# HW_Week2_Linear_Regression

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- Question 1: Let $x_i$ and $y_i$ be real values samples from random variables $X$ and $Y$ and let $\bar{x}$ and $\bar{y}$ be the sample means. Show that

1. $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

$$
\begin{aligned}
0 &= \sum_{i=1}^{n}(x_i - \bar{x}) \\
&= \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x} \\
&= \sum_{i=1}^{n} x_i - \bar{x}\sum_{i=1}^{n} n \\
&= \sum_{i=1}^{n} x_i - n\bar{x} \\
0 &= n\bar{x} - n\bar{x}
\end{aligned}
$$

`END of block`

2. $\sum_{i=1}^{n}(x_i - \bar{x})y_i = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$.

$$
\begin{aligned}
\sum_{i=1}^{n}(x_i-\bar{x})y_i &= \sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) \\
&= \sum_{i=1}^{n} x_iy_i - \sum_{i=1}^{n} x_i\bar{y} - \sum_{i=1}^{n} \bar{x}y_i + \sum_{i=1}^{n} \bar{x}\bar{y} \\
&= \sum_{i=1}^{n} x_iy_i - \bar{y}\sum_{i=1}^{n} x_i - \bar{x}\sum_{i=1}^{n} y_i + \bar{x}\bar{y}\sum_{i=1}^{n} n \\
&= \sum_{i=1}^{n} x_iy_i - \bar{y}\sum_{i=1}^{n} x_i - \bar{x}\sum_{i=1}^{n} y_i + n\bar{x}\bar{y} \\
&= \sum_{i=1}^{n} x_iy_i - \bar{y}\sum_{i=1}^{n} x_i - \bar{x}n\bar{y} + n\bar{x}\bar{y} \\
&= \sum_{i=1}^{n} x_iy_i - \bar{y}\sum_{i=1}^{n} x_i - n\bar{x}\bar{y} + n\bar{x}\bar{y} \\
&= \sum_{i=1}^{n} x_iy_i - \bar{y}\sum_{i=1}^{n} x_i + 0 \\
&= \sum_{i=1}^{n} x_iy_i - \bar{y}n\bar{x} + 0 \\
&= \sum_{i=1}^{n} x_iy_i - n\bar{y}\bar{x} + 0 \\
&= \sum_{i=1}^{n} x_iy_i - \sum_{i=1}^{n} y_i\bar{x} + 0 \\
&= \sum_{i=1}^{n} x_iy_i - \bar{x}\sum_{i=1}^{n} y_i \\
&= \sum_{i=1}^{n} x_iy_i - \bar{x}\sum_{i=1}^{n} y_i \\
&= \sum_{i=1}^{n} x_i\sum_{i=1}^{n} y_i - \bar{x}\sum_{i=1}^{n} y_i \\
&= \sum_{i=1}^{n} y_i(x_i-\bar{x})
\end{aligned}
$$

`END of block`

- Question 2: Let $\mathbf{b}$ be a constant vector and let $\mathbf{x}$ be an variable in $\mathbb{R}^n$. Let $A$ $n \times n$ symmetric matrix. Show the following

1. $\nabla_x \mathbf{b}^T\mathbf{x} = \mathbf{b}$.

$$\nabla_x \mathbf{b}^T \mathbf{x} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_n} \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$$

$$\nabla_x \mathbf{b}^T \mathbf{x} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_n} \end{bmatrix} \begin{bmatrix} b_1 x_1 + b_2 x_2 + b_n x_n \end{bmatrix}$$

$$\nabla_x \mathbf{b}^T \mathbf{x} = \begin{bmatrix} \frac{\partial b_1 x_1 + b_2 x_2 + b_n x_n}{\partial x_1} \\ \frac{\partial b_1 x_1 + b_2 x_2 + b_n x_n}{\partial x_2} \\ \frac{\partial b_1 x_1 + b_2 x_2 + b_n x_n}{\partial x_n} \end{bmatrix}$$

$$\nabla_x \mathbf{b}^T \mathbf{x} = \begin{bmatrix} b_1 \frac{\partial x_1}{\partial x_1} + 0 + 0 \\ 0 + b_2 \frac{\partial x_2}{\partial x_2} + 0 \\ 0 + 0 + b_n \frac{\partial x_n}{\partial x_n} \end{bmatrix}$$

$$\nabla_x \mathbf{b}^T \mathbf{x} = \begin{bmatrix} b_1.1 + 0 + 0 \\ 0 + b_2.1 + 0 \\ 0 + 0 + b_n.1 \end{bmatrix}$$

$$\nabla_x \mathbf{b}^T \mathbf{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_n \end{bmatrix} = b$$

