Scaling in Theropod Dinosaurs: Femoral Bone Dimensions

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Finding topics that inspire students is an important aspect of any physics course. Virtually everyone is fascinated by *Tyrannosaurus rex,* and the excitement of the class is palpable when we explore scaling effects in *T. rex* and other bipedal theropod dinosaurs as part of our discussion of mechanics and elasticity. In this paper, we explore the role of longitudinal stress in the femur bones due to the weight of the dinosaur in determining how the geometry of the femur changes with size of the theropod. This is one area of allometry, the study of how different biological characteristics scale with size.[1]

Only bipedal dinosaurs are studied since their single pair of legs supports all of the animal's weight. Analysis of quadrupedal animals is more difficult since the position of the center of mass of the animal determines the weight supported by either pair of legs. Determining the position of the center of mass of a living (extant) animal can be accomplished by measuring the fraction of the weight supported by each pair of legs. Such a determination in extinct animals is difficult since the positions and sizes of the internal tissues are poorly known and must be estimated using extant animals as guides. The lungs are particularly important since the density of inflated lungs is very low. By working with bipedal animals, this complication is avoided.

Since dinosaurs are extinct, our study is based on the fossilized remains of these magnificent creatures. Such remains are composed almost exclusively of bones and teeth. The leg bones supporting these bipedal animals are the femur (in the thigh) and the tibia and fibula (in the calf). We study the femur since it supports all of the weight of the animal. The important geometric parameters of the femur are its length L and diameter d.

The weight of an animal is related to its volume V via its density ρ. The class is asked about the average density of extant animals. Do they have roughly the same average density? How can one easily tell if various animals have the same average density? This leads to a discussion of the fact that most land animals can float in water, as seen when they swim. We make a list of all the land animals that have been observed swimming. That list usually includes humans, dogs, cats (unhappily), horses, cows, beavers, and various migrating animals that have to cross a river. All of these animals are seen to be able to swim with their head out of water, but not much else. These observations lead us to the conclusion that many extant mammals have a density slightly less than the density of water. This discussion also provides an excellent opportunity to review Archimedes' principle.

Let's examine how the geometry of the animal changes with size by considering two cases: geometric similarity and elastic similarity.

Geometric similarity

In geometric similarity, all of the dimensions of the animal maintain the same proportions to each other for all sizes. For this case, the femoral diameter and length are directly proportional:

$$d \sim L. \tag{1}$$

Elastic similarity

In elastic similarity, it is assumed that the elastic stress in the leg bones is kept constant despite its size. All of the weight of the animal is supported by its leg bones. The weight of the animal is assumed to be proportional to the cube of its femoral length ($W \propto L^3$). The strength of the femur is proportional to its cross-sectional area A and the square of its diameter ($Strength \propto A \propto d^2$). The femoral elastic stress is the weight of the animal divided by the cross-sectional area of the femur. In order to keep this stress constant, L^3 would have to scale as d^2:

$$L^3 \sim d^2 \Rightarrow d \sim L^{3/2}. \tag{2}$$

Models of several dinosaur femora[2] were purchased and are made available to the students for examination and measurement. The length of the femur is easy to measure. The femoral cross-sections are not perfectly circular, and different methodologies for measuring the effective diameter of the femur are discussed. The typical method used by paleontologists is to measure the circumference of the femoral cross section at its narrowest part and divide by π.

The students measure the length L and circumference of the available research casts of theropod femora. Additional data have been obtained by myself during a visit to the Cleveland Museum of Natural History. Donald Henderson of the Royal Tyrrell Museum of Paleontology kindly measured eight additional theropod femora. Data published by Per Christiansen[3] and Peter Larson[4] are also made available to the students. The data for these 87 femora are listed in the appendix. The lengths of the femora range from 8.5 to 134 cm, covering more than an order of magnitude.

