AER525 Fall 2015

Assignment 1

Due: 7-Oct-2015

Question 1 Which of the following matrices are valid rotation matrices and which are not? If not, why?

$$R_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{2}{3} \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 0.6562 & -0.4695 & 0.5908 \\ 0.3489 & 0.8829 & 0.3141 \\ -0.6691 & 0 & 0.7431 \end{bmatrix}, \qquad R_4 = \begin{bmatrix} 0 & -0.4695 & 0 \\ 0.7482 & 0 & -0.4286 \\ 0 & 0.7233 & 0 \end{bmatrix},$$

$$R_5 = \begin{bmatrix} 0.866 & 0 & 0.5 & 1 \\ 0 & 1 & 0 & 1 \\ -0.5 & 0 & 0.866 & 1 \end{bmatrix}, \qquad R_6 = \begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} & \frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix},$$

Question 2 A point ${}^{D}P = [2 \;\; 0 \;\; 2]^T$ is known relative to frame $\{D\}$ (see Figure 1). Frame $\{D\}$ is translated relative to frame $\{C\}$ by 3 units along $\hat{Z}_C$. Frame $\{C\}$ is both rotated about $\hat{Z}_B$ by 10° and translated by 6 units along $\hat{Y}_B$ relative to frame $\{B\}$. Finally, frame $\{B\}$ is rotated relative to frame $\{A\}$ by 15° about $\hat{X}_A$. What is ${}^{A}P$, the point represented in frame $\{A\}$?

Question 3 Show that if R_1, R_2, ..., R_n are all orthogonal matrices, their product is also an orthogonal matrix.

Question 4 A frame $\{B\}$ is rotated relative to a frame $\{A\}$ but they share the same origin. If a vector ${}^{A}G = [2 \;\; 1 \;\; 2]^T$ is known to lie along $\hat{X}_B$ and another vector ${}^{A}H = [-1 \;\; -2 \;\; 2]^T$ is known to lie along $\hat{Z}_B$, what is the equivalent axis of rotation $\hat{K}$ and angle θ about it for which frame $\{B\}$ is relative to frame $\{A\}$?

Question 5 Show the attachment of link frames for the four manipulators shown in Figures 2, 3, 4, and 5 and write their corresponding D-H Tables.

Question 6 Derive the kinematic model of the manipulator in Figure 6 with the end-effector tip being between the fingertips.

