Chapter 4

Forces I

4.1 The Important Stuff

4.1.1 Newton's First Law

With Newton's Laws we begin the study of *how* motion occurs in the real world. The study of the *causes* of motion is called **dynamics**, or **mechanics**. The relation between force and acceleration was given by Isaac Newton in his three laws of motion, which form the basis of elementary physics. Though Newton's formulation of physics had to be replaced later on to deal with motion at speeds comparable to the speed light and for motion on the scale of atoms, it is applicable to everyday situations and is still the best introduction to the fundamental laws of nature. The study of Newton's laws and their implications is often called **Newtonian** or **classical mechanics**.

Particles accelerate because they are being acted on by **forces**. In the absence of forces, a particle will not accelerate, that is, it will move at constant velocity.

The user–friendly way of stating Newton's First Law is:

> Consider a body on which no force is acting. Then if it is at rest it will remain at rest, and if it is moving with constant velocity it will continue to move at that velocity.

Forces serve to *change* the velocity of an object, not to maintain its motion (contrary to the ideas of philosophers in ancient times).

4.1.2 Newton's Second Law

Experiments show that objects have a property called **mass** which measures how their motion is influenced by forces. Mass is measured in kilograms in the SI system.

Newton's Second Law is a relation between the **net force** ($\mathbf{F}$) acting on a mass m and its acceleration $\mathbf{a}$. It says:

$$\boxed{\sum \mathbf{F} = m\mathbf{a}}$$

In words, Newton's Second Law tells us to add up the forces acting on a mass m; this sum, $\sum \mathbf{F}$ (or, $\mathbf{F}_{\rm net}$) is equal to the mass m times its acceleration $\mathbf{a}$.

This is a *vector* relation; if we are working in two dimensions, this equation implies *both* of the following:

$$\sum F_x = ma_x \qquad \text{and} \qquad \sum F_y = ma_y \tag{4.1}$$

The units of force must be $\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}$, which is abbreviated 1 newton (N), to honor Isaac Newton (1642–1727), famous physicist and smart person. Thus:

$$1\,\text{newton} = 1\,\mathrm{N} = 1\,\tfrac{\mathrm{kg\cdot m}}{\mathrm{s}^2} \tag{4.2}$$

Two other units of force which we encounter sometimes are:

$$1\,\text{dyne} = 1\,\tfrac{\mathrm{g\cdot cm}}{\mathrm{s}^2} = 10^{-5}\,\mathrm{N} \qquad \text{and} \qquad 1\,\text{pound} = 1\,\mathrm{lb} = 4.45\,\mathrm{N}$$

4.1.3 Examples of Forces

To begin our study of dynamics we consider problems involving simple objects in simple situations. Our first problems will involve little more than small masses, hard, smooth surfaces and ideal strings, or objects that can be treated as such.

For all masses near the earth's surface, the earth exerts a downward gravitational force which is known as the **weight** of the mass and has a magnitude given by

$$W = mg$$

A taught string (a string "under tension") exerts forces on the objects which are attached to either end. The forces are directed *inward* along the length of the string.) In our first problems we will make the approximation that the string has no mass, and when it passes over any pulley, the pulley's mass can also be ignored. In that case, the magnitude of the string's force on either end is *the same* and will usually be called T, the string's **tension**.

A solid surface will exert forces on a mass which is in contact with it. In general the force from the surface will have a perpendicular (normal) component which we call the **normal force** of the surface. The surface can also exert a force which is parallel; this is a friction force and will be covered in the next chapter.

4.1.4 Newton's Third Law

> Consider two objects A and B. The force which object A exerts on object B is equal and opposite to the force which object B exerts on object A: $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$

This law is popularly stated as the "law of action and reaction", but in fact it deals with the *forces* between two objects.

4.1.5 Applying Newton's Laws

In this chapter we will look at some applications of Newton's law to simple systems involving small blocks, surfaces and strings. (In the next chapter we'll deal with more complicated examples.)

A useful practice for problems involving more than one force is to draw a diagram showing the individual masses in the problem along with the vectors showing the directions and magnitudes of the individual forces. It is so important to do this that these diagrams are given a special name, **free–body diagrams**.

4.2 Worked Examples

4.2.1 Newton's Second Law

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1. A $3.0\,\mathrm{kg}$ **mass undergoes an acceleration given by** $\mathbf{a} = (2.0\mathbf{i} + 5.0\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}^2}$**. Find the resultant force F and its magnitude.** [Ser4 5-7]

Newton's Second Law tells us that the resultant (net) force on a mass m is $\sum \mathbf{F} = m\mathbf{a}$. So here we find:

$$\begin{aligned} \mathbf{F}_{\mathrm{net}} &= m\mathbf{a} \\ &= (3.0\,\mathrm{kg})(2.0\mathbf{i} + 5.0\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}^2} \\ &= (6.0\mathbf{i} + 15.\mathbf{j})\,\mathrm{N} \end{aligned}$$

The *magnitude* of the resultant force is

$$F_{\mathrm{net}} = \sqrt{(6.0\,\mathrm{N})^2 + (15.\,\mathrm{N})^2} = 16.\,\mathrm{N}$$

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2. While two forces act on it, a particle of mass $m = 3.2\,\mathrm{kg}$ **is to move continuously with velocity** $(3\,\frac{\mathrm{m}}{\mathrm{s}})\mathbf{i} - (4\,\frac{\mathrm{m}}{\mathrm{s}})\mathbf{j}$**. One of the forces is** $\mathbf{F}_1 = (2\,\mathrm{N})\mathbf{i} + (-6\,\mathrm{N})\mathbf{j}$**. What is the other force?** [HRW5 5-5]

Newton's Second Law tells us that if **a** is the acceleration of the particle, then (as there are only two forces acting on it) we have:

$$\mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$$

But here the *acceleration* of the particle is *zero*!! (Its velocity does not change.) This tells us that

$$\mathbf{F}_1 + \mathbf{F}_2 = 0 \qquad \Longrightarrow \qquad \mathbf{F}_2 = -\mathbf{F}_1$$

and so the second force is

$$\mathbf{F}_2 = -\mathbf{F}_1 = (-2\,\mathrm{N})\mathbf{i} + (6\,\mathrm{N})\mathbf{j}$$

This was a simple problem just to see if you're paying attention!

