Homework week 6 - Assigment 4

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Question 1:

In a QDA model observation within each class are assumed to be drawn from a normal distribution with class specific means and class specific variances. Suppose we have a single predictor X (i.e., $p = 1$). If we have K classes, $X \sim \mathcal{N}(\mu_k, \sigma_k)$, for $k = 1, \cdots, K$. Show that the decision boundary function $\delta_k(x)$ is quadratic in x.

Consider 1 dimension $X \sim \mathcal{N}(\mu_k, \sigma_k)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_k)^2}{2\sigma_k^2}}$$

I would calculate one covariance matrix per class. I mean, $\sum_k$.

then, assuming I calculate every posterior, I will choose the class with the largest posterior in order to get the decision function.

$$\begin{aligned}
d(x) &= e^{-\frac{(x-\mu_k)^2}{2\sum_k}} .\pi_k \\
&= \log\left(e^{-\frac{(x-\mu_k)^2}{2\sum_k}} \right) + \log(\pi_k) \\
&= -\left[\frac{(x-\mu_k)^2}{2\sum_k} \right] + \log(\pi_k) \\
&= -\left[\frac{1}{2}(x-\mu_k)^2 \sum_k^{-1} \right] + \log(\pi_k) \\
&= -\left[\frac{1}{2}(x^2 - 2x\mu_k + \mu_k^2) \sum_k^{-1} \right] + \log(\pi_k)
\end{aligned}$$

Question 2:

Let $X_1, \cdots, X_n$ be i.i.d random variables with mean μ and variance σ^2 and let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that $E[\bar{X}] = \mu$ and $Var(\bar{X}) = \sigma^2/n$. The random variable $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is called the sample variance. Show that $E[S^2] = \sigma^2$. Can you show that $\bar{X}$ and S^2 are independent (the last part is optional).

Question 3:

Write an 'R' function to compute the FPR and TPR for binary QDA and LDA models at different values of cutoff threshold. Note that the threshold is determined by the prior class probabilities.

```
cutoff.pred <- function(fitmodel, dataset, values, plot = T) {

  pred.t <- predict(fitmodel, newdata = dataset)


  thresh <- seq(0.01, 0.99, by = 0.01)

  TPR <- {}
  FPR <- {}

  for(t in thresh){
  classes.t <- pred.t$class
  per.t <- modelAssessment( values, classes.t)
  TPR <- c(TPR, per.t$TPR)
  FPR <- c(FPR, per.t$FPR)
}
 return(list(TPR, FPR))
}
```

Question 4:

This question involves the development of a classification model to predict whether a given car gets higher or lower bass milage based on the Auto data. For this problem use appropriate predictors (e.g, Auto names should not be used.)

1. Perform a summary statistics on the variables.

```r
library(tidyverse)
```

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.3     v readr     2.1.4
v forcats   1.0.0     v stringr   1.5.0
v ggplot2   3.4.3     v tibble    3.2.1
v lubridate 1.9.2     v tidyr     1.3.0
v purrr     1.0.2
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to becon
```

```r
library(ISLR)
library(MASS)
```

```
Attaching package: 'MASS'

The following object is masked from 'package:dplyr':

    select
```

```r
require(klaR)
```

```
Loading required package: klaR
```

```r
require(pracma)
```

```
Loading required package: pracma

Attaching package: 'pracma'

The following object is masked from 'package:purrr':

    cross
```

```r
data(Auto)

head(Auto)
```

```
  mpg cylinders displacement horsepower weight acceleration year origin
1  18         8          307        130   3504         12.0   70      1
2  15         8          350        165   3693         11.5   70      1
3  18         8          318        150   3436         11.0   70      1
4  16         8          304        150   3433         12.0   70      1
5  17         8          302        140   3449         10.5   70      1
6  15         8          429        198   4341         10.0   70      1
                       name
1 chevrolet chevelle malibu
2         buick skylark 320
3        plymouth satellite
4             amc rebel sst
5               ford torino
6          ford galaxie 500
```