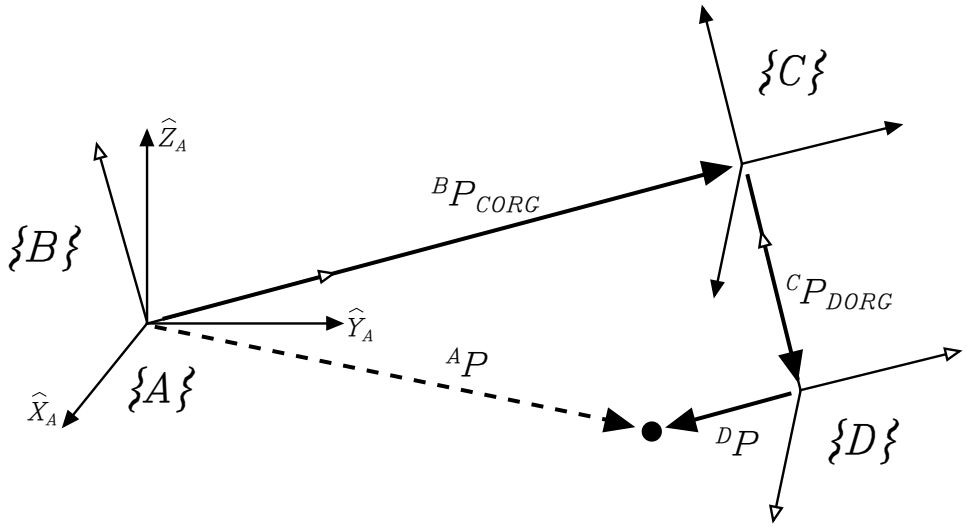

Figure 1: Overview of Question 2

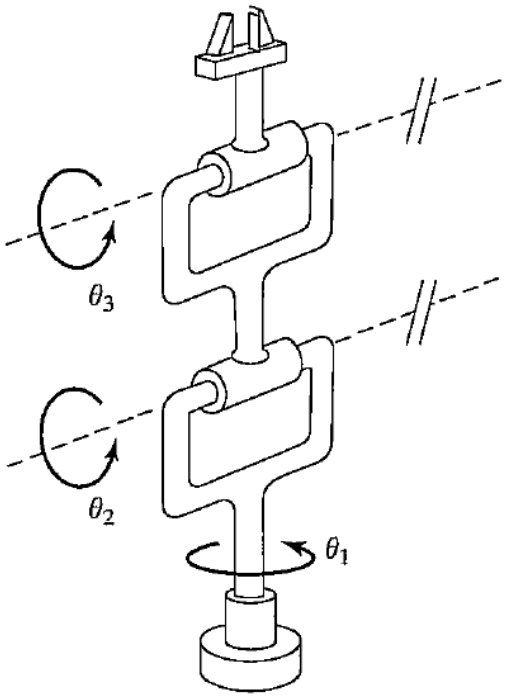

Figure 2: Question 5a

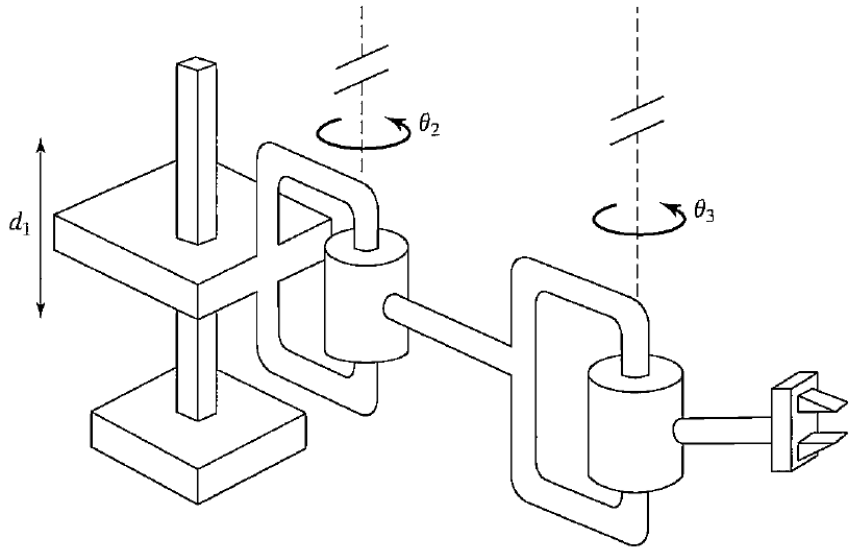

Figure 3: Question 5b

