

UNIVERSITY OF TORONTO  
FACULTY OF APPLIED SCIENCE AND ENGINEERING  
**ROBOTICS (AER 525F)**  
**M.R. EMAMI**  
FINAL EXAMINATION  
December 18, 2014

Note: Rulers may be used in this test.

**2.5 Hours**  
**Exam Type: X**

**Question 1:**

- (a) Explain how a generic 6-d.o.f. manipulator should be designed to have an analytically solvable inverse kinematics model. (5)
- (b) Explain how using a transmission system can help the joint servo controllers in reducing the nonlinear and coupling effects. What are the pay-backs? (5)
- (c) Explain how the inaccuracy of the inverse dynamics model of a manipulator will affect the performance of its model-based control scheme. (5)
- (d) Given a table of desired joint displacements, does the joint-space motion control scheme still require a trajectory planner/generator? Why? (5)

**Question 2:**

For the spatial 4 d.o.f. manipulator shown in the figure ( $0^\circ \leq \theta_1, \theta_2, \theta_4 < 360^\circ$  and  $d_3 \geq 0$ ):

- (a) By using Standard Denavit-Hartenberg convention, define link coordinate frames and link parameters, arrange the D-H table, obtain the homogeneous transformation matrices, and then determine  ${}^0T_4$ . For  $\theta_1 = 0$  and links 2, 3 and 4 upright, check whether your computation is correct. (15)
- (b) Obtain the angular and linear (point  $E$ ) velocities of link 4 relative to and expressed in the base coordinate frame (as functions of joint speeds and manipulator configuration.) (15)
- (c) At each given (static) configuration, how much torque/force is needed at each joint to balance a force at the end-effector point  $E$  along  $\hat{x}_4$  with a unit magnitude? (10)
- (d) A camera is attached to the end-effector at point  $E$  with its view along the direction of  $\hat{x}_4$ . It records the motion of an object as it is heading right toward the camera (i.e., along negative  $\hat{x}_4$ ) with a speed of  $v$ . What is the absolute velocity of the object when it is 1m away from the camera and the manipulator configuration is at  $\theta_1 = \theta_2 = 0$ ? Obtain the answer as a function of  $d_3$ ,  $\theta_4$ ,  $v$ , and joint speeds. (10)

- (e) If joint 4 is fixed so that links 3 and 4 remain aligned at all times, find and show those static configurations where an external force at the end-effector point  $E$  in at least one direction can be totally borne by the structure, i.e., no joint torque/force is needed to balance it. (10)
- (f) If joint 4 is fixed so that links 3 and 4 remain aligned at all times, and joint 1 is fixed ( $\theta_1, \dot{\theta}_1 = 0$ ), and there is no wrench at the end-effector, derive the Lagrangian of the system. Assume links 2, 3 and 4 as rigid bodies with masses  $m_2, m_3$  and  $m_4$ , moments of inertia ( $I_{C_{22}}, I_2, I_3$  and  $I_4$ ), and centres of mass located at  $B, C$  and  $E$ , respectively. (10)
- (g) Considering conditions in (f), obtain the torque equation for joint 2. (5)
- (h) Suppose that a DC brush motor with the following characteristics is available for driving joint 2 (under conditions in (f)) using a gearbox with the ratio of  $\eta$ .

$r_a$ : armature resistance       $b_m$ : motor bearing viscous friction coefficient  
 $K_m$ : torque constant       $K_b$ : back emf constant       $I_m$ : motor inertia

Assuming that the motor inductance and load bearing friction are negligible, find the values of the proportional and derivative gains of a PD controller for joint 2 rotational displacement, if the closed-loop system response is critically damped with the natural frequency of  $\omega_n$ . (5)

