Question 1: Analytical Solutions

A. Two Drunks meeting after n steps of random walk.

Analytical Approach:

We know a random walk of n steps by any of the drunk guys is a random sequence of $L's$ and $R's$.
For e.g, $LLRRLRRL...$ is such a sequence.

Total number of ways to make exactly l left steps out of total n steps (and $n - l$ right steps) = $\binom{n}{l}$
Now for both of the drunk guys to meet at same place after n steps, they both need to make exactly same number of steps towards left and right.

Total number of ways for both of the drunk guys to make exactly l steps towards left = $\binom{n}{l}^2$

Total number of ways for both drunk guys to meet together = $\sum_{l=0}^{n} \binom{n}{l}^2$ = $\binom{2n}{l}$

Total number of possible random walks for each guy = 2^n
Total number of possible random walks for both guy together = $2^n * 2^n$ = 4^n

Probabilty that both the drunk guys meet after walking N random steps = $\frac{\binom{2n}{l}}{4^n}$ (ans)

B. Random walk ending at origin after n steps.

Analytical Approach:

For, the random walk to end at centre, the walk should consist of equal number of left and right steps.

Thus if n is $even$, the number of such possible walks = $\binom{n}{n/2}$
With an $odd\ n$, the number of such walks = 0
Total number of random walks of n steps = 2^n

Thus, if n is odd the probabilty of random walk to end at centre = 0

If n is $even$ the probabilty of random walk to end at centre = $\frac{\binom{n}{n/2}}{2^n}$ (ans)

C. Mean displacement of a Random Walk.

Analytical Approach:

Let's define, X_i be the random variable denoting the ith step out of n steps.
Thus, X_i = -1 if the ith step is toward $left$ and X_i = 1 if ith step is toward $right$.

Therefore final displacement after n steps, d = $\sum_{i=0}^{n} X_i$.

$\implies E[d] = \sum_{i=0}^{n} E[X_i]$

Now, $E[X_i] = \frac{1}{2} * (1) + \frac{1}{2} * (-1) = 0$

$\implies E[d] = n * 0 = 0 \; (ans)$

D. Mean squared displacement of a Random Walk.

Analytical Approach:

Let's define, X_i be the random variable denoting the ith step out of n steps.
Thus, X_i = -1 if the ith step is toward $left$ and X_i = 1 if ith step is toward $right$.

Therefore final displacement after n steps, d = $\sum_{i=0}^{n} X_i$.

$\implies d^2 = (\sum_{i=0}^{n} X_i)^2$

$\implies d^2 = (X_1^2 + X_2^2 + ... + X_n^2) + 2(X_1 X_2 + X_1 X_3 + ... + X_{n-1} X_n)$

$\implies d^2 = \sum_{i=0}^{n} {X_i}^2 + 2 \sum_{i \neq j, i=0, j=0}^{n} X_i X_j$

$\implies E[d^2] = \sum_{i=0}^{n} E[{X_i}^2] + 2 \sum_{i \neq j, i=0, j=0}^{n} E[X_i X_j]$

Now, $E[X_i^2] = \frac{1}{2} * (1)^2 + \frac{1}{2} * (-1)^2 = 1$
Similarly, $E[X_i X_j] = \frac{1}{4} * (1 * 1) + \frac{1}{4} * (-1 * 1) + \frac{1}{4} * (1 * -1) + \frac{1}{4} * (-1 * -1) = 0$

$\implies E[d^2] = n * 1 + 0 = n.$

Thus the mean-squared displacement after n steps = $n \; (ans)$