AER525 Fall 2015

Assignment 1

Due: 7-Oct-2015

Question 1 Which of the following matrices are valid rotation matrices and which are not? If not, why?

$$R_{1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1 \end{bmatrix}, \qquad R_{2} = \begin{bmatrix} 1 & 0 & 0\\ 0 & \frac{2}{3} & \frac{1}{3}\\ 0 & -\frac{1}{3} & \frac{2}{3} \end{bmatrix},$$

$$R_{3} = \begin{bmatrix} 0.6562 & -0.4695 & 0.5908\\ 0.3489 & 0.8829 & 0.3141\\ -0.6691 & 0 & 0.7431 \end{bmatrix}, \qquad R_{4} = \begin{bmatrix} 0 & -0.4695 & 0\\ 0.7482 & 0 & -0.4286\\ 0 & 0.7233 & 0 \end{bmatrix},$$

$$R_{5} = \begin{bmatrix} 0.866 & 0 & 0.5 & 1\\ 0 & 1 & 0 & 1\\ -0.5 & 0 & 0.866 & 1 \end{bmatrix}, \qquad R_{6} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{\sqrt{3}}{4}\\ \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} & \frac{1}{4}\\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix},$$

Question 2 A point ${}^DP = [2 \ 0 \ 2]^T$ is known relative to frame $\{D\}$ (see Figure 1). Frame $\{D\}$ is translated relative to frame $\{C\}$ by 3 units along \hat{Z}_C . Frame $\{C\}$ is both rotated about \hat{Z}_B by 10° and translated by 6 units along \hat{Y}_B relative to frame $\{B\}$. Finally, frame $\{B\}$ is rotated relative to frame $\{A\}$ by 15° about \hat{X}_A . What is AP , the point represented in frame $\{A\}$?

Question 3 Show that if R_1, R_2, \ldots, R_n are all orthogonal matrices, their product is also an orthogonal matrix.

Question 4 A frame $\{B\}$ is rotated relative to a frame $\{A\}$ but they share the same origin. If a vector ${}^AG = [2 \ 1 \ 2]^T$ is known to lie along \hat{X}_B and another vector ${}^AH = [-1 \ -2 \ 2]^T$ is known to lie along \hat{Z}_B , what is the equivalent axis of rotation \hat{K} and angle θ about it for which frame $\{B\}$ is relative to frame $\{A\}$?

Question 5 Show the attachment of link frames for the four manipulators shown in Figures 2, 3, 4, and 5 and write their corresponding D-H Tables.

Question 6 Derive the kinematic model of the manipulator in Figure 6 with the end-effector tip being between the fingertips.

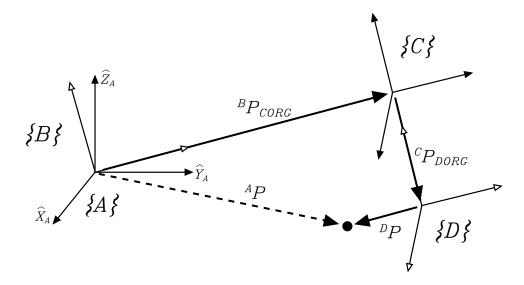


Figure 1: Overview of Question 2

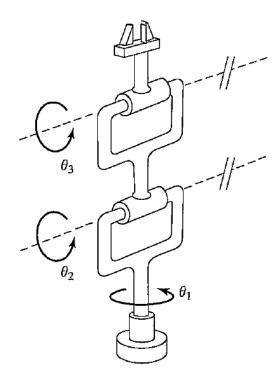


Figure 2: Question 5a

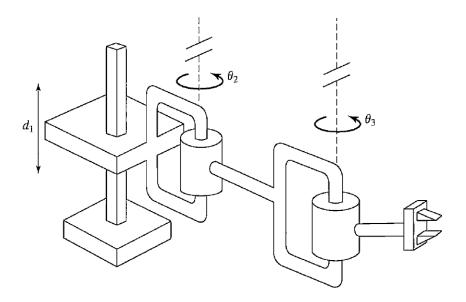


Figure 3: Question 5b

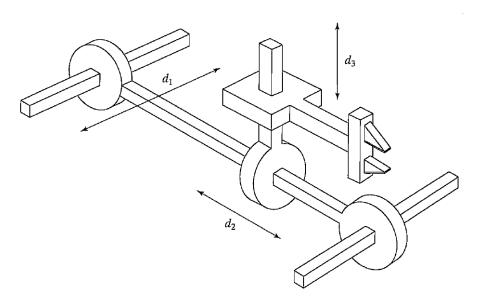


Figure 4: Question 5c

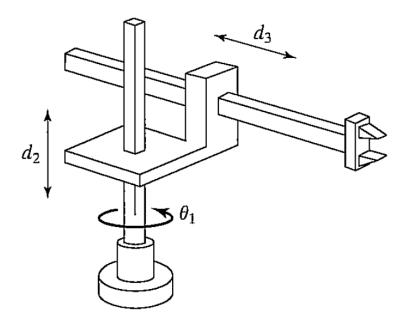


Figure 5: Question 5d

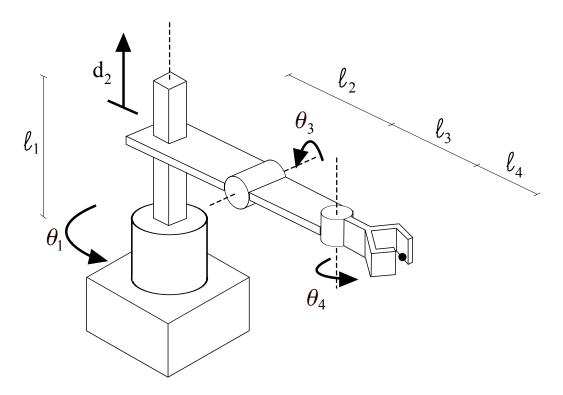


Figure 6: Manipulator for Question 6