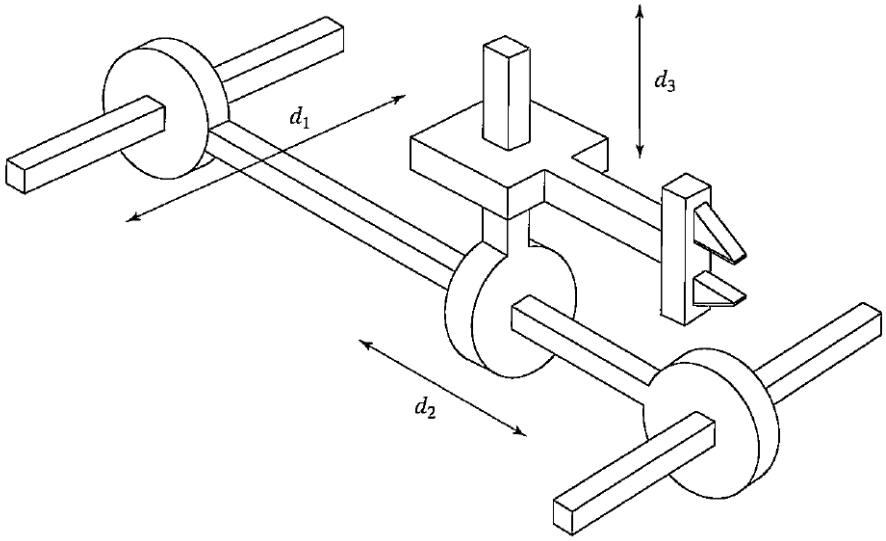

Figure 4: Question 5c

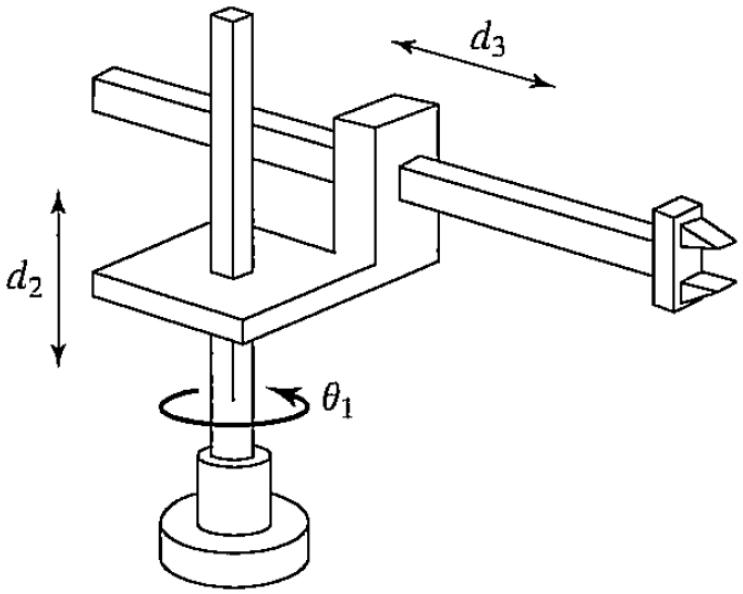

Figure 5: Question 5d

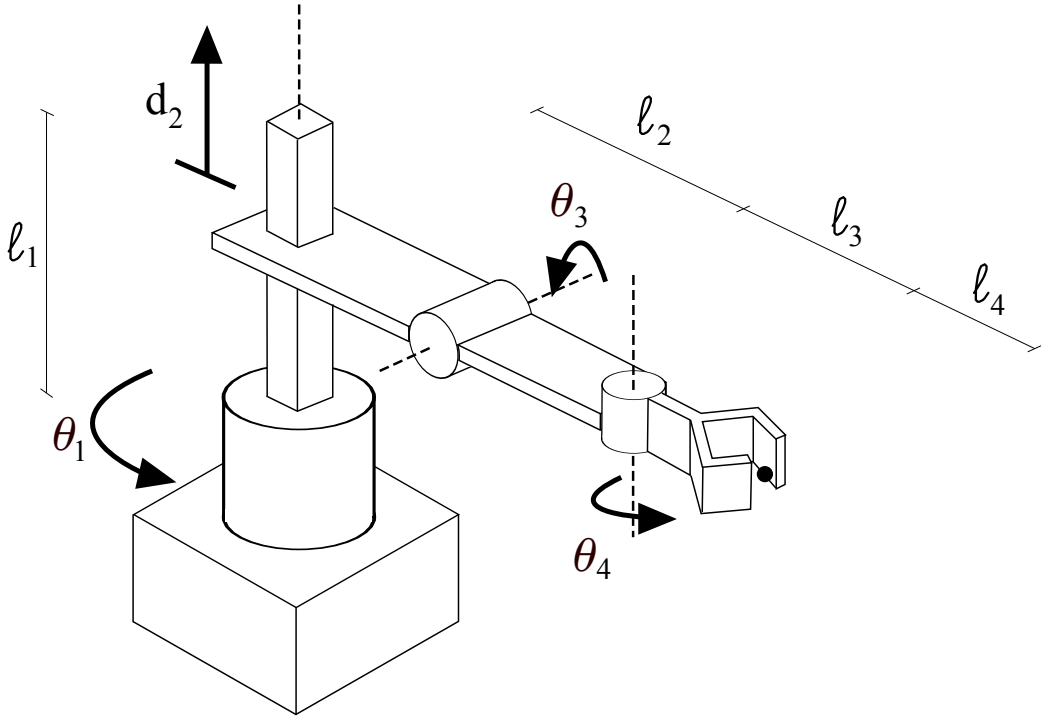

Figure 6: Manipulator for Question 6