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3. A $4.0\,\mathrm{kg}$ object has a velocity of $3.0\mathbf{i}\,\frac{\mathrm{m}}{\mathrm{s}}$ at one instant. Eight seconds later, its velocity is $(8.0\mathbf{i} + 10.0\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}}$. Assuming the object was subject to a constant net force, find (a) the components of the force and (b) its magnitude. [Ser4 5-21]

(a) We are told that the (net) force acting on the mass was *constant*. Then we know that its acceleration was also constant, and we can use the constant–acceleration results from the previous chapter. We are given the initial and final velocities so we can compute the components of the acceleration:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{[(8.0\,\frac{\mathrm{m}}{\mathrm{s}}) - (3.0\,\frac{\mathrm{m}}{\mathrm{s}})]}{(8.0\,\mathrm{s})} = 0.63\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

and

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{[(10.0\,\frac{\mathrm{m}}{\mathrm{s}}) - (0.0\,\frac{\mathrm{m}}{\mathrm{s}})]}{(8.0\,\mathrm{s})} = 1.3\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

We have the mass of the object, so from Newton's Second Law we get the components of the force:

$$F_x = ma_x = (4.0\,\mathrm{kg})(0.63\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 2.5\,\mathrm{N}$$

$$F_y = ma_y = (4.0\,\mathrm{kg})(1.3\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 5.0\,\mathrm{N}$$

(b) The magnitude of the (net) force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2.5\,\mathrm{N})^2 + (5.0\,\mathrm{N})^2} = 5.6\,\mathrm{N}$$

and its direction θ is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{5.0}{2.5} = 2.0 \qquad \Longrightarrow \qquad \theta = \tan^{-1}(2.0) = 63.4^\circ$$

(The question didn't ask for the direction but there it is anyway!)

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4. Five forces pull on the $4.0\,\mathrm{kg}$ box in Fig. 4.1. Find the box's acceleration (a) in unit–vector notation and (b) as a magnitude and direction. [HRW5 5-9]

(a) Newton's Second Law will give the box's acceleration, but we must first find the sum of the forces on the box. Adding the x components of the forces gives:

$$\begin{aligned}\sum F_x &= -11\,\mathrm{N} + 14\,\mathrm{N}\cos 30^\circ + 3.0\,\mathrm{N} \\ &= 4.1\,\mathrm{N}\end{aligned}$$

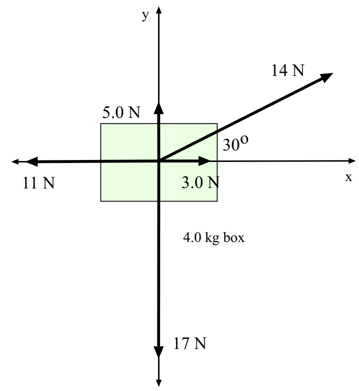

Figure 4.1: Five forces pull on a box in Example 4

(two of the forces have only y components). Adding the y components of the forces gives:

$$\begin{aligned}\sum F_y &= +5.0\,\mathrm{N} + 14\,\mathrm{N}\sin 30^\circ - 17\,\mathrm{N} \\ &= -5.0\,\mathrm{N}\end{aligned}$$

So the net force on the box (in unit-vector notation) is

$$\sum \mathbf{F} = (4.1\,\mathrm{N})\mathbf{i} + (-5.0\,\mathrm{N})\mathbf{j} \ .$$

Then we find the x and y components of the box's acceleration using $\mathbf{a} = \sum \mathbf{F}/m$:

$$\begin{aligned}a_x &= \frac{\sum F_x}{m} = \frac{(4.1\,\mathrm{N})}{(4.0\,\mathrm{kg})} = 1.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \\ a_y &= \frac{\sum F_y}{m} = \frac{(-5.0\,\mathrm{N})}{(4.0\,\mathrm{kg})} = -1.2\,\tfrac{\mathrm{m}}{\mathrm{s}^2}\end{aligned}$$

So in unit–vector form, the acceleration of the box is

$$\mathbf{a} = (1.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2})\mathbf{i} + (-1.2\,\tfrac{\mathrm{m}}{\mathrm{s}^2})\mathbf{j}$$

(b) The acceleration found in part (a) has a magnitude of

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(1.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2})^2 + (-1.2\,\tfrac{\mathrm{m}}{\mathrm{s}^2})^2} = 1.6\,\tfrac{\mathrm{m}}{\mathrm{s}^2}$$

and to find its direction θ we calculate

$$\tan\theta = \frac{a_y}{a_x} = \frac{-1.2}{1.0} = -1.2$$

which gives us:

$$\theta = \tan^{-1}(-1.2) = -50^\circ$$

Here, since a_y is negative and a_x is positive, this choice for θ lies in the proper quadrant.

4.2.2 Examples of Forces

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5. What are the weight in newtons and the mass in kilograms of (a) a $5.0\,\mathrm{lb}$ **bag of sugar, (b) a** $240\,\mathrm{lb}$ **fullback, and (c) a** $1.8\,\mathrm{ton}$ **automobile? (**$1\,\mathrm{ton} = 2000\,\mathrm{lb}$**.)** [HRW5 5-13]

(a) The bag of sugar has a *weight* of 5.0 lb. ("Pound" is a unit of force, or weight.) Then its weight in newtons is

$$5.0\,\mathrm{lb} = (5.0\,\mathrm{lb}) \cdot \left(\frac{4.45\,\mathrm{N}}{1\,\mathrm{lb}}\right) = 22\,\mathrm{N}$$

Then from $W = mg$ we calculate the *mass* of the bag,

$$m = \frac{W}{g} = \frac{22\,\mathrm{N}}{9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 2.3\,\mathrm{kg}$$

(b) Similarly, the weight of the fullback in newtons is

$$240\,\mathrm{lb} = (240\,\mathrm{lb}) \cdot \left(\frac{4.45\,\mathrm{N}}{1\,\mathrm{lb}}\right) = 1070\,\mathrm{N}$$

and then his (her) mass is

$$m = \frac{W}{g} = \frac{1070\,\mathrm{N}}{9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 109\,\mathrm{kg}$$

(c) The automobile's weight is given in tons; express it in newtons:

$$1.8\,\mathrm{ton} = (1.8\,\mathrm{ton})\left(\frac{2000\,\mathrm{lb}}{1\,\mathrm{ton}}\right)\left(\frac{4.45\,\mathrm{N}}{1\,\mathrm{lb}}\right) = 1.6 \times 10^4\,\mathrm{N}\ .$$

Then its mass is

$$m = \frac{W}{g} = \frac{1.6 \times 10^4\,\mathrm{N}}{9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 1.6 \times 10^3\,\mathrm{kg}$$

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6. If a man weighs $875\,\mathrm{N}$ **on Earth, what would he weigh on Jupiter, where the free–fall acceleration is** $25.9\,\frac{\mathrm{m}}{\mathrm{s}^2}$**?** [Ser4 5-12]

The weight of a mass m on the earth is $W = mg$ where g is the free–fall acceleration *on Earth*. The mass of the man is:

$$m = \frac{W}{g} = \frac{875\,\mathrm{N}}{9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 89.3\,\mathrm{kg}$$

His weight *on Jupiter* is found using g_{Jupiter} instead of g:

$$W_{\mathrm{Jupiter}} = mg_{\mathrm{Jupiter}} = (89.3\,\mathrm{kg})(25.9\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 2.31 \times 10^3\,\mathrm{N}$$

The man's weight on Jupiter is $2.31 \times 10^3\,\mathrm{N}$.