```r
summary(Auto)
```

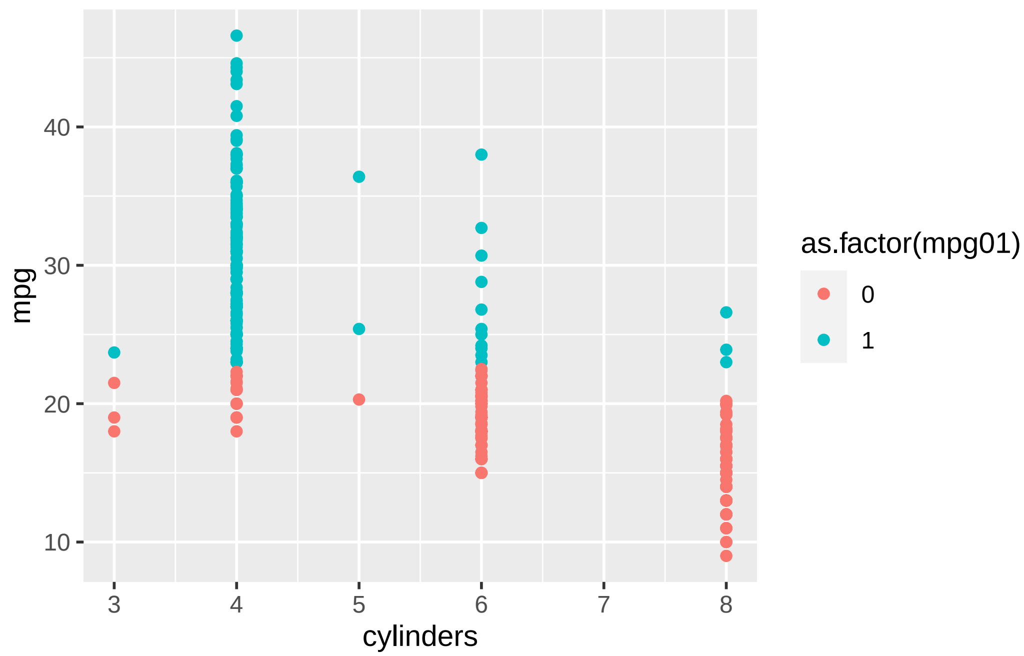
```
      mpg          cylinders      displacement     horsepower        weight    
 Min.   : 9.00   Min.   :3.000   Min.   : 68.0   Min.   : 46.0   Min.   :1613  
 1st Qu.:17.00   1st Qu.:4.000   1st Qu.:105.0   1st Qu.: 75.0   1st Qu.:2225  
 Median :22.75   Median :4.000   Median :151.0   Median : 93.5   Median :2804  
 Mean   :23.45   Mean   :5.472   Mean   :194.4   Mean   :104.5   Mean   :2978  
 3rd Qu.:29.00   3rd Qu.:8.000   3rd Qu.:275.8   3rd Qu.:126.0   3rd Qu.:3615  
 Max.   :46.60   Max.   :8.000   Max.   :455.0   Max.   :230.0   Max.   :5140  
                                                                               
