence physically possible.



Configuration ① corresponds to the first solution of θ_2 (and θ_1). The two segments (angles) in the solution of θ_2 are shown in the figure. The figure also shows that the two circles with centres 1 and 3 can intersect at two points. Therefore, a second solution is possible, but it can only happen if θ_1 rotates for 180° from that of configuration ①, because $d_3 \geqslant 0$. This will create



configuration ②, which corresponds to the second solution of θ_2 (and θ_1). Note that because $d_3 \geqslant 0$, the two possible solutions for θ_1 and θ_2 correspond to each other pairwise, and hence there do not exist 4 feasible combinations.