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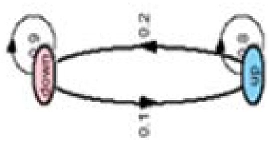
Hidden Markov Models

In the section on supervised learning we discussed estimating regression 'betas' in dynamic systems (e.g. Kalman filter). If these betas change discretely (rather than continuously), we refer to them as 'regimes'. **Hidden Markov Models** (HMM) are similar to the Kalman filter (i.e. similar to other State Space models) where the the probability of the next state only depends on the current state (i.e. hidden state follows a discrete Markov process). HMMs are useful statistical models because in many real world problems, we are interested in identifying some events which are not directly observable (e.g. are we in an up-trending or down-trending market?), but these events can be inferred from other variables that we can observe (e.g. market returns, market volatility, etc.).

Historically, HMMs have been applied extensively in speech recognition in the 1990s and more recently, in bioinformatics such as gene sequence analysis. In finance, it has been used to model market regimes. For inquisitive readers, we recommend the introduction to HMM by Rabiner and Juang (1986). We describe HMMs briefly here, motivated by our use-case of applying it to detect market regimes. The first question is to know the state of the market: Is it trending upwards? Unfortunately, we cannot observe it directly. Hence it is called a hidden state.

A Hidden Markov Model can be formulated as follows: A Markov process for the state of the market (suppose we only have 2 states – Up and Down):

Figure 70: 2-state HMM



	Up	Down
Up	80%	20%
Down	10%	90%

Market state: $\boxed{\text{Up}} \rightarrow \boxed{\text{Down}} \rightarrow \boxed{\text{Up}} \rightarrow \boxed{\text{Up}} \rightarrow \boxed{\text{Down}} \rightarrow \ldots$

Source: J.P.Morgan Macro QDS

This means that if the market is currently in the upward state, then the probability of remaining in the upward state is 80%, and there is 20% of probability of transiting to a downward state. Conditional on the state of the market, assume a distribution for the returns as $(r|\text{state}) \sim N(\mu_{\text{state}}, \sigma^2_{\text{state}})$.

Figure 71: Distributions of log returns conditional on market state

Source: J.P.Morgan Macro QDS

Since we observe the historical sequence of returns, we can infer the likelihood of being in a particular state at each time. Using the Expectation-Maximization algorithm[48] (i.e. the Baum-Welch algorithm, Bilmes (1998)), we can estimate the parameters in the Hidden Markov Model by maximizing the likelihood. The estimated parameters include: the initial probabilities in each state, transition probabilities of the state, the probabilities of being in each state, and mean and volatilities of the returns conditional on each state.

[48] The Expectation Maximization (EM) algorithm and extensions are discussed in Meng and Pedlow (1992), Little and Rubin (2002), Meng and Rubin (1991, 1993), Meng (1994a), van Dyk, Meng and Rubin (1995), Liu and Rubin (1994), Liu, Rubin and Wu (1998), Meng and van Dyk (1997), Liu and Wu (1999) and Liu (2003). Variational Bayes is described in Jordan et al (1999), Jaakkola and Jordan (2000), Blei, Ng and Jordan (2003), Gershman, Hoffman and Blei (2013).

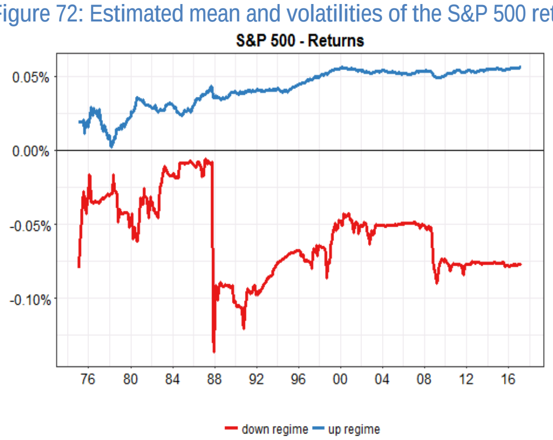
Use of Hidden Markov Models in Factor Timing:

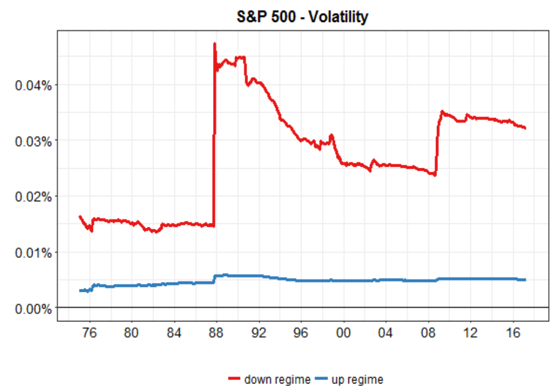
As an illustrative example, we design a market timing trading strategy based on HMM. It is a simple trend following strategy in which we are long the S&P 500 if market is trending higher, and are in cash if the market is down-trending. Using daily returns of the S&P 500 from April 1971, we estimate the above HMM model using the R package "mhsmm" (O'Connell et al 2011). We start the estimation from January 1975 so that we have about 4 decades of daily returns. In general, it is preferable to have more observations in the estimation of HMM so that one can ensure different states (in this case, Up and Down market) have occurred with a significant frequency.