  acceleration        year           origin                      name    
 Min.   : 8.00   Min.   :70.00   Min.   :1.000   amc matador       :  5  
 1st Qu.:13.78   1st Qu.:73.00   1st Qu.:1.000   ford pinto        :  5  
 Median :15.50   Median :76.00   Median :1.000   toyota corolla    :  5  
 Mean   :15.54   Mean   :75.98   Mean   :1.577   amc gremlin       :  4  
 3rd Qu.:17.02   3rd Qu.:79.00   3rd Qu.:2.000   amc hornet        :  4  
 Max.   :24.80   Max.   :82.00   Max.   :3.000   chevrolet chevette:  4  
                                                 (Other)           :365  
```

1.Create a binary variable ‘mpg01‘, that contains 1 if mpg value is above the median, and 0 if mpg is below the median. You may use ‘data.frame()‘ function to create a new data set with the binary response ‘mpg01‘ and other appropriate predictors.

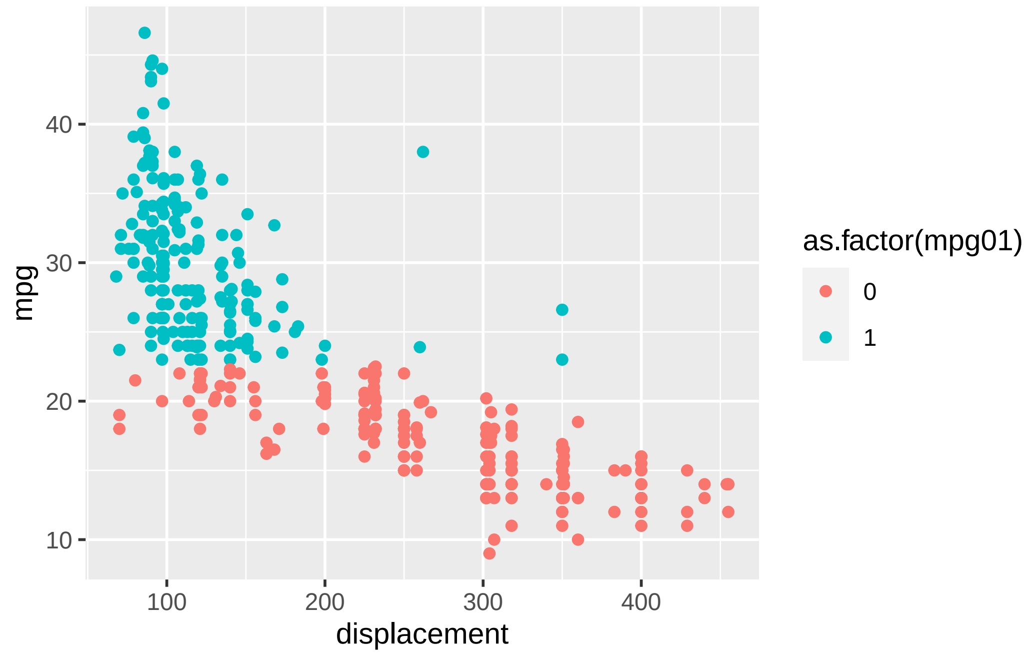
```
auto2 <- Auto |> mutate( mpg01 = ifelse(mpg >= 22.75,1,0 ))
```

2. Explore the data graphically. Color code the points based on response. Which predictor seems most associated with ‘mpg01‘

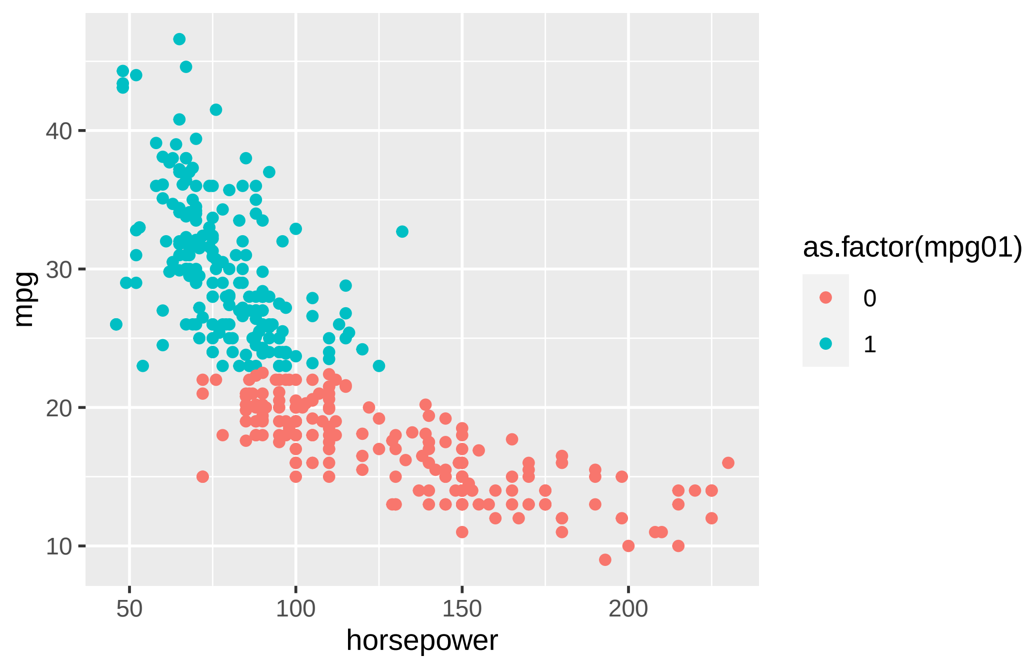
```
ggplot(auto2, aes(y= mpg, x = cylinders)) +
  geom_point(aes(color= as.factor(mpg01)))
```



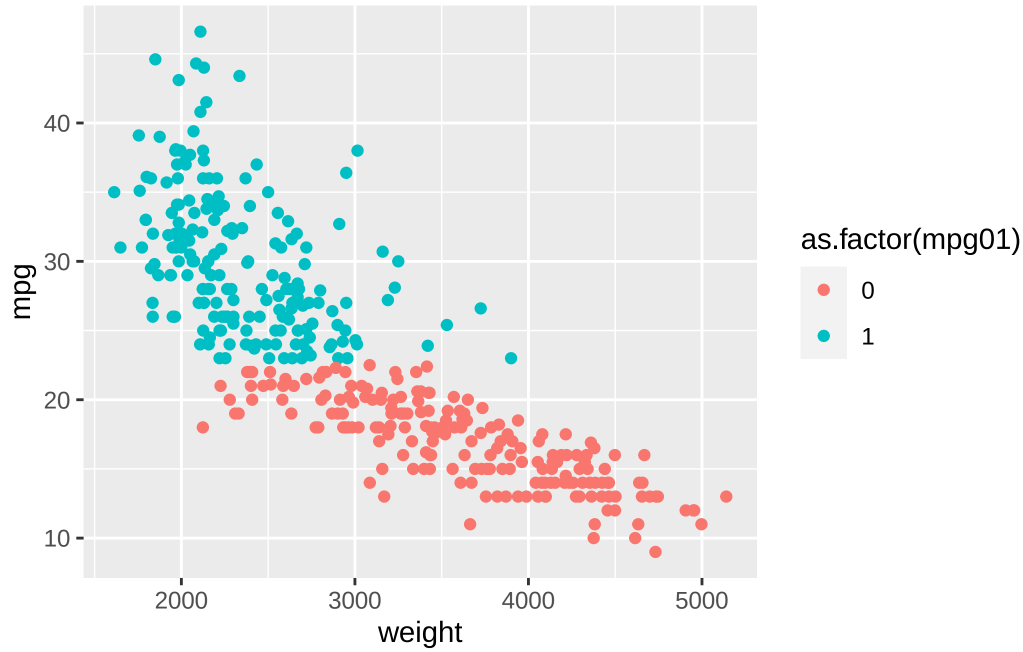