(The statement of the problem is a little deceptive; Jupiter has no solid surface! The planet will indeed pull on this man with a force of $2.31 \times 10^3\,\mathrm{N}$, but there is no "ground" to push back!)

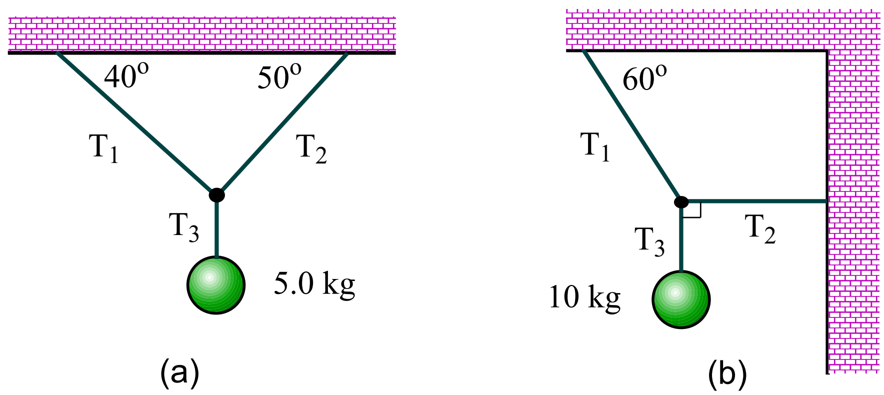

Figure 4.2: Masses suspended by strings, for Example 7.

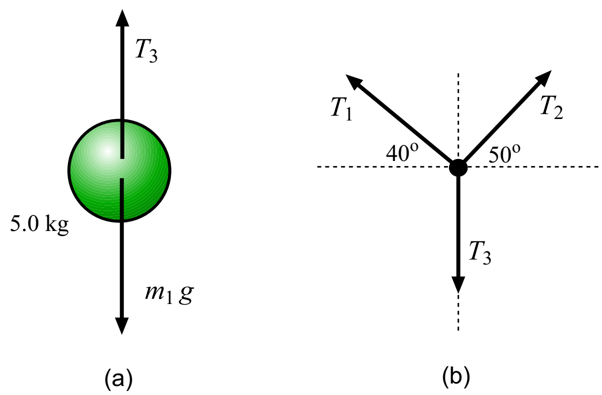

Figure 4.3: Force diagrams for part (a) in Example 7.

4.2.3 Applying Newton's Laws

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7. Find the tension in each cord for the systems shown in Fig. 4.2. (Neglect the mass of the cords.) [Ser4 5-26]

(a) In this part, we solve the system shown in Fig. 4.2(a).

Think of the forces acting on the 5.0 kg mass (which we'll call m_1). Gravity pulls downward with a force of magnitude m_g. The vertical string pulls upward with a force of magnitude T_3. (These forces are shown in Fig. 4.3(a).) Since the hanging mass has *no* acceleration, it must be true that $T_3 = m_1 g$. This gives us the value of T_3:

$$T_3 = m_1 g = (5.0\,\mathrm{kg})(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 49\,\mathrm{N}\ .$$

Next we look at the forces which act at the point where all three strings join; these are shown in Fig. 4.3(b). The force which the strings exert all point *outward* from the joining point and from simple geometry they have the directions shown

Now this point is not accelerating either, so the forces on *it* must all sum to zero. The horizontal components and the vertical components of these forces *separately* sum to zero.

The horizontal components give:

$$-T_1 \cos 40^\circ + T_2 \cos 50^\circ = 0$$

This equation by itself does not let us solve for the tensions, but it does give us:

$$T_2 \cos 50^\circ = T_1 \cos 40^\circ \qquad \Longrightarrow \qquad T_2 = \frac{\cos 40^\circ}{\cos 50^\circ} T_2 = 1.19 T_1$$

The vertical forces sum to zero, and this gives us:

$$T_1 \sin 40^\circ + T_2 \sin 50^\circ - T_3 = 0$$

We already know the value of T_3. Substituting this and also the expression for T_2 which we just found, we get:

$$T_1 \sin 40^\circ + (1.19 T_1) \sin 50^\circ - 49\,\mathrm{N} = 0$$

and now we can solve for T_1. A little rearranging gives:

$$(1.56) T_1 = 49\,\mathrm{N}$$

which gives

$$T_1 = \frac{49\,\mathrm{N}}{(1.56)} = 31.5\,\mathrm{N} \ .$$

Now with T_1 in hand we get T_2:

$$T_2 = (1.19) T_1 = (1.19)(31.5\,\mathrm{N}) = 37.5\,\mathrm{N} \ .$$

Summarizing, the tensions in the three strings for this part of the problem are

$$T_1 = 31.5\,\mathrm{N} \qquad T_2 = 37.5\,\mathrm{N} \qquad T_3 = 49\,\mathrm{N} \ .$$

(b) Now we study the system shown in Fig. 4.2(b).

Once again, the net force on the hanging mass (which we call m_2) must be zero. Since gravity pulls down with a a force $m_2 g$ and the vertical string pulls upward with a force T_3, we know that we just have $T_3 = m_2 g$, so:

$$T_3 = m_2 g = (10\,\mathrm{kg})(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 98\,\mathrm{N} \ .$$

Now consider the forces which act at the place where all the strings meet. We do as in part (a); the horizontal forces sum to zero, and this gives:

$$-T_1 \cos 60^\circ + T_2 = 0 \qquad \Longrightarrow \qquad T_2 = T_1 \cos 60^\circ$$

The vertical forces sum to zero, giving us:

$$T_1 \sin 60^\circ - T_3 = 0$$

But notice that since we know T_3, this equation has only *one* unknown. We find:

$$T_1 = \frac{T_3}{\sin 60^\circ} = \frac{98\,\mathrm{N}}{\sin 60^\circ} = 113\,\mathrm{N} \ .$$