On the last trading day of each month, we re-estimate the HMM model. The Figure below shows the estimated mean and volatilities in the conditional Gaussian distributions. This gives intuitive sense of the meaning of the two market states:

- Up state: Periods of positive returns, with lower volatilities; and
- Down state: Periods of negative returns, with higher volatilities.

Figure 72: Estimated mean and volatilities of the S&P 500 returns, conditional on the up and down state of the market





Source: J.P.Morgan Macro QDS

The probabilities in each state are shown in the figure below. These probabilities will be used to infer the state of the market: if the probability in the upward state is greater than 50%, we take the current state as Up. We also show the latest transition probabilities on the left.

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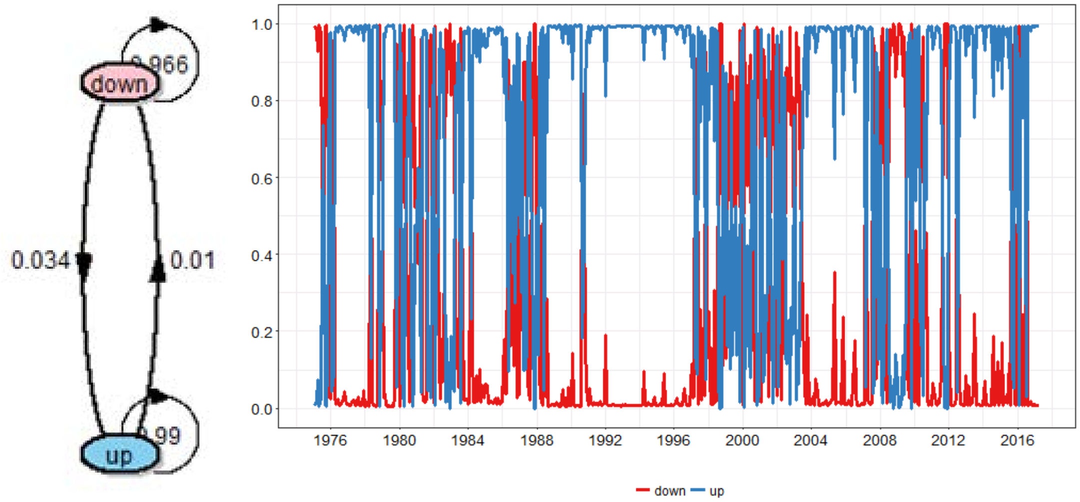
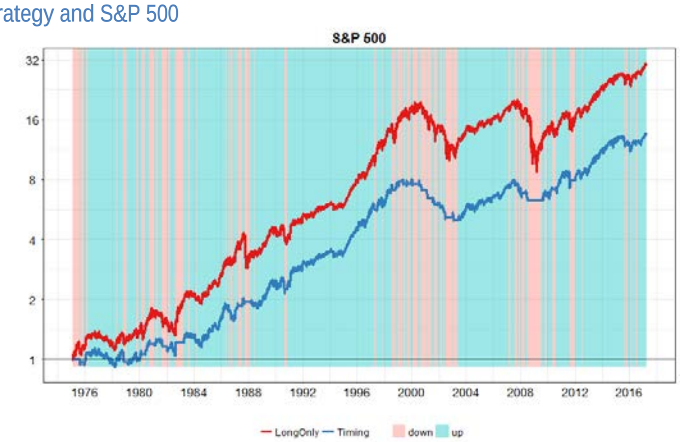
Figure 73: Estimated probabilities in each state (Up or Down market)



Source: J.P.Morgan Macro QDS

Using the HMM estimated on the last trading day of each month, we determine the latest state of the market. The simple trading strategy is as follows: if the market state is Up, we go long the S&P 500 index, and if the market state is Down, we are not invested (zero returns). HMM market timing strategy significantly reduced the drawdown (as compared to long S&P 500), and modestly improved the overall Sharpe ratio. Performance is shown in the figure below.

Figure 74: HMM monthly timing strategy and S&P 500



	Long-only	Timing
CAGR (%)	8.2	6.1
Volatility (%)	16.8	11
Information ratio	0.49	0.56
Max DD (%)	56.8	38.4

Source: J.P.Morgan Macro QDS