```
ggplot(auto2, aes(y= mpg, x = displacement)) +
  geom_point(aes(color= as.factor(mpg01)))
```



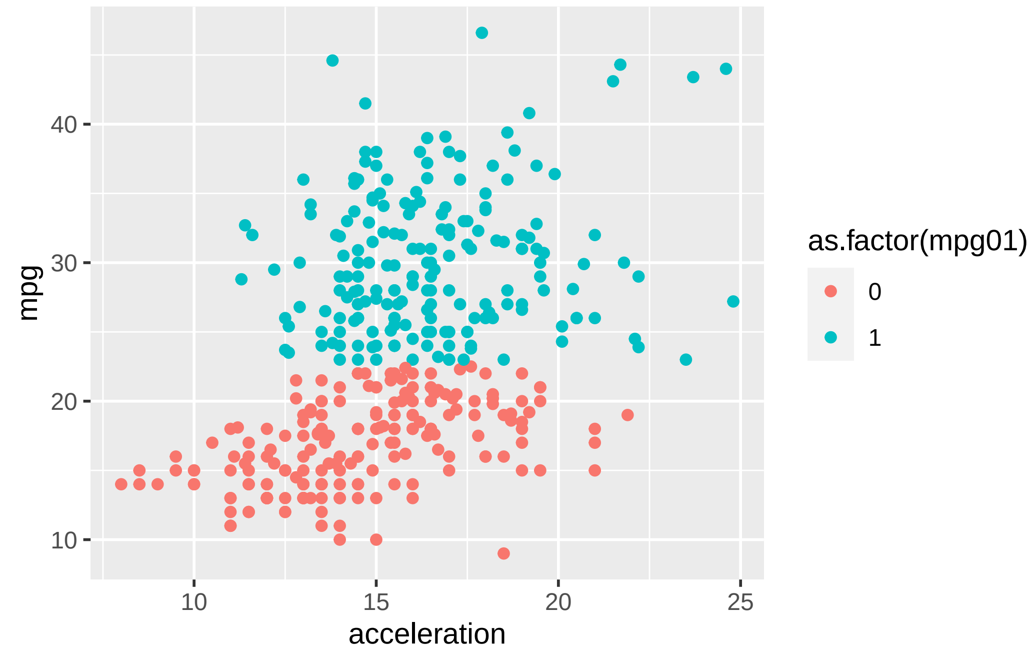

