University of Toronto

Faculty of Applied Science and Engineering

AER 525: ROBOTICS

Final Examination

December 18, 2018

Duration: 2 and 1/2 hrs

Total Marks: 100

Exam Type X (open book)

Calculator Type 2 (non-programmable)

Examiner: *Houman Hakima*

Attempt all five questions. Write your answers in the exam booklet(s) provided.

Question 1

For the manipulator shown in Fig. 1:

(a) Draw the schematic of the manipulator and attach link frames $\{0\}$ to $\{3\}$ according to the Denavit-Hartenberg convention discussed in class. *(5 marks)*

(b) Compute each individual link transformation matrix ${}^0_1T$, ${}^1_2T$, and ${}^2_3T$. *(15 marks)*

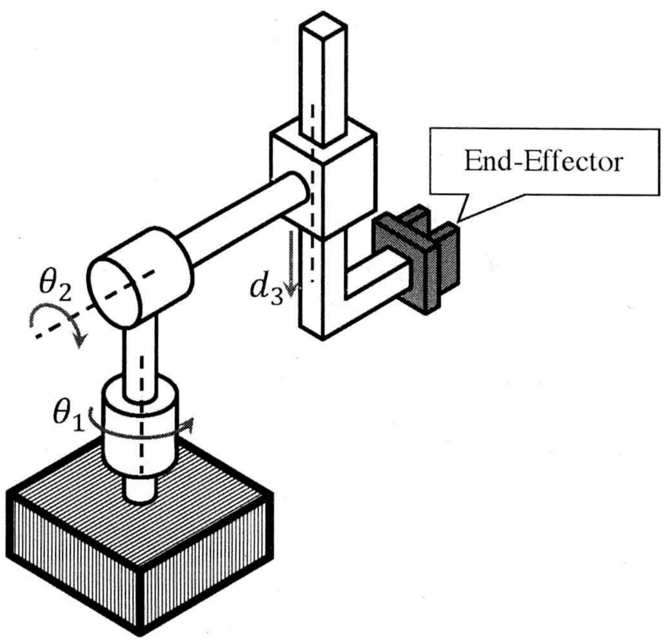

Fig. 1: An *RRP*-type spatial manipulator. Positive direction of motion is shown for each joint.

Question 2

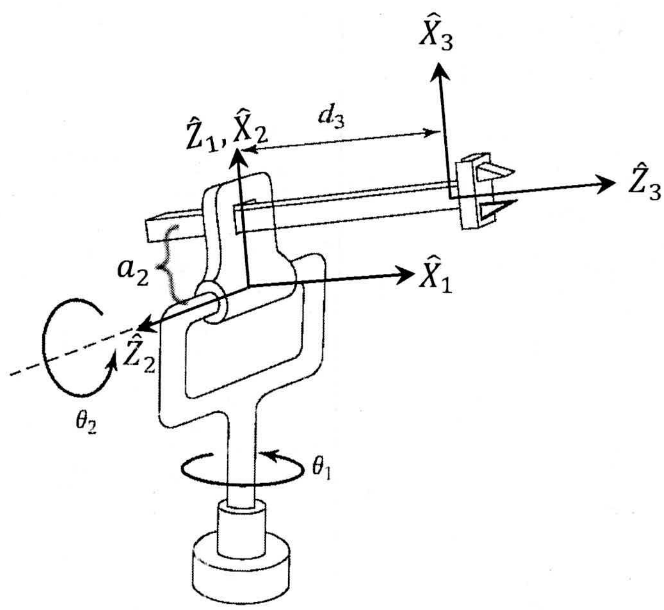

Fig. 2: An *RRP*-type spatial manipulator.

Consider the manipulator shown in Fig. 2. The position of frame $\{3\}$ with respect to frame $\{0\}$ is found to be:

$$^{0}P_{3ORG} = \begin{bmatrix} c_1c_2a_2 + c_1s_2d_3 \\ s_1c_2a_2 + s_1s_2d_3 \\ s_2a_2 - c_2d_3 \end{bmatrix}$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Derive the appropriate 3×3 Jacobian that relates the absolute linear velocity of the origin of frame $\{3\}$ to the joint rates. *(12.5 marks)*

Question 3

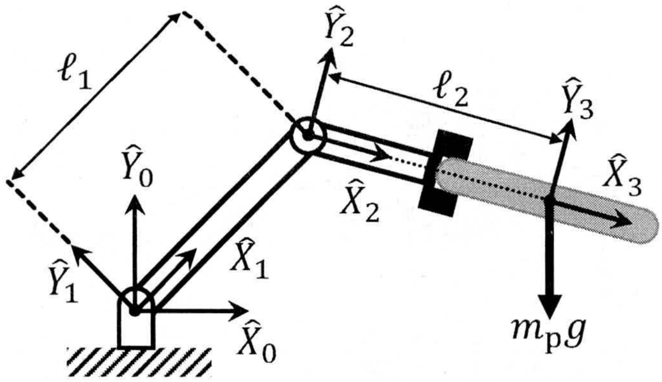

Fig. 3: A rigid planar manipulator holding a large payload. The system is in static equilibrium. The standard gravitational acceleration is denoted by g.