Using this is our expression for T_2 gives:

$$T_2 = T_1 \cos 60^\circ = (113\,\mathrm{N}) \cos 60^\circ = 56.6\,\mathrm{N}$$

Summarizing, the tensions in the three strings for this part of the problem are

$$T_1 = 113\,\mathrm{N} \qquad T_2 = 56.6\,\mathrm{N} \qquad T_3 = 98\,\mathrm{N} \ .$$

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8. A $210\,\mathrm{kg}$ motorcycle accelerates from 0 to $55\,\frac{\mathrm{mi}}{\mathrm{hr}}$ in $6.0\,\mathrm{s}$. (a) What is the magnitude of the motorcycle's constant acceleration? (b) What is the magnitude of the net force causing the acceleration? [HRW5 5-25]

(a) First, let's convert some units:

$$55\,\frac{\mathrm{mi}}{\mathrm{hr}} = (55\,\frac{\mathrm{mi}}{\mathrm{hr}}) \left(\frac{1609\,\mathrm{m}}{1\,\mathrm{mi}}\right) \left(\frac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right) = 24.6\,\frac{\mathrm{m}}{\mathrm{s}}$$

so that the acceleration of the motorcycle is

$$a = \frac{v_x - v_{x0}}{t} = \frac{24.6\,\frac{\mathrm{m}}{\mathrm{s}} - 0}{6.0\,\mathrm{s}} = 4.1\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

(b) Now that we know the acceleration of the motorcycle (and its mass) we know the net horizontal force, because Newton's Law tells us:

$$\sum F_x = ma_x = (210\,\mathrm{kg})(4.1\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 8.6 \times 10^2\,\mathrm{N}$$

The magnitude of the net force on the motorcycle is 8.6×10^2 N.

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9. A rocket and its payload have a total mass of $5.0 \times 10^4\,\mathrm{kg}$. How large is the force produced by the engine (the thrust) when (a) the rocket is "hovering" over the launchpad just after ignition, and (b) when the rocket is accelerating upward at $20\,\frac{\mathrm{m}}{\mathrm{s}^2}$? [HRW5 5-35]

(a) First thing: draw a diagram of the forces acting on the rocket! This is done in Fig. 4.4. If the mass of the rocket is M then we know that gravity will be exerting a force Mg downward. The engines (actually, the gas rushing out of the rocket) exerts a force of magnitude F_{thrust} upward on the rocket.

If the rocket is hovering, i.e. it is motionless but off the ground then it has no acceleration; so, here, a_y=0. Newton's Second Law then says:

$$\sum F_y = F_{\mathrm{thrust}} - Mg = Ma_y = 0$$

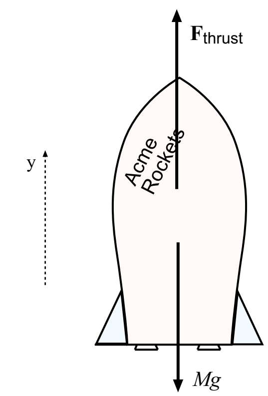

Figure 4.4: Forces acting on the rocket in Example 9

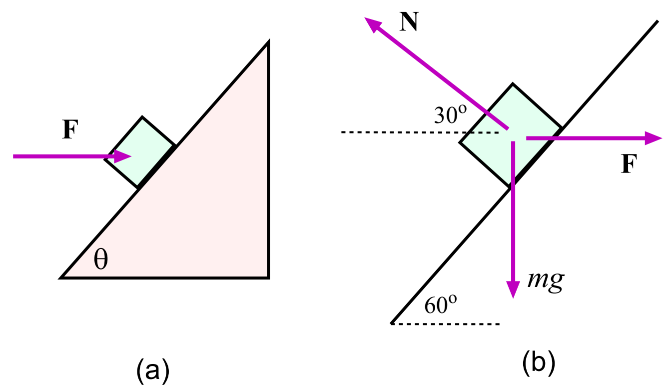

Figure 4.5: (a) Block held in place on a smooth ramp by a horizontal force. (b) Forces acting on the block.

which gives

$$F_{\rm thrust} = Mg = (5.0 \times 10^4\,{\rm kg})(9.80\,\tfrac{\rm m}{\rm s^2}) = 4.9 \times 10^5\,{\rm N}$$

The engines exert an upward force of 4.9×10^5 N on the rocket.

(b) As in part (a), gravity and thrust are the only forces acting on the rocket, but now it has an acceleration of $a_y = 20\,\frac{\rm m}{\rm s^2}$. So Newton's Second Law now gives

$$\sum F_y = F_{\rm thrust} - Mg = Ma_y$$

so that the force of the engines is

$$F_{\rm thrust} = Mg + Ma_y = M(g + a_y) = (5.0 \times 10^4\,{\rm kg})(9.80\,\tfrac{\rm m}{\rm s^2} + 20\,\tfrac{\rm m}{\rm s^2}) = 1.5 \times 10^6\,{\rm N}$$

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10. A block of mass $m = 2.0\,{\rm kg}$ is held in equilibrium on an incline of angle $\theta = 60^\circ$ by the horizontal force F, as shown in Fig. 4.5(a). (a) Determine the value of F, the magnitude of F. (b) Determine the normal force exerted by the incline on the block (ignore friction). [Ser4 5-33]

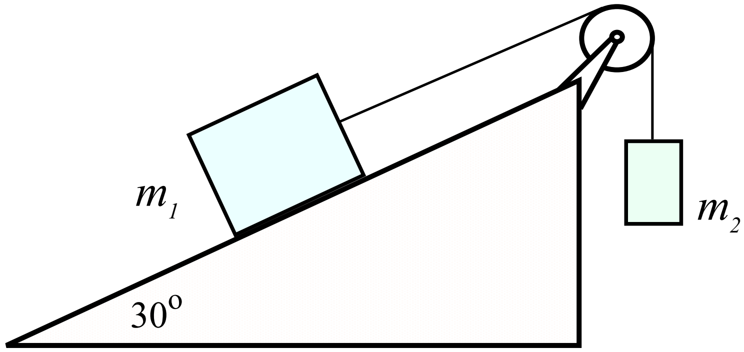

Figure 4.6: Masses m_1 and m_2 are connected by a cord; m_1 slides on frictionless slope.

(a) The first thing to do is to *draw a diagram* of the forces acting on the block, which we do in Fig. 4.5(b). Gravity pulls downward with a force mg. The applied force, of magnitude F, is horizontal. The surface exerts a normal force N on the block; using a little geometry, we see that if the angle of the incline is 60°, then the normal force is directed at 30° above the horizontal, as shown in Fig. 4.5(b). There is no friction force from the surface, so we have shown *all* the forces acting on the block.

Oftentimes for problems involving a block on a slope it is easiest to use the components of the gravity force along the slope and perpendicular to it. For this problem, this would not make things any easier since there is no motion along the slope.

