UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
ROBOTICS (AER 525F)
MID-TERM EXAMINATION
October 18, 2013

Note: Rulers may be used in this test.

Time: 105 Minutes

Question 1:

Describe the following terms briefly (maximum 40 words for each, no formulation required):

- **a)** Manipulator Redundancy **(5)**
- **b)** Degeneracy of a Manipulator's Spherical Wrist **(5)**
- **c)** Dexterous Workspace of a Manipulator **(5)**
- **d)** Coriolis Theorem **(5)**

Question 2:

Consider the human body (up to the shoulders), when standing at a fixed location, as a manipulator, and calculate its number of d.o.f. Ignore the foot toes, but count the hand fingers and thumbs. Assume the base joints for thumbs and fingers as 2 d.o.f., the joints for wrists, shoulders, hips, ankles, and waist as 3 d.o.f., and consider the remaining joints, including the one connecting waist and torso, as 1 d.o.f. joints. How many d.o.f. do the hand fingers and thumbs add to the human manipulator? **(20)**

Question 3:

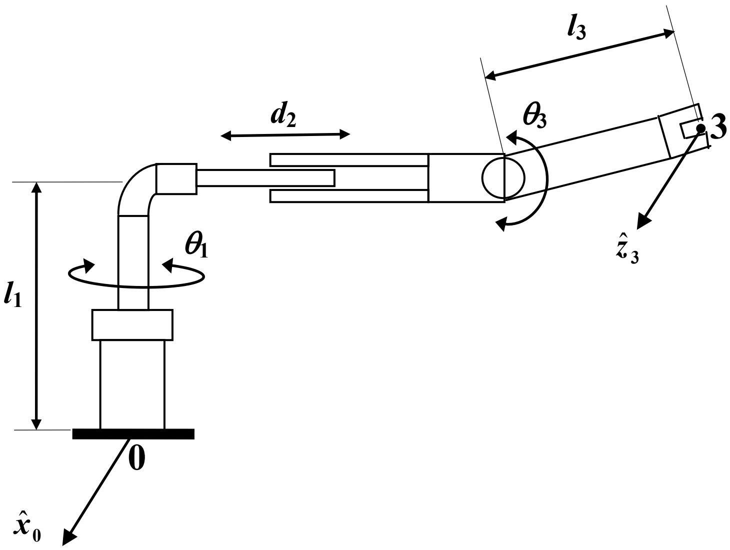
A small camera is positioned at $\begin{bmatrix}3 & 4 & 2\end{bmatrix}^T m$ with respect to a fixed coordinate frame. When the camera has a tilt (pitch) angle of *30*° and a pan (yaw) angle of *60*° (w.r.t. the fixed frame), it detects a light source at $\begin{bmatrix}1 & 0 & 0.5\end{bmatrix}^T m$ with respect to its own coordinates. What is the position of the light source with respect to the fixed coordinate frame? **(10)**

Question 4:

For the spatial 3-d.o.f. manipulator shown in the figure ($0^\circ \le \theta_1, \theta_3 < 360^\circ$, $d_2 \ge 0$):

a) By using Standard Denavit-Hartenberg convention, define link coordinate frames and link parameters, arrange the D-H table, and then determine ${}^0T_3$. For $\theta_1 = 0$ and link 3 vertical down, check whether your computation is correct. **(25)**

b) Having the location of the end-effector point (Point 3), expressed in the base frame, determine the corresponding joint variables. Show all possible solutions using simple sketches. **(25)**



$a_i \equiv$ the length of the common normal between $\hat{z}_{i-1}$ and $\hat{z}_i$ along $\hat{x}_i$ (link length);

$\alpha_i \equiv$ the angle between $\hat{z}_{i-1}$ and $\hat{z}_i$ measured about $\hat{x}_i$ (twist angle);

$d_i \equiv$ the distance from $\hat{x}_{i-1}$ to $\hat{x}_i$ measured along $\hat{z}_{i-1}$ (link offset);

$\theta_i \equiv$ the angle between $\hat{x}_{i-1}$ and $\hat{x}_i$ measured about $\hat{z}_{i-1}$ (joint angle);

$$
{}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\cos q = A \;\Rightarrow\; q = \pm \mathrm{Atan2}\left(\frac{\sqrt{1-A^2}}{A}\right)
$$

$$
-A\sin q + B\cos q = 0 \;\Rightarrow\; \begin{cases} q^1 = \mathbf{Atan2}\left(\dfrac{B}{A}\right) \\ q^2 = q^1 + 180^o \end{cases}
$$

$$
\left({}^A R_B\right)_{XYZ}(\gamma,\beta,\alpha) = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}
$$

(*Roll, Pitch, Yaw*)

$$
d.o.f. = \lambda\,(l-1) - \sum_{i=1}^{n} (\lambda - f_i)
$$

$$
-A\sin q + B\cos q = C \quad\Rightarrow\quad q = \mathrm{Atan}\,2\left(\frac{B}{A}\right) - \mathrm{Atan}\,2\left(\frac{C}{\pm\sqrt{A^2+B^2-C^2}}\right)
$$

$$
\begin{cases} A\cos q_1 + B\cos(q_1+q_2) + C\sin(q_1+q_2) = D \\ A\sin q_1 + B\sin(q_1+q_2) - C\cos(q_1+q_2) = H \end{cases} \;\Rightarrow\; \begin{cases} q_2 = 2\,\mathbf{Atan2}\left(\dfrac{C \pm \sqrt{C^2+B^2-F^2}}{B+F}\right) \\ q_1 = \mathbf{Atan2}\left(\dfrac{HM^2 - DMN}{DM^2 + HMN}\right) \end{cases}
$$

$$
\begin{cases} M = A + B\cos q_2 + C\sin q_2 \\ N = B\sin q_2 - C\cos q_2 \end{cases} \qquad F = \frac{D^2 + H^2 - A^2 - B^2 - C^2}{2A}
$$