```
ggplot(auto2, aes(y= mpg, x = horsepower )) +
  geom_point(aes(color= as.factor(mpg01)))
```



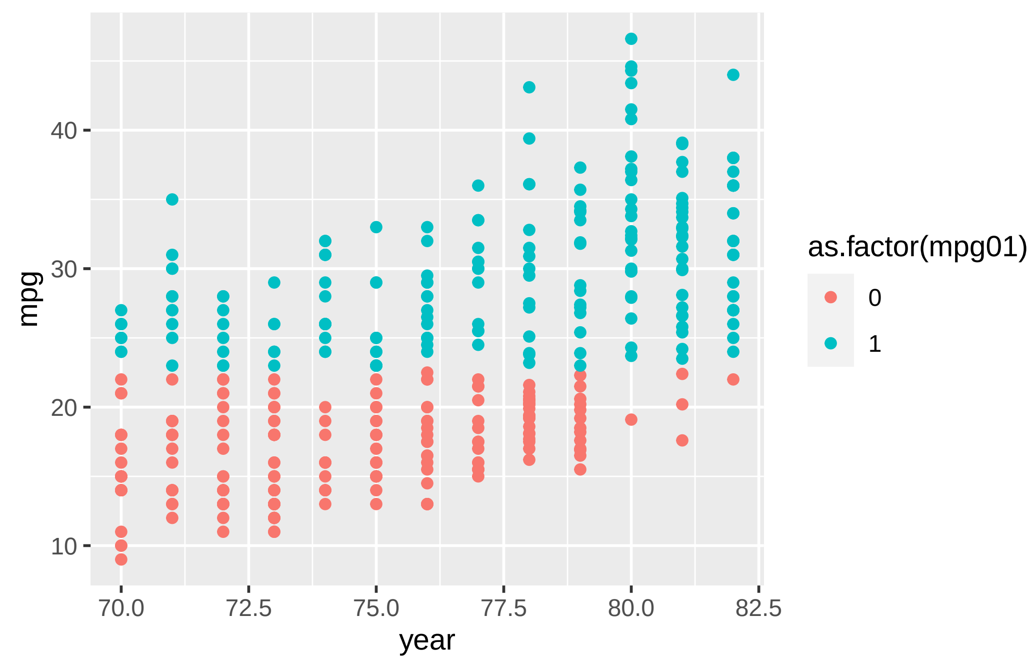
```r
ggplot(auto2, aes(y= mpg, x = weight )) +
  geom_point(aes(color= as.factor(mpg01)))
```



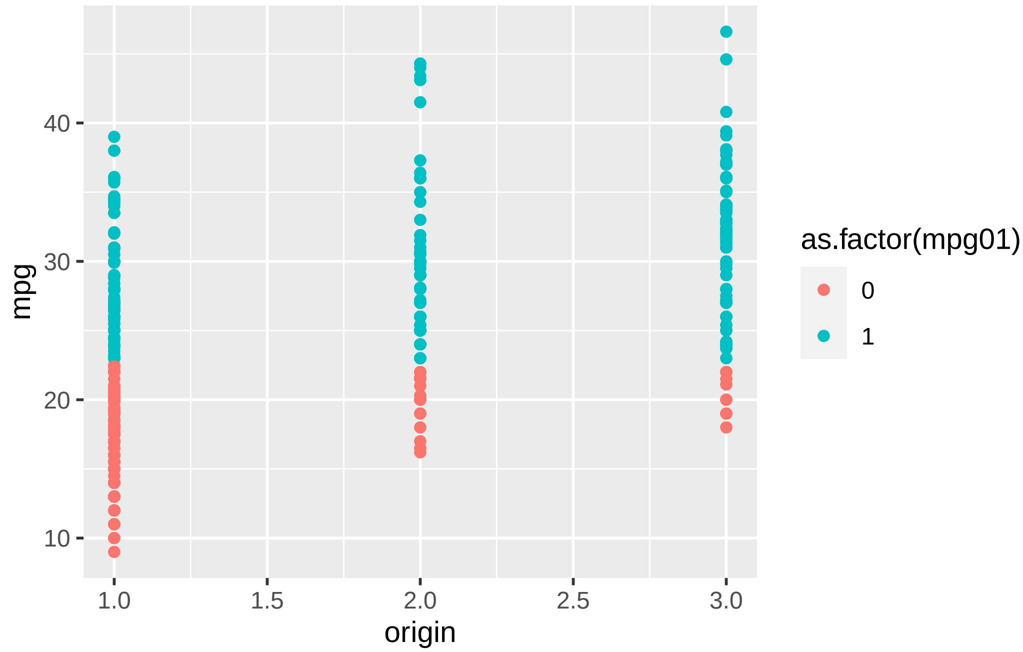