It is important to note that a very wide range of body size should be included in such work in order to verify that the data set does obey a power-law relationship. The range of femora lengths could be increased (particularly for short lengths) by including bipedal mammals such as the kangaroo mouse. Given the enormous variety of quadrupedal animals, an analysis of those animals would cover an even greater range of femoral lengths. Analyzing both bipedal and quadrupedal animals would yield an even larger data set. Such larger data sets could be subdivided in different ways to look for trends among specific groups of animals. The subject of the present study is deliberately chosen to be bipedal theropods and we

 DOI: 10.1119/1.4872416

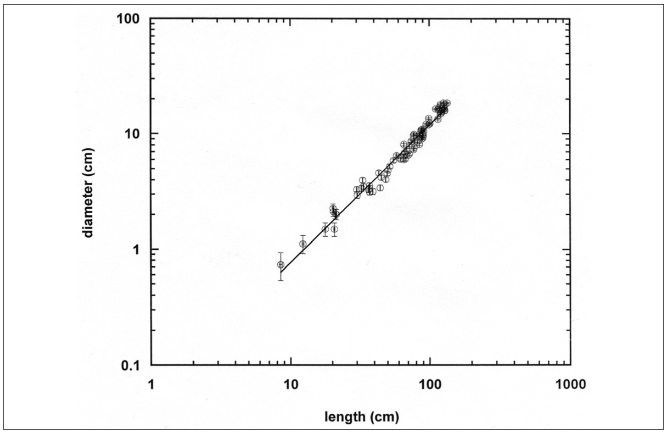

Fig. 1. The diameter of the femur in theropod dinosaurs as a function of femoral length are shown as open circles. The uncertainties are shown by the error bars. The solid line shows a power-law fit to the data.

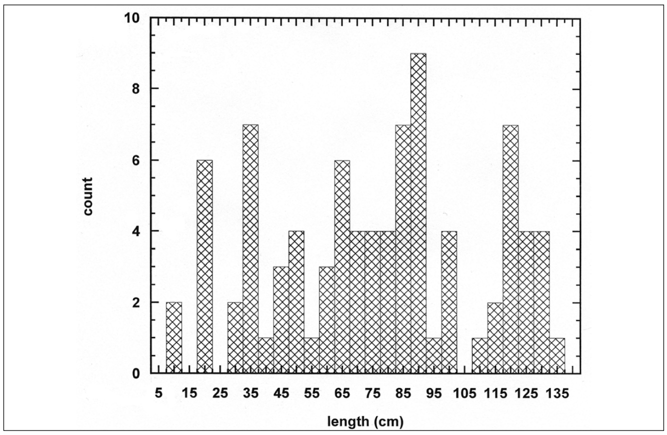

Fig. 2. Histogram of the femoral lengths of our data set. The bin size is 5 cm.

are limited by the fossils that have been recovered to date.

Figure 1 shows the dependence of the diameter on the length for the femur. The data are observed to fall along a straight line when using a log-log plot. This shows that the diameter and length are connected via a power law, consistent with the relationships derived for both similarity cases. Fitting the data, the following relationship is found:

$$d = AL^{\beta}, \tag{3}$$

where $A = (4.90 \pm 0.44)\times 10^{-2}$ cm$^{-.193}$ and $\beta = 1.193 \pm 0.021$. The fit to the data is very good ($R = 0.987$).

Our data do not match either prediction since our measured value of β is intermediate to the predictions of geometric similarity ($\beta = 1$) and elastic similarity ($\beta = 1.5$). This indicates that the geometry of the femur in these theropod dinosaurs is not controlled by either geometric or elastic similarity. Students are usually intrigued by the fact that reality is different from the cases we discussed and excitedly start considering other possible explanations.

An examination of the literature reveals that similar results have been obtained for both extant mammals[5] ($\beta = 1.21$) and extant mammals and birds[6] ($\beta = 1.12$).