Now, the block is in equilibrium, meaning that it has no acceleration and the forces sum to zero. The fact that the vertical components of the forces sum to zero gives us:

$$N \sin 30^\circ - mg = 0 \qquad \Longrightarrow \qquad N = \frac{mg}{\sin 30^\circ}$$

Substitute and get:

$$N = \frac{(2.0\,\mathrm{kg})(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})}{\sin 30^\circ} = 39.2\,\mathrm{N}\ .$$

The horizontal forces also sum to zero, giving:

$$F - N\cos 30^\circ = 0 \qquad \Longrightarrow \qquad F = N\cos 30^\circ = (39.2\,\mathrm{N})\cos 30^\circ = 33.9\,\mathrm{N}\ .$$

The applied force F is 33.9 N.

(b) The magnitude of the normal force was found in part (a); there we found:

$$N = 39.2\,\mathrm{N}\ .$$

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11. A block of mass $m_1 = 3.70\,\mathrm{kg}$ on a frictionless inclined plane of angle $\theta = 30.0^\circ$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_2 = 2.30\,\mathrm{kg}$ hanging vertically, as shown in Fig. 4.6. What are (a) the magnitude of the acceleration of each block and (b) the direction of the acceleration of m_2? (c) What is the tension in the cord? [HRW5 5-58]

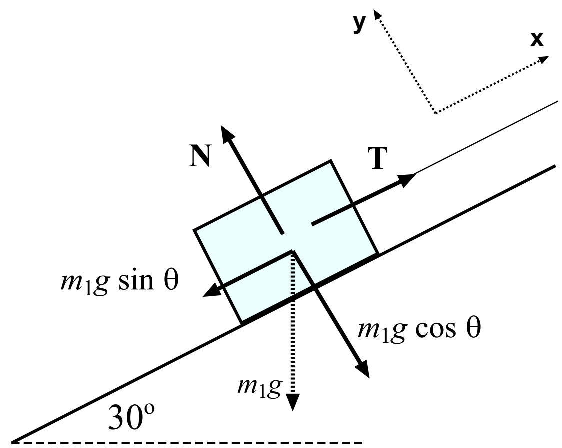

Figure 4.7: The forces acting on m_1

(a) Before thinking about the forces acting on these blocks, we can think about their motion. m_1 is constrained to move along the slope and m_2 must move vertically. Because the two masses are joined by a string, the distance by which m_1 moves up the slope is equal to the distance which m_2 moves downward, and the amount by which m_1 moves down the slope is the amount by which m_2 moves upward. The same is true of their *accelerations*; if it turns out that m_1 is accelerating up the slope, that will be the same as m_2's downward acceleration.

Now we draw "free–body diagrams" and invoke Newton's Second Law for each mass. Consider all the forces acting on m_1. These are shown in Fig. 4.7The force of gravity, with magnitude m_1 pulls straight down. Here, looking ahead to the fact that motion can only occur along the slope it has decomposed into its components perpendicular to the surface (with magnitude $m_1 \cos\theta$) and down the slope (with magnitude $m_1 \sin\theta$). The normal force of the surface has magnitude N and points... normal to the surface! Finally the string pulls *up* with slope with a force of magnitude T, the tension in the string.

Suppose we let x be a coordinate which measures movement *up* the slope. (Note, we are not saying that the block *will move up the slope, this is just a choice of coordinate.* Let y be a coordinate going perpendicular to the slope. We know that there is no y acceleration so the components of the forces in the y direction must add to zero. This gives:

$$N - m_1 g\cos\theta = 0 \qquad \Longrightarrow \qquad N = m_1 g\cos\theta$$

which gives the normal force should we ever need it. (We won't.) Next, the sum of the x forces gives $m_1 a_x$, which will *not* be zero. We get:

$$T - m_1 g\sin\theta = m_1 a_x \tag{4.3}$$

Here there are two unknowns, T and a_x.

The free–body diagram for mass m_2 is shown in Fig. 4.8. The force of gravity, $m_2 g$ pulls downward and the string tension T pulls upward. Suppose we use a coordinate x' which points straight *down*. (This is a little unconventional, but you can see that there is a

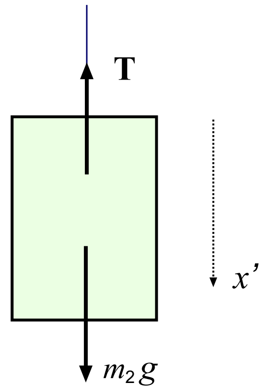

Figure 4.8: The forces acting on m_2. Coordinate x' points downward.

connection with the coordinate x used for the motion of m_1. Then the sum of forces in the x' direction gives $m_2 a_{x'}$:

$$m_2 g - T = m_2 a_{x'}$$

Now as we argued above, the accelerations are equal: $a_x = a_{x'}$. This gives us:

$$m_2 g - T = m_2 a_x \tag{4.4}$$

At this point, the physics is done and the rest of the problem is doing the math (algebra) to solve for a_x and T. We are first interested in finding a_x. We note that by adding Eqs. 4.3 and 4.4 we will eliminate T. Doing this, we get:

$$(T - m_1 g \sin\theta) + (m_2 g - T) = m_1 a_x + m_2 a_x$$

this gives:

$$m_2 g - m_1 g \sin\theta = (m_1 + m_2) a_x$$

and finally:

$$a_x = \frac{(m_2 - m_1 g \sin\theta) g}{m_1 + m_2}$$

Substituting the given values, we have:

$$\begin{aligned} a_x &= \frac{(2.30\,\mathrm{kg} - 3.70\,\mathrm{kg}\sin 30^\circ)(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})}{(3.70\,\mathrm{kg} + 2.30\,\mathrm{kg})} \\ &= +0.735\,\frac{\mathrm{m}}{\mathrm{s}^2} \end{aligned}$$

So $a_x = +0.735\,\frac{\mathrm{m}}{\mathrm{s}^2}$. What does this mean? It means that the acceleration of m_1 *up* the slope and m_2 *downwards* has magnitude $0.735\,\frac{\mathrm{m}}{\mathrm{s}^2}$. The plus sign in our result for a_x is telling us that the acceleration *does* go in the way we (arbitrarily) set up the coordinates. If we had made the opposite ("wrong") choice for the coordinates then our acceleration would have come out with a minus sign.

(b) We've already found the answer to this part in our understanding of the result for part (a). Mass m_1 accelerates *up* the slope; mass m_2 accelerates vertically *downward*.

