UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
ROBOTICS (AER 525F)
MID-TERM EXAMINATION
October 17, 2014

Note: Rulers may be used in this test.

Time: 105 Minutes

Question 1:

Describe the following terms briefly (maximum 40 words for each, no formulation required):

a) Pieper's Theorem **(5)**

b) Redundant Manipulator **(5)**

Question 2:

Draw a schematic diagram and show the link coordinate frames of a manipulator with the following Standard DH table and root frame.

Link	a_i	α_i **(deg)**	d_i	θ_i **(deg)**
1	0	0	d_1 (D_1)	0
2	0	– 90	D_2	θ_2 (– 90)
3	L_3	– 90	– D_3	θ_3 (0)
4	0	– 90	D_4	θ_4 (90)
5	0	– 90	0	θ_5 (180)
6	0	0	D_5	θ_6 (0)

$$ {}^{R}T_0 = \begin{bmatrix} 0 & 1 & 0 & D_3 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$ **(15)**

Question 3:

Based on the course notation for the differentiation of vector $\hat{q}$, indicate whether each of the following four terms is correct or incorrect. If correct, explain the term; if incorrect, explain why. **(10)**

a) $d_B\hat{q}$ **b)** $d_B\left({}^{A}q\right)$ **c)** ${}^{A}\dot{q}$ **d)** ${}^{A}\left(d_B\hat{q}\right)$

Question 4:

For a vector $\hat{p}$, describe how to relate ${}^{A}\dot{p}$ and ${}^{B}\dot{p}$, where $\{A\}$ and $\{B\}$ are two arbitrary coordinate frames (write the formulation): **(15)**

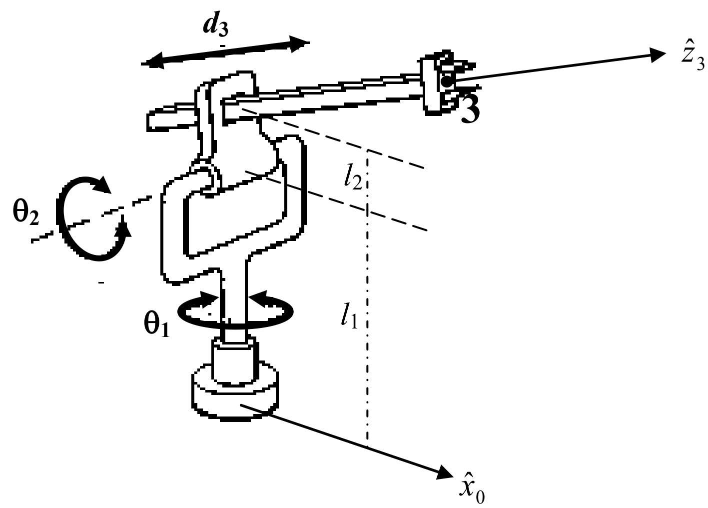
Question 5:

For the 3-d.o.f. spatial manipulator shown in the figure ($0^\circ \le \theta_1 < 360^\circ$, $-90^\circ \le \theta_2 < +270^\circ$, and $d_3 \ge 0$):

a) Draw a schematic of the manipulator, and by using the Standard Denavit-Hartenberg convention define link coordinate frames and link parameters, arrange the DH table, and then determine ${}^0T_3$. Check the results for when $\theta_1 = \theta_2 = 0$. **(20)**

b) What is the expression of $\hat{x}_0$ in the end-effector coordinate frame, i.e., ${}^3x_0$? **(10)**

c) Having the position of the end-effector point (Point 3), expressed in the base frame, determine the corresponding joint variables. Discuss the number and feasibility of all possible solutions. **(20)**



$a_i \equiv$ the length of the common normal between $\hat{z}_{i-1}$ and $\hat{z}_i$ along $\hat{x}_i$ (link length);

$\alpha_i \equiv$ the angle between $\hat{z}_{i-1}$ and $\hat{z}_i$ measured about $\hat{x}_i$ (twist angle);

$d_i \equiv$ the distance from $\hat{x}_{i-1}$ to $\hat{x}_i$ measured along $\hat{z}_{i-1}$ (link offset);

$\theta_i \equiv$ the angle between $\hat{x}_{i-1}$ and $\hat{x}_i$ measured about $\hat{z}_{i-1}$ (joint angle);

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos q = A \;\Rightarrow\; q = \pm \text{Atan}\,2\left(\frac{\sqrt{1-A^2}}{A}\right)$$

$$^A\Omega_B = {}^A\dot{T}_B\, {}^AT_B^{-1} = \left[\begin{array}{c:c} {}^A\tilde{\omega}_{AB} & {}^A_B v_{AB} \\ \hdashline 0 & 0 \end{array}\right] \qquad {}^A\tilde{\omega}_{AB} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$-A\sin q + B\cos q = 0 \;\Rightarrow\; \begin{cases} q^1 = \mathbf{Atan2}\left(\dfrac{B}{A}\right) \\ q^2 = q^1 + 180^o \end{cases}$$

$$-A\sin q + B\cos q = C \quad \Rightarrow \quad q = \text{Atan2}\left(\frac{B}{A}\right) - \text{Atan2}\left(\frac{C}{\pm\sqrt{A^2+B^2-C^2}}\right)$$

$$\frac{d_A\hat{p}}{dt} = \frac{d_B\hat{p}}{dt} + \hat{\omega}_{AB} \times \hat{p}$$