Consider the planar manipulator shown in Fig. 3. The end effector is holding a large payload with mass m_p. The force acting on $\{3\}$ (located at the mass centre of the payload) is the weight of the payload acting in the $-\hat{Y}_0$ direction.

(a) Assuming that the robot and payload are in static equilibrium, develop the expressions for ${}^3\mathcal{F}_3$, i.e., the generalized force exerted on the payload by the robot expressed in $\{3\}$. *(5 marks)*

(c) Obtain the expressions for the joint torques τ_1 and τ_2 that are required to maintain static equilibrium. *(15 marks)*

(d) Suppose that the required torque from joint 1 must be $\tau_1 = 1.0$ N·m (Newton meter) to maintain static equilibrium. However, the maximum torque the actuator can provide is only 0.2 N·m. With the help of a simple diagram, propose a transmission system that has the minimum "gear" ratio to account for the torque offset. Assume that the transmission system is ideal, i.e., no backlash or friction in the system. *(2.5 marks)*

Question 4

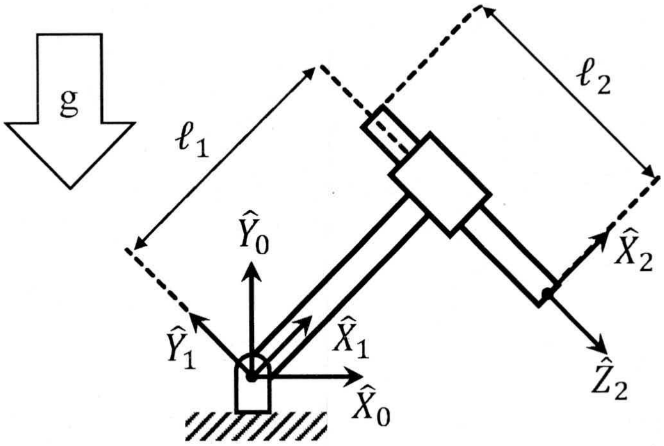

Fig. 4: A planar *RP*-type manipulator.

Consider the two-link manipulator shown in Fig. 4. For each link, the mass distribution is represented by a point-mass in the middle of the link. For link 1, the mass of link 1 is m_1 and the mass of link 2 is m_2. The following link transformation matrices are given:

$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1_2T = \begin{bmatrix} 1 & 0 & 0 & \ell_1 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(a) Sketch the approximate reachable workspace of the manipulator. The tip of the manipulator is the origin of frame $\{2\}$. The range of motion of joint 1 is $0° \leq \theta_1 \leq 180°$. *(2.5 marks)*

Derive the followings:

(b) Position of each link's centre of mass (CoM) relative to frame $\{0\}$. *(2.5 marks)*

(c) Absolute angular velocity of frame $\{1\}$ described in $\{1\}$, and the absolute angular velocity of frame $\{2\}$ described in $\{2\}$. *(5 marks)*

(d) The absolute linear velocity of each link's CoM. *(5 marks)*

(e) Absolute linear acceleration of frame $\{1\}$ described in $\{1\}$, and the absolute linear acceleration of frame $\{2\}$ described in $\{2\}$. *(10 marks)*

(f) Total potential energy of the manipulator. *(5 marks)*

Question 5

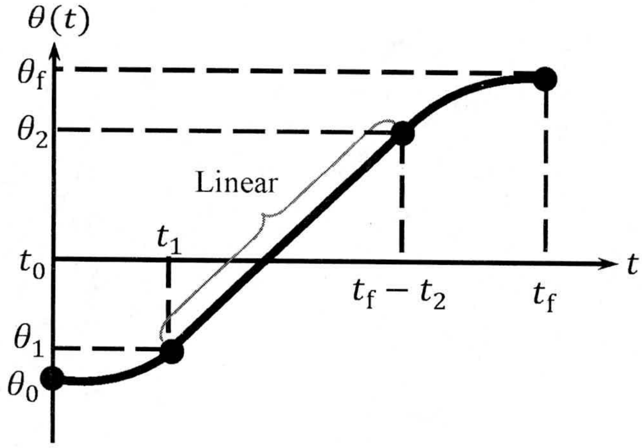

Fig. 5: Trajectory is designed using a linear function with two parabolic blends.

For the rest-to-rest joint trajectory shown in Fig. 5, the following parameters are prescribed:

$t_0 = 0$	
$t_1 = 2$ s	(duration of the first blend)
$t_2 = 3$ s	(duration of the second blend)
$t_f = 10$ s	(duration of the entire maneuver)
$\theta_1 = -10°$	(position at the end of first blend)
$\theta_2 = 15°$	(position at the beginning of second blend)

Answer the followings:

(a) Calculate the initial and final joint values, i.e., θ_0 and θ_f, respectively. *(10 marks)*

(b) Plot the time histories of the joint velocity and acceleration for the duration of maneuver (two separate plots). *(5 marks)*