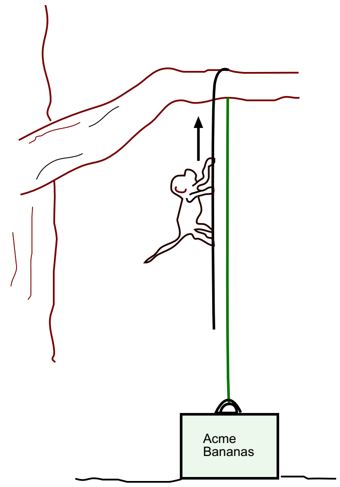

Figure 4.9: Monkey runs up the rope in Example 12.

(c) To get the tension in the string we may use either Eq. 4.3 or Eq. 4.4. Using 4.4 gives:

$$m_2 g - T = m_2 a_x \qquad \Longrightarrow \qquad T = m_2 g - m_2 a_x = m_2(g - a_x)$$

Substituting everything,

$$T = (2.30\,\mathrm{kg})(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2} - (0.735\,\tfrac{\mathrm{m}}{\mathrm{s}^2})) = 20.8\,\mathrm{N}$$

---

12. A $10\,\mathrm{kg}$ monkey climbs up a massless rope that runs over a frictionless tree limb (!) and back down to a $15\,\mathrm{kg}$ package on the ground, as shown in Fig. 4.9. (a) What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground? If, after the package has been lifted the monkey stops its climb and holds onto the rope, what are (b) the monkey's acceleration and (c) the tension in the rope? [HRW5 5-64]

(a) Before we do *anything* else, let's understand what forces are acting on the two masses in this problem. The free–body diagrams are shown in Fig. 4.10. The monkey holds onto the rope so it exerts an *upward* force of magnitude T, where T is the tension in the rope. Gravity pulls down on the monkey with a force of magnitude mg, where m is the mass of the monkey. These are all the forces. Note that they will *not* cancel since the problem talks about the monkey having an acceleration and so the net force on the monkey will *not* be zero.

The forces acting on the box are also shown. Gravity pulls downward on the box with a force of magnitude Mg, M being the mass of the box. The rope pulls upward with a force T, If the box is resting on the ground, the ground will be pushing upward with some force F_{ground}. (Here, the ground cannot pull downward.) However when the box is *not* touching the ground then F_{ground} will be zero.

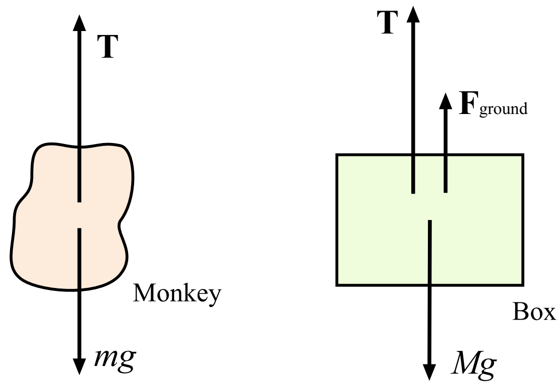

Figure 4.10: The forces acting on the two masses in Example 12.

In the first part of the problem, the monkey is moving along the rope. It is *not* stuck to any point of the rope, so there is no obvious relation between the acceleration of the monkey and the acceleration of the box. Suppose we let $a_{y,\text{monkey}}$ be the vertical acceleration of the monkey and $a_{y,\text{box}}$ be the vertical acceleration of the box. Then from our free–body diagrams, Newton's Second Law gives the acceleration of the monkey:

$$T - mg = ma_{y,\text{monkey}}$$

When the box is on the ground its acceleration is zero and then $T + F_{\text{gr}} = mg$. But when the box is off then ground then we have:

$$T - Mg = Ma_{\text{box}} \qquad \text{(Box off the ground)}$$

In the first part of the problem we are solving for the condition that the monkey climbs just *barely* fast enough for the box to be lifted off the ground. If so, then the ground would exert no force but the net force on the box would be so small as to be virtually zero; the box has a *very, very* tiny acceleration upwards. From this we know:

$$T - Mg = 0 \qquad \Longrightarrow \qquad T = Mg$$

and substituting this result into the first equation gives

$$Mg - mg = ma_{\text{monkey}} \qquad \Longrightarrow \qquad a_{\text{monkey}} = \frac{(M-m)g}{m}$$

Substituting the given values,

$$a_{\text{monkey}} = \frac{(15\,\text{kg} - 10\,\text{kg})(9.80\,\frac{\text{m}}{\text{s}^2})}{10\,\text{kg}} = 4.9\,\frac{\text{m}}{\text{s}^2}$$

The monkey must pull himself upwards so as to give himself an acceleration of $4.9\,\frac{\text{m}}{\text{s}^2}$. Anything less and the box will remain on the ground.

(b) Next, suppose that after climbing for while (during which time the box has been rising off the ground) the monkey grabs onto the rope. What new condition does this give us?

Now it is true that the distance that the monkey moves *up* is the same as the distance which the box moves *down*. The same is true of the velocities and accelerations of the monkey and box, so in this part of the problem (recalling that I defined both accelerations as being in the upward sense),

$$a_{\rm monkey} = -a_{\rm box} \ .$$

This condition is not true in general, but here it *is* because we are told that the monkey is holding fast to the rope.

If you recall the example of the Atwood machine from your textbook or lecture notes, this is the same physical situation we are dealing with here. We expect the less massive monkey to accelerate upwards and the more massive box to accelerate downwards. Let's use the symbol a for the monkey's vertical acceleration; then the box's vertical acceleration is $-a$ and our equations are:

$$T - mg = ma$$

and

$$T - Mg = M(-a) \ .$$

At this point the physics is done and the rest is math (algebra) to solve for the two unknowns, T and a. Since the first of these equations gives $T = mg + ma$, substituting this into the second equation gives:

$$mg + ma - Mg = -Ma \qquad \Longrightarrow \qquad ma + Ma = Mg - mg$$

which gives:

$$(M+m)a = (M-m)g \qquad \Longrightarrow \qquad a = \frac{(M-m)}{(M+m)}g$$

Plugging in the numbers gives

$$a = \frac{(15.0\,\mathrm{kg} - 10.0\,\mathrm{kg})}{(15.0\,\mathrm{kg} + 10.0\,\mathrm{kg})}(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 2.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \ .$$

When the monkey is holding tight to the rope and both masses move freely, the monkey's acceleration is $2.0\,\frac{\mathrm{m}}{\mathrm{s}^2}$ upwards.