```r
ggplot(auto2, aes(y= mpg, x = acceleration )) +
  geom_point(aes(color= as.factor(mpg01)))
```



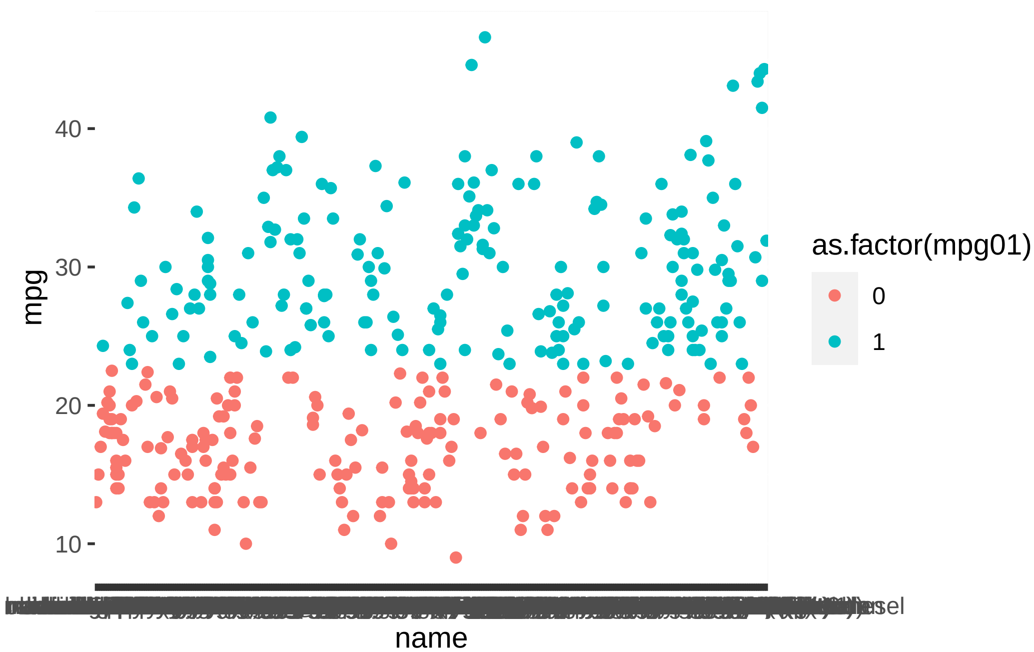

```r
ggplot(auto2, aes(y= mpg, x = year)) +
  geom_point(aes(color= as.factor(mpg01)))
```



```r
ggplot(auto2, aes(y= mpg, x = origin )) +
  geom_point(aes(color= as.factor(mpg01)))
```




```r
ggplot(auto2, aes(y= mpg, x = name )) +
  geom_point(aes(color= as.factor(mpg01)))
```



3. Split the data randomly to train and test sets (use $2/3$ of the data from training and $1/3$ for testing. You can use ‘sample()‘ function to randomly select the indices.)

```
lenght.Test <- ceiling(nrow(auto2)* 0.33)

lenght.Train <- nrow(auto2)- lenght.Test

row.test <- sample(1: nrow(auto2), lenght.Test, replace = F)

auto.test <- auto2[row.test,]