The "typical" homework assignment never deals with possible biases in data, making this activity another learning opportunity for the students. Since paleontology is an observational science, only those fossils that have been discovered can provide data for our analysis. Does our data contain any bias due to incomplete sampling? Examining Fig. 1, it appears that there are fewer short femora than long ones. The fact that the axes in this figure are logarithmic makes such a visual determination difficult. Figure 2 shows a histogram of femoral lengths for our data set. Though no severe bias is revealed, there are gaps in the data and there are fewer short femora than long ones. It is beyond the scope of our physics course, but these observations about our data provide motivation for the students to learn about analyzing incomplete data sets via the appropriate statistical techniques. More importantly, it helps the students learn to keep asking questions as they explore the solution to a problem.

Since the femoral geometry is not controlled by either geometric or elastic similarity, the class then discusses other possibilities. In elastic similarity, the bones were loaded with longitudinal stress. Bones (and any long solid object) are weaker in the transverse direction than in the longitudinal direction. A discussion of the stresses on the leg bones during locomotion reveals that transverse stresses are always present while the animal is moving. In a future article, we will describe how we engage our students in the investigation of the role of transverse stresses on the geometry of the leg bones in theropod dinosaurs.

Acknowledgment

The assistance of Dr. Donald Henderson of the Royal Tyrrell Museum of Paleontology is gratefully acknowledged. The hospitality of the staff of the Cleveland Museum of Natural History is also gratefully acknowledged.

References

1. B. K. Ahlborn, *Zoological Physics* (Springer-Verlag, Berlin, 2006); J. T. Bonner, *Why Size Matters: From Bacteria to Blue Whales* (Princeton University Press, Princeton, NJ, 2006); A. Shingleton, "Allometry: The study of biological scaling," *Nat. Educ. Knowl.* **3** (10) 2, (2010).
2. Purchased from Gaston Design, Inc. (http://www.gastondesign.com). Other possible sources of bone models include Bone Clones, Inc. (http://www.boneclones.com/). An Internet search will reveal other sources of model bones.
3. P. Christiansen, "Long bone scaling and limb posture in non-avian theropods: Evidence for differential allometry," *J. Vertebr. Paleontol.* **19**, 666–680 (1999).
4. P. Larson, "Variation and sexual dimorphism in Tyrannosaurus rex," in *Tyrannosaurus rex: The Tyrant King,* edited by Peter Larson and Kenneth Carpenter (Indiana University Press, 2008), pp. 103–128.
5. P. Christiansen and G. S. Paul, "Limb bone scaling, limb proportions, and bone strength in neoceratopsian dinosaurs," *Gaia* **16**, 13–29 (2001).
6. A. A. Biewener, "Biomechanical consequences of scaling," *J. Exp. Biol.* **208**, 1665–1676 (2005).

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Appendix

Table I. The species, length, and circumference for 87 theropod femora. Source of measurements: a = research casts owned by author; b = measurements made on research casts at the Cleveland Museum of Natural History; c = measurements made by Donald Henderson at the Royal Tyrrell Museum of Paleontology; d = measurements from Ref. 3; and e = measurements from Ref. 4.