(c) Now that we have the acceleration a for this part of the problem, we can easily substitute into our results in part (b) and find the tension T. From $T - mg = ma$ we get:

$$T = mg + ma = m(g+a) = (10.0\,\mathrm{kg})(9.80\,\tfrac{\mathrm{m}}{\mathrm{s}^2} + 2.0\,\tfrac{\mathrm{m}}{\mathrm{s}^2}) = 118\,\mathrm{N} \ .$$

The tension in the rope is 118 N.

---

13. A mass M is held in place by an applied force F and a pulley system as shown in Fig. 4.11. The pulleys are massless and frictionless. Find (a) the tension in each section of rope, T_1, T_2, T_3, T_4, and T_5, and (b) the magnitude of F. [Ser4 5-65]

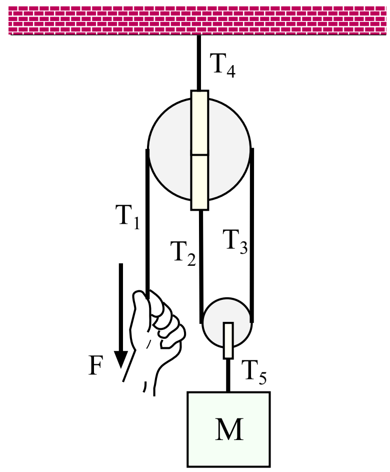

Figure 4.11: Crudely-drawn hand supports a mass M by means of a rope and pulleys

(a) We note first that the mass M (and therefore everything else) is motionless. This simplifies the problem considerably! In particular, every mass in this problem has no acceleration and so the total force on each mass is zero.

We have *five* rope tensions to find here, so we'd better start writing down some equations, fast! Actually, a few of them don't take much work at all; we know that when we have the (idealized) situation of massless rope passing around a frictionless massless pulley, the string tension is the *same* on both sides. As shown in the figure, it is a *single* piece of rope that wraps around the big upper pulley and the lower one, so the tensions T_1, T_2 and T_3 must be the *same*:

$$T_1 = T_2 = T_3$$

Not bad so far!

Next, think about the forces acting on mass M. This is pretty simple... the force of gravity Mg pulls down, and the tension T_5 pulls upward. That's all the forces but they sum to zero because M is motionless. So we must have

$$T_5 = Mg \ .$$

Next, consider the forces which act on the small pulley. These are diagrammed in Fig. 4.12(a). There is a downward pull of magnitude T_5 from the rope which is attached to M and also upward pulls of magnitude T_2 and T_3 from the long rope which is wrapped around the pulley. These forces must sum to zero, so

$$T_2 + T_3 - T_5 = 0$$

But we already know that $T_5 = Mg$ and that $T_2 = T_3$ so this tells us that

$$2T_2 - Mg = 0$$

which gives

$$T_2 = \frac{Mg}{2} \qquad \Longrightarrow \qquad T_3 = T_2 = \frac{Mg}{2} \ .$$

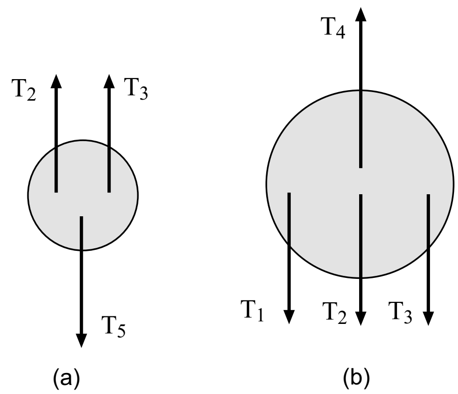

Figure 4.12: (a) Forces on the small (lower) pulley. (b) Forces on the large (upper) pulley.

We also have: $T_1 = T_2 = Mg/2$.

Next, consider the forces on the large pulley, shown in Fig. 4.12(b). Tension T_4 (in the rope attached to the ceiling) pulls upward and tensions T_1, T_2 and T_3 pull downward. These forces sum to zero, so

$$T_4 - T_1 - T_2 - T_3 = 0 \ .$$

But T_4 is the only unknown in this equation. Using our previous answers,

$$T_4 = T_1 + T_2 + T_3 = \frac{Mg}{2} + \frac{Mg}{2} + \frac{Mg}{2} = \frac{3Mg}{2}$$

and so the answers are:

$$T_1 = T_2 = T_3 = \frac{Mg}{2} \qquad T_4 = \frac{3Mg}{2} \qquad T_5 = Mg$$

(b) The force **F** is the (downward) force of the hand on the rope. It has the same magnitude as the force of the *rope on the hand*, which is T_1, and we found this to be $Mg/2$. So $F = Mg/2$.

---

14. Mass m_1 on a frictionless horizontal table is connected to mass m_2 through a massless pulley P_1 and a massless fixed pulley P_2 as shown in Fig. 4.13. (a) If a_1 and a_2 are the magnitudes of the accelerations of m_1 and m_2 respectively, what is the relationship between these accelerations? Find expressions for (b) the tensions in the strings and (c) the accelerations a_1 and a_2 in terms of m_1, m_2 and g. [Ser4 5-46]

(a) Clearly, as m_2 falls, m_1 will move to the right, pulled by the top string. But how do the magnitudes of the displacements, velocities and accelerations of m_2 and m_1 compare? They are not necessarily the same. Indeed, they are *not* the same.

Possibly the best way to show the relation between a_1 and a_2 is to do a little math; for a very complicated system we would have to do this anyway, and the practice won't hurt.

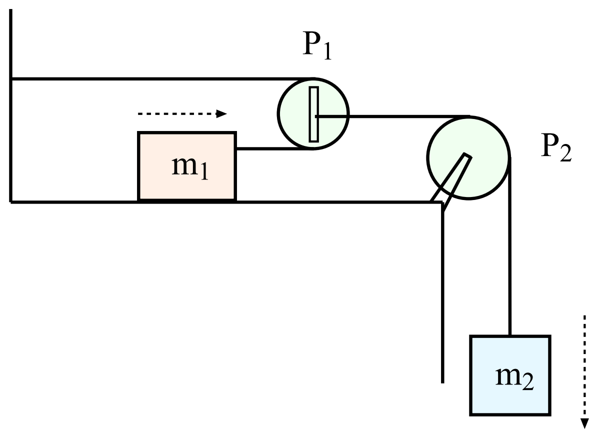

Figure 4.13: System of masses and pulleys for Example 14.

