AER525 MIDTERM SOLUTIONS (2013 FALL)

#1) (a) Manipulator Redundancy:

When the manipulator has more no. of d.o.f. (joint space dimension) than independent variables considered for the end-effector pose (task space dimension), the manipulator is said to be redundant.

(b) Degeneracy of Manipulator's spherical wrist:

When two joint axis of the spherical wrist become colinear, degeneracy occurs and the manipulator loses a degree of freedom.

Example: Pitch angle = 0 in Roll/Pitch/Yaw Wrist

(c) Dextrous Workspace:

It is the loci of end-effector points which can be reached at any arbitrary end-effector orientation.

(d) Coriolis theorem:

The theorem says that the difference between the differentiation of a vector quantity ~~is~~ with respect to two different frames is equal to the cross product of their relative angular velocity and the vector itself.

P.T.O

#2) Approach: Find # of links l, & D, the # of limiting d.o.f by joints

l = no of links
= 2 (hands) × [4 (fingers) × 3 + 1 (thumb) × 3 + 1 (palm)]
+ 2 (arms) × [1 (forearm) + 1 (upper arm)]
+ 1 (upper torso)
+ 1 (lower torso)
+ 1 (waist)
+ 2 (legs) × [1 (thigh) + 1 (lower leg)]
+ 1 (foot)
= 44 links

Note: 1) The thumb's base link is embedded in the palm.
2) Based on the definition of a joint, there must be a link between the waist and hips.

D = no. of limiting d.o.f by joints
= [2 wrists + 2 shoulders + 2 hips + 2 ankles + 1 waist] × (6 − 3)
+ [2 × 8 (fingers) + 2 × 2 (thumb) + 2 (elbows) + 2 (knees) + 1 (torso)] × (6 − 1)
+ [2 × 1 (thumb) + 4 × 2 (fingers)] × (6 − 2)

$= (9 \times 3) + (25 \times 5) + (10 \times 4)$
$= 27 + 125 + 40 = 192$

$\Rightarrow$ d.o.f $= 6(l-1) - D = 6(44-1) - 192 = 66$

without fingers and thumbs,
$l = 14$, $D = 52$ $\Rightarrow$ d.o.f $= 26$

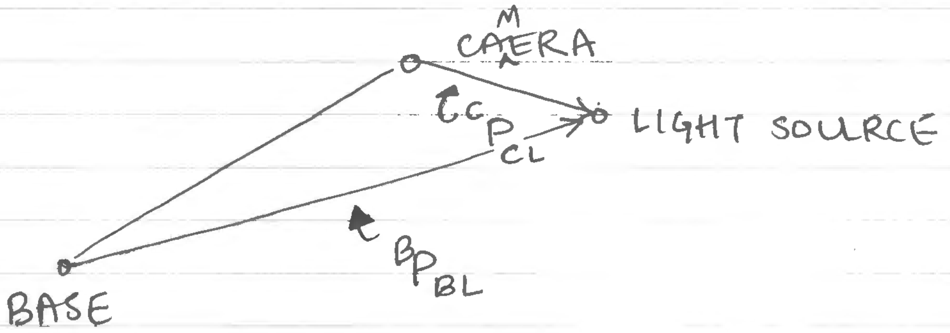
so, fingers and thumbs add 40 d.o.f to the human manipulator.

#3) Base frame : B
Camera frame : C
Light Source : L

$c60 = \frac{1}{2}$ $\quad s60 = \frac{\sqrt{3}}{2}$
$c30 = \frac{\sqrt{3}}{2}$ $\quad s30 = \frac{1}{2}$

$$ {}^{B}R_{C} = \begin{bmatrix} c60c30 & -s60 & c60s30 \\ s60c30 & c60 & s60s30 \\ -s30 & 0 & c30 \end{bmatrix} $$

$$ {}^{B}T_{C} = \begin{bmatrix} \sqrt{3}/4 & -\sqrt{3}/2 & 1/4 & 3 \\ 3/4 & 1/2 & \sqrt{3}/4 & 4 \\ -1/2 & 0 & \sqrt{3}/2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

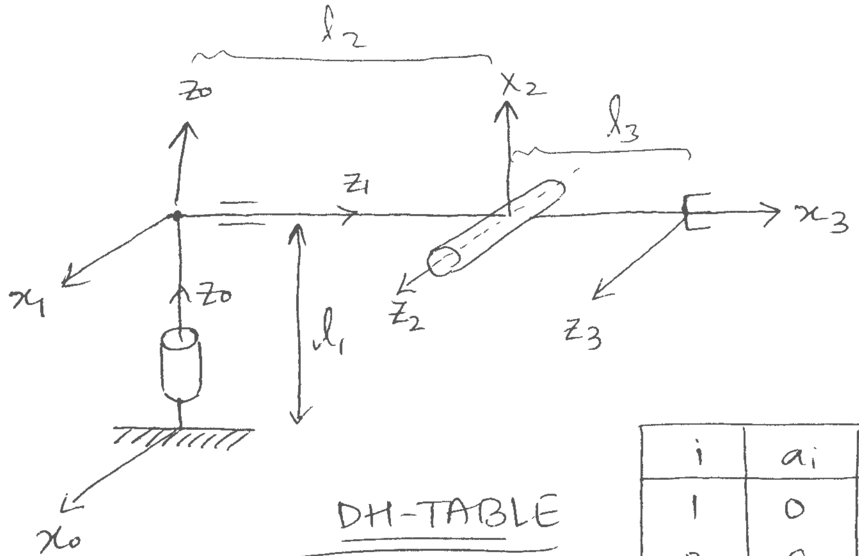


Given ${}^{C}P_{CL} = [1 \quad 0 \quad 0.5]^T$

$$ \begin{bmatrix} {}^{B}P_{BL} \\ \hline 1 \end{bmatrix} = {}^{B}T_{C} \begin{bmatrix} {}^{C}P_{CL} \\ \hline 1 \end{bmatrix} = {}^{B}T_{C} \begin{bmatrix} 1 \\ 0 \\ 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/4 + 1/8 + 3 \\ 3/4 + \sqrt{3}/8 + 4 \\ -1/2 + \sqrt{3}/4 + 2 \\ 1 \end{bmatrix} $$

or $$ {}^{B}P_{BL} = \left[\frac{25+2\sqrt{3}}{8}, \frac{38+\sqrt{3}}{8}, \frac{12+2\sqrt{3}}{8} \right]^T $$