auto.train <- auto2[-row.test,]
```

4. Perform LDA on the training data in order to predict ‘mpg01‘ on the test set. You will need to use the ‘newdata‘ argument in the ‘predict()‘ function. What is the test error rate? Plot the ROC curve and compute the AUC.

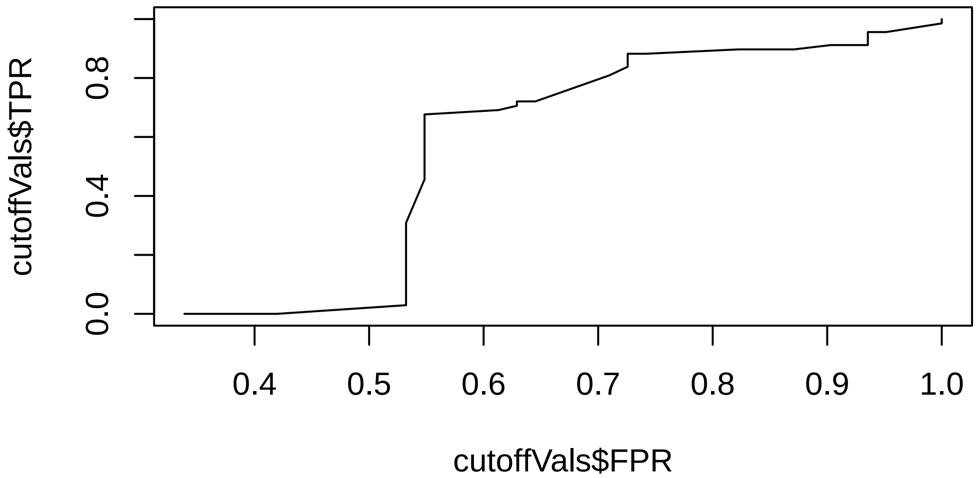
```
modelAssessment <- function(obs, pred){
  TP <- length(which(obs == 1 & pred == 1))
  TN <- length(which(obs != 1 & pred != 1))
  FP <- length(which(obs != 1 & pred == 1))
  FN <- length(which(obs == 1 & pred != 1))
```

```r
  sens <- TP / (TP + FN)
  spec <- TN / (FP + TN)
  accu <- (TP + TN) / (TP + TN + FP + FN)

  TPR <- sens
  FPR <- 1 - spec

  L <- list(sens = sens, spec = spec, accu = accu, TPR = TPR, FPR = FPR)

  return(L)
}

colnames(auto2)
```

```
[1] "mpg"          "cylinders"    "displacement" "horsepower"   "weight"
[6] "acceleration" "year"         "origin"       "name"         "mpg01"
```

```r
## LDA
fit.lda <- lda(mpg01~displacement+horsepower, data = auto.train)
pred.lda <- predict(fit.lda, newdata= auto.train)

lda.lab <- pred.lda$class
lda.per <- modelAssessment(auto.train$mpg01,lda.lab)

### fit test using my fuction in the exercise 2

cutoff.pred <- function(fitmodel, dataset, values) {

  pred.t <- predict(fitmodel, newdata = dataset)

  thresh <- seq(0.01, 0.99, by = 0.01)

  TPR <- {}
  FPR <- {}

  major <-apply(pred.t$posterior,1, max)

  for(t in thresh){
  classes.t <- ifelse( major >= t, 1, 0)
  per.t <- modelAssessment( values, classes.t)
```

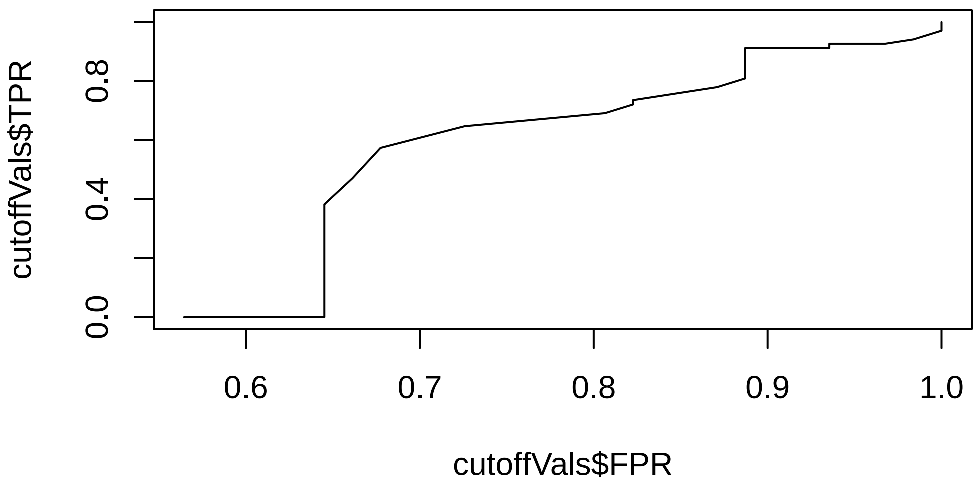
```
  TPR <- c(TPR, per.t$TPR)
  FPR <- c(FPR, per.t$FPR)
}
 return(list(TPR = TPR, FPR = FPR))
}


cutoffVals <- cutoff.pred(fitmodel = fit.lda, dataset = auto.test, values = auto.test$mpg0