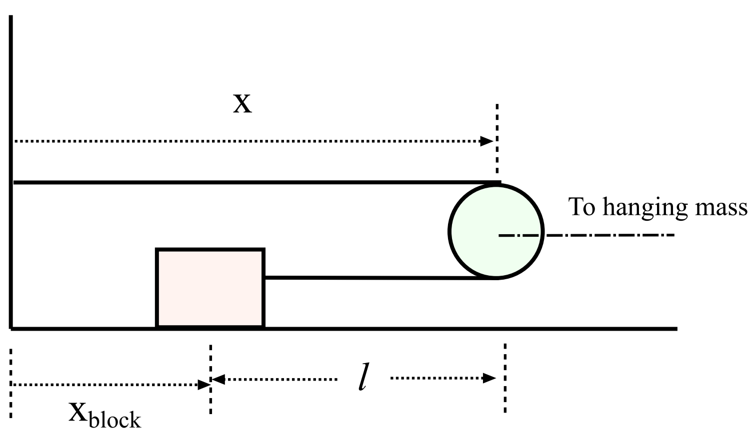

Figure 4.14: Some geometry for Example 14.

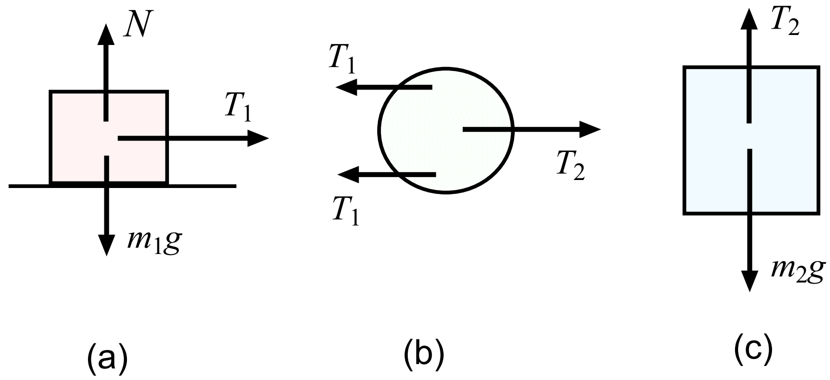

Figure 4.15: Forces on the masses (and moving pulley) in Example 14. (a) Forces on m_1. (b) Forces on the moving (massless) pulley. (c) Forces on m_2.

Focus on the upper mass (m_1) and pulley P_1, and consider the lengths labelled in Fig. 4.14. x measures the distance from the wall to the moving pulley; clearly the position of m_2 is also measured by x. ℓ is the length of string from m_1 to the pulley. $x_{\rm block}$ measures the distance from the wall to m_1. Then:

$$x_{\rm block} = x - \ell \ .$$

This really ignores the bit of string that wraps around the pulley, but we can see that it won't matter.

Now the *total* length of the string is $L = x + \ell$ and it does *not* change with time. Since we have $\ell = L - x$, we can rewrite the last equation as

$$x_{\rm block} = x - (L - x) = 2x - L$$

Take a couple time derivatives of this, keeping in mind that L is a constant. We get:

$$\frac{d^2 x_{\rm block}}{dt^2} = 2\frac{d^2 x}{dt^2}$$

But the left side of this equation is the acceleration of m_1 and the right side is the (magnitude of the) acceleration of m_2. The acceleration of m_1 is twice that of m_2:

$$a_1 = 2a_2$$

We can also understand this result by realizing that when m_2 moves down by a distance x, a length $2x$ of the string must go from the "underneath" section to the "above" section in Fig. 4.14. Mass m_1 follows the end of the string so it must move forward by a distance $2x$. Its displacement is always twice that of m_2 so its acceleration is always twice that of m_2.

(b) Now we try to get some information on the forces and accelerations, and we need to draw free–body diagrams. We do this in Fig. 4.15. Mass m_1 has forces $m_1 g$ acting downward, a normal force from the table N acting upward, and the string tension T_1 pulling to the right. The vertical forces cancel since m_1 has only a horizontal acceleration, a_1. Newton's Second Law gives us:

$$T_1 = m_1 a_1 \tag{4.5}$$

The pulley has forces acting on it, as shown in Fig. 4.15(b). The string wrapped around it exerts its pull (of magnitude T_1) both at the top and bottom so we have *two* forces of magnitude T_1 pulling to the left. The second string, which has a tension T_2, pulls to the right with a force of magnitude T_2.

Now this is a slightly subtle point, but the forces on the pulley must add to *zero* because the pulley is assumed to be *massless*. (A net force on it would give it an *infinite* acceleration!) This condition gives us:

$$T_2 - 2T_1 = 0 \tag{4.6}$$

Lastly, we come to m_2. It will accelerate downward with acceleration a_2. Summing the downward forces, Newton's Second Law gives us:

$$m_2 g - T_2 = m_2 a_2 \tag{4.7}$$

For good measure, we repeat the result found in part (a):

$$a_1 = 2a_2 \tag{4.8}$$

In these equations, the unknowns are T_1, T_2, a_1 and a_2... four of them. And we have four equations relating them, namely Eqs. 4.5 through 4.8. The *physics* is done. We just do *algebra* to finish up the problem.

There are many ways to do the algebra, but I'll grind through it in following way: Substitute Eq. 4.8 into Eq. 4.5 and get:

$$T_1 = 2m_1 a_2$$

Putting this result into Eq. 4.6 gives

$$T_2 - 2T_1 = T_2 - 4m_1 a_2 = 0 \qquad \Longrightarrow \qquad T_2 = 4m_1 a_2$$

and finally using this in Eq. 4.7 gives

$$m_2 g - 4m_1 a_2 = m_2 a_2$$

at which point we can solve for a_2 we find:

$$m_2 g = a_2(4m_1 + m_2) \qquad \Longrightarrow \qquad a_2 = \frac{m_2 g}{(4m_1 + m_2)} \tag{4.9}$$

Having solved for one of the unknowns we can quickly find the rest. Eq. 4.8 gives us a_1:

$$a_1 = 2a_2 = \frac{2m_2 g}{(4m_1 + m_2)} \tag{4.10}$$

Then Eq. 4.8 gives us T_1:

$$T_1 = m_1 a_1 = \frac{2m_1 m_2 g}{(4m_1 + m_2)} \tag{4.11}$$

Finally, since Eq. 4.6 tells us that $T_2 = 2T_1$ we get

$$T_2 = \frac{4m_1m_2g}{(4m_1+m_2)} \tag{4.12}$$

Summarizing our results from Eqs. 4.9 through 4.12, we have:

$$T_1 = \frac{2m_1m_2g}{(4m_1+m_2)} \qquad T_2 = \frac{4m_1m_2g}{(4m_1+m_2)}$$

for the tensions in the two strings and:

(c)

$$a_1 = \frac{2m_2g}{(4m_1+m_2)} \qquad a_2 = \frac{m_2g}{(4m_1+m_2)}$$

for the accelerations of the two masses.