a11 Part 1

by ___________________

Using the rules we provide build the complete trees that allow you to infer the types of the following expressions. You should produce both the completed tree, and identify each rule as you use it. We provide an example below. Note that the rules on the next page may be slightly different from those used in class.

$$
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\dfrac{\emptyset,\ y : Nat,\ x : Nat(y) = Nat}{\emptyset,\ y : Nat,\ x : Nat \vdash y : Nat}\,Var
}{\emptyset,\ y : Nat,\ x : Nat \vdash (sub1\ y) : Nat}\,sub1
}{\emptyset,\ y : Nat \vdash (\lambda\ (x)\ (sub1\ y)) : Nat \to Nat}\,Abstr
\qquad
\dfrac{}{\emptyset,\ y : Nat \vdash 4 : Nat}\,Num-Axiom
}{\emptyset,\ y : Nat \vdash ((\lambda\ (x)\ (sub1\ y))\ 4) : Nat}\,App
}{\emptyset,\ y : Nat \vdash (zero?\ ((\lambda\ (x)\ (sub1\ y))\ 4)) : Bool}\,zero?
}{\emptyset \vdash (\lambda\ (y)\ (zero?\ ((\lambda\ (x)\ (sub1\ y))\ 4))) : Nat \to Bool}\,Abstr
$$

1. $(((\lambda\ (x)\ (\lambda\ (y)\ (sub1\ y)))\ 5)\ 6)$
2. $(\lambda\ (!)\ (\lambda\ (n)\ (if\ (zero?\ n)\ 1\ (*\ n\ (!\ (sub1\ n))))))$
3. $(fix\ (\lambda\ (k1)\ (\lambda\ (n)\ (if\ (zero?\ n)\ 1\ (k1\ (sub1\ n))))))$

Typing Rules

The following are the typing rules for the λ-calculus and some additional forms. The variables we use in our expressions do have meanings. Where e, and subscripted versions thereof, represent arbitrary terms of the language, b represents only booleans, n represents only natural numbers, and x represents only variables. These induce restrictions on the application of these rules.

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}\ Var$$

$$\frac{}{\Gamma \vdash n : Nat}\ Nat - Axiom$$

$$\frac{}{\Gamma \vdash b : Bool}\ Bool - Axiom$$

$$\frac{\Gamma,\ x\ : \tau_1 \vdash e : \tau}{\Gamma \vdash (\lambda\ (x)\ \ e) : \tau_1 \rightarrow\ \tau}\ Abstr$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow\ \tau \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash (e_1\ e_2) : \tau}\ App$$

$$\frac{\Gamma \vdash e_1 : Nat \qquad \Gamma \vdash e_2 : Nat}{\Gamma \vdash (+\ e_1\ e_2) : Nat}\ +$$

$$\frac{\Gamma \vdash e_1 : Nat \qquad \Gamma \vdash e_2 : Nat}{\Gamma \vdash (*\ e_1\ e_2) : Nat}\ *$$

$$\frac{\Gamma \vdash e : Nat}{\Gamma \vdash (sub_1\ e) : Nat}\ sub1$$

$$\frac{\Gamma \vdash e : Nat}{\Gamma \vdash (zero?\ e) : Bool}\ zero?$$

$$\frac{\Gamma \vdash e_1 : Bool \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma \vdash e_3 : \tau}{\Gamma \vdash (if\ e_1\ e_2\ e_3) : \tau}\ if$$

$$\frac{\Gamma,\ x\ :\ \tau \vdash e : \tau}{\Gamma \vdash (fix\ (\lambda\ (x)\ e)) : \tau}\ fix$$