plot(cutoffVals$FPR, cutoffVals$TPR, type = 'l')
```



5. Perform QDA on the training data in order to predict ‘mpg01‘ on the test set. What is the test error rate? Plot the ROC curve and compute the AUC.

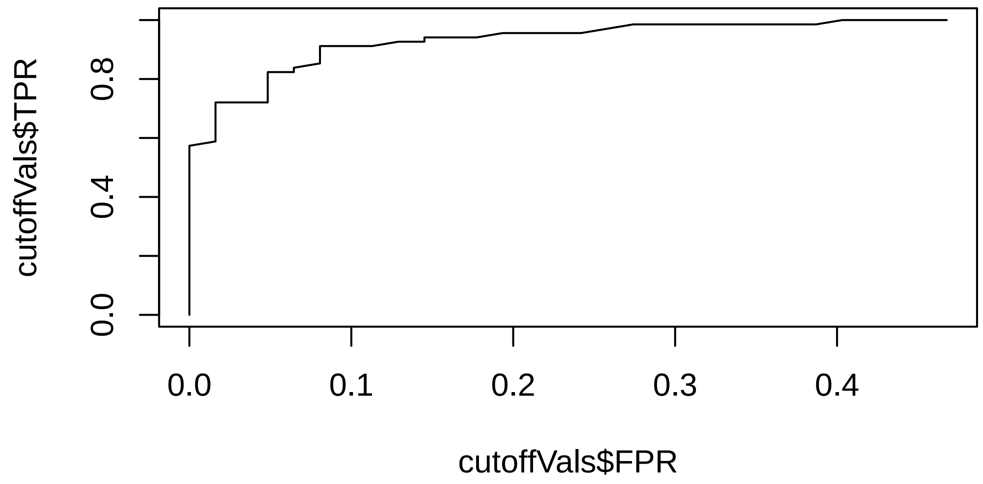
```
## QDA
fit.qda <- qda(mpg01~displacement+horsepower, data = auto.train)
pred.qda <- predict(fit.qda, newdata= auto.train)

qda.lab <- pred.qda$class
qda.per <- modelAssessment(auto.train$mpg01,qda.lab)

### fit test using my fuction in the exercise 2

cutoff.pred <- function(fitmodel, dataset, values) {

  pred.t <- predict(fitmodel, newdata = dataset)
```

```r
  thresh <- seq(0.01, 0.99, by = 0.01)

  TPR <- {}
  FPR <- {}

  major <-apply(pred.t$posterior,1, max)

  for(t in thresh){
  classes.t <- ifelse( major >= t, 1, 0)
  per.t <- modelAssessment( values, classes.t)
  TPR <- c(TPR, per.t$TPR)
  FPR <- c(FPR, per.t$FPR)
}
 return(list(TPR = TPR, FPR = FPR))
}


cutoffVals <- cutoff.pred(fitmodel = fit.qda, dataset = auto.test, values = auto.test$mpg0

plot(cutoffVals$FPR, cutoffVals$TPR, type = 'l')
```



Perform a logistic regression on the training data in order to predict ‘mpg01‘ on the test set. Perform a summary statistic on the fit object. What are the important predictors if any? Plot the ROC curve and compute the AUC.

```r
## Logistic regression
fit.logit <- glm(mpg01~displacement+horsepower, data = auto.train, family = 'binomial')
pred.logit <- predict(fit.logit, newdata = auto.train, type = 'response')
logit.lab <- ifelse(pred.logit >= 0.5, 1, 0)
logit.per <- modelAssessment(auto.train$mpg01, logit.lab)


### fit test using my fuction in the exercise 2

cutoff.pred <- function(fitmodel, dataset, values) {

  pred.t <- predict(fitmodel, newdata = dataset, type = 'response')

  thresh <- seq(0.01, 0.99, by = 0.01)

  TPR <- {}
  FPR <- {}

  for(t in thresh){
  classes.t <- ifelse( pred.t >= t, 1, 0)
  per.t <- modelAssessment( values, classes.t)
  TPR <- c(TPR, per.t$TPR)
  FPR <- c(FPR, per.t$FPR)
}
 return(list(TPR = TPR, FPR = FPR))
}


cutoffVals <- cutoff.pred(fitmodel = fit.logit, dataset = auto.test, values = auto.test$mp

plot(cutoffVals$FPR, cutoffVals$TPR, type = 'l')
```


cutoffVals$TPR
0.0
0.4
0.8
0.0
0.1
0.2
0.3
0.4
cutoffVals$FPR