Species	Length (cm)	Circumference (cm)	Species	Length (cm)	Circumference (cm)
Tyrannosaurus rex[a]	128.9	50.2	*Struthiomimus altus*[d]	48.6	12.7
Acrocanthosaurus atokensis[a]	115.1	42.1	*Struthiomimus altus*[d]	37.2	9.9
Allosaurus[a]	84.3	30.7	*Ornithomimidae*[d]	37.1	11.2
Teratophoneus[a]	74.1	27.1	*Ornithomimidae*[d]	37.2	10.6
Struthiomimus sedens[a]	62.2	18.9	*Sauronrnithoides*[d]	8.5	2.3
Utahraptor[a]	58.1	20.4	*Alectrosaurus olseni*[d]	66.1	21.5
Falcarius[a]	33.2	12.5	*Albertosaurus*[d]	90.1	31.3
Conchoraptor[a]	20.3	7.1	*Gorgosaurus libratus*[d]	65.4	18.8
Velociraptor mongoliensis[a]	20.4	6.7	*Gorgosaurus libratus*[d]	89.3	28.7
Allosaurus[b]	81.7	29.5	*Gorgosaurus libratus*[d]	90.5	29.8
Allosaurus[b]	77.7	31.1	*Gorgosaurus libratus*[d]	78.1	24.1
Nanotyrannus[b]	71.8	21.1	*Tyrannosaurus bataar*[d]	85.4	31.2
Nanotyrannus[b]	77.0	23.0	*Tyrannosaurus bataar*[d]	77.1	28.1
Tyrannosaurus rex[c]	124	52	*Tyrannosaurus rex*[d]	128.9	49.7
tyrannosaurid[c]	95	38	*Tyrannosaurus rex*[d]	127.3	54.0
tyrannosaurid[c]	67	19	*Tyrannosaurus torosus*[d]	100.6	38.9
Gorgosaurus[c]	77	30.5	*Tyrannosauridae*[d]	59.9	19.8
Gorgosaurus[c]	89	31	*Tyrannosauridae*[d]	100.7	38.2
Gorgosaurus[c]	86	30	*Tyrannosauridae*[d]	70.2	20.5
Gorgosaurus[c]	88	34	*Microvenator celer*[d]	12.3	3.5
Albertosaurus[c]	65	20	*Oviraptor philoceratops*[d]	30.3	9.3
Coelophysis bauri[d]	20.7	4.7	*Deinonychus antirrhopus*[d]	33.6	11.0
Dilophosaurus wetherilli[d]	55.1	18.5	*Saurornitholestes langstoni*[d]	21.4	6.3
Elaphrosaurus bambergi[d]	51.9	16.5	*Theropoda sp.*[d]	68.8	22.8
Allosaurus fragilis[d]	87.4	29.3	*Theropoda sp.*[d]	65.6	25.4
Allosaurus fragilis[d]	98.1	37.5	*Theropoda sp.*[d]	17.8	4.7
Allosaurus fragilis[d]	88.4	34	*Theropoda sp.*[d]	36.5	10.4
Allosaurus fragilis[d]	90.8	33.1	*Tyrannosaurus rex*[e]	120	54.5
Allosaurus fragilis[d]	89.9	34.5	*Tyrannosaurus rex*[e]	126	58
Allosaurus fragilis[d]	84.8	25.5	*Tyrannosaurus rex*[e]	115	51
Allosaurus fragilis[d]	87.2	33.8	*Tyrannosaurus rex*[e]	123.2	48.3
Sinraptor dongi[d]	88.4	28.3	*Tyrannosaurus rex*[e]	134	58
Dryptosaurus aquilunguis[d]	78.7	27	*Tyrannosaurus rex*[e]	118	52.7
Ornitholestes hermanni[d]	21	6.4	*Tyrannosaurus rex*[e]	111	51.5
Anserimimus planinychus[d]	43.3	14.5	*Tyrannosaurus rex*[e]	119	49.4
Archaeornithomimus asiaticus[d]	32.6	10.6	*Tyrannosaurus rex*[e]	118	51.2
Archaeornithomimus asiaticus[d]	30.1	10.3	*Tyrannosaurus rex*[e]	120	56
Dromiceionimus brevitertius[d]	39.2	10.0	*Tyrannosaurus rex*[e]	121	47
Dromiceionimus brevitertius[d]	45.1	13.2	*Tyrannosaurus rex*[e]	99	42.5
Gallimimus bullatus[d]	67.3	21.6	*Tyrannosaurus rex*[e]	120	46.7
Gallimimus bullatus[d]	50.4	15.3	*Tyrannosaurus rex*[e]	127.5	51.4
Ornithomimus edmontonensis[d]	44.3	10.7	*Tyrannosaurid*[e]	129.5	56
Ornithomimus edmontonensis[d]	49.7	14.3	*Nanotyrannus*[e]	72	25
			Gorgosaurus[e]	82.5	27