UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 2015

LIEP Program - Mechanical Engineering

AER525H1S - ROBOTICS

Exam Type: X

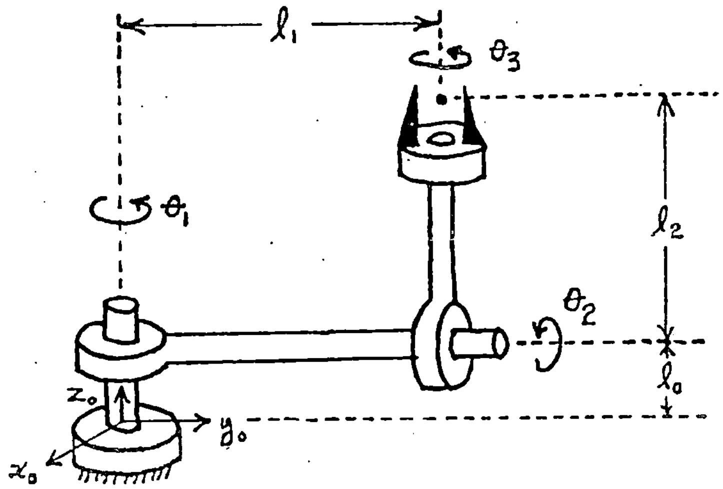
Examiner - C. J. Damaren

**Total Marks: 60**

Attempt all questions. The following notations are in effect:

$$c_i = \cos\theta_i,\quad s_i = \sin\theta_i,\quad c_{ij} = \cos(\theta_i + \theta_j),\quad s_{ij} = \sin(\theta_i + \theta_j)$$

1. Consider the three-link rigid robot manipulator in the figure below:



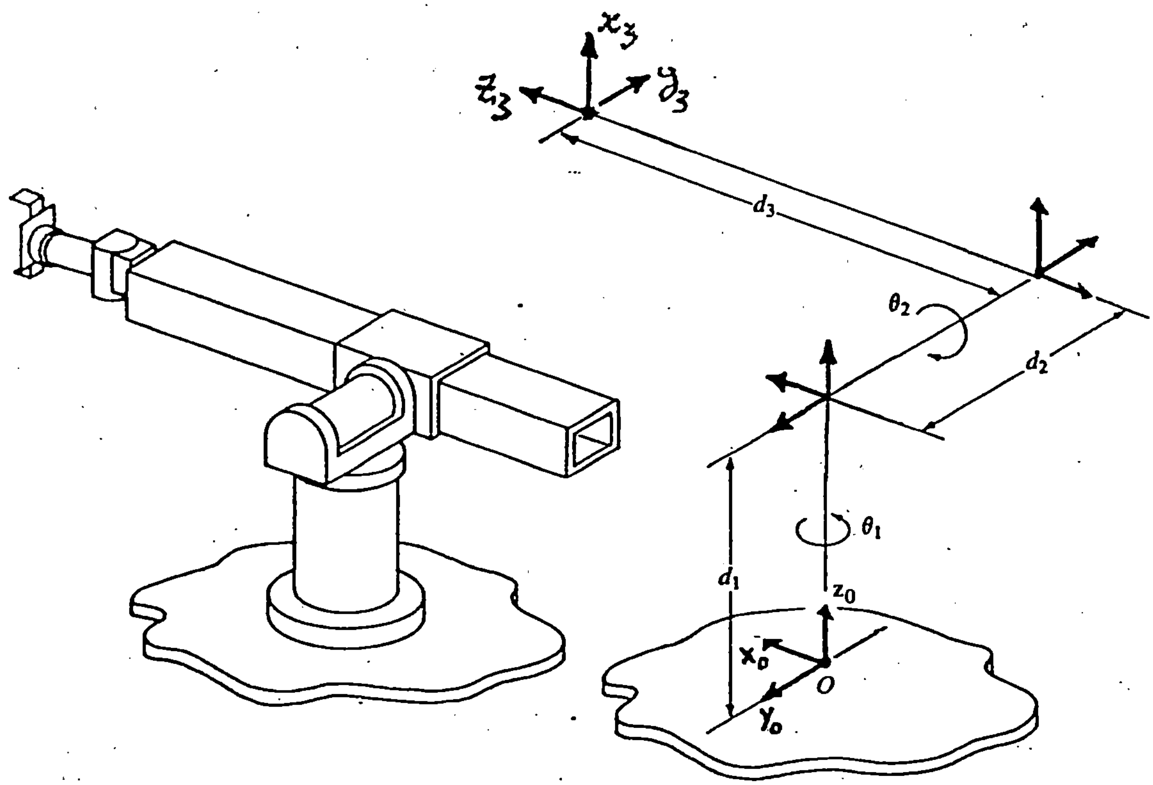

(a) Locate reference frames {1}, {2}, and {3} according to the Denavit-Hartenburg convention. What are the D-H parameters for each link?

