## AER 525 - ROBOTICS Assignment 6 (Problems 4 and 5 will be marked.)

- 1. (a) Derive the equations of motion for the two link manipulator in fig. 1, using the Lagrangian Formulation. Assume that the gravity acts vertically downward. (Note that  $\tilde{I}_i$  is the moment of inertia about the center of mass of link i).
  - (b) Derive the equations of motion using Newton-Euler Formulation and verify if the result is equivalent to that of problem 1(a).

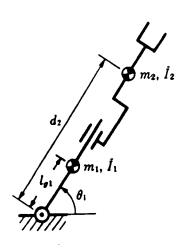


Fig. 1 A two link manipulator

2. Derive the equations of motion of the parallel drive two-link manipulator shown in fig. 2. Assume that the torque  $\tau_1$  is applied between the base and link A, and torque  $\tau_2$  is applied between the base and link B and that gravity acts vertically downward.

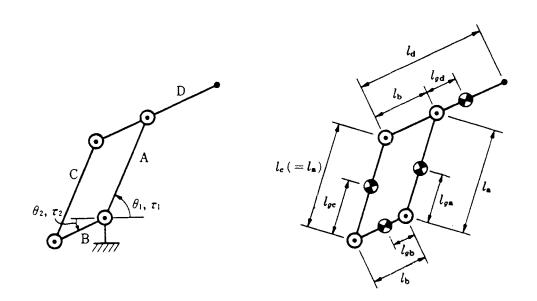


Fig. 2 A parallel two link manipulator

- 3. Consider the two degree of freedom manipulator shown in fig. 3 below. When the manipulator grasps an unknown mass particle M at the tip of link 2, the mass properties of link 2 change from known values of  $m_2$ ,  $l_{c2}$ ,  $l_2$  to  $m_2^*$ ,  $l_{c2}^*$ ,  $l_2^*$  where  $m_2^*$  =  $m_2$  + M. It is required to identify the unknown mass properties by experiments.
  - (a) The unknown mass M is modelled as a point mass, and the centroids are assumed to be located on the center line of each link. Derive the distance  $l_{c2}^*$  and the centroidal moment of inertia  $I_2^*$  as functions of unknown mass.
  - (b) Assume that the generalised coordinates are  $\theta_1$  and  $\theta_2$ , the actuators exert torques  $\tau_1$  and  $\tau_2$ , and the manipulator with unknown mass moves at angular velocities  $\dot{\theta}_1$ ,  $\dot{\theta}_2$  and angular accelerations  $\ddot{\theta}_1$  and  $\ddot{\theta}_2$ . Determine the unknown mass M at the tip of link 2 from this set of data.

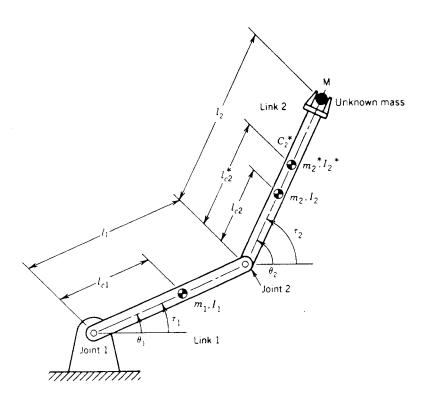


Fig. 3 A two degree of freedom manipulator

4. Figure 4 shows a 2 d.o.f manipulator. The end-point of the manipulator is in contact with a smooth surface and while applying a normal force  $f_N$  to the surface, the manipulator is moving in a constant speed  $v_t$  along the tangential direction. Compute the required joint torques  $\tau_1$  and  $\tau_2$  in the case shown in figure.

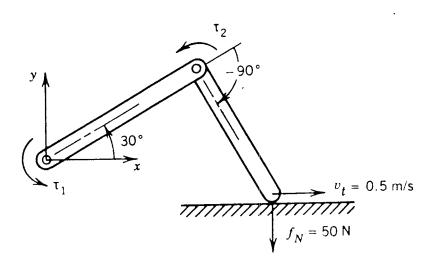


Fig. 4 A two degree of freedom manipulator in constrained motion

5. Figure 5 shows a manipulator with a gripper transferring an object whose mass properties are unknown. In order to calculate the mass properties, a wrist sensor detecting linear forces  $f_u$  and  $f_v$  and moment  $N_w$  is attached to the arm tip as shown in figure. Joint angles, joint velocities and accelerations as well as forces and moment at the wrist sensor, are measured while moving the object. From the data measured by the wrist sensor, the joint position sensors, the velocity sensors, find the total mass  $M_o$  of the object and the gripper, their total moment of inertia  $I_o$  and the distance  $I_o$  between their total mass center and the wrist sensor. Is the set of joint accelerations data neccessary to determine the mass properties?

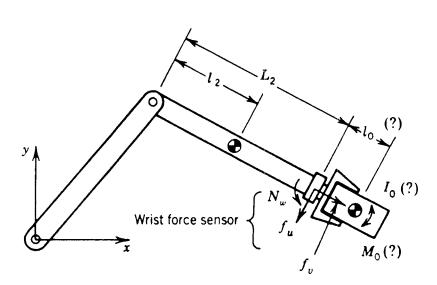


Fig. 5 A two degree of freedom manipulator transferring an object with unknown mass properties.