#4) (a)



DH-TABLE

i	a_i	α_i	d_i	θ_i
1	0	$-90°$	l_1	$\theta_1(0°)$
2	0	$-90°$	$d_2(l_2)$	$-90°$
3	l_3	0	0	$\theta_3(270°)$

$$^0T_1 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

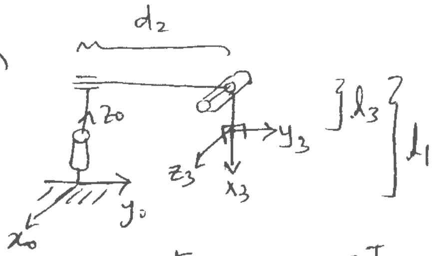
$$^2T_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & l_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & l_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_2 = \begin{bmatrix} 0 & S\theta_1 & C\theta_1 & -d_2S\theta_1 \\ 0 & -C\theta_1 & S\theta_1 & d_2C\theta_1 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = \begin{bmatrix} S\theta_1S\theta_3 & S\theta_1C\theta_3 & C\theta_1 & {}^0P_{03a} \\ -C\theta_1S\theta_3 & -C\theta_1C\theta_3 & S\theta_1 & {}^0P_{03b} \\ C\theta_3 & -S\theta_3 & 0 & {}^0P_{03c} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, $$\begin{bmatrix} {}^0P_{03a} \\ {}^0P_{03b} \\ {}^0P_{03c} \end{bmatrix} = \begin{bmatrix} l_3S\theta_1S\theta_3 - d_2S\theta_1 \\ -l_3S\theta_3C\theta_1 + d_2C\theta_1 \\ l_3C\theta_3 + l_1 \end{bmatrix}$$

Inspection



$${}^0x_3 = [0 \quad 0 \quad -1]^T$$
$${}^0y_3 = [0 \quad 1 \quad 0]^T$$
$${}^0z_3 = [1 \quad 0 \quad 0]^T$$

$${}^0P_{03} = \begin{bmatrix} 0 \\ d_2 \\ l_1 - l_3 \end{bmatrix}$$

, $$^0T_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_2 \\ -1 & 0 & 0 & l_1 - l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as predicted.

(b) $P_x = l_3 \sin\theta_1 \sin\theta_3 - d_2 \sin\theta_1$ — ①
$P_y = -l_3 \cos\theta_1 \sin\theta_3 + d_2 \cos\theta_1$ — ②
$P_z = l_3 \cos\theta_3 + l_1$ — ③

→ Solving equation 3,

$$\cos\theta_3 = \frac{P_z - l_1}{l_3}$$

$$\sin\theta_3 = \pm\sqrt{1 - c^2\theta_3}$$

$$\theta_3 = \text{Atan2}\left(\frac{\sin\theta_3}{\cos\theta_3}\right) \Rightarrow 2 \text{ answers}$$

→ Knowing θ_3,

$$P_x = (l_3 s\theta_3 - d_2)\sin\theta_1$$
$$P_y = -(l_3 s\theta_3 - d_2)\cos\theta_1$$

Square and add,

$$P_x^2 + P_y^2 = (l_3 s\theta_3 - d_2)^2$$

$$d_2 = l_3 s\theta_3 \pm \sqrt{P_x^2 + P_y^2}$$

θ_3 has full range $([0, 360))$

∴ for $\sin\theta_3 < 0 \xRightarrow{d_2 \geq 0}$ d_2 has one answer
i.e $d_2 = l_3 \sin\theta_3 + \sqrt{P_x^2 + P_y^2}$

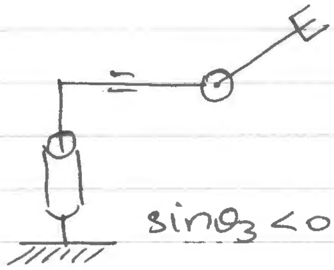
for $\sin\theta_3 \geq 0 \Rightarrow$ if $|l_3 \sin\theta_3| \geq \sqrt{P_x^2 + P_y^2}$
then d_2 has two answers
otherwise it has only one.

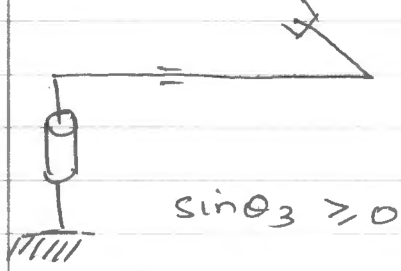
→ Knowing θ_3 & d_2,

$$\left.\begin{aligned} \sin\theta_1 &= \frac{P_x}{l_3 \sin\theta_3 - d_2} \\ \cos\theta_1 &= \frac{P_y}{d_2 - l_3 \sin\theta_3} \end{aligned}\right\} \theta_1 = \text{Atan2}(\sin\theta_1 / \cos\theta_1)$$

$\Rightarrow$ 1 answer.

$\Rightarrow$





if $|l_3 \sin\theta_3| \geq |\sqrt{P_x^2 + P_y^2}|$ then there exists one more configuration,