**(6 marks)**

(b) Express the position (${}^0P_{3ORG}$) and orientation (${}^0_3\mathbf{R}$) of the end-effector in terms of the angles $\theta_1$, $\theta_2$, and $\theta_3$.

**(8 marks)**

2. Consider the first three joints of the Stanford manipulator shown below:



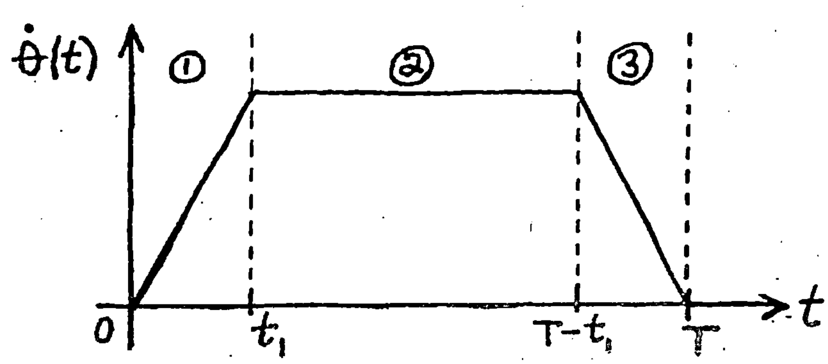

On the midterm examination, we showed that

$$^0P_{3\mathrm{ORG}} = \begin{bmatrix} d_3 c_1 s_2 + d_2 s_1 \\ d_3 s_1 s_2 - d_2 c_1 \\ -d_3 c_2 + d_1 \end{bmatrix}$$

Derive an appropriate $3 \times 3$ Jacobian matrix $^0\mathbf{J}$.

**(10 marks)**

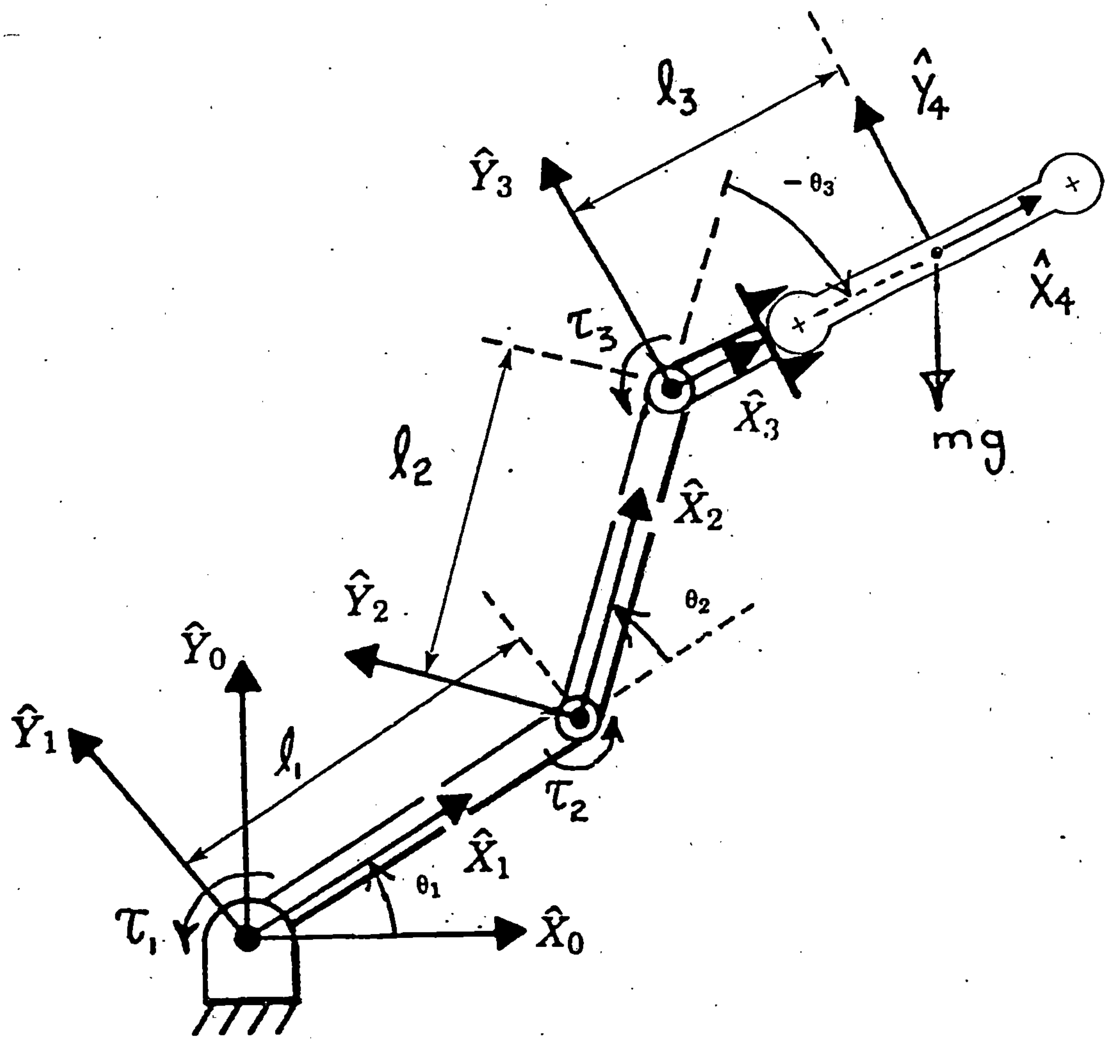
3. It is desired to plan a joint trajectory $\theta(t)$ that satisfies the end conditions $\theta(0) = 0$ and $\theta(T) = \theta_d$ where the desired angle, $\theta_d$, and the terminal time, $T$ are prescribed. The required rate profile is a trapezoidal function of time:




If in regions ① and ③, the joint acceleration satisfies $|\ddot{\theta}| = \alpha$ where $\alpha$ is a prescribed constant, determine the value of $t_1$ in terms of $\alpha$, $\theta_d$, and $T$.

**(6 marks)**

4. Consider the planar three-link rigid arm shown in the figure below. It is grasping a large rigid payload with mass $m$ in a gravitational field with acceleration due to gravity $g$. The forces acting on $\{4\}$ (located at the mass centre of the payload) are equivalent to $mg$ (the weight of the payload) acting in the direction $-\hat{Y}_0$.




(a) Let

$$^4\mathcal{F}_4 = [F_x\ F_y\ F_z\ N_x\ N_y\ N_z]^T = [F_x\ F_y\ 0\ 0\ 0\ N_z]^T$$

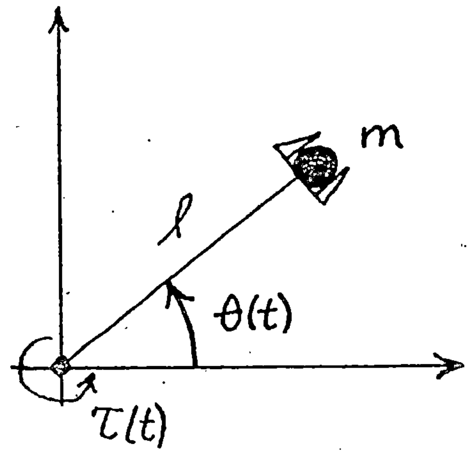
be the generalized force exerted on the payload by the robot, expressed in $\{4\}$. Develop an expression for $^4\mathcal{F}_4$ as a function of the quantity $mg$, the robot configuration, and the link parameters. Assume that the robot and payload are in static equilibrium.

(4 marks)

(b) Using recursive methods, obtain expressions for the joint torques $\tau_i$, $i = 1, 2, 3$, required to maintain static equilibrium.

(10 marks)

5. Consider the single link rigid robot in the figure below. The payload mass is modelled as a point mass and the link is modelled as massless with a length of $\ell = 1$ m.




(a) Write down the differential equation governing the manipulator plus payload system.

**(2 marks)**

(b) It is desired to regulate the position of the arm in the vicinity of the constant configuration $\theta_d = 0$. Assuming that $m = 2$ kg, design a proportional-derivative controller which yields critical damping $\zeta = 1$ and an undamped natural frequency of $\omega_n = 2$ rad/s.

**(5 marks)**

(c) Assume that in addition to the applied motor torque, $\tau(t)$, there is a constant frictional torque $\tau_d = 0.01$ N $\cdot$ m. Calculate

$$e_{ss} = \lim_{t \to \infty} [\theta(t) - \theta_d]$$

**(2 marks)**

(d) In an effort to reduce $e_{ss}$, the following controller is proposed:

$$\tau(t) = -K_d \dot{e}(t) - K_p e(t) - K_i \int_0^t e(\tau)\, d\tau, \quad e(t) = \theta(t) - \theta_d$$

where $K_d = 12$ N$\cdot$m$\cdot$s/rad, $K_p = 22$ N$\cdot$m/rad, and $K_i = 12$ N$\cdot$m/(rad$\cdot$s). Assuming that the solution for the error is of the form

$$e(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + C_3 e^{s_3 t}$$

($C_1$, $C_2$, and $C_3$ are constants) calculate the values of $s_1$, $s_2$, and $s_3$ ($m = 2$ kg).
**(7 marks)**