`END of block`

2. $\nabla_x \mathbf{x}^T \mathbf{b} = \mathbf{b}$.

$$\nabla_x \mathbf{x}^T \mathbf{b} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_n \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T \mathbf{b} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_1 b_1 + x_2 b_2 + x_n b_n \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T \mathbf{b} = \begin{bmatrix} \frac{\partial x_1 b_1 + x_2 b_2 + x_n b_n}{\partial x_1} \\ \frac{\partial x_1 b_1 + x_2 b_2 + x_n b_n}{\partial x_2} \\ \frac{\partial x_1 b_1 + x_2 b_2 + x_n b_n}{\partial x_n} \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T \mathbf{b} = \begin{bmatrix} b_1 \frac{\partial x_1}{\partial x_1} + 0 + 0 \\ 0 + b_2 \frac{\partial x_2}{\partial x_2} + 0 \\ 0 + 0 + b_n \frac{\partial x_n}{\partial x_n} \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T \mathbf{b} = \begin{bmatrix} b_1.1 + 0 + 0 \\ 0 + b_2.1 + 0 \\ 0 + 0 + b_n.1 \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_n \end{bmatrix} = b$$

`END of block`

3. $\nabla_x \mathbf{x}^T A \mathbf{x} = 2A\mathbf{x}$.

$$\nabla_x \mathbf{x}^T A \mathbf{x} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_n \end{bmatrix} \begin{bmatrix} x_1 1 & x_1 2 & x_1 m \\ x_2 1 & x_2 2 & x_2 m \\ x_n 1 & x_n 2 & x_n m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T A \mathbf{x} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix} \begin{bmatrix} x_1 1 & x_1 2 & x_1 m \\ x_2 1 & x_2 2 & x_2 m \\ x_n 1 & x_n 2 & x_n m \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T A \mathbf{x} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_1^2 + x_2^2 + x_n^2 \end{bmatrix} \begin{bmatrix} x_1 1 & x_1 2 & x_1 m \\ x_2 1 & x_2 2 & x_2 m \\ x_n 1 & x_n 2 & x_n m \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T A \mathbf{x} = \begin{bmatrix} \frac{\partial x_1^2}{\partial x_1} + \frac{\partial x_2^2}{\partial x_1} + \frac{\partial x_n^2}{\partial x_1} \\ \frac{\partial x_1^2}{\partial x_2} + \frac{\partial x_2^2}{\partial x_2} + \frac{\partial x_n^2}{\partial x_2} \\ \frac{\partial x_1^2}{\partial x_n} + \frac{\partial x_2^2}{\partial x_n} + \frac{\partial x_n^2}{\partial x_n} \end{bmatrix} \begin{bmatrix} x_1 1 & x_1 2 & x_1 m \\ x_2 1 & x_2 2 & x_2 m \\ x_n 1 & x_n 2 & x_n m \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T A \mathbf{x} = \begin{bmatrix} 2x_1 + 0 + 0 \\ 0 + 2x_2 + 0 \\ 0 + 0 + 2x_n \end{bmatrix} \begin{bmatrix} x_1 1 & x_1 2 & x_1 m \\ x_2 1 & x_2 2 & x_2 m \\ x_n 1 & x_n 2 & x_n m \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T A \mathbf{x} = 2 \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix} \begin{bmatrix} x_1 1 & x_1 2 & x_1 m \\ x_2 1 & x_2 2 & x_2 m \\ x_n 1 & x_n 2 & x_n m \end{bmatrix}$$

$$\nabla_x \mathbf{x}^T A \mathbf{x} = 2xA = 2Ax$$